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Examining the Oldest-Old Mortality in the U.S.: A Forecast Reconciliation Approach

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Examining the Oldest-Old Mortality in the U.S.: A Forecast Reconciliation Approach

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ABSTRACT

Understanding the heterogeneity in regional-level mortality experience is of fundamental importance. This paper analyzes the state-level mortality rates for 50 U.S. states as well as the District of Columbia at age 80 and above via a novel forecast reconciliation approach. Based on mortality data from 1990–2017, we project the 10-year-ahead mortality rates at national and state levels up to 2027. We find that the geographical heterogeneity in the old-age mortality experience is likely to continue and the mortality improvement rates will to slow in the next decade.

1 BACKGROUND

With rapid economic growth and medical advancement, life expectancy in the U.S. has continuously improved in recent decades. The increasing trend in life expectancy has led to a considerable amount of longevity risk faced by insurance companies, pension providers, government agencies and individuals. Numerous studies have been conducted to understand and analyze the U.S. national-level mortality trends from a demographic, actuarial or epidemiology point of view (see, e.g., Rice and Feldman 1983; Lee and Carter 1992; Preston and Wang 2006). However, relatively little is known about the disparities in mortality experience among individual U.S. states, and more importantly, whether these state-level mortality trends will converge or diverge in the future. Recent research has found large geographical inequalities in mortality experience within the U.S. (see, e.g., Wang et al. 2013; Dwyer-Lindgren et al. 2016). These findings have urged the need to model and forecast mortality on not only the national level, but also the state level, in an integrated manner.

In this paper, we propose a novel forecast reconciliation approach to jointly projecting national-level and state-level age-gender-specific mortality rates. We focus on the mortality experience of the so-called “oldest old” (age 80 and over) across 51 U.S. states.[1] We have chosen to investigate this age range since its mortality experience plays an important role in overall longevity improvement, especially in an era of population aging. In Section 2, we compare the state-level oldest-old mortality rates between 1990 and 2017. To better visualize the geographical heterogeneity in mortality, we plot estimates of both mortality rate and percentage change in mortality rate on maps. In Section 3, we introduce the cutting-edge trace minimization forecast reconciliation approach (Wickramasuriya, Athanasopoulos and Hyndman 2019) in the context of mortality forecasting. The approach is then applied in Section 4, to forecast the 10-year-ahead U.S. mortality rates up to 2027. Based on our results, we find that the heterogeneity in mortality experience across U.S. states is likely to persist in the future. Moreover, even though almost all states are still expecting a decrease in mortality rates of the oldest old, the overall improvement rate seems to slow down in the next decade.

1 For ease of exposition, in this paper we will refer to the District of Columbia as a “state.”

2 STATE-LEVEL OLDEST-OLD MORTALITY RATES

To compute mortality rates for age groups 80 to 100+, we collect U.S. state-level death and population data from 1990–2017 from two main sources:

- **National Center for Health Statistics (NCHS).** The NCHS records information of all individual deaths in the U.S. since 1959, including gender, date of birth, month of death and geographical identifier. We collect state-level death data from NCHS for the period 1990–98.
- **Centers for Disease Control and Prevention (CDC) Wide-ranging Online Data for Epidemiologic Research (WONDER) database.** The CDC WONDER database provides a rich query system for the analysis of public health data. We collect state-level death data for the period 1999–2017 via "Underlying Cause of Death, 1999–2017 Request," and collect state-level mid-year population data for the period 1990–2017 via "Bridged-Race Population Estimates, 1990–2017 Request."[2]

The crude mortality rate m is calculated by:

$$m = \frac{D}{E}, \tag{1}$$

where D represents the number of deaths and E represents the corresponding population exposure. We use the mid-year population estimates obtained from the CDC WONDER database to approximate the population exposure.

Based on the collected data, we calculate the crude mortality rates of age group 80 to 100+ for 51 U.S. states from 1990–2017, as well as the corresponding percentage change in mortality rates between 1990 and 2017. Table 1 shows mortality experience in 2017 for the "top" five and "bottom" five states in terms of crude rate and percentage change, with top ranked states having the lowest crude rate or the smallest value of percentage change (largest mortality improvement). We can see that overall, the District of Columbia ranked highest, with a crude rate of 7.99% and percentage change of –24.19%. Hawaii, Florida, Arizona and Alaska also stood out as states with low mortality rates for ages 80 and above. On the other hand, high rates of oldest-old mortality were found in the Southeast and Midwest regions in the United States including West Virginia, Tennessee, Kentucky and Indiana. In terms of mortality improvement, besides highly urbanized states such as the District of Columbia and New York, and Alaska, Wyoming and California also experienced a substantial decrease in the oldest-old mortality rates. Nevertheless, the worst morality improvement rates were found in several sparsely populated states including Maine, Utah, Rhode Island, South Dakota and Idaho. As an exception, Idaho was the only state that experienced worsened mortality rates from 1990 to 2017.

2 Note that the state-level population data is only available for ages 80 to 85+. However, as we would need to aggregate both death data and exposure data for ages 80 to 100+ to obtain mortality forecasts for the oldest old, this does not affect our final estimates.

