LIVING to 100

SOCIETY OF ACTUARIES
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1C - Teaching Session: Death is Certain, Survival Isn't

2020 Living to 100 Symposium

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Session 1C: Death Is Certain, Survival Isn't

13th January 2020

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Introductions

- **Stefan Ramonat**, FSA, FCIA
    - Consultant, Mercer Canada
    - Survival analysis; Mercer Mortality Model
- **Kai Kaufhold**, Aktuar DAV, FSAS
    - Partner, NMG Consulting
    - Lead for Longevity & Prediction Consulting

- **Who are you?**

Session Agenda

1. Why Worry about Uncertainty?
2. Introductory Example: CPM2014 Canadian Pensioner Mortality
3. Technical Background
4. Case Study: Pensioner Mortality Pool
5. Other Applications for Measuring Uncertainty

Why Worry about Uncertainty?

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Uncertainty is important, because …

Motivation	Reason
Research	Statistical significance / credibility
Regulation	Prudential margins / solvency capital
Risk	Uncertainty of financial outcome → Economic price for taking risk

Introductory Example: CPM2014

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2014 Canadian Pensioner Mortality Table

CPM2014 data publicly available

- Males, ages 55 – 95
- 63,541 deaths

“Classic” estimation error calculation

- Standard deviation around base rates under $\boldsymbol{Poisson}$ distribution:

$$\sigma = {}^{1}/_{sqrt(deaths)} = {}^{1}/_{sqrt(63{,}541)} = 0.397\%$$

- Assume $\boldsymbol{Normal}$ distribution for confidence intervals

$$95\%\ confidence\ interval = \pm 1.96 \times 0.397\% = \pm 0.778\%$$

Who cares about mortality rates $q_x \pm \Delta q_x$?

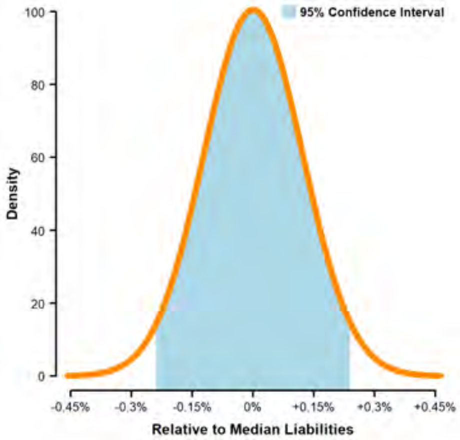
Risk of mis-estimating the quantity of interest, e.g.

- Reserves / liabilities: ${}_tV$
- Total cost of insurance: $A_{\overline{x:n}|}$
- Expected claims at time t: ${}_tp_x q_{x+t}$

Error on mortality rates → error on pensioner liabilities

- 95% C.I. on the base rates of $\Delta q_x = \pm 0.778\%$
- 95% C.I. around the pensioner liabilities $\Delta_t V = \pm 0.24\%$

  discounting @ 3%, truncated to age 95 (end of fitting range)



Comparing Different Graduation Methods

Crude rates: $\mu_x \approx {}^{d_x}/_{E_x^{central}}$

Mechanical smoothing: $P - splines\ (10yr\ knots, 7\ coeff.)$

Gompertz-Makeham(r,s): $\mu_x = e^r + e^{s_1 + s_2 x + s_3 x^2}$

Makeham-Perks: $\mu_x = \frac{e^{\epsilon} + e^{\alpha + \beta x}}{1 + e^{\alpha + \beta x}}$

Mis-estimation Risk in Different Methods

C.I.'s around pension liabilities truncated at age 95 (to avoid extensions) using weighted exposures at each age. Interest 3%.

Method	80% C.I.	90% C.I.	95% C.I.	99% C.I.
"Classic" Approach	(-0.16%, 0.16%)	(-0.20%, 0.20%)	(-0.24%, 0.24%)	(-0.31%, 0.32%)
MIS-ESTIMATION RESULTS (10,000 Simulations Each)				
Crude Rates	(-0.16%, 0.16%)	(-0.21%, 0.20%)	(-0.25%, 0.24%)	(-0.32%, 0.32%)
P-splines	(-0.16%, 0.16%)	(-0.20%, 0.20%)	(-0.24%, 0.24%)	(-0.32%, 0.31%)
GM(1,3)	(-0.16%, 0.16%)	(-0.20%, 0.21%)	(-0.25%, 0.25%)	(-0.32%, 0.32%)
Makeham-Perks	(-0.16%, 0.16%)	(-0.21%, 0.21%)	(-0.25%, 0.25%)	(-0.32%, 0.33%)

Spot the difference!

So What?

Mis-estimation can help answer these questions:

- How much additional uncertainty is created by incorporating **new risk factors**?
- How does the **heterogeneity** (e.g. by benefit amount) affect uncertainty?
- Uncertainty when model is applied to **subset/external population**?
- How does the **length of the study period** influence the uncertainty?

