SOCIETY OF ACTUARIES®

Pub-2010 Public Retirement Plans Mortality Tables Report

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Pub-2010 Public Retirement Plans Mortality Tables Report

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Caveat and Disclaimer

CONTENTS

Preface: Substantive Revisions Made to this Report Subsequent to 1/22/2019 Release

February 2019 Updates

A few Disabled Retiree annuity factors in Table 11.10 (Amount-Weighted Safety Disabled Retirees) and Table 11.11 (Amount-Weighted Non-Safety Disabled Retirees) have been changed. For males and females, the age-55 annuity factors for RP-2000 Disabled projected with Scale BB and for RP-2006 Disabled projected with MP-2017 were incorrect and are now updated, along with the corresponding percentage changes of moving to the Pub-2010 tables. All other annuity factors in the table remain unchanged.

Section 1: Executive Summary

1.1 Purpose

As a result of comments received on the prior RP-2014 Mortality Tables (RP-2014) study, which included only data from private pension plans, the Society of Actuaries (SOA) and the Retirement Plans Experience Committee (RPEC or "the Committee") initiated a mortality study of public pension plans in January 2015. The primary focus of this study was a comprehensive review of recent mortality experience of public retirement plans in the United States. The objectives of this study were the following:

1. Develop mortality tables based exclusively on public-sector pension plan experience.
2. Provide new insights into the composition of gender-specific pension mortality by factors such as job category (e.g., Teachers, Public Safety, General), salary/benefit amount, health status (i.e., healthy or disabled), geographic region and duration since event.

1.2 Summary of Data Collected

The final dataset upon which this study has been based includes approximately 46 million life-years of exposure and 580 thousand deaths from public pension systems across the United States. Data were received from a total of 35 different public pension systems that collectively submitted information for 78 plans, and the vast majority of the collected data was included in the study. In an effort to study potential variations in mortality by job category, contributors were asked to identify plan members as teachers, public safety personnel or general employees.

The mortality experience collected comes from calendar years 2008–2013.[1] Based on a weighted average of the exposures included in the study, the rates in the tables should be considered to be one-year mortality probabilities as of July 1, 2010.

1.3 Mortality Tables Developed

The following gender-specific tables were developed on both an amount-weighted and headcount-weighted basis:

- Employee Tables (ages 18 through 80)
    - Teachers
        - Total Teacher dataset
        - Above-Median Income (based on salary)
        - Below-Median Income (based on salary)
    - Public Safety
        - Total Public Safety dataset
        - Above-Median Income (based on salary)
        - Below-Median Income (based on salary)
    - General Employees

[1] Contributors were asked to submit data for a five-year period ended in 2013. Many non-calendar-year plans were included in the study; the final dataset includes partial years of exposure in 2008 and 2013 for these plans.

        - Total General dataset
        - Above-Median Income (based on salary)
        - Below-Median Income (based on salary)
- Retiree Tables (through age 120, with beginning age differing by job category)[2]
    - Corresponding table types were developed as for Employees, with above- and below-median splits determined by retirement benefit amount rather than salary
- Disabled Retiree Tables (ages 18 through 120)
    - Public Safety
    - Non-Safety (for Teachers and General)
- Contingent Survivor Tables[3] (ages 45 through 120)
    - Total contingent survivor dataset
    - Above-Median Income (based on benefit amount)
    - Below-Median Income (based on benefit amount)

For completeness, the Committee also developed gender-specific Juvenile tables covering ages 0 through 17. Although studied by the Committee, tables by geographic region were ultimately not developed.[4]

The names for each of the amount-weighted mortality tables presented in this report are PubT-2010, PubS-2010 and PubG-2010, respectively, for the total Teacher, total Public Safety and total General employee populations. The corresponding names for the headcount-weighted tables are PubT.H-2010, PubS.H-2010 and PubG.H-2010. For Disabled Retirees, the Teachers and General data were combined into a Non-Safety group, and the corresponding Disabled Retiree tables are named PubNS-2010.[5] Wherever applicable, the above-median and below-median versions of a given table are designated by the letter (A) or (B), respectively, immediately following the corresponding total population table name; e.g., PubT-2010(A) for the amount-weighted Above-Median Teachers tables. Collectively, the set of all tables presented in this report is named Pub-2010. The Pub-2010 Mortality Tables can be found on the SOA website at the following link: https://www.soa.org/research-reports/2019/pub-2010-retirement-plans/.

It should be noted that with the exception of the tables for Contingent Survivors, none of the mortality tables presented in this report reflects the combined experience of members from all three job categories. See Subsection 13.1 for RPEC's rationale for this decision.

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[2] Teacher tables cover ages 55 through 120, Public Safety tables cover ages 45 through 120, and General tables cover ages 50 through 120.
[3] The Contingent Survivor tables were based on data from all three job categories combined.
[4] See Subsection 13.2 for details.
[5] In the Excel file accompanying this report, the PubNS-2010 rates are not explicitly labeled as such. They can be found in the "Disabled Retiree" columns on the "PubT-2010" and "PubG-2010" tabs. Note that the Disabled Retiree rates are identical between these two tabs.

1.4 Impact on Deferred-to-62 Annuity Values

Tables 1.1., 1.2 and 1.3 present comparisons of deferred-to-62 annuity values calculated as of July 1, 2018,[6] for Teachers, Public Safety and General members, respectively, to those calculated using mortality tables and projection scales previously published by the SOA:[7]

- RP-2000 base mortality rates projected generationally using one-dimensional Scale BB
- RP-2006[8] base mortality rates projected generationally using Scale MP-2017
- RP-2006 White Collar (WC) base mortality rates projected generationally using Scale MP-2017

All of the deferred annuity values shown in the following tables were developed using amount-weighted mortality rates, a pre-retirement discount rate of 7.0% and a post-retirement discount rate of 5.0%. The 7.0% rate was chosen to be broadly representative of discount rates recently used in the funding valuations of public-sector retirement plans, and the "spread" of 2.0% broadly representative of recent post-retirement cost-of-living adjustments.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubT-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubT-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.2406**	*5.7%*	*6.3%*	*3.7%*
	Age 35	2.2720	2.2594	2.3191	**2.4113**	*6.1%*	*6.7%*	*4.0%*
	Age 45	4.4101	4.3812	4.5028	**4.6932**	*6.4%*	*7.1%*	*4.2%*
	Age 55	8.6263	8.5443	8.7849	**9.1655**	*6.3%*	*7.3%*	*4.3%*
	Age 65	13.0772	12.9595	13.3331	**13.9245**	*6.5%*	*7.4%*	*4.4%*
	Age 75	9.8517	9.6858	10.0300	**10.5286**	*6.9%*	*8.7%*	*5.0%*
	Age 85	6.3586	6.0423	6.2543	**6.6215**	*4.1%*	*9.6%*	*5.9%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1867**	*5.8%*	*7.9%*	*2.8%*
	Age 35	2.1668	2.1251	2.2369	**2.3018**	*6.2%*	*8.3%*	*2.9%*
	Age 45	4.1995	4.1143	4.3391	**4.4721**	*6.5%*	*8.7%*	*3.1%*
	Age 55	8.2051	8.0345	8.4670	**8.7317**	*6.4%*	*8.7%*	*3.1%*
	Age 65	12.3695	12.2340	12.8373	**13.2171**	*6.9%*	*8.0%*	*3.0%*
	Age 75	8.9093	8.9690	9.4431	**9.7232**	*9.1%*	*8.4%*	*3.0%*
	Age 85	5.3409	5.4378	5.6904	**5.8822**	*10.1%*	*8.2%*	*3.4%*