Table 1

MORTALITY EXPERIENCE FOR SELECTED U.S. STATES IN 2017

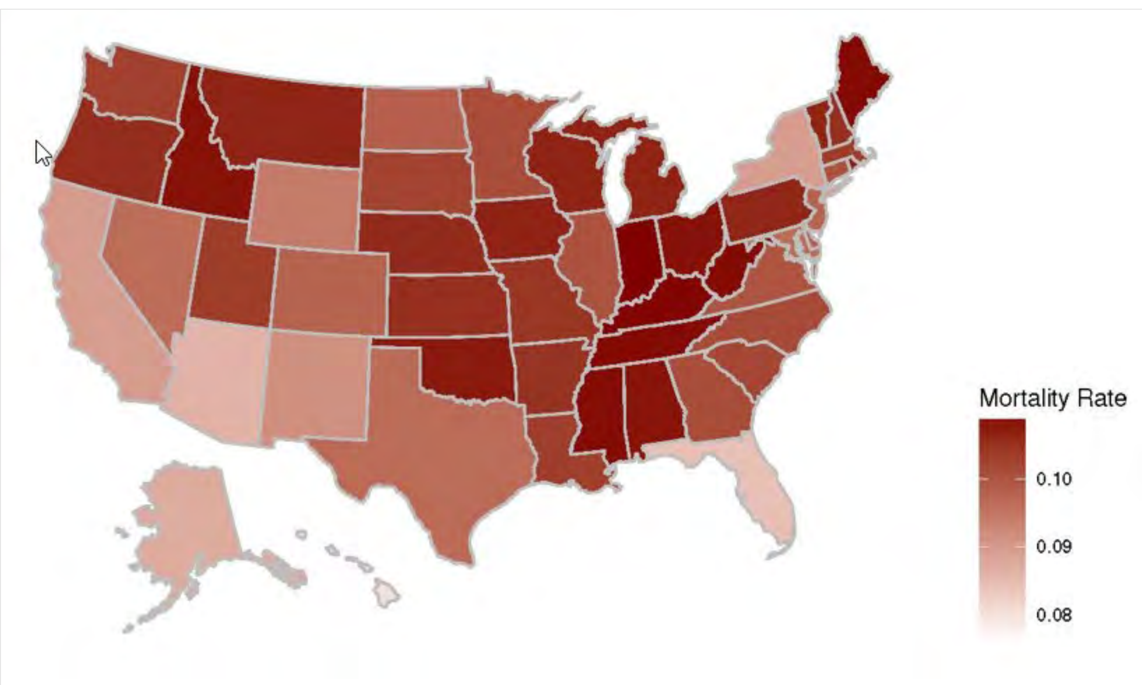
Rank	State	Crude Rate	Rank	State	Percentage Change
1	District of Columbia	7.99%	1	District of Columbia	−24.19%
2	Hawaii	8.03%	2	New York	−18.69%
3	Florida	8.47%	3	Alaska	−17.58%
4	Arizona	8.68%	4	Wyoming	−17.13%
5	Alaska	8.76%	5	California	−16.03%
⋮	⋮	⋮	⋮	⋮	⋮
47	West Virginia	10.91%	47	Maine	−1.23%
48	Maine	10.91%	48	Utah	−1.02%
49	Tennessee	10.92%	49	Rhode Island	−0.39%
50	Kentucky	10.95%	50	South Dakota	−0.38%
51	Indiana	10.98%	51	Idaho	4.91%

Sources: National Center for Health Statistics (NCHS) and Centers for Disease Control and Prevention (CDC). Wide-ranging Online Data for Epidemiologic Research (WONDER). Accessed March 8, 2019.

Figures 1 and 2 illustrate the geographical variations in the mortality rates and the percentage changes, respectively. We can observe a very clear geographic pattern in these figures: Southern states have considerable survival disadvantages for their oldest-old age groups. Overall, there is a tendency for better mortality experience in urban states and states with favorable weather or a large population of elderly. On the other hand, the worst old-age mortality experience is mostly in rural states, states with lower socio-economic profiles and states with extremely cold winters. Our findings are consistent with those by Dwyer-Lindgren et al. (2016), Holman (2017) and Andreev, Gu and Dupre (2017).