Technical Background

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Components of Risk

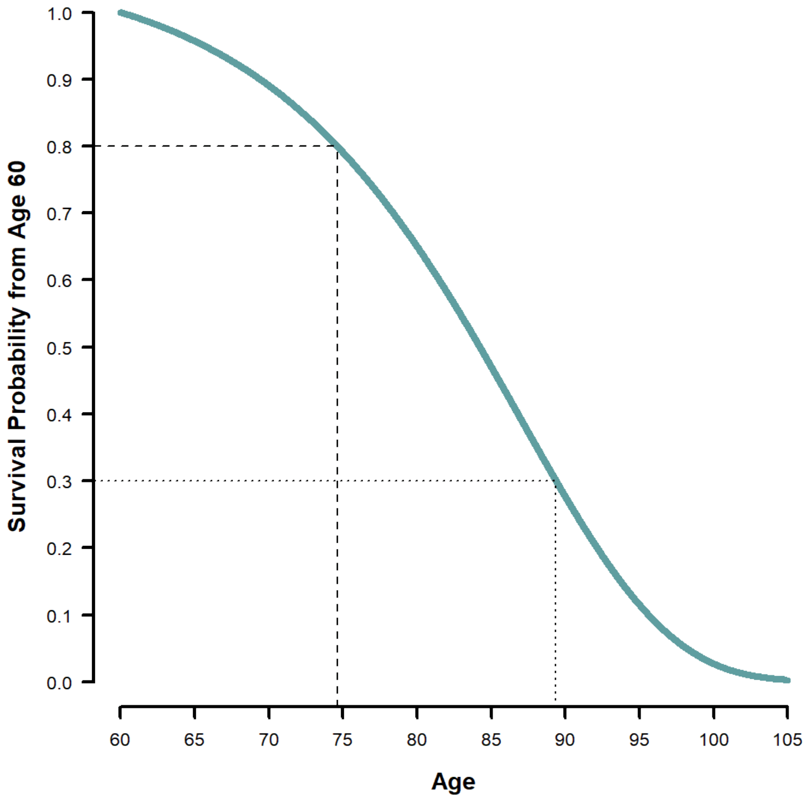
Type of Uncertainty	Description	Application
Stochastic risk (process risk)	Each individual experiences random events	Run-off simulation
Mis-estimation risk (estimation error)	Probabilities estimated from finite amount of data	Bootstrapping, parameter perturbation
Trend risk	Change in probabilities over time, can be due to changes in business mix as well as true secular trend	Include trend in graduation, stochastic mortality projection models
Catastrophe risk (out of scope)	Large single event	Jump processes, over-dispersion, Strickler-Method
Basis risk	Model derived from different dataset	Portfolio-specific table versus industry table, Limited Fluctuation Credibility Theory, multi-population mortality projection models

Process Risk: Run-off Simulation

Survivorship function:

$$S_t = {}_tp_x = e^{-\int_0^t \mu_{x+s} ds}$$

- Each survival probability ${}_tp_x$ corresponds to a time-lived t
- **Inverse transform:** substitute ${}_tp_x$ with randomly-generated $U(0,1)$ variable and then solve for t
- In example, a realized $U(0,1)$ of 0.3 would lead to a time-lived of 29.375 years from age 60



Run-off Simulation: Pseudo-code

For $i \ = \ 1, \ldots, n$ # *number of simulations*

For $j \ = \ 1, \ldots, N$ # *number of individual lives*

Draw $U(0,1)$ random Number r;

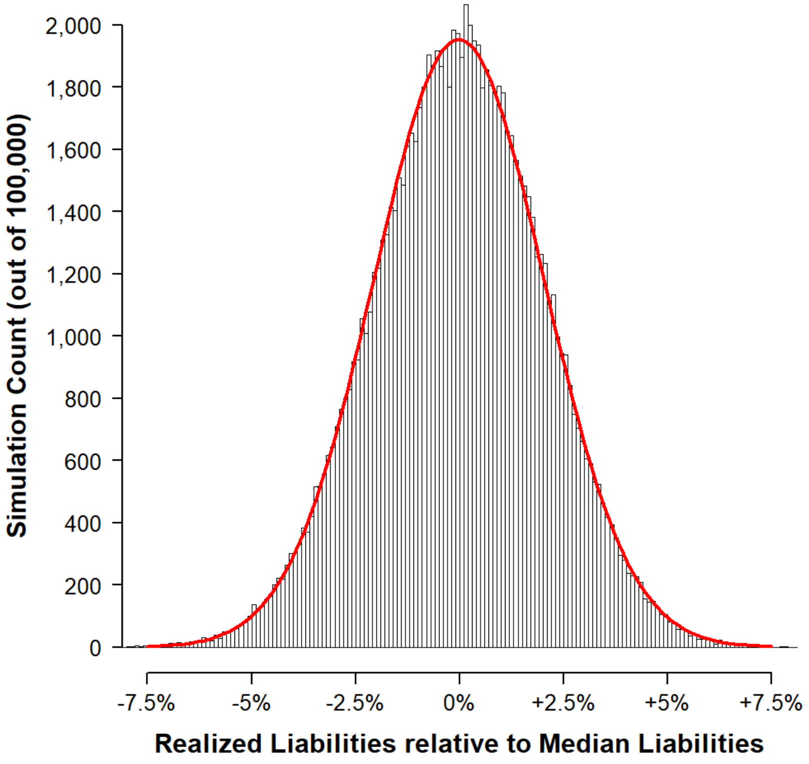
Calculate remaining time lived $t = \ S^{-1}(r)$;

Calculate liabilities (or similar) from simulated t;

Return total liabilities, reserves etc.;