Table 1.1: Teachers

[6] See Subsection 13.6 for a discussion of calculating annuity factors as of July 1, 2018.
[7] Employee mortality rates were assumed for all ages younger than 62, and Healthy Annuitant rates (Retiree rates for the Pub-2010 tables) were assumed for all ages 62 and older.
[8] The RP-2006 Mortality Tables are based on the same data used to construct the RP-2014 Mortality Tables, but as of 2006, the base year of the RP-2014 study. These were computed by backing out mortality improvement from 2007-2014 from the RP-2014 rates. The SOA formally published these tables in July 2018.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubS-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubS-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.1820**	*0.7%*	*1.3%*	*-1.2%*
	Age 35	2.2720	2.2594	2.3191	**2.2919**	*0.9%*	*1.4%*	*-1.2%*
	Age 45	4.4101	4.3812	4.5028	**4.4517**	*0.9%*	*1.6%*	*-1.1%*
	Age 55	8.6263	8.5443	8.7849	**8.6740**	*0.6%*	*1.5%*	*-1.3%*
	Age 65	13.0772	12.9595	13.3331	**13.0713**	*0.0%*	*0.9%*	*-2.0%*
	Age 75	9.8517	9.6858	10.0300	**9.7245**	*-1.3%*	*0.4%*	*-3.0%*
	Age 85	6.3586	6.0423	6.2543	**6.1480**	*-3.3%*	*1.8%*	*-1.7%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1330**	*1.0%*	*3.1%*	*-1.8%*
	Age 35	2.1668	2.1251	2.2369	**2.1949**	*1.3%*	*3.3%*	*-1.9%*
	Age 45	4.1995	4.1143	4.3391	**4.2582**	*1.4%*	*3.5%*	*-1.9%*
	Age 55	8.2051	8.0345	8.4670	**8.2939**	*1.1%*	*3.2%*	*-2.0%*
	Age 65	12.3695	12.2340	12.8373	**12.4434**	*0.6%*	*1.7%*	*-3.1%*
	Age 75	8.9093	8.9690	9.4431	**8.9533**	*0.5%*	*-0.2%*	*-5.2%*
	Age 85	5.3409	5.4378	5.6904	**5.3471**	*0.1%*	*-1.7%*	*-6.0%*

Table 1.2: Public Safety

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubG-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubG-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.2085**	*3.0%*	*3.5%*	*1.0%*
	Age 35	2.2720	2.2594	2.3191	**2.3444**	*3.2%*	*3.8%*	*1.1%*
	Age 45	4.4101	4.3812	4.5028	**4.5554**	*3.3%*	*4.0%*	*1.2%*
	Age 55	8.6263	8.5443	8.7849	**8.8853**	*3.0%*	*4.0%*	*1.1%*
	Age 65	13.0772	12.9595	13.3331	**13.4541**	*2.9%*	*3.8%*	*0.9%*
	Age 75	9.8517	9.6858	10.0300	**10.0760**	*2.3%*	*4.0%*	*0.5%*
	Age 85	6.3586	6.0423	6.2543	**6.2831**	*-1.2%*	*4.0%*	*0.5%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1344**	*1.1%*	*3.2%*	*-1.7%*
	Age 35	2.1668	2.1251	2.2369	**2.1955**	*1.3%*	*3.3%*	*-1.9%*
	Age 45	4.1995	4.1143	4.3391	**4.2605**	*1.5%*	*3.6%*	*-1.8%*
	Age 55	8.2051	8.0345	8.4670	**8.3168**	*1.4%*	*3.5%*	*-1.8%*
	Age 65	12.3695	12.2340	12.8373	**12.5732**	*1.6%*	*2.8%*	*-2.1%*
	Age 75	8.9093	8.9690	9.4431	**9.1604**	*2.8%*	*2.1%*	*-3.0%*
	Age 85	5.3409	5.4378	5.6904	**5.5437**	*3.8%*	*1.9%*	*-2.6%*

Table 1.3: General

The amount-weighted deferred annuity values for Teachers are consistently larger than those for Public Safety and General, and, in fact, they are considerably larger than even those developed using the White Collar version of the projected RP-2006 table. The deferred annuity values for Public Safety members tended to be similar to those developed using RP-2000 (projected with Scale BB). For General members, close matches to previously released tables were less obvious, with deferred annuity values generally within about 1% of RP-2006 WC values for females, and with values for males falling between the projected RP-2006 and RP-2006 WC values.

The corresponding deferred annuity comparisons using headcount-weighted mortality rates are roughly similar to those using amount-weighted rates shown above. Compared to their amount-weighted counterparts, headcount-weighted deferred-to-62 annuities are generally about 0.5% to 1.5% lower for females and about 1.0% to 3.5% lower for males, depending in both cases on job category and age.[9]

Multivariate analysis indicated that salary (for Employees) and benefit amount (for nondisabled Annuitants) were the most statistically significant predictors of mortality differences within individual gender/job classifications. As a result, the Committee produced Above-Median and Below-Median versions of the Employee, Retiree and Contingent Survivor tables. In general, the impact of moving from the total dataset table to either the Above- or Below-Median tables is considerably smaller for Teachers than for Public Safety or General, and the impact for males in each of the three job categories is considerably larger than that for females.[10]

Finally, it should be noted that the Retiree mortality rates were used for ages 62 and above in the calculation of the above Pub-2010 annuity factors. The RP-2000 and RP-2006 datasets utilized "Healthy Annuitant" mortality rates for ages 62 and above, which combined mortality experience for Retirees and Contingent Survivors. This complicates direct comparisons of those older tables to the Pub-2010 tables. See Subsection 11.4 for an annuity factor comparison using the Pub-2010 Contingent Survivor mortality rates and Subsection 12.4 for a discussion of the application of Contingent Survivor tables.

1.5 Application of Pub-2010 Tables

The Committee encourages all stakeholders in the financial viability of U.S. public-sector retirement plans to carefully review the findings presented in this report. The Pub-2010 tables should be considered as part of the relevant "assumption universe" described in Actuarial Standard of Practice No. 35, *Selection of Demographic and Other Noneconomic Assumptions for Measuring Pension Obligations* (ASOP 35) for the measurement of public plan obligations [ASB 2014]. In conjunction with knowledge of the individual characteristics and recent experience of the covered group, actuaries could use the Pub-2010 tables (possibly blended or otherwise adjusted using appropriate credibility techniques) as relevant benchmarks for the development of the ASOP 35 assumption universe for mortality.

For example, the statistical analyses summarized in this report support the observation that members with higher amounts (salary for Employees and benefit amount for Nondisabled Annuitants) tend to have lower rates of mortality than those with lower amounts. Consistent with the principles of ASOP 35 and subject to other relevant criteria, knowledge that the population being valued falls predominantly in the above (or below) median amount category could indicate that the corresponding Above-Median (or Below-Median) tables developed in this report could be considered as alternative benchmark tables to the corresponding "total population" tables.

The Committee believes that for most pension-related actuarial applications, the Pub-2010 mortality rates (including those for Disabled Retirees) should be projected with an appropriate mortality improvement scale, and that generational projection should be considered as an approach to projecting

[9] See Subsection 11.1.3 for details.
[10] See Subsection 11.2 for details.

future mortality rates. In all cases, the selection of a mortality improvement assumption must satisfy the applicable requirements of ASOP 35.

1.6 Acknowledgments

The SOA would like to thank RPEC, and especially the Public Plans Subcommittee, for their support, guidance, direction and feedback throughout the project.

Members of the Retirement Plans Experience Committee:
(Members of the Public Plans Subcommittee are denoted with an asterisk)

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- Erika Schulty, SOA Research Associate

Special Recognition of Others Not Formally on RPEC

Thirty-five public pension systems across the United States provided data for this project, and RPEC would like to thank these systems for making their data available for study. In particular, the Committee would like to express its gratitude to those who worked to compile the submissions and respond to data questions, many of which were detailed and required extensive research. In some cases, this work was performed by the systems' retained actuarial consultants, and RPEC would like to thank these actuarial firms and their staff for contributing their time as well. For confidentiality reasons, the pension systems and actuarial firms have not been listed by name.

The Committee would like to express its gratitude to the National Association of State Retirement Administrators (NASRA) for their promotion of the study and their assistance with distributing the data request. Participation in the study was significantly enhanced thanks to their efforts.

The Committee would like to thank Aon for their efforts to compile and process the data, especially given that the data collection phase was extended well beyond the original timeline of the study. Aon's flexibility to provide the necessary output to the Committee was a key factor in the completion of the study.