Figure 1

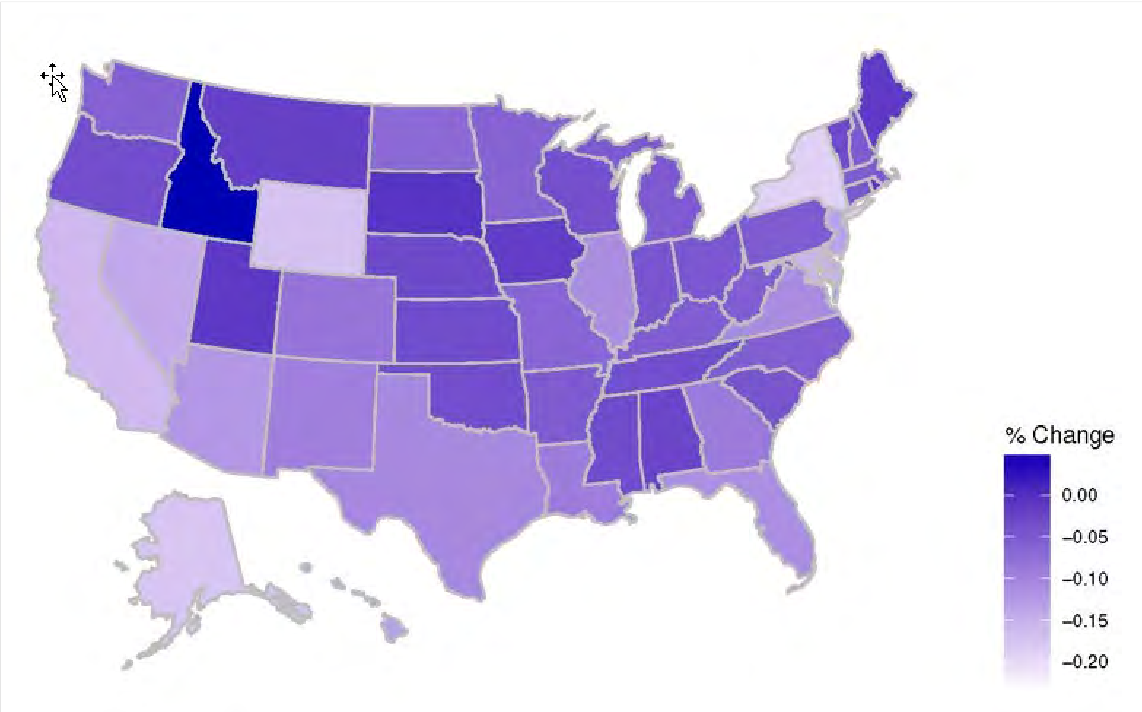
U.S. STATE-LEVEL OLDEST-OLD MORTALITY RATES IN 2017



Source: Centers for Disease Control and Prevention (CDC). Wide-ranging Online Data for Epidemiologic Research (WONDER). Accessed March 8, 2019.

Figure 2

U.S. STATE-LEVEL PERCENTAGE CHANGE IN OLDEST-OLD MORTALITY RATES, 1990–2017



Sources: National Center for Health Statistics (NCHS) and Centers for Disease Control and Prevention (CDC). Wide-ranging Online Data for Epidemiologic Research (WONDER). Accessed March 8, 2019.

3 A HIERARCHICAL RECONCILIATION APPROACH

Forecast reconciliation refers to the process of adjusting hierarchical time series forecasts such that the underlying aggregation constraints are met. Literature has been fast growing on forecast reconciliation, in particular during the last few decades (see, e.g., Dangerfield and Morris 1992; Kahn 1998; Zellner and Tobias 2000; Athanasopoulos, Ahmed and Hyndman 2009; Hyndman et al. 2011; Wickramasuriya, Athanasopoulos and Hyndman 2019).[3] The advantages of forecast reconciliation are twofold. First, the reconciliation process ensures forecast coherency that helps people to make aligned decisions. In reality, it is almost impossible for independently projected forecasts at different levels to add up in a manner consistent with the underlying hierarchical structure. Therefore, reconciliation becomes a useful tool to reduce the discrepancy resulting from conflicting forecasts. Second, by incorporating information at all levels into the forecasting process, reconciliation improves the overall forecast accuracy in the hierarchy (see, e.g., Athanasopoulos, Ahmed and Hyndman 2009; Capistran, Constandse and Ramos-Francia 2010; Borges, Penya and Fernandez 2013; Syntetos et al. 2016). It is worth noting that the application of a forecast reconciliation approach in mortality modeling is not a new phenomenon; several earlier attempts have been made in reconciling cause-of-death mortality rates and regional infant mortality rates (see, e.g., Shang and Haberman 2017; Li et al. 2019).