Analyze distribution of results

Run-off Simulation: Example



Parametric survival model

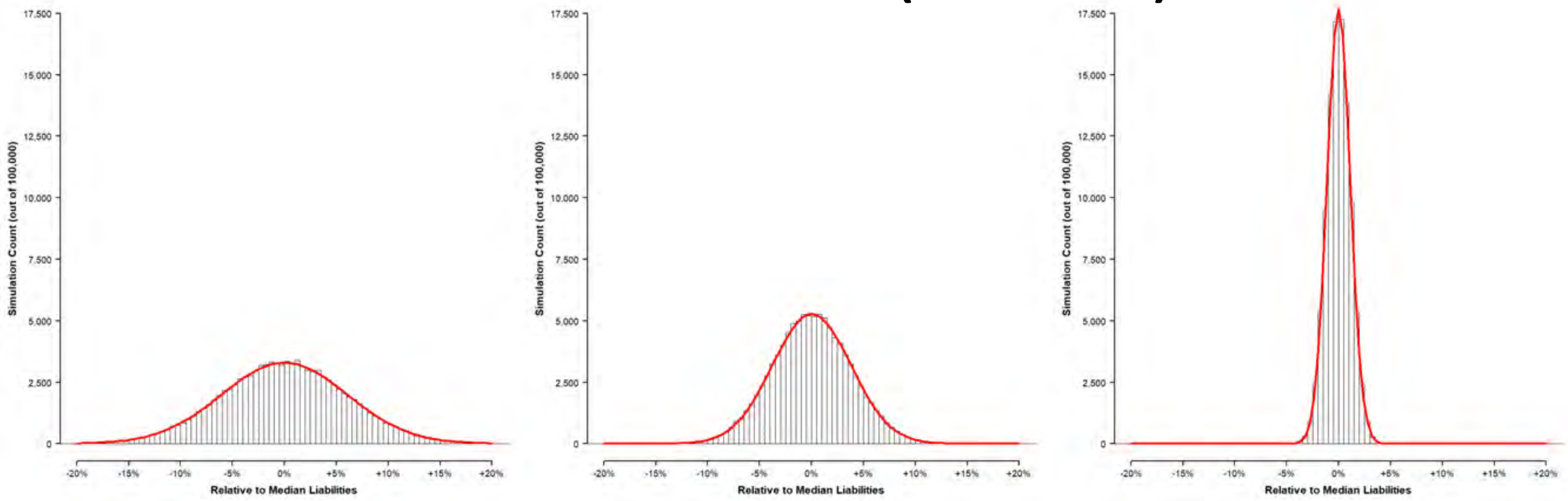
- 500 pensioners
- 100,000 simulations
  - Run in under 2 minutes
- 95% Confidence Interval: (-4.1%, +3.9%)
- Use of the parametric survival model speeds up simulation and provides for greater flexibility, though similar procedure possible for standard table

Run-off + Mis-estimation Together

- The run-off simulation can be extended so that at each iteration the underlying mortality basis is varied according to variance-covariance matrix
- On the next slide, results based on three sample datasets (each random subsets of a larger one), with about 500, 1,500, and 15,000 individuals alive at the end of the period
- Two key points to note:
  - Process and mis-estimation risk both diminish in importance as the size of the group increases
  - There is a diversification effect such that the combined effect is much less than the sum

The remainder of the presentation will focus on the effects of the mis-estimation risk in isolation as it is the one less familiar, though ideally the process, mis-estimation, trend, etc. risks would be considered simultaneously

Run-off + Mis-Estimation (Cont'd)



Lives | Deaths	500	120	1,500	335	15,000	3,410
Run-off 95% CI	(-4.3%, +4.3%)		(-2.3%, +2.3%)		(-0.7%, +0.7%)	
Mis-Est. 95% CI	(-12.8%, +13.3%)		(-7.0%, +7.1%)		(-2.1%, +2.1%)	
Combined 95%	(-13.5%, +14.0%)		(-7.4%, +7.5%)		(-2.2%, +2.2%)	

Mis-estimation Risk: The Long Way Round

Bootstrap Method

Assume deaths by age-group are $Poisson$-distributed

a. Randomly generate simulated deaths by drawing $Poisson$-distributed random numbers, for each age-group

b. Re-graduate table from simulated experience data according to chosen model

c. Simulate run-off and calculate liabilities

d. Repeat steps a to c n times

Mis-estimation Risk: ML Estimation

Use Maximum Likelihood Estimation instead of re-graduating in each run

Individual Level-Data (*Survival Model*):

$$Likelihood \propto \prod_i {}_{t_i}p_{x_i}\,\left(\mu_{x_i+t_i}\right)^{d_i}$$

$$where\ d_i = \begin{cases} 1\ if\ individual\ i\ dies\ at\ time\ t_i \\ 0\ if\ person\ i\ survives \end{cases}$$

Grouped Data (*Poisson Assumption*)

$$Likelihood \propto \prod_x e^{-E_x\mu_{x+1/2}}\,\left(\mu_{x+1/2}\right)^{d_x}$$

$$where\ d_x = deaths\ at\ age\ x\ and\ E_x = central\ exposure\ for\ age\ x$$

Mis-estimation Risk: MLE Theory

Fitting with MLE allows some key properties to be invoked

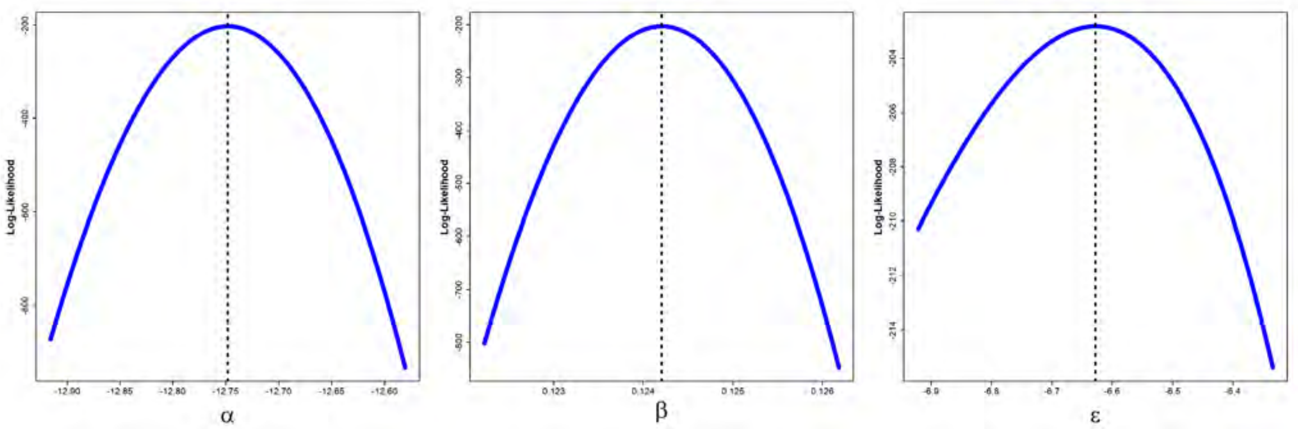
- Parameter estimates (asymptotically) follow multi-variate $Normal$ distribution with ML estimates as mean
- The variance-covariance matrix of the parameter estiamtes can be estimated via the inverse of the observed **Fisher Information**
  - The Fisher Information is the negative of the **Hessian** matrix of 2nd partial derivatives of the log-likelihood function evaluated at the MLE
  - The Hessian can be evaluated either analytically or numerically
  - Hessian can be returned by R's **`optim`** function or other functions
  - Covariance matrix returned directly from R models like **`glm`** via **`vcov`**