Michelle Xia, PhD, and Lei Hua, ASA, PhD, of Northern Illinois University performed the multivariate analysis for this study, and the Committee greatly appreciates their deep technical knowledge that enabled the Committee to make important decisions regarding table creation. This phase of the project required many iterations and custom models, and their consistent, prompt attention to Committee requests was extremely helpful.

Philip Adams, FSA, CERA, MAAA, a volunteer who performed graduations for the 2015 Valuation Basic Tables, joined the Public Plans Subcommittee for the table development phase of the project. Philip was an invaluable source of expertise on Generalized Additive Models (GAMs) and volunteered his own time to set up and run subroutines to perform graduations on many tables. The Committee is very grateful to Philip for all his efforts.

Section 2: Background and Process

2.1 Reason for Study

The Society of Actuaries (SOA) published the RP-2014 Mortality Tables Report in October 2014 [SOA 2014a]. In addition to the private pension plan data with which those tables were ultimately constructed, RPEC also collected data from three very large public/federal retirement plans for that study. Multivariate analysis indicated that (1) the overall mortality experience for the combined public/federal plans was significantly different from that of the combined private-sector plans, and (2) the mortality experience for each of those three plans was significantly different from that of the other two. As a result, the RP-2014 research team decided to exclude the public/federal plan data from that study.

Following the publication of the RP-2014 exposure draft, RPEC received questions regarding the potential appropriateness of the RP-2014 Mortality Tables for public-sector plans, given the exclusion of the public plan data. The final RP-2014 report included language stating RPEC's view that "it would not necessarily be inappropriate" for actuaries to consider one of the RP-2014 tables as a benchmark for a specific public plan. However, RPEC also recommended in the accompanying "Response to Comments" document that "the SOA initiate a separate study of public plan mortality experience, with the expectation that the study results would include separate tables for (1) public safety, (2) teachers and (3) other public entities" [SOA 2014b]. This report represents the culmination of that recommended study.

2.2 RPEC's Process

RPEC formed a subcommittee to study public-sector retirement plan mortality in February 2015. This group generally met once per week via conference call during most stages of the project. These meetings were not open to the public. Status updates of the subcommittee's progress on this study were shared with all RPEC members once per month. The RPEC Industry Advisory Group (RIAG) was formed in the fall of 2016 with the purpose of collecting industry feedback regarding ongoing RPEC projects. Updates regarding this study were provided to the RIAG periodically (about three times per year).

In a departure from the RP-2014 process in which smaller "subteams" were created to focus on particular aspects of the project, all members of the Public Plan Subcommittee participated in each of the associated project subtasks. The following three subprojects required the services of external resources:

- For data collection, processing and validation, RPEC engaged the services of Aon; see Section 3 for details.
- For multivariate analysis of the final datasets, RPEC engaged a research team of Lei Hua, ASA, PhD, and Michelle Xia, PhD, both from Northern Illinois University; see Section 4 for details.
- For the graduation of raw mortality rates, RPEC enlisted the help of Philip Adams, FSA, CERA, MAAA, a volunteer who performed graduations for the 2015 Valuation Basic Tables. In addition to providing valuable expertise in this area, Philip performed all the graduations that formed the basis for the mortality tables summarized in this report; see Section 5 for details.

2.3 Naming Conventions

2.3.1 Member Status

RPEC has used the following terms throughout this report to describe various subgroups of plan members:

- *Employee:* A nondisabled plan member who is actively employed[11] (including those in plans that no longer have ongoing benefit accruals).
- *Retiree:* A formerly active member in benefit receipt who was not deemed disabled at the date of retirement.
- *Contingent Survivor:* A surviving beneficiary[12] (of a formerly active or retired member) who is older than age 17 and in benefit receipt.
- *Disabled Retiree:* A retired member in benefit receipt who was deemed disabled as of the date of retirement.
- *Juvenile:* A member's surviving beneficiary who is under the age of 18.

The term *Nondisabled Annuitant* is used when it is not necessary to distinguish between a Retiree and a Contingent Survivor, and the term *Annuitant* is used when it not necessary to distinguish between any member in payment status.

2.3.2 Type of Public-Sector Employment

Early in the multivariate analysis phase of this project, it became clear that members with certain types of public-sector employment exhibited overall mortality patterns different from those who had other types of jobs. As a result, RPEC decided to analyze mortality experience separately for each of the following three *job categories:*

- *Teachers:* School teachers and college/university professors, excluding all other school/university staff.
- *Public Safety:* Police officers, firefighters and correctional officers. The name of this job classification has been shortened to *Safety* throughout most of this report.
- *General:* All other types of public plan members not specifically designated as Teacher or Safety, including members classified as general employees, non-faculty school/university staff, judges, members of the military, officials holding executive offices, and those submitted with miscellaneous or unknown job categories.

RPEC originally considered the development of a set of "combined" public retirement plan mortality tables, which would reflect the aggregated mortality experience of all three job categories. The Committee ultimately concluded that it would not be appropriate to develop such tables given (1) the different mortality patterns exhibited by each of the three job categories and (2) the unequal sizes of the job category datasets.[13] See Subsection 13.1 for additional comments regarding this issue.

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[11] Consistent with the RP-2014 tables, terminated members (both nonvested and vested but not yet in payment status) were excluded from this study.
[12] Because of the difficulty in obtaining reliable information for beneficiaries while the Retiree is still alive, exposures and deaths for Contingent Survivors were counted starting with the Retiree's death.
[13] See Appendix B.

The previous paragraph notwithstanding, there were two instances when data for different job categories were combined. As explained in Subsection 6.3, data for all three job categories were combined to create the Contingent Survivor tables. The Teachers and General datasets were aggregated for purposes of constructing the Disabled Retiree mortality tables applicable to members in either of those two job categories, as described in Subsection 8.1.

2.3.3 Mortality Table Names

RPEC wanted the names of the individual tables presented in this report to clearly identify various important features reflected in those tables, specifically:

- The tables were developed with data provided exclusively by public-sector systems.
- Members in each job category generally exhibited mortality patterns different from that of the other two job categories.
- The central year of the study's observation period began in 2010.[14]
- The research team developed two full sets of mortality tables, one set with amount-weighted rates and the other with headcount-weighted rates.

In an attempt to capture all these features succinctly, RPEC adopted the following naming convention. The amount-weighted mortality tables for the total Teacher, total Safety and total General employee *populations* are denoted *PubT-2010, PubS-2010* and *PubG-2010,* respectively. The corresponding names for the headcount-weighted tables are PubT.H-2010, PubS.H-2010 and PubG.H-2010. For Disabled Retirees, the Teachers and General data were combined into a Non-Safety group, and the corresponding Disabled Retiree tables are named PubNS-2010. Where applicable, the above-median and below-median versions of a given table are designated by the letter (A) or (B), respectively, immediately following the corresponding total population table name, e.g., PubT.H-2010(A) for the headcount-weighted Above-Median Teachers tables.

[14] The central year of the study, as computed by a weighted average of the calendar year of exposures, is approximately July 1, 2010, through June 30, 2011. See Subsection 12.1 for additional details.

Section 3: Data Collection and Validation

3.1 Overview

Below is a list of the phases involved in the development of the final dataset that generated the raw mortality rates for this study:

1. Data collection
2. Review for reasonableness and completeness
3. Data consolidation and validation
4. Month-by-month death pattern review
5. Actual-to-expected ("A/E") ratio analysis

3.2 Data Collection

As this was the SOA's first mortality experience study to focus solely on U.S. public pension plans, the first step in the data collection process was to identify a list of potential data contributors. Using the database on the National Association of State Retirement Administrators (NASRA) web site (www.nasra.org) and suggestions from public-sector actuaries on the Committee, the SOA prepared a large list of public pension systems to which the data request would be distributed. NASRA assisted the SOA with ensuring the list of systems was as comprehensive as possible and helped by promoting the study and distributing the data request packages.

Public-sector retirement systems often have separate plans (or special provisions) for different job classifications, e.g., Teachers, Safety and General employees. Therefore, the data request was designed to collect enough information so that variables such as job category could be included in the multivariate analysis.