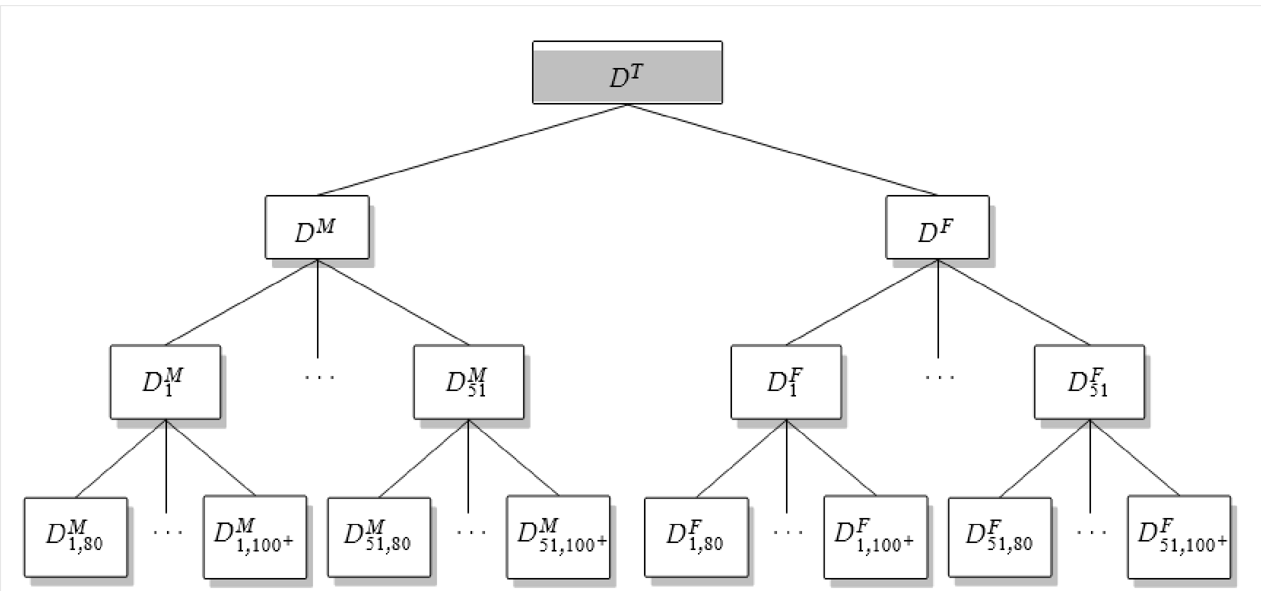
In this paper, we adopt the trace minimization (MinT) method proposed by Wickramasuriya, Athanasopoulos and Hyndman (2019) to reconcile both death counts and population exposure in a hierarchical setting. Our approach is different from the approach used in Shang and Haberman (2017), which reconciled the ratio of death counts to total

3 For an overview of major forecast reconciliation methods, see Hyndman and Athanasopoulos (2014).

exposure but ignored the hierarchical structure inherent in the population exposure. We decide to first reconcile death and population forecasts separately, and then produce mortality forecasts based on these coherent forecasts. In doing so, information on number of deaths and population exposure at all levels are taken into account in the forecasting process.

We illustrate the reconciliation process for death data as follows.[4] As can be seen in Figure 3, death data naturally forms a three-level hierarchical structure. At the top level (which we refer to as level 0), we have the national total number of deaths D^T. This number can be divided into total number of male deaths D^M and female deaths D^F at level 1. At level 2, we further categorize gender-specific deaths by state, where D_i^M denotes the number of male deaths in state i. Finally, we have gender-state-age-specific death counts at the bottom level (level 3), where $D_{i,x}^M$ denotes the number of male deaths in state i at age x. Therefore, we have 21 × 51 × 2 = 2142 time series at level 3, 51 × 2 = 102 time series at level 2, two time series at level 1 and one time series at level 0. These sum up to 2247 series in total.

Figure 3
THREE-LEVEL HIERARCHICAL TREE FOR NUMBER OF DEATHS



For the hierarchical structure in the death counts, we have the following aggregation constraints at all times:

$$\sum_{x=85}^{100+} D_{i,x}^j = D_i^j, \forall i \in [1,51], j = M, F, \quad (2)$$

$$\sum_{i=1}^{51} D_i^j = D^j, j = M, F, \quad (3)$$

$$D^M + D^F = D^T. \quad (4)$$

4 Note that the same approach described in this section can also be applied to population exposure.

To introduce the MinT reconciliation method, we first express the aforementioned aggregation constraints in a matrix form and formally define the following notation:

- Define $y = (D^T,\ D^M,\ D^F, D_1^M, \dots,\ D_{51}^M,\ D_1^F, \dots,\ D_{51}^F, D_{1,80}^M, \dots,\ D_{1,100+}^M, \dots,\ D_{51,80}^M,$ $\dots, D_{51,100+}^M,\ D_{1,80}^F, \dots,\ D_{1,100+}^F, \dots, D_{51,80}^F, \dots,\ D_{51,100+}^F)'$

  as a vector that contains observations at all levels in the hierarchy, namely level 0, level 1, level 2 and level 3;
- Define $b = (D_{1,80}^M, \dots,\ D_{1,100+}^M, \dots,\ D_{51,80}^M, \dots,\ D_{51,100+}^M,\ D_{1,80}^F, \dots, D_{1,100+}^F, \dots, D_{51,80}^F, \dots, D_{51,100+}^F)'$

  as a vector that contains observations at the bottom level (level 3) only.

The two vectors can then be linked by the equation

$$y_t = Sb_t, \tag{5}$$

where y_t and b_t represent the value of y and b at time t, respectively. S is a "summing matrix" of dimension 2247 × 2142, which aggregates gender-state-age-specific death counts at the bottom level to obtain death counts at higher levels. It is given by

$$S = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \dots & \dots & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \dots & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \dots & \mathbf{1} \\ \mathbf{1} & \dots & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{1} & \dots & \mathbf{1} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{1} & \dots & \mathbf{1} \\ \\ & & & & \boldsymbol{I_{2142}} & & & & & \end{pmatrix}$$

where I_{2142} denotes an identity matrix of dimension 2142 × 2142. Therefore, the aggregation constraints in equations (2)–(4) are reflected in the structure of matrix S.

