LIVING to 100

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Jan. 13–15
Lake Buena Vista, FL

6A – Aging Measurement and Mortality Modeling 2

Short- and Long-Term Dynamics of Cause-specific Mortality Rates using the Cointegration Analysis

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SOA Living to 100 Symposium
Lake Buena Vista 2020

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Motivation

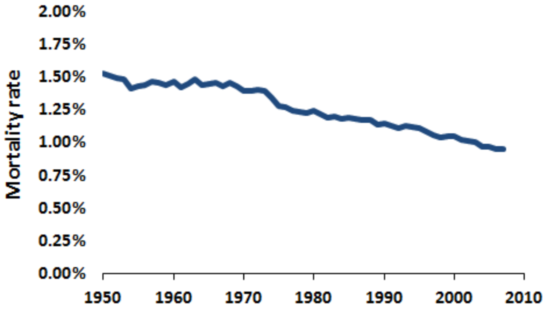
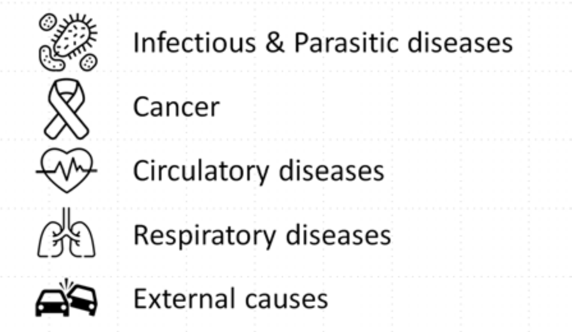


Figure: Total age-standardized mortality rates, US males, WHO

Motivation



→ International Classification of Diseases

Motivation

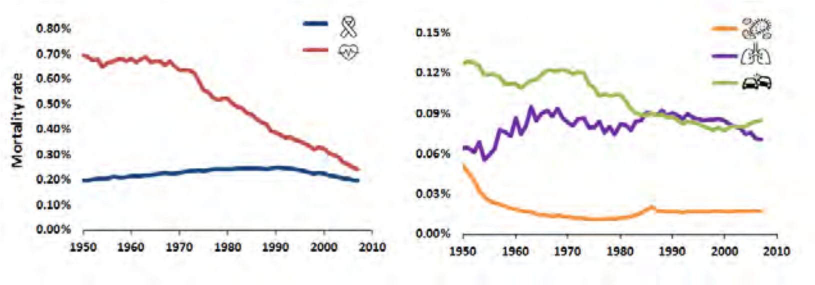


Figure: Cause-specific age-standardized mortality rates, US males, WHO

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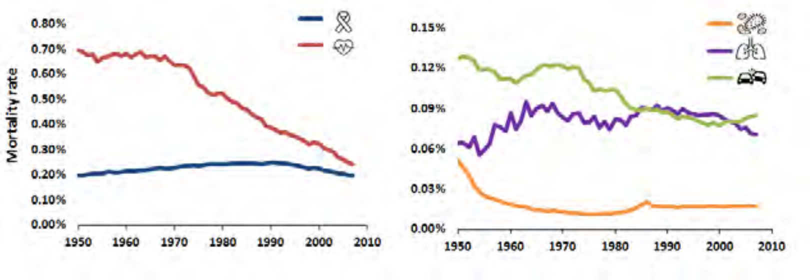

Figure: Cause-specific age-standardized mortality rates, US males, WHO

→ Build a model which preserves the information on different causes of death

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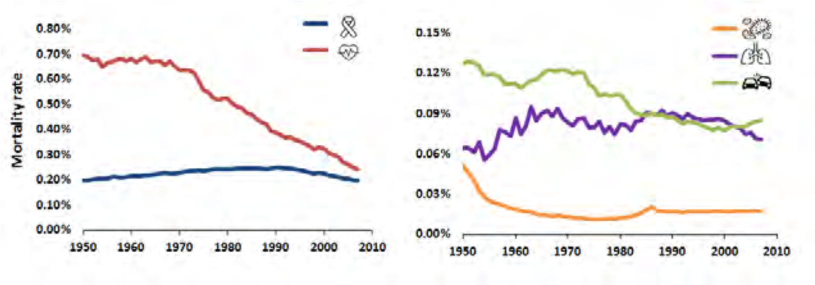

Figure: Cause-specific age-standardized mortality rates, US males, WHO

→ Build a model which preserves the information on different causes of death

→ and takes into account their dependency.

Datasets

(World Health Organization database)

Datasets

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→ Identify patterns/trends and compare them across datasets

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Data variables

$$\mathbf{y}_t = \begin{pmatrix} log(m_t^{IP}) \\ log(m_t^{Cancer}) \\ log(m_t^{Circulatory}) \\ log(m_t^{Respiratory}) \\ log(m_t^{External}) \end{pmatrix}$$

- Data dimenstion: $n = 5$, $t = 1$, ..., 55-65 years

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- **Cointegration** relation = linear combination of non-stationary variables that is stationary
- Information on the long-term equilibrium between the causes

Model

Error Correction Model representation for the cause-specific mortality rates vector $\mathbf{y}_t$:

$$\Delta \mathbf{y}_t = \mathbf{c} + \mathbf{dt} + \alpha\beta'\mathbf{y}_{t-1} + \sum_{n=1}^{p} \mathbf{\Gamma}_i \Delta \mathbf{y}_{t-i} + \epsilon_t$$

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  - α is a loading matrix

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- Long-term dynamics via the cointegrated term $\alpha\beta'\mathbf{y}_{t-1}$
- Short-term dynamics via the matrices $\mathbf{\Gamma}_i$