The formal data request package consisted of the following seven documents (three of which had two separate versions depending on the data submission layout chosen by the contributor):

1. A cover letter outlining the goals of the study, an approximate timetable and the required file formats
2. A plan-level information questionnaire, which requested details regarding the format of the submission and characteristics of the plan
3. A document containing instructions for completing the plan-level information questionnaire
4. A member-level information worksheet, which showed the information that must be provided for each member and denoted the situations for which each field is required
5. A document containing instructions for completing the member-level information worksheet
6. An Excel file showing a sample submission
7. A file that summarized the list of acceptable inputs for some categorical data fields

To maintain confidentiality of the submitted data, the data-collection and data-processing phases of the project were coordinated by SOA staff working directly with an outside data compiler, Aon.[15] Aon performed validation checks on the data, compiled data statistics, computed experience analytics and, per approval by the Committee, imputed missing information where needed. In many cases, Aon made direct contact with the data contributors (coordinated through and including SOA staff) to address specific issues with data submissions.

The SOA intended to collect experience data for calendar years 2009–2013. However, many public-sector plans have non-calendar-year valuation cycles. Plans that track snapshots of member data at a time during the year other than January 1 (or December 31) were instructed to provide data for a consecutive five-year period ended in calendar year 2013. This resulted in the collection of some experience for the 2008 calendar year, which was included in the study. Overall, the SOA received raw data for 78 different public pension plans, which were contributed by 35 different public pension systems from across the country.

3.3 Review for Reasonableness and Completeness

Prior to processing the data, Aon reviewed each data submission to determine whether the format was in accordance with the data request specifications and whether all required information was provided. High-level checks provided at this stage of the process included the following:

- Confirmation that all critical data fields were populated and had valid entries per the format requested for most members
- Review of record identifiers to assess feasibility of linking data across multiple years (where necessary)
- Review of record pairs with duplicate identifiers to confirm that the correct data could be determined

In the event that this initial review revealed issues with processing a given submission, the SOA followed up with the contributor to determine whether the problems could be resolved. In most cases, contributors were able to send clarifications or additional information that enabled Aon to process the submission. Approximately 99% of the submitted life-years of data were retained through this "reasonableness and completeness" phase of the validation process.[16]

3.4 Data Consolidation and Validation

The Committee requested member-level data in accordance with one of the following two layouts:

1. One record per member for the entirety of the study period, including annual updates of member status (i.e., Employee, terminated, Retiree, Contingent Survivor, Disabled Retiree or deceased), salary[17] (for Employees) and pension amount (for Annuitants).

[15] The contract between Aon and the SOA included confidentiality requirements that restricted the distribution of confidential information to other parties.
[16] See row (b) of Table A.1 in Appendix A.
[17] RPEC attempted to collect information on the types of compensation included in salary by plan. However, the definitions received were very diverse and did not lend themselves to quantifiable adjustments that could be used to ensure a consistent salary definition throughout the study. RPEC was therefore unable to reflect these varying definitions of salary in its analysis.

2. Six annual snapshots of census data with a unique identifier for members that would allow information from different years to be linked across the study period.

In the event the second layout was chosen, Aon used the provided unique identifiers to link together each snapshot of a member's data throughout the study period to make the record of each member's experience as complete as possible. The use of consolidated records facilitated accurate counting of exposures and review of key data fields for internal consistency.

Aon's review of individual records resulted in the identification of various issues for each contributor. Some issues could be resolved by making standardized assumptions, but others required record-specific analysis and often data questions for the contributor. Situations in which missing or invalid data were resolved via an assumption included the following:

Missing Dates (e.g., Date of Termination, Date of Retirement, Date of Death)

The event date was assumed to be on the member's birthday during the 12-month period of the change in status. This approach distributes imputed status change dates uniformly throughout the calendar year, rather than clustering them among a small number of dates, such as the beginning of a month or quarter. The missing date methodology is best illustrated by an example:

- 1/1/2010 Status: Employee
- 1/1/2011 Status: Retiree
- Date of Birth: 6/1/1951
- Date of Retirement: (blank)

In this situation, the assumed Date of Retirement would be 6/1/2010.

Missing Salary or Total Monthly Pension

If the member had a reasonable amount provided for a different year, that year's amount would be used. If no valid amount existed in any year for that member, the plan's average for that member's job category and status group (Employee, Contingent Survivor, primary Annuitant) would be imputed.

Common situations that required data questions to contributors included the following:

Annuitants Ceasing Payment

In some cases, annuitants would disappear from the data from one snapshot date to the next. Listings of these records were sent to the contributor for clarification, and although many of these instances turned out to be deaths, some cases were found to be erroneous records or the end of a temporary annuity.

Missing Gender or Date of Birth

Records with either a missing Gender or Date of Birth were sent to the contributor in an attempt to obtain the missing information. All records still missing a Gender or Date of Birth after this extra attempt to collect these data were excluded from the study.

Large Monthly Pension Amounts

Unusually large monthly pension values were sent to contributors for confirmation. Many of those values were confirmed to be legitimate, but in some circumstances, it was determined that the contributor had incorrectly provided annual amounts or lump sum distributions, and these records were corrected.

Unclear Disability Status

Per the data request instructions, a status of Disabled Retiree should apply for the entire duration of a member's retirement if and only if they were disabled under the terms of the plan at the time of retirement. Some plans submitted member statuses that transitioned from Disabled Retiree to Retiree and vice versa, which should not have been possible under the requested definition. These records were sent to contributors to determine which of the two statuses should apply.

In some cases, plans tracked Disabled Retirees only prior to a certain age, and attainment of that age would cause a member to transition from Disabled Retiree to Retiree in the data. For these plans, all instances of Retiree were changed to Disabled Retiree if we were provided at least one Disabled Retiree status for the member, unless the contributor directed us otherwise. However, because some members may have attained the age that triggers this change prior to the study's observation period, it is likely that there were some Disabled Retirees classified as healthy Retirees for these plans. The effect of this categorization issue is likely minimal, and the Committee did not observe abnormally high Retiree mortality rates for plans with this issue.

Ideally, the Committee would have preferred to include the type of disability (e.g., in the line of duty or not) in the analysis. However, a large portion of the data submissions had unknown or missing information for Disability Type, which made this field unreliable for purposes of this study.

Status Progression Inconsistencies

For situations with a small number of discrepancies between the status progressions provided and the associated non-death event dates, the Committee proceeded by trusting the status progression when the event date occurred more than 15 months prior to the date of the reported status change (with 15 months being used rather than 12 to account for potential reporting lag). See the below examples.

<u>Example 1</u>

- 1/1/2010 Status: Employee
- 1/1/2011 Status: Employee
- 1/1/2012 Status: Retiree
- Date of Retirement: 12/1/2010

In this situation, because the Date of Retirement was within 15 months of the date at which the statuses indicated a movement to Retiree status (i.e., 1/1/2012), the Date of Retirement of 12/1/2010 was treated as the beginning of the member's exposure as a

Retiree, and the member was considered to be retired from that date, including as of 1/1/2011 when the member was indicated to be an Employee.

Example 2

- 1/1/2010 Status: Employee
- 1/1/2011 Status: Employee
- 1/1/2012 Status: Retiree
- Date of Retirement: 6/1/2009
- Date of Birth: 5/1/1952

In this situation, the reported Date of Retirement was not within 15 months of the date at which the statuses indicated a movement to Retiree status (i.e., 1/1/2012). The difference is greater than could typically be attributed to a lag in updating the member's status information. Therefore, the Date of Retirement was assumed to be on 5/1/2011, the member's birthday during the 12-month period of the change in status (in accordance with the procedure for "Missing Dates," above).

When there were large numbers of such discrepancies, questions were asked to the contributors to help assess the reliability of the information for a given date field or the status progressions. In most cases, the status progressions were confirmed to be more reliable than the dates. In those instances when the contributor did not provide additional assistance in resolving the discrepancies, the Committee's default decision was to rely upon the status progression provided by the contributor.

The only exception to the above rule was in connection with the Committee's handling of the Date of Death field. Any reasonable Date of Death provided by the contributor was always given greater credence than the associated status progression.

Invalid Status Progressions

In some cases, Aon discovered counterintuitive status progressions that occurred in more than just a small subset of records. The two most common cases were

1. Status implying Employee or Annuitant exposure after a deceased status and
2. Records shifting between statuses designated for a Contingent Survivor and statuses designated for an original member.