Although Equation (5) holds for all observed values, it is unlikely to hold for independently obtained forecasts in the hierarchy. Let $\hat{y}_{T+h}$ denote the unreconciled h-step-ahead forecasts at all levels, and let $\tilde{y}_{T+h}$ denote the correspondingly reconciled h-step-ahead forecasts, which satisfy all aggregation constraints. Any linear reconciliation method, according to Wickramasuriya, Athanasopoulos and Hyndman (2019), can be expressed as

$$\tilde{y}_{T+h} = SP\hat{y}_{T+h}, \tag{6}$$

where in our example P is a 2142 × 2247 matrix. The selection of P is not unique and is a key step in forecast reconciliation.

Hyndman et al. (2011) provided an optimal combination method to estimate P such that the reconciled forecasts are unbiased given that the unreconciled forecasts are also unbiased. Wickramasuriya, Athanasopoulos and Hyndman (2019) extended the work by Hyndman et al. (2011) and further improved the method by proposing an alternative estimator of P. Wickramasuriya, Athanasopoulos and Hyndman (2019) selected P to be the matrix that minimizes the trace of the covariance matrix of the in-sample reconciled forecast errors, which is $VAR[y_{t+h} - \tilde{y}_{t+h}|y_1, y_2, \ldots, y_t]$. Therefore, the method is referred to as the trace minimization method.[5]

Based on the MinT approach, the reconciliation matrix P is given by

$$P = (S'W_h^{-1}S)^{-1}S'W_h^{-1}, \quad (7)$$

where W_h represents the covariance matrix of the h-step-ahead in-sample unreconciled forecast errors, which is $VAR[y_{t+h} - \hat{y}_{t+h}|y_1, y_2, \ldots, y_t]$.[6]

Wickramasuriya, Athanasopoulos and Hyndman (2019) also showed that the reconciled forecasts will be at least as accurate as the unreconciled forecasts. Moreover, the reconciliation process takes into account the dependence structure across different levels of the hierarchy, which is particularly beneficial when dependence is ignored in the process of obtaining unreconciled forecasts for each individual time series. In our case, with a very large number of time series in the hierarchy, it is extremely challenging to use a joint modeling approach to producing forecasts, due to the curse of dimensionality. Therefore, the MinT reconciliation method becomes a particularly effective tool when we obtain unreconciled forecasts based on independent models.

4 PROJECTING MORTALITY IN 2027 VIA MINT APPROACH

In this section, we apply the MinT reconciliation method described in Section 3 to both death counts and population exposure. Based on the U.S. historical data from 1990 to 2017, we obtain the 10-year-ahead unreconciled forecasts by independent autoregressive integrated moving average (ARIMA) models at all levels. The Akaike information criterion is used to select the optimal ARIMA model for each time series in the hierarchy (Akaike 1974). Once reconciled forecasts for both the number of deaths and population exposure are computed, we use these forecasts to calculate the 10-year-ahead mortality rates at state level and national level.

Table 2 shows mortality forecasts in 2027 for the top five and bottom five states, in terms of crude mortality rate and the percentage change in mortality. Compared to the figures for 2017 shown in Table 1, the top five states with the lowest crude mortality rates remain unchanged: District of Columbia, Florida, Arizona, Hawaii and Alaska continue to experience the most favorable oldest-old mortality experience in the country. Florida is also predicted to have a big mortality improvement over the next 10 years and overtake Hawaii to have the second lowest old-age mortality rates. On the other hand, Indiana and West Virginia remain in the bottom five states with the highest crude rates. In terms of percentage change in mortality, most states except South Dakota are predicted to have a reduction in the oldest-old mortality rates over the next decade. However, for many states, including Ohio, Iowa, Pennsylvania and Connecticut, the projected mortality improvement rates are rather marginal and insignificant.

5 For a detailed explanation of the method, see Wickramasuriya, Athanasopoulos and Hyndman (2019).
6 In this paper, we have used the "shrinkage" estimator of W_h as described in Wickramasuriya, Athanasopoulos and Hyndman (2019), Section 2.4.

Table 2
MORTALITY EXPERIENCE FOR SELECTED U.S. STATES IN 2027

Rank	State	Crude Rate	Rank	State	Percentage Change
1	District of Columbia	7.38%	1	Georgia	−10.98%
2	Florida	7.64%	2	Nevada	−10.28%
3	Arizona	7.89%	3	South Carolina	−10.16%
4	Hawaii	7.93%	4	Wyoming	−9.97%
5	Alaska	8.06%	5	Florida	−9.74%
⋮	⋮	⋮	⋮	⋮	⋮
47	Iowa	10.65%	47	Ohio	−0.43%
48	Indiana	10.67%	48	Iowa	−0.37%
49	Rhode Island	10.72%	49	Pennsylvania	−0.16%
50	West Virginia	10.74%	50	Connecticut	−0.05%
51	Ohio	10.83%	51	South Dakota	0.06%