Mis-estimation Risk: MLE Example

Example: Maximum likelihood estimates and log-likelihood profiles from graduated CPM2014 data using Makeham-Perks survival model

- Profile log-likelihoods calculated over 95% confidence interval around parameter estimate by keeping all other parameters constant
- Quadratic nature of profile log-likelihoods consistent with asymptotic normality

α	-12.748283
β	-0.124205
ε	-6.627853



Mis-estimation Risk: Covariance Matrix

Variance-Covariance matrix following from example, along with corresponding **Correlation Matrix**

- Note the strong correlations between the parameters, underlying the vital importance of considering the covariances in the context of mis-estimation

	α	β	ε
α	0.007314	-0.000086	-0.010850
β	-0.000086	0.000001	0.000125
ε	-0.010850	0.000125	0.022452

	α	β	ε
α	100.0%	-99.8%	-84.7%
β	-99.8%	100.0%	82.6%
ε	-84.7%	82.6%	100.0%

Mis-estimation: Cholesky Decomposition

	α	β	ε
α	0.085525	-0.001007	-0.126858
β	—	0.000067	-0.043477
ε	—	—	0.066852

R's **`chol`** function produces an upper-triangular matrix, as illustrated on the left with the Cholesky decomposition corresponding to the variance-covariance matrix for the Makeham-Perks model from above

Mis-estimation Simulation: Pseudo-code

$\boldsymbol{p_0}$: $Maximum\ Likelihood\ Estimates, with\ k\ parameters$

$\bar{\bar{A}}$: $Upper\ triangular\ Cholesky\ decomposition\ of\ cov\ matrix$ *# "square root" of matrix*

For $i = 1, \ldots, n$ *# number of simulations*

- Draw random vector $\vec{r}$ from $k - variate\ N(0,1)$ distributions
- Take the matrix product of $\bar{\bar{A}}^T$ and random vector $\vec{r}$
- Create a new, perturbed set of parameters $\overrightarrow{p_i} = \overrightarrow{p_0} + \bar{\bar{A}}^T\ \vec{r}$
- Calculate liabilities (or similar) based on perturbed parameter set

Return total liabilities, reserves etc.;

The distribution of results can then be analyzed

Mis-estimation & Parametric Models

- The “parameters” estimated in MLE context could be very generic, such as coefficients on splines
- However, fully parametric models offer powerful advantages:
    1. Multiple risk factors can be incorporated simultaneously
    2. Automatic extensions to high/low ages
    3. Trend over period can be captured directly by parameters

Multiple Risk Factors in Survival Models

Survival Model	Parametrization
Gompertz (log-linear)	$\mu_x = e^{\alpha+\beta x}$
Makeham	$\mu_x = e^{\epsilon} + e^{\alpha+\beta x}$
Perks (logistic)	$\mu_x = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$
Makeham-Perks	$\mu_x = \frac{e^{\epsilon} + e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$
Beard	$\mu_x = \frac{e^{\epsilon} + e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x+\varrho}}$

Estimating Parameters using MLE

$$LL = -\sum_i H(\alpha_i, \beta_i, \epsilon_i, \varrho_i) + \sum_i \log(\mu(\alpha_i, \beta_i, \epsilon_i, \varrho_i))$$

$$\alpha_i = \alpha_{i,baseline} + \sum_j \alpha_{ij} Z_{ij}$$

$$Z_{ij} = \begin{cases} 1 \; if \; i \; belongs \; to \; group \; j \\ 0 \quad\quad\quad\quad\quad\quad\quad otherwise \end{cases}$$

Example: Incorporating Multiple Factors

Subset of Mercer Mortality Model for Canada: Build up a model with multiple risk factors to illustrate how mis-estimation uncertainty evolves

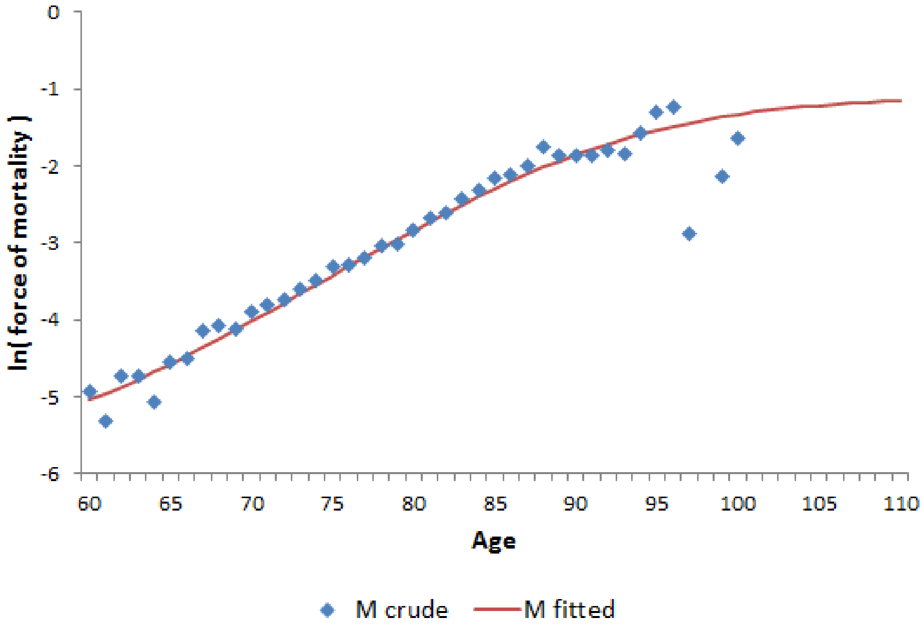
- Base model only includes the time-varying Makeham-Perks parameters
- Liabilities calculated using a small subset: female blue collar retirees with large pensions