In both cases, the Committee consulted contributors with a listing of affected records to attain the proper resolution. In a small number of cases, answers were not provided and assumptions needed to be made. In the first case above, the Committee trusted the Date of Death provided or, if none was provided, assumed that the first provision of a deceased status was correct and subsequent nondeceased statuses were errors. For the second case, crossovers between Contingent Survivor and original member statuses were generally treated as Contingent Survivors throughout. Note that this does not include legitimate movements from original member to Contingent Survivor status under data layout no. 1, because these represent a normal death progression.

3.5 Month-by-Month Death Pattern Review

Before reviewing aggregate plan statistics, the Committee looked at the distribution of deaths by month for each plan in the study. The original rationale for doing this was to see whether there were plans for which there was a lag in death reporting near the end of the study. In the event that a plan's death count notably dropped in the last month or two of the study period, the Committee wanted to confirm whether the lower counts were due to deaths not yet being reflected in the plan's database by the time of the last snapshot date.

When the Committee analyzed the individual plan reports, a number of other irregularities were discovered. Some plans had entire years of abnormally high or low death counts. Other plans reported an excessive concentration of deaths in a single month, which was typically indicative of the contributor providing Dates of Death that were defaulted to a single date in a given year rather than reflective of the actual Dates of Death. Inquiries on all these issues (including the potential reporting lags) were sent to the contributor for further clarification.

In response to the Committee's questions, some contributors were able to provide an updated register of death records to correct the problem. Others acknowledged that the unusual death patterns in some months or years were inaccurate for a known reason. This resulted in those time periods being excluded from the study for those contributors. If no response was received for a reporting lag question, the Committee generally excluded months at the end of the study period for which the death count was less than 1.5 standard deviations below the average for that calendar month across the study period. In the case of an over-concentration of deaths in a single month, the Committee effectively treated the Date of Death as missing and reallocated it to the member's birthday in the same 12-month period as described above.

3.6 Actual-to-Expected (A/E) Ratio Analysis

Once the final data for each plan were deemed complete, the Committee began reviewing total mortality results for each plan. The "expected" number of deaths for this analysis was calculated on a year-by-year basis using the RP-2014 (aggregate) mortality rates at the base year of 2006, projected using Scale MP-2015 to the appropriate year in the observation period. Scale MP-2015 was chosen because it was the most recently released mortality improvement scale in the SOA-published "MP" series when experience analytics were calculated. For each status/job category combination, an exposure-weighted average A/E ratio was developed, which was used to normalize all plan A/E ratios in that subgroup such that the average A/E ratio was 100%. This was done to ensure an appropriate basis of comparison for determining outlier A/E ratios.

The Committee then developed approximate 95% confidence intervals for the normalized A/E ratios for each plan/status/job category combination. If the low end of the 95% confidence interval was greater than 110% or if the high end of the interval was less than 90%, the plan was flagged for additional examination. For example, assume that the Employees in Plan X produced a normalized A/E ratio of 0.63, with a corresponding 95% confidence interval of 0.50 to 0.76. Since 0.76 (the high end of the confidence interval) is less than 0.90, Plan X would have been flagged for additional examination.

Mortality statistics for flagged plan/status/job category combinations were sent to contributors for confirmation. In most cases, contributors were either able to provide confirmation of their statistics or corrected data that resulted in that data subgroup no longer being flagged. Outliers that could not be confirmed by the contributor were dropped from the study. In total, less than 5% of the total dataset was excluded as a result of the Committee's A/E analysis.[18] In addition, the crude A/E ratio for the aggregated excluded data was less than 45%, which seemed indicative of issues with the reliability of death tracking within these subgroups.

3.7 Summary of the Final Dataset

Table 3.1 presents a summary of the final dataset by status, gender and job category. This includes only data used in the development of the final Pub-2010 mortality tables.[19] A reconciliation of excluded data can be found in Appendix A.

		Teachers		**Safety**		**General**		**Total All Job Categories**	
		Exposures	*Deaths*	*Exposures*	*Deaths*	*Exposures*	*Deaths*	*Exposures*	*Deaths*
Employee	Female	5,428,981	3,879	364,054	259	10,357,789	11,142	16,150,824	15,280
	Male	1,987,854	2,404	1,592,218	1,472	6,426,369	13,067	10,006,441	16,943
	Total	7,416,835	6,283	1,956,272	1,731	16,784,158	24,209	26,157,265	32,223
Retiree	Female	2,838,037	51,420	114,915	1,001	6,909,604	168,619	9,862,556	221,040
	Male	1,530,452	35,466	935,939	16,787	4,702,559	152,722	7,168,951	204,975
	Total	4,368,490	86,886	1,050,854	17,788	11,612,163	321,341	17,031,507	426,015
Disabled Retiree	Female	96,944	2,809	30,671	259	446,291	13,952	573,905	17,020
	Male	34,391	1,329	327,523	6,787	364,034	14,236	725,948	22,352
	Total	131,335	4,138	358,194	7,046	810,325	28,188	1,299,853	39,372
Contingent Survivor	Female	194,350	7,801	129,776	4,527	1,078,401	52,761	1,402,527	65,089
	Male	67,003	3,330	3,415	118	242,108	13,107	312,526	16,555
	Total	261,353	11,131	133,191	4,645	1,320,509	65,868	1,715,054	81,644
Total	Female	8,558,312	65,909	639,415	6,046	18,792,086	246,474	27,989,813	318,429
	Male	3,619,700	42,529	2,859,096	25,164	11,735,070	193,132	18,213,866	260,825
	Total	12,178,013	108,438	3,498,511	31,210	30,527,155	439,606	**46,203,679**	**579,254**

Table 3.1: Summary of Exposures and Deaths

It should be noted that it was not the Committee's intention to create stand-alone tables for each of the subgroups listed in Table 3.1. As described in Subsection 4.7, certain job category-specific subgroups with relatively small exposure and death counts were combined with other subgroups (of the same status), as long as the underlying raw mortality experience was similar.

3.8 Determination of Amount-Based Quartiles and Medians

To analyze results by benefit amount (Annuitants) and annualized salary (Employees), the data were divided into four amount quartiles, with unique breakpoints determined within each status, gender, year and job category. Data provided with missing amounts were excluded from this process. These splits were performed on the original seriatim data, meaning that breakpoints were determined to split the number of records evenly between the four quartiles. As some records generated more exposure than others (e.g., a person may have a partial year of exposure due to being hired or terminated during a particular

[18] See row (e) of Table A.1 in Appendix A.

[19] A relatively small amount of validated data fell outside of the age ranges used in the graduation phase of the table construction process and therefore was excluded from Table 3.1. See Sections 6–8 for the details of graduation age ranges.

calendar year), this meant that the number of life-years of exposure in each quartile was not exactly equal, though the distribution was reasonably even.

After reviewing the multivariate analysis, the Committee came to the conclusion that splitting the data into above- and below-median segments rather than into four quartiles would be a more effective way to present the effects of benefit amount and salary.[20] Thus, the data for the top two quartiles within each status and job category were combined to create the above-median datasets, and the data for the bottom two quartiles formed the below-median datasets. Records provided without an amount were included in the full dataset (with an imputed amount per Subsection 3.4) but were not part of either the above-median or below-median groups. A summary of the annualized quartile breakpoints[21] by gender, job category and status is shown in Table 3.2.

