Robust strategies for target benefit pension plans under default risk for ARC2020

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Abstract

In this paper, we consider the optimal investment and benefit payment problem for a target benefit pension (TBP) with default risk and model uncertainty. The pension fund is invested in a risk-free asset, a stock and a defaultable bond. The objective is to maximize the wealth and benefit excess from the target value or minimize the wealth and benefit gap from the target value with CARA utility. Applying stochastic control approach, we establish the Hamilton-Jacobi-Bellman equations for both the post-default case and the pre-default case, respectively. Robust optimal investment strategies and benefit payment adjustment strategies are derived explicitly for the two cases. We also consider the no-ambiguity model for degenerate case and compare the results under two cases. Numerical analyses are provided to illustrate the effects of parameters on the optimal strategies and demonstrate properties of the strategies.

Highlights

1.We take model uncertainty and default risk into account for optimal investment problem of a target benefit plan for the first time. 2.We compare the optimal amount invested in the stock and the defaultable bond for the pre-default case and the post-default case, respectively. 3.The investment and benefit payment strategy for no ambiguity case is derived and compared with our strategy.

Formulation for TBP

$n(t)$ is the amount of new entrants aged a at time t. The amount of those who attain age x at time t is $n(t-(x-a))s(x)$, $x > a$. For a retiree age x at time t ($x \geq b$), we define his assumed salary at retirement as $\widetilde{L}(x,t) = L(t)e^{-\alpha(x-b)}$, $t \geq 0$, $x \geq b$. Individual pension payment rate at time t for those aged x: $B(x,t) = f(t)\widetilde{L}(x,t)e^{\eta(x-b)} = f(t)L(t)e^{-(\alpha-\eta)(x-b)}$, $x \geq b$. Aggregate pension benefit rate for all the retirees at time t: $B(t) = \int_b^c n(t-x+a)s(x)B(x,t)\mathrm{d}x = I(t)f(t)L(t)$, $t \geq 0$. Individual contribution rate for an active member aged x: $C(x,t) = c_0 e^{\alpha t}$, $a \leq x < b$. Aggregate contribution rate of all active members at time t: $C(t) = \int_a^b n(t-x+a)s(x)C(x,t)\mathrm{d}x = C_1(t)\cdot e^{\alpha t}$.

Financial Market

Defaultable bond $\mathrm{d}P(t,T_1) = P(t-,T_1)\Big[r\mathrm{d}t + (1-Z(t))\delta(1-\Delta)\mathrm{d}t - (1-Z(t-))\zeta \mathrm{d}M^{\mathrm{P}}(t)\Big]$,

Objective function

Maximize the wealth and benefit excess from the target value or minimize the wealth and benefit gap from the target value with CARA utility.

Optimal control problem with model uncertainty

Dynamics of pension fund under probability measure P (reference model):

$$\begin{cases} \mathrm{d}X(t) = \pi_1(t)\dfrac{\mathrm{d}S(t)}{S(t)} + \pi_2(t)\dfrac{\mathrm{d}P(t)}{P(t-)} + (X(t)-\pi_1(t)-\pi_2(t))\dfrac{\mathrm{d}S_0(t)}{S_0(t)} \\ \qquad + (C(t)-B(t))\mathrm{d}t, \\ X(0) = x_0. \end{cases}$$

$$\sup_{\pi\in\Pi} \mathrm{E}^{\mathrm{P}}\Big[-\frac{\lambda}{m}\mathrm{e}^{-m(X(T)-x_0\mathrm{e}^{rT})}\cdot \mathrm{e}^{-rT} - \int_0^T \frac{1}{m}\mathrm{e}^{-m(B(s)-B^*\mathrm{e}^{\beta s})}\cdot \mathrm{e}^{-rs}\mathrm{d}s\Big], \tag{1}$$

Dynamics of pension fund under probability measure Q (a family of probability measures equivalent to P):

$$\begin{cases} \mathrm{d}X^Q(t) = \Big[rX(t) + \mu\pi_1(t) + (1-Z(t))\delta\pi_2(t) + C_1(t)e^{\alpha t} \\ \qquad - I(t)f(t)L(t) - \phi_1(t)\pi_1(t)\sigma\Big]\mathrm{d}t \\ \qquad + \pi_1(t)\sigma \mathrm{d}W^Q(t) - \pi_2(t)(1-Z(t-))\zeta \mathrm{d}Z(t), \\ X(0) = x_0. \end{cases} \tag{2}$$

