increases by ΔS. The delta of one share of the stock is 1.0, so that the long position in 1,200 shares has a delta of +1,200. The delta of the trader's overall position in our example is, therefore, zero. The delta of the stock position offsets the delta of the option position. A position with a delta of zero is referred to as *delta neutral*.

It is important to realize that, since the delta of an option does not remain constant, the trader's position remains delta hedged (or delta neutral) for only a relatively short period of time. The hedge has to be adjusted periodically. This is known as *rebalancing*. In our example, by the end of 1 day the stock price might have increased to \$110. As indicated by Figure 19.2, an increase in the stock price leads to an increase in delta. Suppose that delta rises from 0.60 to 0.65. An extra $0.05 \times 2{,}000 = 100$ shares would then have to be purchased to maintain the hedge. A procedure such as this, where the hedge is adjusted on a regular basis, is referred to as *dynamic hedging*. It can be contrasted with *static hedging*, where a hedge is set up initially and never adjusted. Static hedging is sometimes also referred to as "hedge-and-forget."

Delta is closely related to the Black–Scholes–Merton analysis. As explained in Chapter 15, the Black–Scholes–Merton differential equation can be derived by setting up a riskless portfolio consisting of a position in an option on a stock and a position in the stock. Expressed in terms of Δ, the portfolio is

-1: option

$+\Delta$: shares of the stock.

Using our new terminology, we can say that options can be valued by setting up a delta-neutral position and arguing that the return on the position should (instantaneously) be the risk-free interest rate.

Delta of European Stock Options

For a European call option on a non-dividend-paying stock, it can be shown (see Problem 15.17) that the Black–Scholes–Merton model gives

$$\Delta(\text{call}) = N(d_1)$$

Figure 19.3 Variation of delta with stock price for (a) a call option and (b) a put option on a non-dividend-paying stock ($K = 50$, $r = 0$, $\sigma = 25\%$, $T = 2$).