		Income Percentile Amounts ($) by Gender, Job Category, and Status					
		Females			**Males**		
Job Category	Percentile	Employees	Retirees	Contingent Survivors*	Employees	Retirees	Contingent Survivors*
Teachers	25th	41,308	13,967	5,197	44,929	22,225	3,252
	50th	58,385	28,536	11,036	62,660	37,789	7,282
	75th	74,530	46,536	20,710	79,960	55,424	14,651
Safety	25th	44,027	13,429	5,197	53,536	23,511	3,252
	50th	61,775	29,243	11,036	72,154	36,909	7,282
	75th	86,380	43,912	20,710	95,673	53,973	14,651
General	25th	20,228	5,139	5,197	30,218	9,069	3,252
	50th	34,686	11,872	11,036	45,773	21,239	7,282
	75th	50,161	23,846	20,710	65,651	35,912	14,651

**The Pub-2010 tables only distinguish Contingent Survivor mortality by benefit amounts, not job category. The percentiles shown for Contingent Survivors are for the entire Contingent Survivor population for each gender.*

Table 3.2: Quartile Breakpoints

It should be emphasized that the above income percentiles effectively represent values as of the central year of the study, July 1, 2010 – June 30, 2011. They are presented here to provide additional context for the above- and below-median subsets of the study data. An actuary seeking to use these percentiles as benchmarks to characterize members of a given plan should consider what adjustments, if any, are necessary to make them more temporally relevant to the application date.

[20] See Subsection 13.4 for more details on this decision.
[21] The figures in Table 3.2 represent an average of the quartile breakpoints across calendar years.

Section 4: Multivariate Analysis

4.1 Overview

Mortality studies almost always assume different tables will be created by gender, and in the case of pension data, separate tables within gender for active employee members versus those receiving retirement annuities. In addition, disabled retirees are usually segregated for analysis because they typically have higher mortality rates than healthy members, at least during a period of years following disability.

Several other potentially predictive variables were collected for this study, including job category (Teachers, Safety, General) and geographical region (Northeast, Midwest, South, West).[22] In addition, Retirees (and Employees) within each gender were grouped by benefit amount (or salary) into quartiles, smallest to largest (Q1, Q2, Q3, Q4), and regression data also contained indicators for calendar year (2008–2013) and duration.

RPEC engaged Michelle Xia, PhD, and Lei Hua, ASA, PhD, two researchers at Northern Illinois University (NIU), to investigate which predictor variables appeared statistically significant as well as evaluate each variable’s contribution to estimation. In addition to the NIU analysis, RPEC performed supplementary multivariate analysis using pivot tables and graphs that is also included in the discussion below.

4.2 Nature of Analyses

The NIU analysis based modeling on a Poisson or Negative Binomial distribution, with Negative Binomial used when data dispersion was higher than suitable for a Poisson assumption.

The underlying shape of log-mortality was expressed as a polynomial in age (nearest birthday), with differences for male versus female. The other variables (such as job category or income quartile) were expressed as indicators. The significance of each variable was assessed based on Type III analysis[23] using the likelihood ratio test, after adjusting for all other variables in the model. A small number of two-way interaction terms were also explored among these variables, such as age and benefit quartile, and age and job category. These interaction terms often showed statistical significance, indicating that separate tables—as opposed to simpler loading factors—might be desirable if enough data exist for those splits.

4.3 Summary and Conclusions for Active Employees

4.3.1 Variations by Job Category

After controlling for other potentially explanatory variables such as income quartile (and its interaction with age), regression analysis showed job category to be a statistically significant predictor. Interaction terms among age and job category indicated that the differences between job categories were not simple scalar multiple relationships but differently shaped curves. Practitioners have indicated tables by job

[22] These are the four broad regions defined by the U.S. Census Bureau in the 2010 U.S. Census.
[23] Rather than producing individual tests of significance for each coefficient of a class variable (such as amount quartile, job category or geographic region), the Type III analysis tests the significance of the null hypothesis that all the coefficients in the entire set are equal to zero. This allows easy comparisons and rankings of all the variables in the model, both class and numeric.

category would be especially useful in the public plan context. Many retirement programs specifically cover Teachers, Safety or General employee populations.

Even when plans cover multiple job categories, there are potentially different benefit provisions that apply to separate job categories. Individually reasonable assumptions differentiated by job category are important for estimating program costs. Overall, Teachers had the lowest mortality, while Safety employees tended to have higher mortality than General employees.

4.3.2 Variations by Amount

Employee experience data showed that income quartile was generally the most significant explanatory variable among the region, quartile and year indicators, even exceeding job category. Higher compensation was correlated with lower mortality.

Income quartile was found to be a significant variable in the RP-2014 study, and tables were created for top-quartile and bottom-quartile groups (within gender). Practitioners have expressed difficulty in using quartile-based tables, except in limited situations such as supplemental plans that provide benefits in excess of qualified plan limits and are clearly high-paid, top-quartile members, or alternatively low-paid groups that are easily identifiable as bottom-quartile.

The regressions run by NIU indicated a significant difference between below-median (quartiles Q1 and Q2) and above-median (Q3 and Q4) mortality. Differences between each of the bottom two quartiles (Q1 and Q2) were less stable in models that contained more explanatory variables, but differences between above- and below-median remained strong.

4.3.3 Variations by Amount within Job Category

Regressions including both income quartile and job category (including their interactions with age) continued to show income quartile as a statistically significant variable, especially the above- or below-median distinction. Separate regressions by job category and gender (e.g., male Safety) reinforced that pattern, across all combinations of job and quartile.

4.3.4 Variations by Duration

Duration since hire (i.e., service) was considered to be potentially explanatory. An early regression run implied mortality was significantly lower for employees with five or more years since hire. However, this is likely due to correlations between increased service and other more relevant factors influencing mortality. A more robust model that included income quartile and job category (including their interactions with age), but excluded duration, proved easier to interpret and more in accordance with expectations. The use of duration (measured from hire date at a given employer) as a predictive variable is further complicated by job mobility. Hence, no duration-based mortality adjustments were developed for the Employee tables.

4.3.5 Variations by Geographical Region

Information about each member's broad geographic region (Northeast, Midwest, South or West) was included in the multivariate analysis. Despite showing some degree of statistical significance, concerns with respect to that covariate's predictive effectiveness convinced the Committee not to create separate mortality tables by geographic region; see Subsection 13.2 for further details.

4.4 Summary and Conclusions for Retirees

4.4.1 Retiree and Contingent Survivor Experience

An initial regression of the data for Nondisabled Annuitants indicated that Contingent Survivor mortality tended to be higher than Retiree mortality, and the difference was statistically significant. This initial regression included (controlled for) the following additional potentially explanatory variables: age, gender, region, income quartile and year. As a result, subsequent analyses were performed by segregating Retiree experience from that of Contingent Survivors.

This does not necessarily imply that separate sets of mortality rates are warranted, or even possible, for every subset of Contingent Survivor data, especially in the case of male beneficiaries where the datasets were considerably smaller than those for female beneficiaries.

4.4.2 Variations by Job Category

Regression analysis also showed job category to be a significant explanatory variable for Retirees. As with benefit quartile, interaction terms among age and job category indicated the differences were not simple factor relationships but actually different curve shapes. In addition, practitioners have indicated tables by job category would be especially useful for in the public plan context. Even when plans cover multiple job categories, there are potentially different provisions that apply to separate job categories. Individually reasonable assumptions that differentiate are important for estimating program costs.

Overall, retired Teachers had the lowest mortality by a significant margin at most ages. Safety Retirees tended to have slightly higher mortality than General Retirees.

4.4.3 Variations by Amount

After disaggregation from the Contingent Survivor experience, the multivariate analysis of the Retiree data showed that benefit quartile was generally the most significant explanatory variable among the region, job category, quartile and year indicators. Individual regressions were also performed by gender and job category, and these also confirmed income quartile to be significant. Consistent with a number of earlier studies of retirement plan mortality experience, higher benefit amounts were correlated with lower levels of mortality. Interaction terms among age and quartile indicated that the effects of income quartile varied significantly by age.

For similar reasons as stated for Employees in Subsection 4.3.2, the Committee focused on above- and below-median splits for table creation purposes.

4.4.4 Variations by Amount within Job Category

Regressions by gender for each job category (e.g., male Safety) continued to show benefit amount quartile as a statistically significant variable, with a more pronounced effect for males than for females.

4.4.5 Variations by Duration

Duration since retirement was also considered during regression analysis. Results were mixed. One problem, perhaps related to data sources and collection, was that mortality experience for duration zero was extremely low. Since data were collected from annual pension valuations, an employee who retired and then died soon after may have ended up being reported or tabulated as an employee who died, without the intermediate retirement status being recorded. There were also few first-year retirees in the experience data, and the treatment of experience data at duration zero had no material impact on the final results.