$$\sup_{\pi\in\Pi}\inf_{Q\in\mathcal{Q}} \mathrm{E}^{\mathrm{Q}}\Big[\Big\{-\frac{\lambda}{m}\mathrm{e}^{-m(X(T)-x_0\mathrm{e}^{rT})}\cdot\mathrm{e}^{-rT} + \int_0^T -\frac{1}{m}\mathrm{e}^{-m(B(s)-B^*\mathrm{e}^{\beta s})}\cdot\mathrm{e}^{-rs}\mathrm{d}s + \int_0^T \Psi(u,X(u),Z(u),\phi_1(u),\phi_2(u))\Big\}\Big],$$

$$\begin{cases} \Psi(t,x,z,\phi_1(t),\phi_2(t)) = \dfrac{R_1(t)}{\kappa_1(t,x,z)} + \dfrac{R_2(t,z)}{\kappa_2(t,x,z)}, \\ R_1(t) = \dfrac{\phi_1(t)^2}{2}, \qquad R_2(t,z) = h^{\mathrm{P}}(\phi_2(t)\ln\phi_2(t) - \phi_2(t)+1)(1-z), \\ \kappa_1(t,x,z) = -\dfrac{\rho_1}{mW(t,x,z)}, \qquad \kappa_2(t,x,z) = -\dfrac{\rho_2}{mW(t,x,z)}. \end{cases}$$

Solution

$$\sup_{\pi_1,\pi_2,f}\inf_{Q\in\mathcal{Q}}\Big\{V_t + V_x[rx+\mu\pi_1+\delta(1-z)\pi_2 + C_1(t)e^{\alpha t} - I(t)fL - \phi_1\pi_1\sigma] + \frac{1}{2}V_{xx}\pi_1^2\sigma^2 + [V(t,x-\pi_2\zeta,1) - V(t,x,0)]\phi_2 h^P(1-z) - \frac{1}{m}\mathrm{e}^{-m(I(t)fL-B^*\mathrm{e}^{\beta t})}\cdot\mathrm{e}^{-rt} - \frac{\phi_1^2}{2\rho_1}mV - \frac{h^P(\phi_2\ln\phi_2-\phi_2+1)(1-z)}{\rho_2}mV\Big\} = 0.$$

$$\pi_1^*(t) = \frac{\mu}{(m+\rho_1)P(t)\sigma^2},\ \pi_2^*(t) = \frac{1}{m\zeta P(t)}\cdot\Big[\ln\frac{\delta}{\zeta h^P\phi_2^*(t)} - (Q_1(t)-Q_2(t))\Big], \qquad t\in[0,\tau\wedge T) \tag{3}$$

$$f^*(t) = \begin{cases} \dfrac{1}{I(t)L(t)}\cdot\Big(-\dfrac{\ln\lambda P(t) - mP(t)x + Q_2(t) + rt}{m} + B^*\mathrm{e}^{\beta t}\Big), t\in[0,\tau\wedge T) \\ \dfrac{1}{I(t)L(t)}\cdot\Big(-\dfrac{\ln\lambda P(t) - mP(t)x + Q_1(t) + rt}{m} + B^*\mathrm{e}^{\beta t}\Big), t\in[\tau\wedge T, T] \end{cases} \tag{4}$$

$\phi_1^*(t) = \dfrac{\rho_1\mu}{\sigma(\rho_1+m)}$, $\phi_2^*(t)$ satisfies equation $\dfrac{mh^{\mathrm{P}}}{\rho_2}\phi_2^*(t)\ln\phi_2^*(t) + \phi_2^*(t)h^{\mathrm{P}} - \dfrac{\delta}{\zeta} = 0$ for $t\in[0,\tau\wedge T)$.

Table

	pre-default case				post-default case			
Proportion	$t=0$	$t=2$	$t=4$	$t=6$	$t=0$	$t=2$	$t=4$	$t=6$
Defaultable Bond	0.3432	0.3487	0.3457	0.3305	0	0	0	0
Stock	0.0042	0.0037	0.0032	0.0028	0.0042	0.0038	0.0033	0.0029
Bank Account	0.6526	0.6476	0.6511	0.6667	0.9958	0.9962	0.9967	0.9971

Conclusion

This paper considers the optimal investment and benefit payment problem for a target benefit pension (TBP) with model uncertainty and default risk. We find that: 1.The optimal amount invested in the defaultable bond is much larger than that invested in the stock. 2.A higher model uncertainty aversion level results in the less investment in financial market. 3.The parameters of stock and defaultable bond have effects on the optimal benefit payment adjustment strategy.