Model	Δ Liabilities	95% Mis-Estimation C.I.
Base	—	±0.47%
+ Gender	+4.34%	±0.50%
+ Retiree vs Surviving Spouse	+1.31%	±0.55%
+ Collar Classification (3-level factor)	-3.91%	±0.80%
+ Pension Bandings (3-level factor)	+ 3.88%	±1.05%

Extrapolation to Older Ages

Makeham-Beard Model

- Natural extrapolation to older ages
- German pensioners
- Source: Richards, Kaufhold, & Rosenbusch (2013)



Trend Risk with Parametric Models

By introducing a trend parameter impact and uncertainty can be measured

Example: Makeham-Perks

$$\mu_{x,y} = \frac{e^{\epsilon} + e^{\alpha+\beta x+\delta(y-2000)}}{1 + e^{\alpha+\beta x+\delta(y-2000)}}$$

Trend parameter δ, along with corresponding entries in covariance matrix, is fitted with Maximum Likelihood Estimation.

Note: age x and calendar year y

Example: Trend Uncertainty

- Subset of Mercer Mortality Model for Canada
  - 138,750 annuitants over 2007 – 2017
  - 37,700 actual deaths
- The model form used is based on the “Hermite II” from Richards (2019), with the time trend component extended to allow a single change in the strength of the trend (all fitted using the experience)

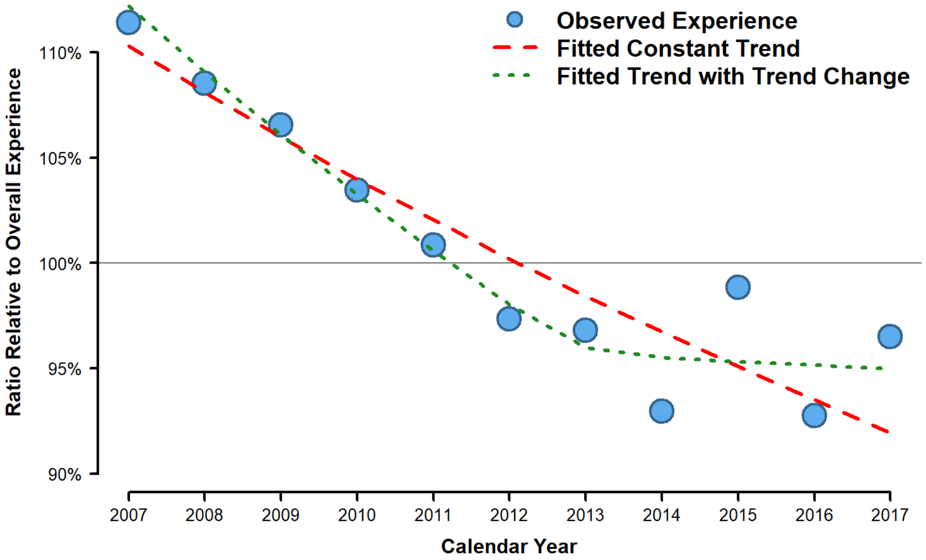
Example: Trend Uncertainty (Cont’d)

Two versions of fitted trend

1. With the (age-varying) trend constant over the period
2. With a single trend change, centred at $y_{change} = 2013.49$ as fitted to the experience

- Version with the trend change fits the experience better
- With the trend change, the estimated liabilities **decrease by 1.3%**, but the width of the 95% mis-estimation confidence interval in terms of liabilities **increases by 0.3%**

→ **Increasing uncertainty requires greater margin**



Basis Risk: A Balancing Act

- Portfolio-specific mortality tables always preferable
- BUT: what if insufficient data available?
- May need to rely on pools of industry data

Advantages of Data Pools	Disadvantages of Pooling Data
Larger data volumes	Data quality defined by worst source
Cross-section of industries	Data may not be relevant to business
	Anonymization → may be multiple policies/benefits per life
	Limited availability of risk factors

Trade-off: Model and Mis-estimation Risk

- Mis-estimation is a diversifiable risk, decreasing as experience grows
- However, whenever a pool is referred to instead of a group's own experience, there is potential for significant basis risk (a.k.a. model risk)

    ➔ Unlike mis-estimation, basis risk / model risk is difficult to quantify

- Basis risk can be mitigated to the extent possible by ensuring all key available risk factors are considered in a model

    ➔ Increasing model complexity often increases mis-estimation risk

- Goal is to restrict the mis-estimation risk to an acceptable range while ensuring the data and model form are sufficiently relevant

Case Study: Pensioner Mortality Pool

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Mercer Mortality Model for Canada

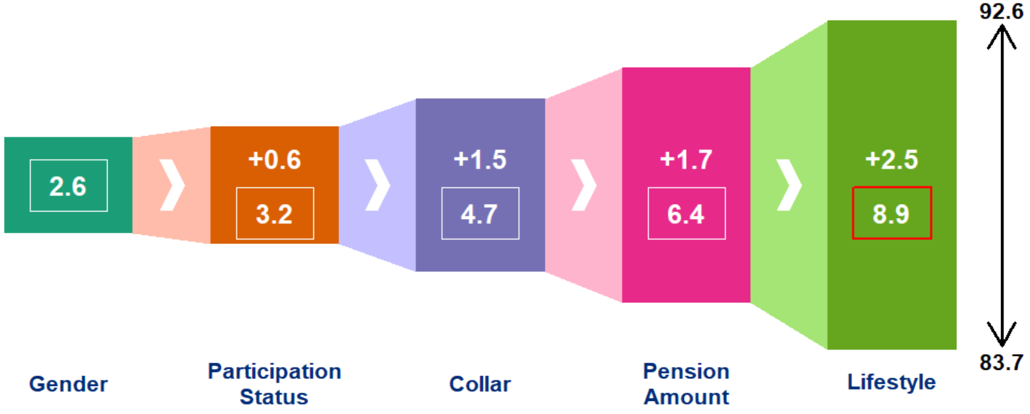
- "Mercer Mortality Model" is based on a dataset of mortality experience data from many large Canadian pension plans