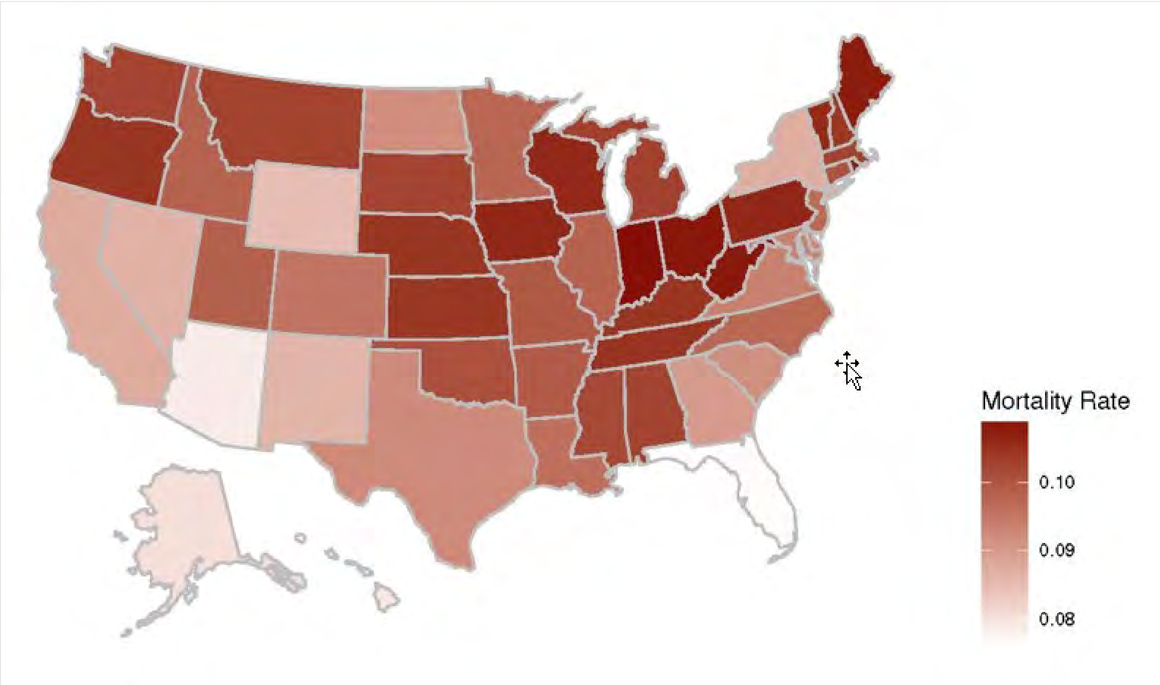
Note: The numbers in the table are based on the modeling results of the proposed method, using data gathered from Centers for Disease Control and Prevention (CDC), Wide-ranging Online Data for Epidemiologic Research (WONDER), accessed March 8, 2019.

We plot the mortality forecasts and percentage change forecasts for all 51 states in Figures 4 and 5. Figure 4 shows that the old-age survival disadvantage across Southern states continues to exist. The highest mortality rate forecasts are generally observed in Rust Belt states such as Pennsylvania, West Virginia, Ohio and Indiana. Also, based on Figure 5, we observe marginal mortality improvement in a large number of states. It is anticipated that the mortality improvement rate will slow down in the next 10 years for a majority of states in the U.S., including those with extremely good mortality experience such as Hawaii.

To provide a complete picture of the future mortality projections, we also plot the national-level forecasts for the period 2018–27 in Figure 6. It can be seen that males are predicted to experience a more rapid mortality improvement compared to females. However, we also observe a slowdown in the decreasing trend of mortality rates, especially in the case of female mortality rates. In fact, Figure 6 shows that the slowdown may have begun in the early 2010s. In addition, based on our forecasts, we find that the future total mortality improvement is more likely to result from improvement in male mortality rates.