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Algorithm

Following procedure was applied to every dataset :

- Test formally for unit roots
- Test for the number of lags in a vector ECM
- Test for the number of cointegration relations r and the form of the deterministic elements (Johansen, 1994)
- Calculate the matrices α, β and $\mathbf{\Gamma}_i$
- Check the residuals for autocorrelation and normality

Example of a vector ECM - US males

The following model was chosen as the best describing the data:

$$\Delta \mathbf{y}_t = \mathbf{c} + \mathbf{d}\mathbf{t} + \mathbf{\Gamma_1} \Delta \mathbf{y}_{t-1} + \alpha\beta' \mathbf{y}_{t-1} + \epsilon_t =$$

$$= \begin{bmatrix} \mathbf{-2.772} \\ -0.150 \\ -0.243 \\ \mathbf{-2.057} \\ 0.156 \end{bmatrix} + \begin{bmatrix} \mathbf{0.010} \\ 0.000 \\ 0.000 \\ \mathbf{0.005} \\ 0.000 \end{bmatrix} \mathbf{t} + \begin{bmatrix} -0.121 & -0.707 & -0.188 & 0.174 & 0.323 \\ -0.004 & 0.015 & \mathbf{-0.166} & -0.008 & \mathbf{0.133} \\ -0.043 & -0.121 & 0.034 & -0.089 & 0.196 \\ -0.135 & -0.294 & -0.085 & \mathbf{-0.381} & \mathbf{1.119} \\ 0.042 & -0.323 & 0.198 & \mathbf{-0.146} & 0.227 \end{bmatrix} \Delta \mathbf{y}_{t-1}$$

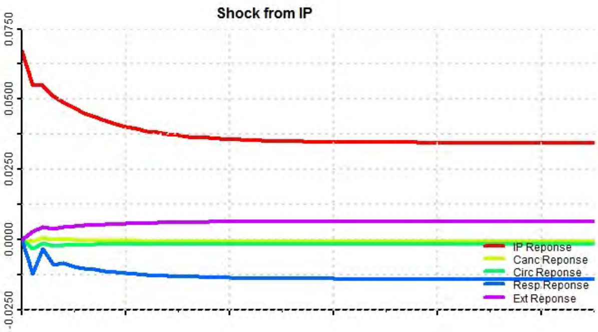
$$+ \begin{bmatrix} \mathbf{-0.033} \\ -0.002 \\ -0.003 \\ \mathbf{-0.026} \\ 0.002 \end{bmatrix} \begin{bmatrix} 1.772 & -5.499 & -18.602 & 13.217 & 14.132 \end{bmatrix} \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t$$

Significant coefficients (at 5% significance level) are in bold.

Impulse-response analysis (1)

- What is the response of every cause-specific mortality rate to a shock from a particular cause (e.g. IP)?

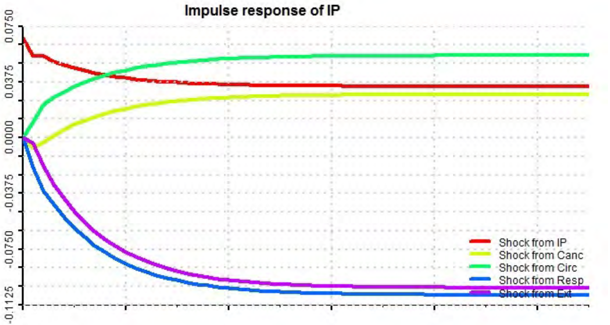
→ Vector of starting values: $\mathbf{y}_0 = (\text{sdt.dev(IP)}, 0, 0, 0, 0)$



→ All causes show weak reaction to the shock from IP mortality rate

Impulse-response analysis (2)

- Alternative way: how sensitive is a particular cause-specific mortality rate (e.g. IP) to a random shock from other causes (one at a time)?



→ IP mortality rate shows important reaction to the shocks from every other cause

Short- vs. long-term dynamics

What drives this behavior of the IP mortality rate?

- Not the short term: the corresponding coefficients in the $\mathbf{\Gamma}_1$ matrix are not significant
- Long-term: the cointegration relation enters the equation for the IP mortality rate with a significant coefficient α_1

Overview : impulse-response analysis

- Same procedure applied to the rest of the datasets
- High-level summary of responses of the mortality rate Y to the shock given to the rate X:

X \ Y	IP	Cancer	Circulatory	Respiratory	External
IP	—	Low	Low	Low	Low
Cancer	High	—	Med	High	High
Circulatory	High	Low	—	High	Med
Respiratory	Med	Low	Low	—	Low
External	High	Low	Low	High	—

Overview: the short term

$\mathbf{\Gamma}_1$ coefficients which are significantly different from zero, significance level of 5%:

Dataset	ΔIP_t	$\Delta Canc_t$	$\Delta Circ_t$	$\Delta Resp_t$	ΔExt_t
US/M	-	Circ,Ext	-	Resp,Ext	Resp
JP/M	-	Canc	Resp	Circ,Resp	-
FR/M	-	-	Resp	Circ,Resp	Circ,Resp
E&W/M	Ext	-	-	IP,Circ, Resp	Ext
AU/M	IP	Canc, Ext	Circ	-	Ext
US/F	Canc,Ext	Ext	Canc	Resp	Canc,Resp
JP/F	IP,Resp	-	Resp	-	Circ,Resp,Ext
FR/F	Canc,Circ	-	-	Canc,Circ,Resp	IP
E&W/F	-	-	Resp	Resp	Resp
AU/F	IP,Resp	Canc	Circ	Circ,Resp	-

Overview: the long term

Equations to which the long-term component (i.e., cointegration relation) enters with a coefficient α_i significantly different from zero, significance level of 5%:

Country		Males	Females
US	α_i	ΔIP_t, $\Delta Resp_t$	ΔIP_t, $\Delta Canc_t$,$\Delta Circ_t$, $\Delta Resp_t$
JP	α_{1i}	ΔIP_t, $\Delta Canc_t$	$\Delta Circ_t$, $\Delta Resp_t$
	α_{2i}	ΔIP_t, $\Delta Canc_t$, $\Delta Resp_t$	ΔIP_t, $\Delta Canc_t$,$\Delta Circ_t$, $\Delta Resp_t$
FR	α_i	ΔIP_t, $\Delta Canc_t$, $\Delta Resp_t$	-
	α_i	-	ΔIP_t, $\Delta Canc_t$, $\Delta Resp_t$
E&W	α_i	$\Delta Canc_t$, $\Delta Resp_t$, ΔExt_t	ΔIP_t, $\Delta Circ_t$, ΔExt_t
AU	α_i	-	ΔIP_t, $\Delta Circ_t$, ΔExt_t
	α_i	ΔIP_t, $\Delta Circ_t$, $\Delta Resp_t$	-

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Concluding remarks

In the short term

- Development of Circulatory, Respiratory, and External mortality rates depends on other cause-specific mortality rates;
- IP and Cancer mortality rates seem to be less impacted by other causes.

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- IP and Respiratory mortality rates are the most impacted by the cointegration relation;
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- External causes seem to be totally independent from it.

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Objective

- Set more informed assumptions on the future development of mortality.

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Objective

- Set more informed assumptions on the future development of mortality.

Next steps

- Study common stochastic trends shared by the cause-specific mortality rates.

Bibliography

- S. Arnold and M. Sherris. International cause-specific mortality rates: new insights from a cointegration analysis. ASTIN Bulletin, January 2016, pp 1- 30.
- S. Johansen : The role of the constant and linear terms in cointegration analysis of nonstationary variables, Econometric Reviews, 1994, 13:2, 205-229
- J. D. Hamilton. Time Series Analysis. Princeton University Press, 1994.
- H. Lütkepohl. New Introduction to Multiple Time Series Analysis. Crown Publishing Group, 2005.
- W. Kuo, C. Tsai, W.-K. Chen. An empirical study on the lapse rate: The cointegration approach. The Journal of Risk and Insurance, 2003, 70(3), 489-508

Thank you for your attention!

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2020 Living to 100 Symposium

Rotation of the Age Pattern of Mortality Improvements in EU Member States

Péter Vékás, Ph.D. (Corvinus University of Budapest)

January 15, 2020

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Introduction

- If human lifetimes are three years longer than expected (in line with underestimations in the past), costs of aging will increase by 50% of GDP in advanced economies and 25% of GDP in emerging economies (IMF, 2012).
- Mortality forecasting is invaluable for actuaries.
- Model assumptions may make a huge difference in the long run!

Contents

1. Time-invariance assumption of the Lee–Carter (1992) model
2. Rotation of the age pattern of mortality improvements (Li–Lee–Gerland, 2013)
3. Proposed methodology to assess rotation (Vékás, 2019)
4. Results on EU data

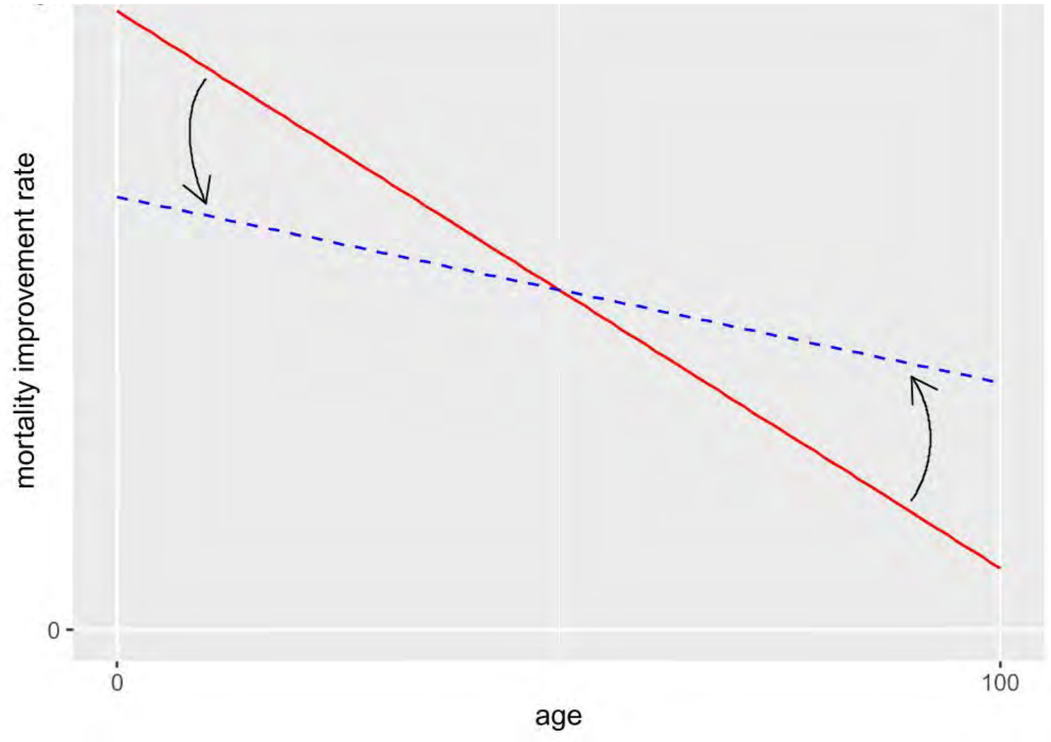
Lee–Carter (1992) model

- "Gold standard" of mortality forecasting.
- Log-mortality rates modeled as
  $\ln m_{xt} = a_x + b_x k_t + \varepsilon_{xt}$.
- As k_t declines over time, the coefficients b_x regulate the rates of improvement by age.
- Age-specific improvement rates b_x assumed to be independent of time (not b_{xt})!