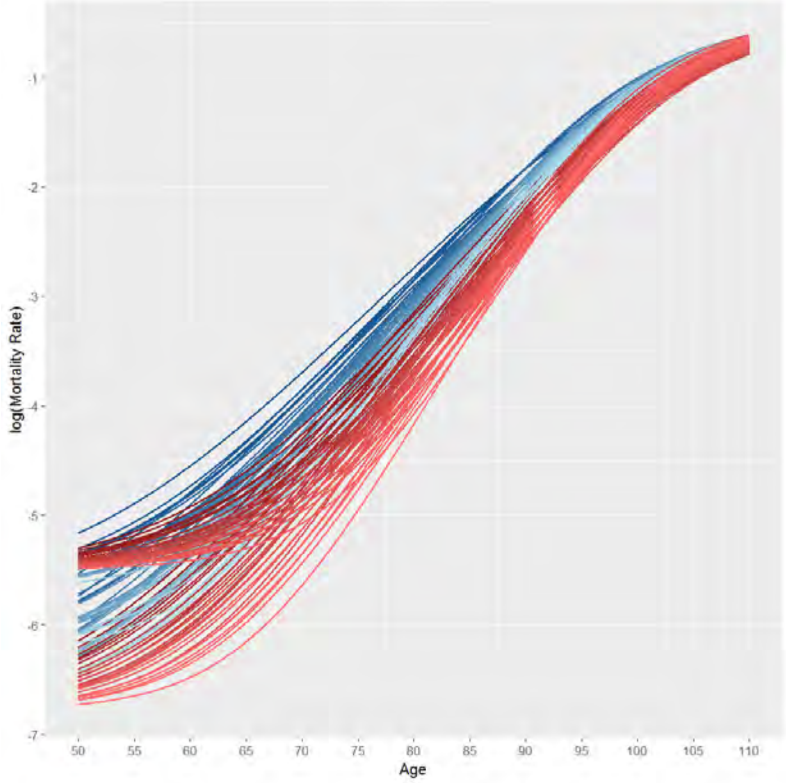
- Main risk characteristics available:
  - Pension amount
  - "Collar" of employment: white/blue/"mixed"
  - Postal codes
- Data suitable for analysis at individual level, with precise information on age & time and commencement as well as exposure period

More on the Mercer Mortality Model

Mercer Mortality Model currently explains an 8.9-year differential in remaining life expectancy at age 65,

100+ unique mortality curves by combination of factors





Example from Mercer Pool

- Subset: male, white-collar pensioners, residing in the province of Québec
  - 1,650 deaths over 10-year experience period
- Model captures financial effects of pension amount and incorporates mortality improvement trend up to the end of the observation period
  - Liabilities $4.77bn (@ 3% interest), ≈ 92.5% of industry table

Example from Mercer Pool (Cont’d)

- Mis-estimation risk for the group-specific result
  - 95% confidence interval: ±2.35% liabilities
  - Loadings on industry table ±7.5%
- Depending on the purpose, result may be considered sufficiently accurate, or not
- **Alternatively: Borrow credibility from combined Mercer dataset**

Example from Mercer Pool (Cont'd)

Alternatives using Mercer Pool

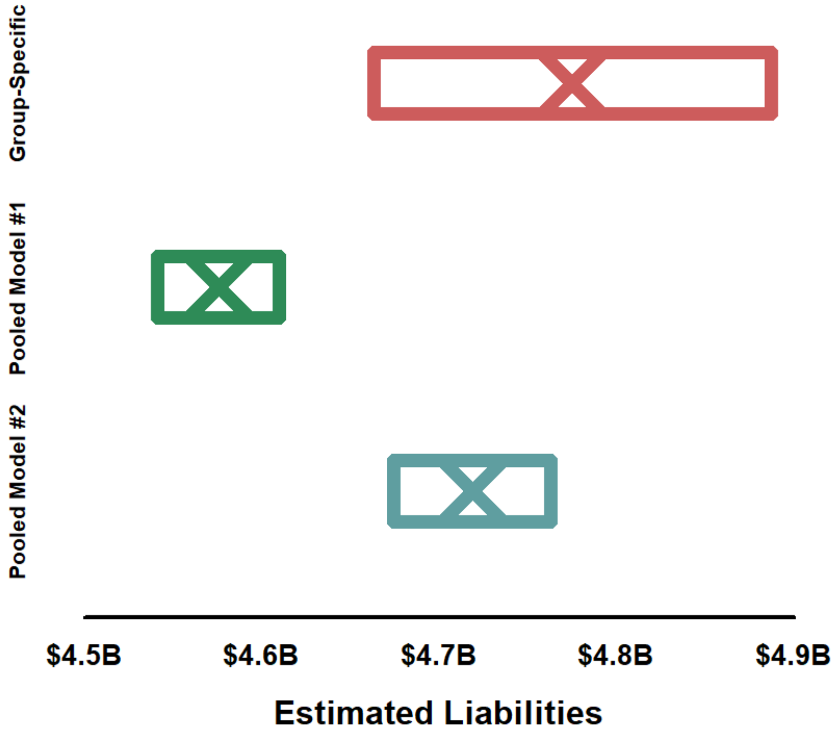
- **Pooled Model #1**: Using pension as sole rating factor
  - Liabilities of $4.58B, 4.2% lower than group-specific result
  - 95% mis-estimation confidence interval ±0.75% around this result
- **Pooled Model #2:** Add collar (white/blue/mixed) as factor
  - Liabilities estimated at $4.72B, 0.8% lower than group-specific
  - 95% mis-estimation confidence interval ±0.95% around this result

Example from Mercer Pool (Cont'd)

Overview of results

1. Portfolio-specific results show widest confidence band
2. Simplest Pooled Model has tightest confidence band, but no overlap with portfolio-specific result
3. More complex Pooled Model overlaps with portfolio-specific model

Which model would you choose?