Figure 4

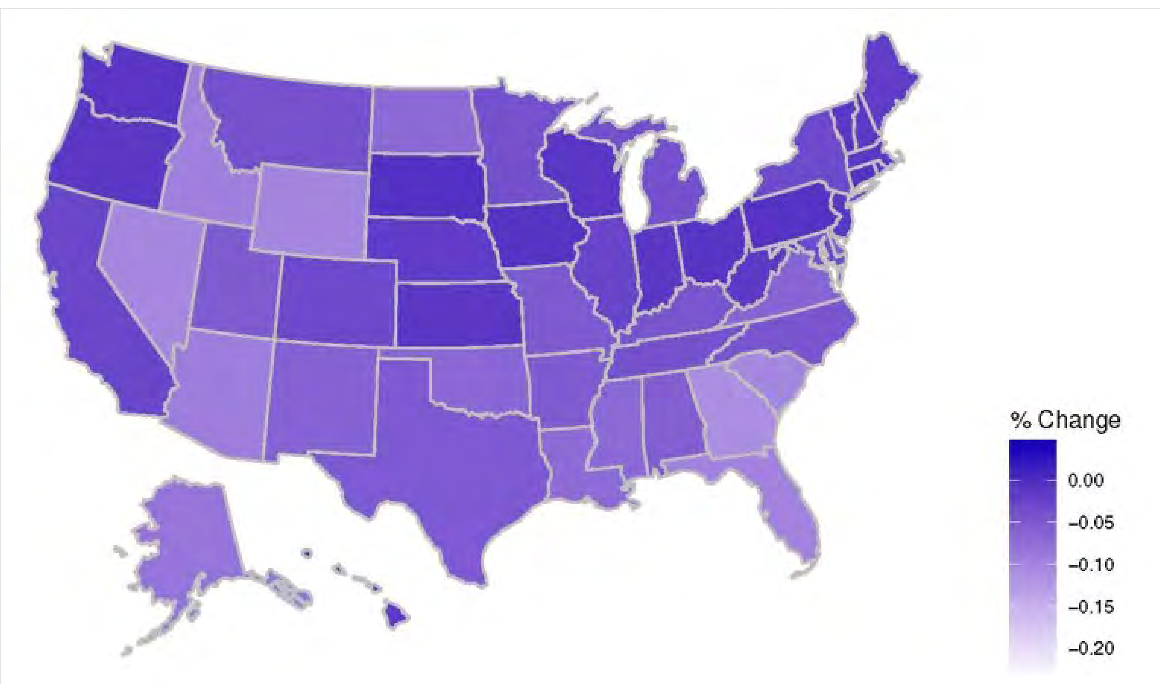
U.S. STATE-LEVEL OLDEST-OLD MORTALITY FORECASTS IN 2027



Note: The information in this figure is based on the modeling results of the proposed method.

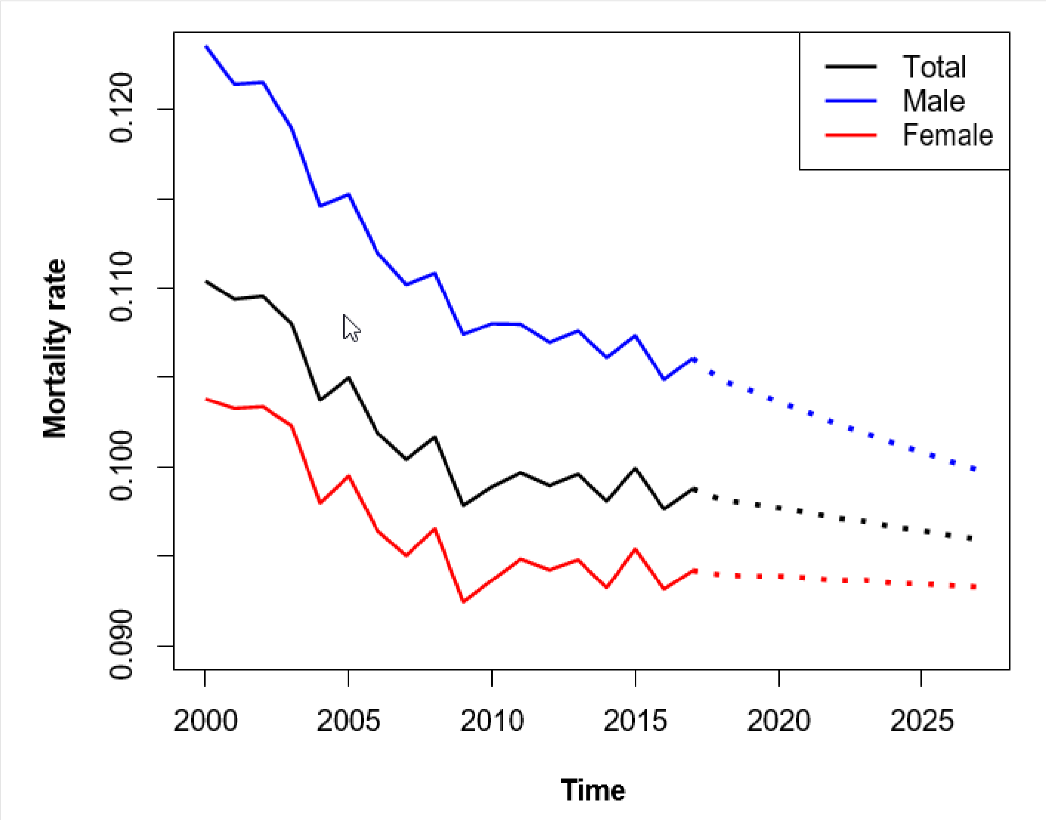
Figure 5

U.S. STATE-LEVEL PERCENTAGE CHANGE FORECAST IN OLDEST-OLD MORTALITY RATES 2017–27



Note: The information in this figure is based on the modeling results of the proposed method, using data gathered from Centers for Disease Control and Prevention (CDC), Wide-ranging Online Data for Epidemiologic Research (WONDER), accessed March 8, 2019.

Figure 6
OLDEST-OLD MORTALITY RATE FORECASTS IN THE U.S. FOR THE PERIOD 2018–27



Note: The information in this figure is based on the modeling results of the proposed method, using data gathered from Centers for Disease Control and Prevention (CDC), Wide-ranging Online Data for Epidemiologic Research (WONDER), accessed March 8, 2019.