Rotation

- *Rotation of the age pattern of mortality improvements* (Li–Lee–Gerland, 2013): mortality improvements tend to slow down in younger ages and speed up in older ages.
- Possible reasons:
  - little room left for spectacular advances in preventing child mortality,
  - improved, costly medical technology to cope with serious illness and extend life.

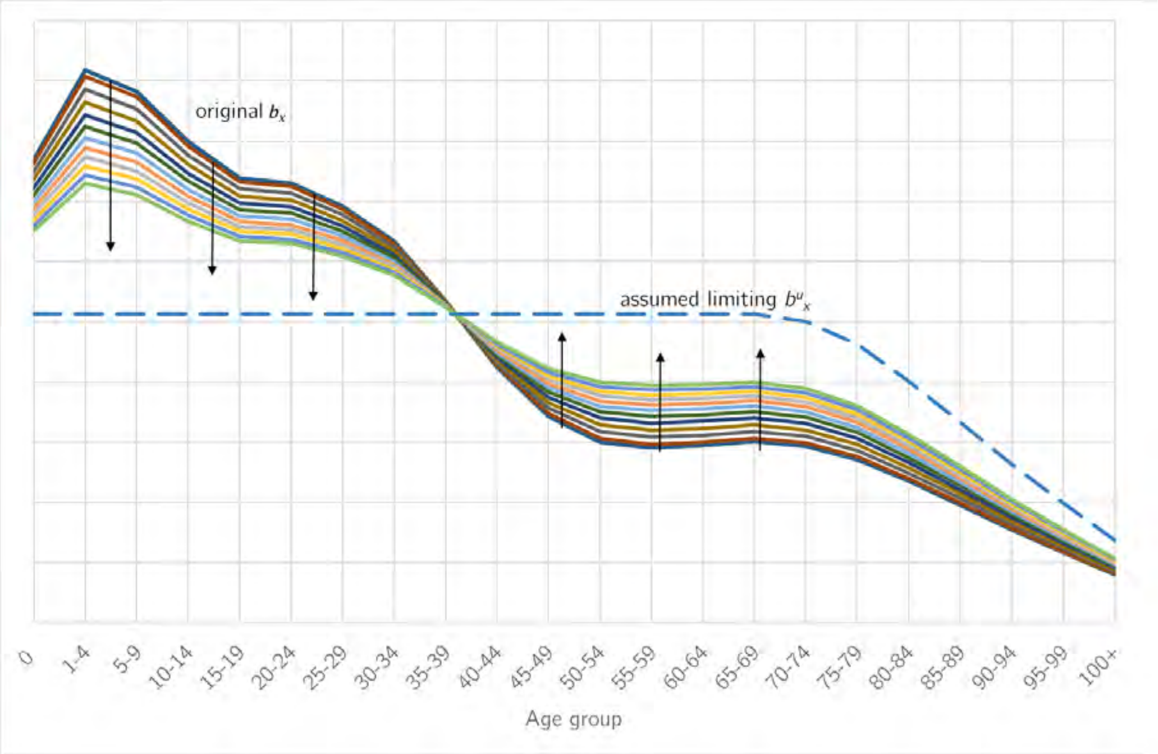
Illustration



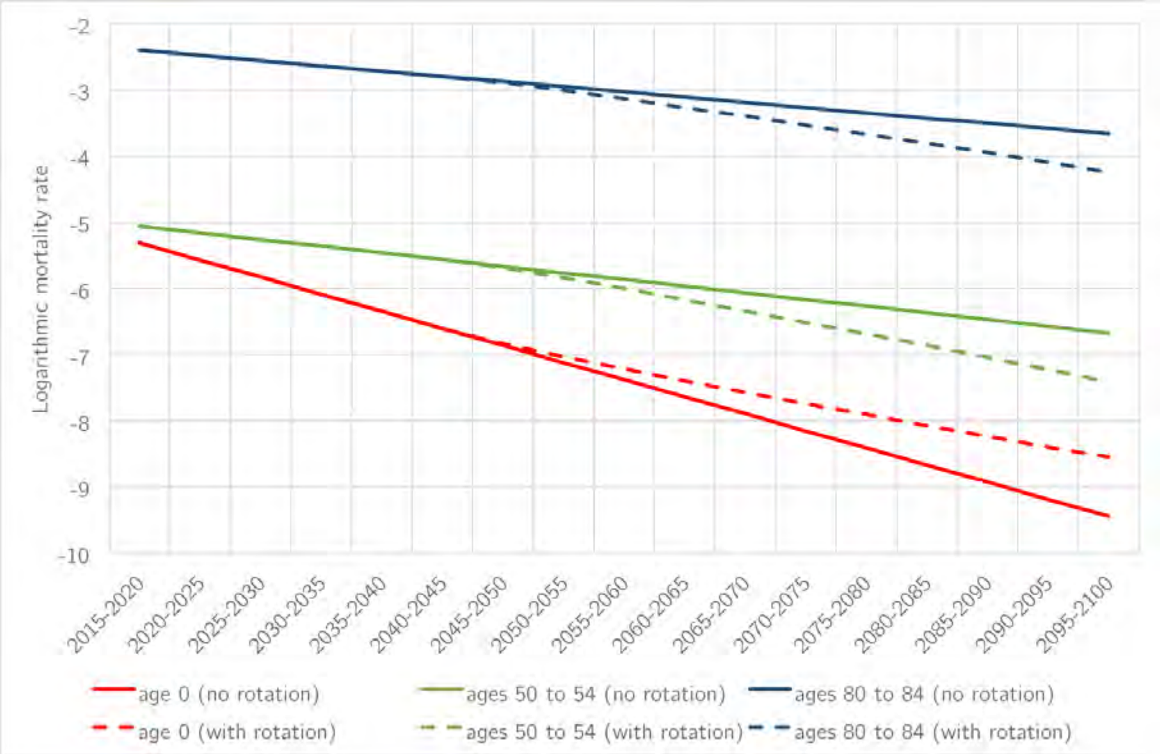
Rotation in theory and practice

- Plenty of sporadic evidence for rotation in the literature, mostly based on *ad hoc* methods.
- Li–Lee–Gerland (2013) have created the LC model including rotation (LCR variant).
- Applied by Vékás (2018) on Hungarian data to see the long-term impact of rotation.

Rotation of b_x in LCR model
(Hungary, 2018-2100)

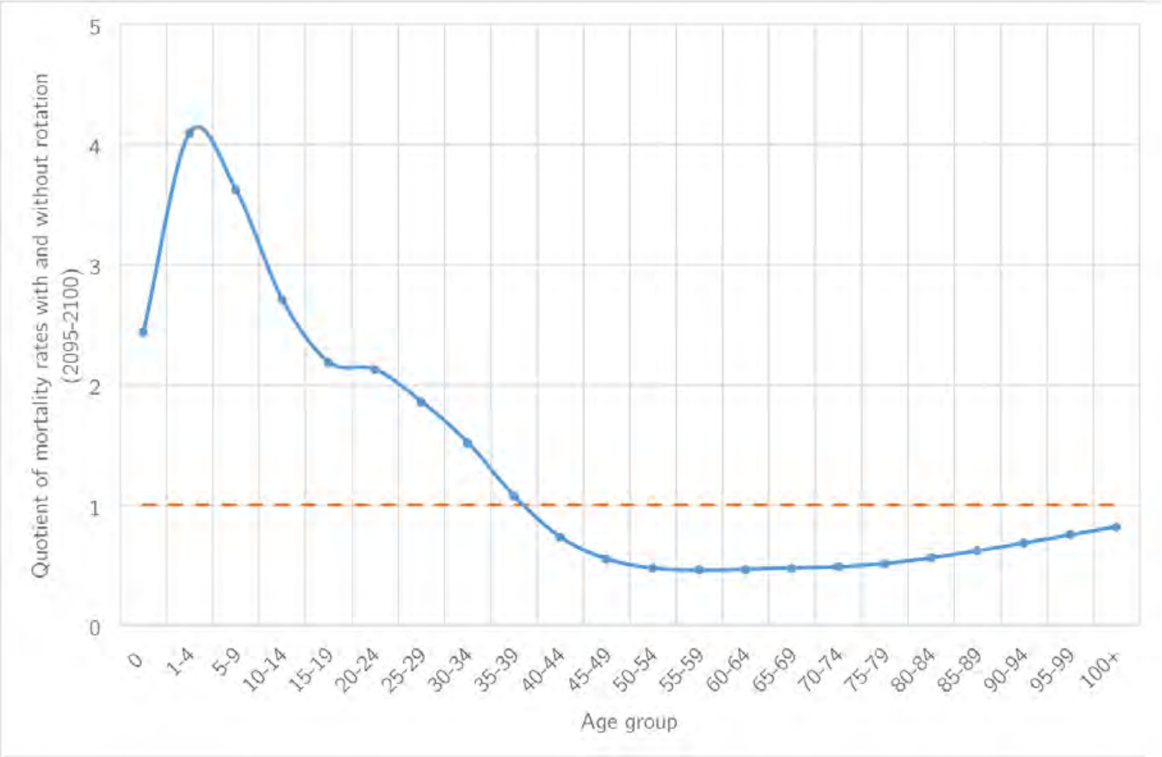


Projection with and without rotation
(Hungary, 2018-2100)



Substantial impact!

(Hungary, rates in 2100 with/without rotation)



To rotate or not to rotate?

- Forecasts ignoring rotation *systematically underestimate longevity risk*!