Example from Mercer Pool (Cont'd)

- Differences between the three results highlight significance of basis risk / model risk

1. Group-Specific Result	Avoids basis risk, at the price of large mis-estimation range ➔ Mis-estimation range could define level of margin
2. Pooled Model #1	Narrowest mis-estimation range, but result differs significantly from group-specific ➔ Mis-estimation confirms that this result significantly outside reasonable range
3. Pooled Model #2	Result reasonably close to group-specific, though on edge of mis-estimation range ➔ Mis-estimation could be used to blend this and group-specific results

- Note: Pooled result may, or may not become closer to group-specific one when additional factors (e.g., postal code) added

Other Applications for Measuring Uncertainty

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Overview of Risk Applications

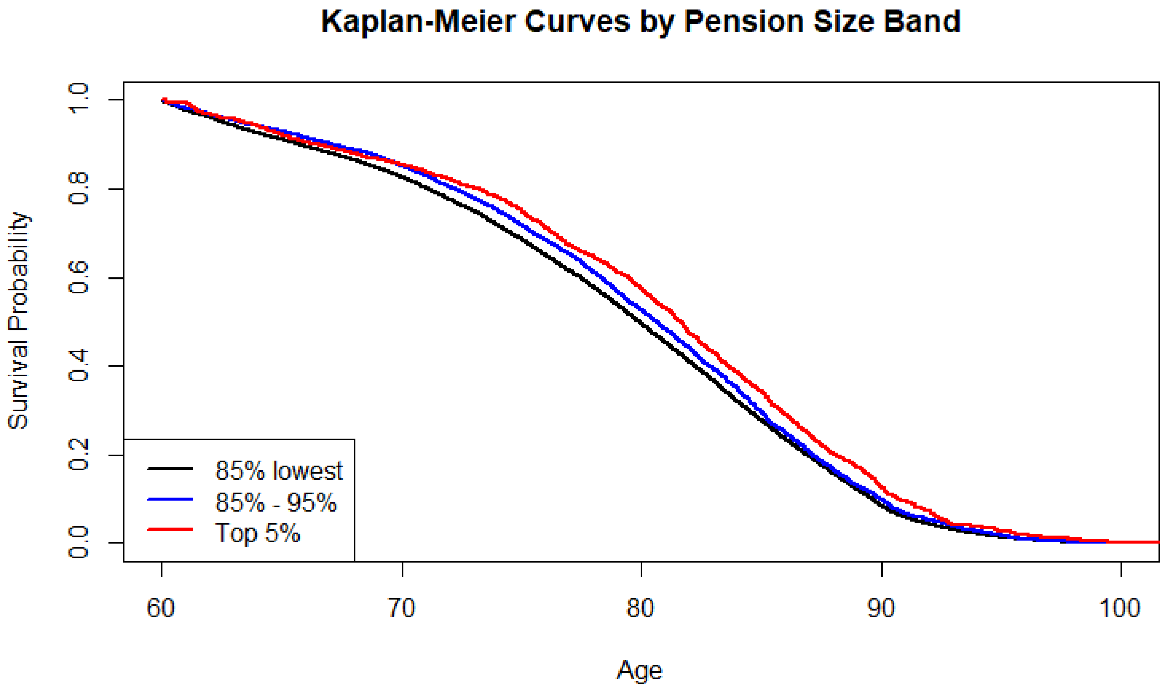
1. Model choice criteria (as seen in preceding examples)
2. Prudential margins / reserving for insurance
3. Capital assumptions (economic & regulatory capital)
4. Risk management / risk-adjusted strategy
5. Quantitative retention management

Modelling Volatility of the Business

- Adding risk factors (generally) increases mis-estimation risk, but lowers model risk – why again?
- Statistically significant risk factors represent **heterogeneity** within the portfolio of risks
- Heterogeneity increases liabilities, and volatility!

Benefit Amount for a German Pension Plan

- Three statistically significant size bands
- Top band only 5% of lives, but 15% of pensions
- On average 35% reduction in mortality rates for Band 3
- Mis-estimation risk increases