5 CONCLUSIONS

The geographical heterogeneity in mortality experience, especially among older age groups, remains an important issue to be addressed and investigated. In this paper, we propose a forecast reconciliation approach to predict U.S. oldest-old mortality rates. Coherent mortality forecasts are produced using the MinT reconciliation method by Wickramasuriya, Athanasopoulos and Hyndman (2019). Based on our forecasts, we find that the observed less favorable mortality experience in the Southern states are likely to continue in the next decade. We also find that the future mortality improvement rate will tend to slow down across a majority number of states.

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REFERENCES

Akaike, Hirotugu. 1974. A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control* 19, no. 6:716–23.

Andreev, Kirill, Danan Gu and Matthew Dupre. 2017. Regional Mortality in the United States at Ages 80 and Older: An Analysis of Direct Estimates, 1959–2011. Society of Actuaries' *Living to 100 Symposium Monograph*.

Athanasopoulos, George, Roman A. Ahmed and Rob J. Hyndman. 2009. Hierarchical Forecasts for Australian Domestic Tourism. *International Journal of Forecasting* 25, no. 1:146–66.

Borges, Cruz E., Yoseba K. Penya and Ivan Fernandez. 2013. Evaluating Combined Load Forecasting in Large Power Systems and Smart Grids. *IEEE Transactions on Industrial Informatics* 9, no. 3:1570–77.

Capistran, Carlos, Christian Constandse and Manuel Ramos-Francia. 2010. Multi-Horizon Inflation Forecasts Using Disaggregated Data. *Economic Modelling* 27, no. 3:666–77.

Dangerfield, Byron J., and John S. Morris. 1992. Top-Down or Bottom-Up: Aggregate Versus Disaggregate Extrapolations. *International Journal of Forecasting* 8, no. 2:233–41.

Dwyer-Lindgren, Laura, Amelia Bertozzi-Villa, Rebecca W. Stubbs, Chloe Morozoff, Michael John Kutz, Chantal Huynh, C., Ryan M. Barber et al. 2016. U.S. County-Level Trends in Mortality Rates for Major Causes of Death, 1980–2014. *Journal of the American Medical Association* 316, no. 22:2385–2401.

Holman, R. Jerome. 2017. U.S. Population Mortality Rate Study: Variation by Age Group, Cause of Death and Region from 2000–2015. Society of Actuaries study.

Hyndman, Rob J., and George Athanasopoulos. 2014. *Forecasting: Principles and Practice*. Heathmont, Vic.: OTexts. *https://otexts.com/fpp2/index.html*.

Hyndman, Rob J., Roman A. Ahmed, George Athanasopoulos and Han Lin Shang. 2011. Optimal Combination Forecasts for Hierarchical Time Series. *Computational Statistics & Data Analysis* 55, no. 9:2579–89.

Kahn, Kenneth B. 1998. Revisiting Top-Down Versus Bottom-Up Forecasting. *The Journal of Business Forecasting* 17, no. 2:14.

Lee, Ronald D., and Lawrence R. Carter. 1992. Modeling and Forecasting U.S. Mortality. *Journal of the American Statistical Association* 87, no. 419:659–71.

Li, Han, Hong Li, Yang Lu and Anastasios Panagiotelis. 2019. A Forecast Reconciliation Approach to Cause-of-Death Mortality Modeling. *Insurance: Mathematics and Economics* 86:122–33.

Preston, Samuel H., and Haidong Wang. 2006. Sex Mortality Differences in the United States: The Role of Cohort Smoking Patterns. *Demography* 43, no. 4:631–46.

Rice, Dorothy P., and Jacob J. Feldman. 1983. Living Longer in the United States: Demographic Changes and Health Needs of the Elderly. *The Milbank Memorial Fund Quarterly. Health and Society*: 362–96.

Shang, Han Lin and Steven Haberman. 2017. Grouped Multivariate and Functional Time Series Forecasting: An Application to Annuity Pricing. *Insurance: Mathematics and Economics* 75:166–79.

Syntetos, Aris. A., Zied Babai, John E. Boylan, Stephan Kolassa, and Konstantinos Nikolopoulos. 2016. Supply Chain Forecasting: Theory, Practice, Their Gap and the Future. *European Journal of Operational Research* 252, no. 1:1–26.

Wang, Haidong, Austin E. Schumacher, Carly E. Levitz, Ali H. Mokdad and Christopher J. L. Murray. 2013. Left Behind: Widening Disparities for Males and Females in U.S. County Life Expectancy, 1985–2010. *Population Health Metrics* 11, no. 1:8.

Wickramasuriya, Shanika L., George Athanasopoulos and Rob J. Hyndman. 2019. Optimal Forecast Reconciliation for Hierarchical and Grouped Time Series Through Trace Minimization. *Journal of the American Statistical Association* 114, no. 526: 804–819.

Zellner, Arnold, and Justin Tobias. 2000. A Note on Aggregation, Disaggregation and Forecasting Performance. *Journal of Forecasting* 19, no. 5:457–65.