- Errors may be tremendous in the long run.
- It is crucial to assess whether there is rotation.
- Vékás (2019) proposes a methodology to measure and statistically test rotation and applies it on historical data of 28 EU countries.

Mortality improvement rates and acceleration rates

- Mortality improvement rates (x: age, t: period, c: country, g: gender)

$$r_{xt}^{cg} = -\log\left(\frac{m_{x,t+1}^{cg}}{m_{xt}^{cg}}\right)$$

- Long-term acceleration = slope of linear trend of mortality improvement rates over time:

$$\beta_x^{cg} = \frac{\sum_{t=1}^{12}(r_{xt}^{cg} - \bar{r}_x^{cg})(t-\bar{t})}{\sum_{t=1}^{12}(t-\bar{t})^2}$$

Strength of rotation

- Measured by Spearman's ρ between acceleration and age, weighted by population sizes of age groups:

$$\rho^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg} (\text{rank}(\beta_{x_i}^{cg}) - \mu^{cg})(i - \nu^{cg})}{\sqrt{\sum_{i=1}^{22} P_{x_i}^{cg} (\text{rank}(\beta_{x_i}^{cg}) - \mu^{cg})^2} \sqrt{\sum_{i=1}^{22} P_{x_i}^{cg} (i - \nu^{cg})^2}}$$