Comparing durations less than five years to those five and above, there did seem to be an increase in mortality associated with longer duration from retirement. In addition, an indicator variable for retirement age less than 62 was tested. It showed higher mortality for those who retired earlier. However, both of these effects were far less explanatory than benefit quartile or job category. In addition, early retirement effects would generally be incorporated into graduated rates for early retirement ages. Any select period effect remaining afterwards would likewise be included in the experience for all mortality rates at later ages.

4.4.6 Variations by Geographical Region

Similar to the Employee dataset, geographic region presented some statistical significance for Nondisabled Annuitants, but its value as a predictor was limited. However, it can be noted that mortality experience from the South was slightly higher than the Northeast, which was slightly higher than the Midwest. Mortality in the West was observed to be the lowest among the four regions. These generalities were not utilized in table development primarily because the effects of income and job category were much greater. Further details can be found in Subsection 13.2.

4.4.7 Variations by Experience Year

Mortality across years was fairly level. There was a slight indication of mortality improvement over the period, but it is doubtful the size and continuity of these experience data was sufficient to parse mortality improvement trends. Mortality improvement trends are probably better explored via national data sets such as those available from Centers for Medicare and Medicaid Services, Centers for Disease Control and the Social Security Administration. Crude mortality rates developed from raw tabulations of this information are posted on the SOA website [SOA 2017].

4.5 Summary and Conclusions for Contingent Survivors

4.5.1 Contingent Survivor Experience

As noted above, an initial regression of the data for annuitants indicated that Contingent Survivor mortality tended to be higher than Retiree mortality, particularly at younger ages, and the difference was statistically significant. Several potentially predictive factors were considered in the regression models, and most appeared to have some significant impact. Benefit quartile was the most consistently significant predictor across multiple models. Job category was relatively less significant than benefit quartile.

As with the Retiree analysis discussed above, duration (since the primary member's death) and geography were somewhat problematic. Geography issues are identical to the Retiree case. It was hoped duration might provide useful information. However, an initial regression indicated mortality was higher for durations five and above. This is contrary to past research indicating that widow(er) mortality is significantly higher the year or two following a spouse's death [Frees 1996]. Reporting anomalies similar to those described in Subsection 4.4.5 may have led to these counterintuitive results, and the Committee was therefore not comfortable incorporating duration effects into the tables.

4.6 Summary and Conclusions for Disabled Retirees

Among Disabled Retirees, regression analysis showed Safety members with significantly lighter mortality compared to Teachers or General members. This is probably due to generally less restrictive definitions of disability, consistent with the demands of Safety occupations. In addition to the Safety job category, benefit quartile, geographic region and duration also demonstrated some statistical significance.

Being in the lower two benefit quartiles was associated with higher mortality, as was the case for every other experience category studied. Lower duration since disability correlated to higher levels of mortality. This matches expectations that the period following initial disability is a critical time, and that the longer one survives post disability, the less of a factor that disability is likely to be on mortality.

4.7 Determination of Tables to Be Developed

Determining which tables would be produced involved three key criteria:

- The significance of specific predictive variables within the Employee, Retiree, Contingent Survivor and Disabled Retiree categories,
- The perceived reliability and usefulness of specific predictive variables to the actuarial community, and
- The availability of sufficient experience data within a specific segment of the data to either produce graduated rates or estimate a reasonable and simple structure of loads relative to a less differentiated table.

It should first be noted that quartiles were re-determined within each job category before graduation. This was done because the original quartiles determined across combined job categories were skewed relative to the salary and benefit amount distributions within each job category.

The Pub-2010 tables described below can be found on the SOA website at the following link: https://www.soa.org/research-reports/2019/pub-2010-retirement-plans/.

4.7.1 Employee and Retiree Rates

For Employees and Retirees, it was decided that separate male and female tables would be produced by job category (Teachers, Safety and General). In addition, Above-Median (Q3 and Q4) and Below-Median (Q1 and Q2) versions were estimated within each job category. It was decided that a combined table including all three job categories should not be produced; see Subsection 13.1 for further details.

However, for Retirees, there was one context in which the combined experience across all job categories was utilized. For advanced ages above 100, graduated rates were constructed using a blend of all the

Nondisabled Annuitant rates. This was done in part to address limited sample size at those extreme ages. It was also felt that, within a gender, the most salient characteristic of a member above age 100 is probably their age, and not whether (many decades previously) they happened to have been in a certain profession. Even income level loses predictive value at extreme ages.

4.7.2 Contingent Survivor Rates

As discussed previously, the job category of the primary Employee was generally a less significant predictor for the mortality of the Contingent Survivor. In addition, population sizes were smaller for survivor beneficiaries, particularly for a couple of the job categories. Benefit quartile, however, did carry predictive value. Consistent with the Employee and Retiree tables, separate rates were developed for above- and below-median Contingent Survivors.

4.7.3 Disabled Retiree Rates

Consistent with the notes above, it was decided that Safety members should have a separate set of Disabled Retiree rates. Female data were relatively sparse, but the graduated rates appeared reasonable, both consistent in shape with the male Disabled Retirees, and appropriate in level compared to female below-median Retirees.

The Disabled Retiree dataset for Teachers was not large, and the mortality experience for that group was similar to that of General Disabled Retirees, especially at ages older than 60. It was decided to combine Teacher and General experience and produce Disabled Retiree rates for that combined “Non-Safety” population. Unlike Safety occupations, presumably public-sector plans covering Teachers and General employees apply definitions of disability closer to “any occupation” or “permanent and total disability,” similar to those typically found in the private sector. Unlike the other three status groups, Above- and Below-Median tables were not produced for Disabled Retirees because of their lower predictive value and the thinness of certain gender/job category subsets (particularly female Safety).

Duration was also mentioned previously as a potentially significant factor. There was not enough Safety experience to explore the relationship between duration (since disability) and mortality within that population. The combined Teacher and General data did seem to show declining mortality over at least the first five years of duration. However, no meaningful conclusions could be drawn regarding the first year (duration 0) due to the issues discussed previously. Furthermore, a significant portion of the experience data did not specify duration. Removing all the data of unknown duration eliminated too much experience at later ages.

Mortality rates below retirement ages, typically below 65 or 70, do include some influence from members who were recently disabled. On average this should approximately take into account the effect of higher mortality incidence within several years following disability. Select-and-ultimate rates would not only be hard to model reliably with the given dataset, they are also more difficult to apply in practice—assuming the information is available—and only have a noticeable effect for a small portion of the post-disability lifetime. Especially outside of the Safety job category, disability is an infrequent contingency typically comprising only a very small part of plan obligations. Therefore, the Committee decided not to model duration-based select and ultimate rates.

Section 5: Graduation of Raw Rates

RPEC developed raw mortality rates under two bases: (1) amount-weighted rates, which reflected annualized salary for Employees and annualized retirement plan benefits for Retirees, Contingent Survivors and Disabled Retirees and (2) headcount-weighted rates. As is typical with empirical datasets, each set of gender-/age-specific raw mortality rates developed by the Committee exhibited a certain degree of random fluctuations around a smooth trend curve.

The objective of any graduation methodology is to smooth observed experience in a way that maintains an appropriate degree of fit with the underlying raw dataset. RPEC developed smoothed mortality rates under both the Whittaker-Henderson (Type B) with the "Lowrie variation" (W-H-L) methodology that was used in the SOA's RP-2014 study and a technique based on Generalized Additive Model (GAM) methodology.

Central to both the W-H-L and GAM graduation methodologies is the concept of an "objective function" that needs to be minimized. Both of the W-H-L and GAM objective functions include two components: one that measures the overall fit and the other that measures the overall smoothness of the graduated values. For example, given a set of raw mortality rates, q_x, over the age range x_{min} to x_{max}, with weights w_x, the GAM objective function used by RPEC was equivalent to[24]

$$\sum_x w_x\left(\ln q_x - f(x)\right)^2 + \lambda \int_{x_{min}}^{x_{max}} [f''(x)]^2 dx$$

RPEC used the "mgcv" package in R, a widely used language and environment for statistical computing and graphics, to solve for the minimizing function, *f(x)*. Further information on the GAM graduation methodology used by RPEC can be found in Appendix C.