Modelling Volatility

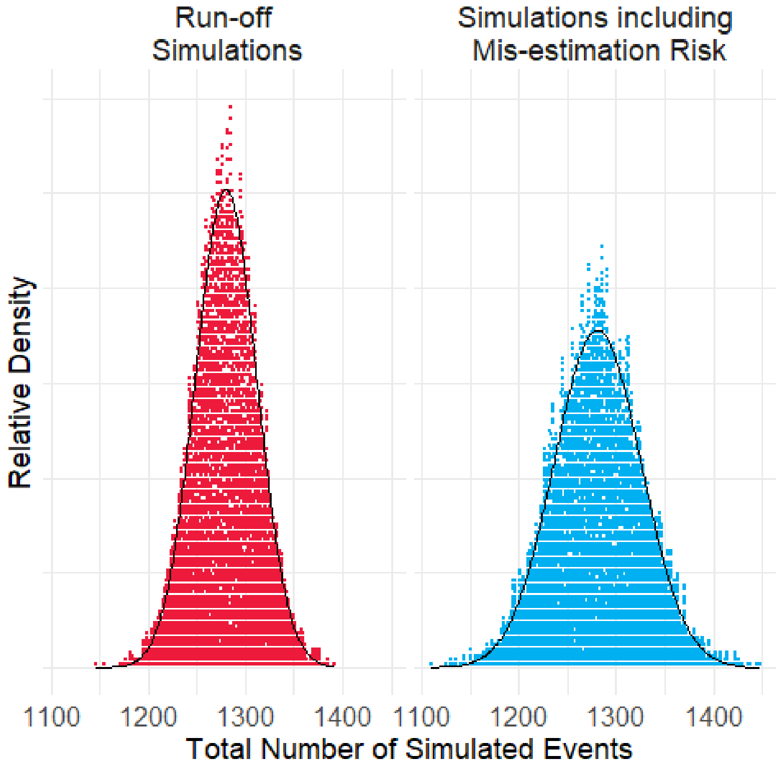
Profit Margins

PBR Valuation Margins

Solvency Capital Internal Model

Risk Management

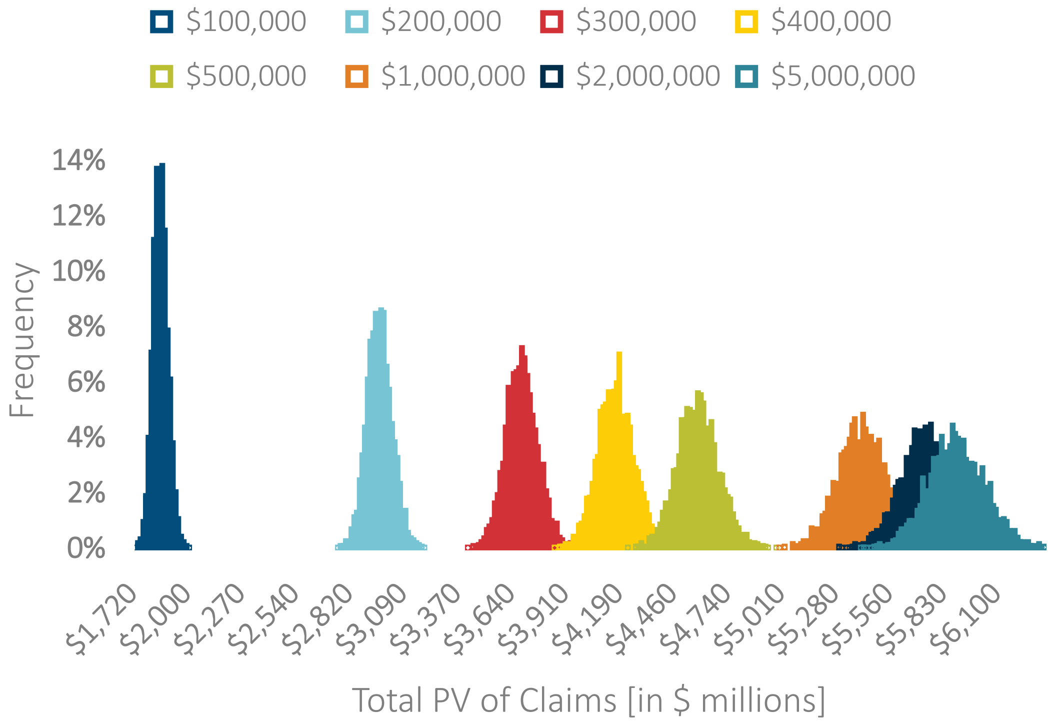
Optimal Reinsurance Risk Retention



Quantitative Retention Management

Example: US Term Life

- Different levels of reinsurance retention
- Reduction in retention reduces volatility, but also lowers profits



Quantitative Retention Management (2)

SOA Research

- Reinsurance impacts reserve margins for smaller companies
- Reinsurance impacts capital margins for all companies

Retention	Best-Estimate Liabilities	Capital	Capital Margin	Return on EC
$150m	$5.99bn	$65.0m	1.09%	12.0%
$5m	$5.96bn	$58.7m	0.98%	13.2%
$1m	$5.48bn	$45.7m	0.83%	15.4%
$500k	$4.63bn	$34.9m	0.75%	16.3%
$100k	$1.87bn	$12.1m	0.65%	11.2%

Base Return on Economic Capital = 12%; Cost of R/I = 20% of expected profit

References

Richards, S. J. (2016), 'Mis-estimation risk: measurement and impact', *British Actuarial Journal* **21**(III), 429 – 457.
(Available at https://www.actuaries.org.uk/documents/sessional-event-mis-estimation-risk-measurement-and-impact)

Kaufhold, K. F. and Lennartz, H. W. (2016), *Optimizing Risk Retention*, SOA Research Paper.
(Available at https://www.soa.org/Files/Research/Projects/research-2016-quantitative-retention.pdf)

Ramonat S. J. and Kaufhold, K. F. (2018), *A Practitioner's Guide to Statistical Mortality Graduation,* SOA Research Paper.
(Available at https://www.soa.org/resources/tables-calcs-tools/2018-stat-mort-graduation/)

References (Cont'd)

Richards, S. J. (2012), 'A handbook of parametric survival models for actuarial use', *Scandinavian Actuarial Journal* **2012**(4), 233 – 257.
(Accessible via https://www.tandfonline.com/doi/abs/10.1080/03461238.2010.506688)

Richards, S. J., K. Kaufhold and S. Rosenbusch (2013), 'Creating portfolio-specific mortality tables: a case study', *European Actuarial Journal* **2013** Vol. 3 Issue 2, pp 295 – 319.
(Accessible via https://link.springer.com/article/10.1007/s13385-013-0076-6)

Richards, S. J. (2019), 'A Hermite-spline model of post-retirement mortality', *Scandinavian Actuarial Journal*.
(Accessible via https://www.tandfonline.com/doi/abs/10.1080/03461238.2019.1642239)

CIA Pension Experience Subcommittee – Research Committee (2014), *Final Report – Canadian Pensioners' Mortality*, Canadian Institute of Actuaries.

(Available at http://www.cia-ica.ca/docs/default-source/2014/214013e.pdf)
(Underlying data available from tool at https://www.cia-ica.ca/publications/publication-details/214013T5)

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