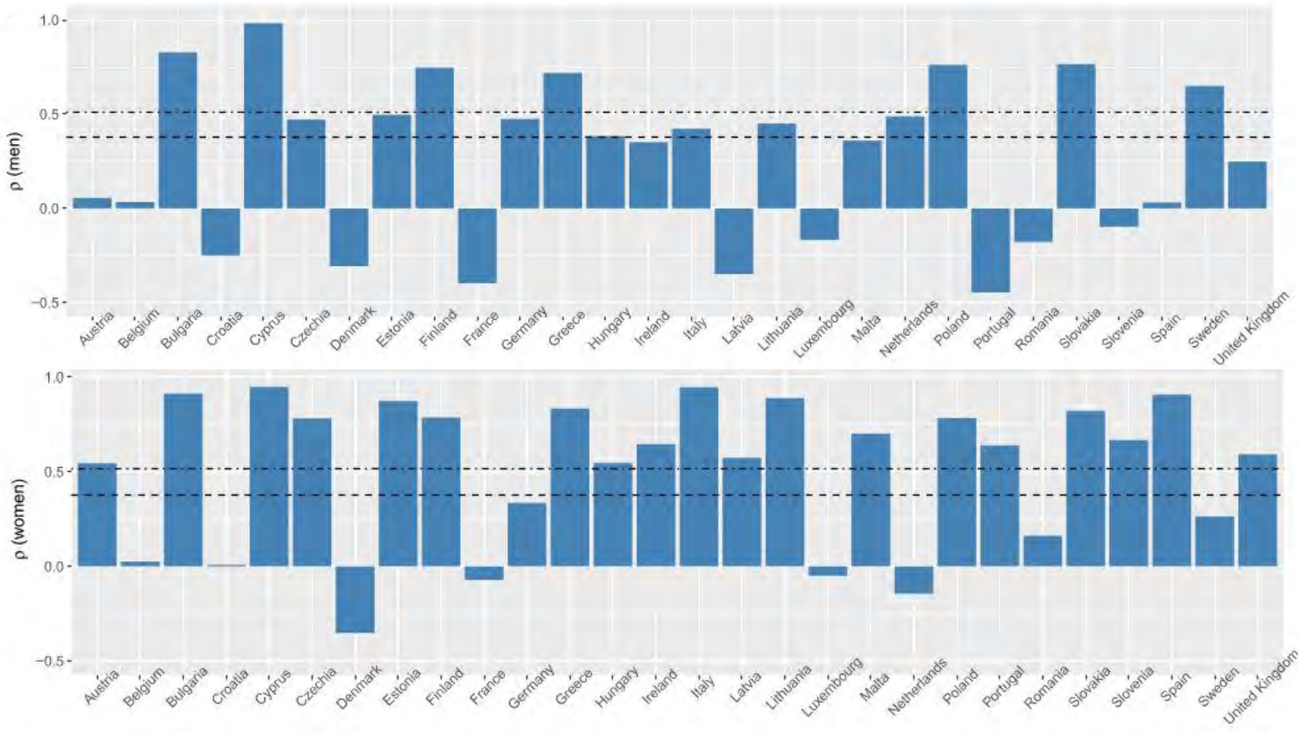
$$((c,g) \in \{c_1, c_2, \ldots, c_{28}\} \times \{M, W\}),$$

$$\mu^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg} \text{rank}(\beta_{x_i}^{cg})}{\sum_{i=1}^{22} P_{x_i}^{cg}} \quad \text{and} \quad \nu^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg} i}{\sum_{i=1}^{22} P_{x_i}^{cg}}$$

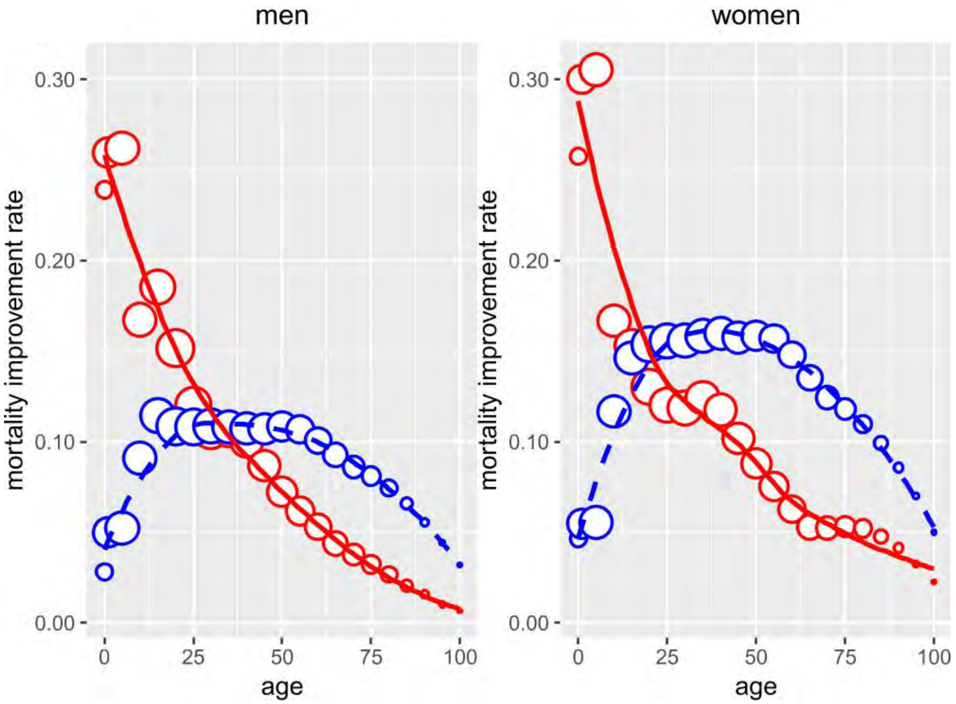
Data

- UN World Population Prospects 2017
- Mortality rates, life expectancies at birth and population counts
- 22 age groups, both genders
- 13 periods (1950–1955 up to 2010–2015)
- 28 member states of the EU

Strength of rotation by country and gender



Strongest rotation: Cyprus (earliest vs. latest periods)



Potential predictors of rotation

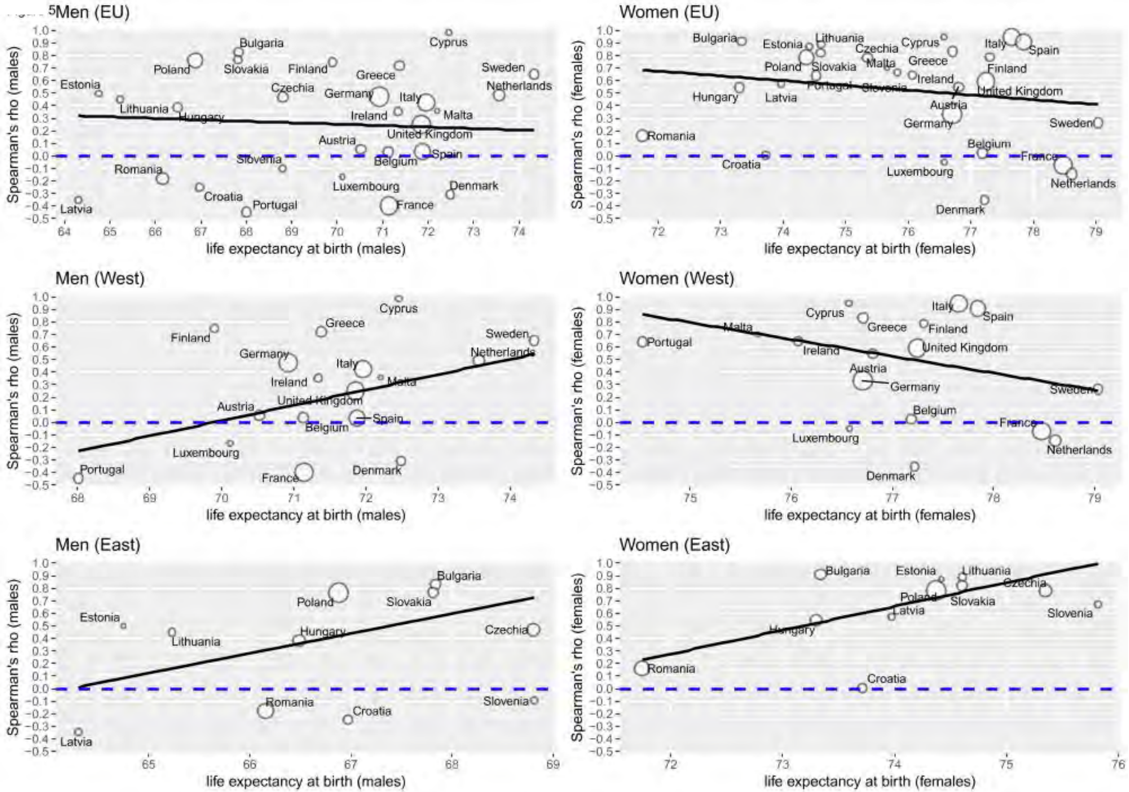
- Gender
- Former political bloc (East or West)
- e_0 (Li–Lee–Gerland, 2013: rotation is more prevalent in low-mortality countries)
- Possibly also e_{60}, or improvement of e_0 between 1950 and 2015

Impact of gender and political bloc

- Significantly more rotation in women's data!
- More rotation in the former Eastern bloc, but difference is not significant.

Gender (region)	$\rho^b(g)$	*p*-value	
Men (EU)	0.246		
Women (EU)	0.498		
Gender difference (EU)	-0.252	0.001	**
Men (West)	0.199		
Women (West)	0.466		
Gender difference (West)	-0.267	0.016	*
Men (East)	0.43		
Women (East)	0.619		
Gender difference (East)	-0.189	0.011	*
Regional difference (Men)	-0.231	0.202	
Regional difference (Women)	-0.153	0.396	

Rotation only correlated with e_0 in the East



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Conclusions

- New, simple, data-driven methodology to assess rotation and its relationships with other variables.
- Rotation is far from universal: only in some member states.
- More rotation in women’s data.

Conclusions

- Somewhat more in rotation in former Eastern bloc.
- Only related to e_0 in the East.
- Methodology may be used to decide whether to use LC model or LCR variant.

Thank you very much for your attention!

References

- Bohk-Ewald C, Rau R (2017) Probabilistic mortality forecasting with varying age-specific survival improvements. Genus J Popul Sci 73(1):15. https://doi.org/10.1186/s41118-016-0017-8
- Bongaarts J (2005) Long-range trends in adult mortality: Models and projection methods. Demography. 42(1):23–49. https://doi.org/10.1353/dem.2005.0003
- Booth H, Tickle L (2008) Mortality modelling and forecasting: a review of methods. Ann Actuar Sci. 3(1–2):3–43. https://doi.org/10.1017/S1748499500000440
- Booth H, Maindonald J, Smith L (2002) Applying Lee-Carter under Conditions of Variable Mortality Decline. Population Studies 56(3):325–336. https://doi.org/10.1080/00324720215935
- Cairns AJG, Blake D, Dowd K, Coughlan GD, Khalaf-Allah M (2011) Bayesian stochastic mortality modelling for two populations. ASTIN Bull 41(1):29–59
- Carter LR, Prskawetz A (2001) Examining structural shifts in mortality using the Lee -Carter method (working paper). Max Planck Institute for Demographic Research. https://www.demogr.mpg.de/Papers/Working/wp-2001-007.pdf
- Christensen K, Doblhammer G, Rau R, Vaupel JW(2009) Ageing populations: the challenges ahead. Lancet 374(9696):1196–1208. https://doi.org/10.1016/S0140-6736(09)61460-4
- Cleveland WS, Devlin SJ (1988) Locally-weighted regression: an approach to regression analysis by local fitting. J Am Stat Assoc 83(403):596–610. https://doi.org/10.2307/2289282

References

- De Beer J, Janssen F (2016) A new parametric model to assess delay and compression of mortality. Popul Health Metr 14:46. https://doi.org/10.1186/s12963-016-0113-1
- Dion P, Bohnert N, Coulombe S, Martel L (2015) Population Projections for Canada (2013 to 2063),Provinces and Territories (2013 to 2038): technical report on methodology and assumptions. Technical report, Statistics Canada. https://www150.statcan.gc.ca/n1/en/catalogue/91-620-X
- Haberman S, Renshaw A (2012) Parametric mortality improvement rate modelling and projecting. Insur Math Econ 50(3):309–333. https://doi.org/10.1016/j.insmatheco.2011.11.005
- Horiuchi S, Wilmoth JR (1995) The aging of mortality decline. In: Annual meeting of the population Association of America, San Francisco, CA
- Hyndman RJ, Ullah MS (2007) Robust forecasting of mortality and fertility rates: a functional data approach. Comput Stat Data Anal 51(10):4942–4956. https://doi.org/10.1016/j.csda.2006.07.028
- Hyndman RJ, Booth H, Yasmeen F (2013) Coherent mortality forecasting: the product-ratio method with functional time series models. Demography 50(1):261–283. https://doi.org/10.1007/s13524-012-0145-5
- IMF (2012). Global Financial Stability Report. Chapter 4: The financial impact of longevity risk. International Monetary Fund, Washington D.C. http://www.imf.org/external/pubs/ft/gfsr/2012/01/pdf/text.pdf
- Lee RD, Carter LR (1992) Modeling and forecasting US mortality. J Am Stat Assoc 87:659–671. https://doi.org/10.2307/2290201

References

- Kannisto V, Lauritsen J, Thatcher AR, Vaupel JW (1994) Reductions in mortality at advanced ages: several decades of evidence from 27 countries. Popul Dev Rev 20(4):793–810. https://doi.org/10.2307/2137662
- Lee R (2000) The Lee–Carter method for forecasting mortality, with various extensions and applications. North Am Actuar J 4(1):80–93. https://doi.org/10.1080/10920277.2000.10595882
- Lee R, Miller T (2001) Evaluating the performance of the Lee–Carter method for forecasting mortality. Demography 38(4):537–549. https://doi.org/10.1353/dem.2001.0036
- Li H, Li JS (2017) Optimizing the Lee–Carter approach in the presence of structural changes in time and age patterns of mortality improvements. Demography 54(3):1073–1095. https://doi.org/10.1007/s13524-017-0579-x
- Li N, Gerland P (2011) Modifying the Lee–Carter method to project mortality changes up to 2100. In: Annual meeting of the population Association of America, Washington, DC
- Li N, Lee R (2005) Coherent mortality forecasts for a group of populations: an extension of the Lee–Carter method. Demography 42(3):575–594. https://doi.org/10.1353/dem.2005.0021
- Li N, Lee R, Gerland P (2013) Extending the Lee–Carter method to model the rotation of age patterns of mortality-decline for long-term projection. Demography 50(6):2037–2051. https://doi.org/10.1007/s13524-013-0232-2
- Mitchell D, Brockett P, Mendoza-Arriaga R, Muthuraman K (2013) Modeling and forecasting mortality rates. Insur Math Econ 52(2):275–285. https://doi.org/10.1016/j.insmatheco.2013.01.002

References

- Pinto da Costa J (2015) Rankings and preferences – new results in weighted correlation and weighted principal component analysis with applications. Springer, Berlin. ISBN 978-3-662-48343-5
- Pitacco E, Denuit M,Haberman S, Olivieri A(2009) Modelling longevity dynamics for pensions and annuity business. Oxford University Press, Oxford ISBN: 9780199547272
- R Development Core Team (2008) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna
- Rau R, Soroko E, Jasilionis D, Vaupel JW(2008) Continued reductions in mortality at advanced ages. Popul Dev Rev 34:747–68. https://doi.org/10.1111/j.1728-4457.2008.00249.x
- RussolilloM,Giordano G, Haberman S (2011) Extending theLee–Carter model: a three-way decomposition. Scand Actuar J 2011(2):96–117. https://doi.org/10.1080/03461231003611933
- Ševčíková H, Li N, Kantorová V, Gerland P, Raftery AE (2016) Age-specific mortality and fertility rates for probabilistic population projections. In: Schoen R (ed.) Dynamic demographic analysis, vol 39. The Springer Series on Demographic Methods and Population Analysis. Springer, Switzerland, pp 69–89
- Tuljapurkar S, Li N, Boe C (2000) A universal pattern of mortality change in the G7 countries. Nature 405(6788): 789–792
- United Nations Population Division (2018). World Population Prospects 2017 (maintained by Ševčíková, H.). https://CRAN.R-project.org/package=wpp2017