Comparisons of the smoothed rates developed under the W-H-L and GAM methodologies indicated that the two techniques produced very similar results. Given the closeness of the two sets of graduated rates and the large number of distinct tables that required graduation, the Committee ultimately decided to proceed with the GAM methodology, which allowed RPEC to generate graduated rates very efficiently using the readily available R packages.

The specific age ranges used in the GAM graduation for each type of table (Employee, Retiree, Contingent Survivor and Disabled Retiree) and each job classification (Teacher, Safety and General) are reflected in the dataset summaries presented in Appendix B. The range of ages used to develop the amount-weighted rates and the corresponding headcount-weighted rates are the same.

[24] Note that for the sake of exposition, some details are omitted. The theory of GAMs is based on maximum likelihood, and the binomial likelihood with log link was used for the RPEC graduations. It can be shown that the likelihoods in the GAM framework are equivalent to optimizing this objective function via Iteratively Reweighted Least Squares (IRLS). Under IRLS, the formula provided here has two adjustments. For each iteration, the weights are updated to reflect the fitted model (to that point), and a special residual, in this case $(q_x - \hat{q}_x)/\sqrt{\hat{q}_x}$, is added to the error $\ln q_x - f(x)$. Each iteration is then a weighted least squares calculation.

Section 6: Construction of Retiree and Contingent Survivor Tables

6.1 Overview

The following outlines the steps that RPEC took to construct the Retiree and Contingent Survivor mortality tables:

1. The development of mortality rates at ages 100 and above, starting with the graduated gender-specific rates derived from data for the aggregated (i.e., all three job categories combined) Nondisabled Annuitant (Retirees plus Contingent Survivors) dataset.

2. For each table weighting (amount and headcount), and separately for each job category, graduated gender-specific Retiree mortality rates were developed starting at various job category-specific early retirement ages through age 95.

3. For each table weighting, graduated gender-specific Contingent Survivor mortality rates (for all three job categories combined) were developed for ages 50 through 95. Contingent Survivor mortality rates for ages 45 through 49 were based on scaled versions of the corresponding aggregated Retiree rates for those ages.

4. For each of the 12 sets of Retiree rates (two genders, two weightings and three job categories) and each of the four sets of Contingent Survivor rates (two genders and two weightings), RPEC used quintic polynomials to interpolate mortality rates smoothly between ages 90 and 100.

5. For each of the 16 separate mortality tables described in step 4, two additional tables were developed, representing subpopulations of the corresponding dataset, bifurcated based on whether the underlying annuity amount was above- or below-median for that population.[25]

As a result, RPEC ended up constructing a total of 36 separate Retiree mortality tables and 12 separate Contingent Survivor tables.

6.2 Development of Mortality Rates at Ages 100 and Above

At an early stage of the table construction process, RPEC decided that the gender-specific mortality rates at ages 100 and above for all annuitants in all three job categories should coincide, as there was insufficient experience to support the contrary. For each of the four combinations of gender and weighting, RPEC developed mortality rates at ages 100 through 119 using projection methodology originally developed by Kannisto [Kannisto 1992]. Each of the Kannisto projections was based on the graduated mortality rates at ages 89 and 90 from the corresponding aggregated (all three job categories

[25] For example, in addition to the amount-weighted Male Retiree Teachers table, an amount-weighted Male Retiree Teachers Above-Median table and an amount-weighted Male Retiree Teachers Below-Median table were produced. The Male Retiree Teachers dataset was split into the Above- and Below-Median subpopulations using the median annual retirement benefit for Male Retiree Teachers of $37,789; see Subsection 3.8.

combined) Nondisabled Annuitant database. The resulting annual mortality rates were capped at 0.5, and the annual mortality rate at age 120 was set equal to 1.0.

6.3 Graduation of Mortality Rates for Retirees and Contingent Survivors

Analysis of the final Retiree dataset indicated significant differences in the distribution of exposures and deaths among the three job categories. For example, virtually all Retiree exposures and deaths under age 50 were concentrated within the Safety subpopulation. Because of these differences, the Committee decided to start the graduation processes at different ages for each of the three job category subpopulations: age 55 for Teachers, age 45 for Safety and age 50 for General. The graduation processes for all Retiree subgroups ended at age 95.

Although some exposure/death variations by job category were identified within the final Contingent Survivor dataset, those differences were less significant than those within the Retiree dataset. That fact, along with the relatively small size of the Contingent Survivor dataset for males, persuaded RPEC not to construct separate Contingent Survivor tables for each of the three job categories. Hence the graduation processes for the Contingent Survivor rates were based on the total (all three job categories combined) dataset, starting at age 50 and continuing through age 95.

6.4 Further Extensions of Mortality Rates for Retirees and Contingent Survivors

At this point, the Committee had developed smoothed mortality rates for all Retiree and Contingent Survivor subpopulations through age 95 using the GAM graduation methodology and for ages 100 and above using Kannisto's methodology. Given the relatively small amount of data at ages greater than 90, RPEC decided to complete each of these tables by fitting a quintic polynomial to rates at the following six ages: 88, 89 and 90 (from the GAM graduation) and 100, 101 and 102 (from the Kannisto projection).

The Committee decided that it would also be useful to develop Contingent Survivor rates down to age 45. The mortality rates for ages 45 through 49 were computed by extending backwards smoothly from the graduated rates at age 50 using a constant multiple of the corresponding aggregated (all three job categories) Retiree rates for ages 45 through 49. Each of these scaling factors was based on the ratio of the age-50 Contingent Survivor mortality rate for the table being extended to the corresponding age-50 aggregate Retiree rate. For example, each of the amount-weighted Contingent Survivor mortality rates for females between ages 45 and 49 was set equal to 1.45782 times the corresponding amount-weighted aggregate Retiree rate, where the scaling factor was calculated as the ratio of the corresponding rates at age 50.[26]

6.5 Development of Above-Median and Below-Median Annuitant Tables

As discussed in Subsection 4.3, the multivariate analysis performed on the nondisabled subpopulations revealed clear evidence for variations in mortality experience based on benefit amount for both Retirees

[26] RPEC constructed aggregate Retiree tables solely for the purpose of extending the Contingent Survivor tables from age 50 down to age 45. See Subsection 13.1 for a discussion of why aggregate Retiree tables were not published as part of this report.

and Contingent Survivors. As a result of these findings, the Committee concluded that it would be important to construct separate sets of tables based on datasets bifurcated on "Above-Median" and "Below-Median" subpopulations. For purposes of table development, above- or below-median benefit amount designations were determined separately for each gender and job category.

With the exception of the tables for female Safety Retirees discussed later in this subsection, RPEC constructed Above-Median and Below-Median tables by applying a piecewise-linear weighted regression (PLWR) methodology to the appropriate "total subpopulation" table constructed as described in the previous three subsections. This PLWR methodology consisted of fitting a continuous two-segment linear exposure-weighted regression (with the piecewise-linear bend point at age 65) to the age-by-age ratios[27] of actual deaths to expected deaths, where

- Actual deaths (by amount or headcount, as appropriate) were counted separately based on their above- or below-median designations and

- Expected deaths (by amount or headcount, as appropriate) were calculated by multiplying the age-specific exposures for the appropriate above- or below-median designation times the corresponding mortality rate developed for the "total subpopulation" being bifurcated.

One particularly nice feature of the PLWR methodology is that the total number of predicted deaths (based the rates derived from the regression curve) always equals the total number of actual deaths for that subpopulation. In the relatively rare instances when either the resulting PLWR Above-Median factor exceeded 1.0 or the PLWR Below-Median factor dropped below 1.0, the offending factor was set equal to 1.0.

As an example, Figure 6.1 presents the raw A/E values and the resulting piecewise-linear weighted regression curve that was used to develop the amount-weighted Below-Median rates for male Safety Retirees. In particular, the age-45 and age-95 mortality rates for Below-Median male Safety Retirees were approximately 1.95 and 1.01 times as large, respectively, as the corresponding "total subpopulation" male Safety rates.

[27] The PLWR regressions were based on ratios through age 90.

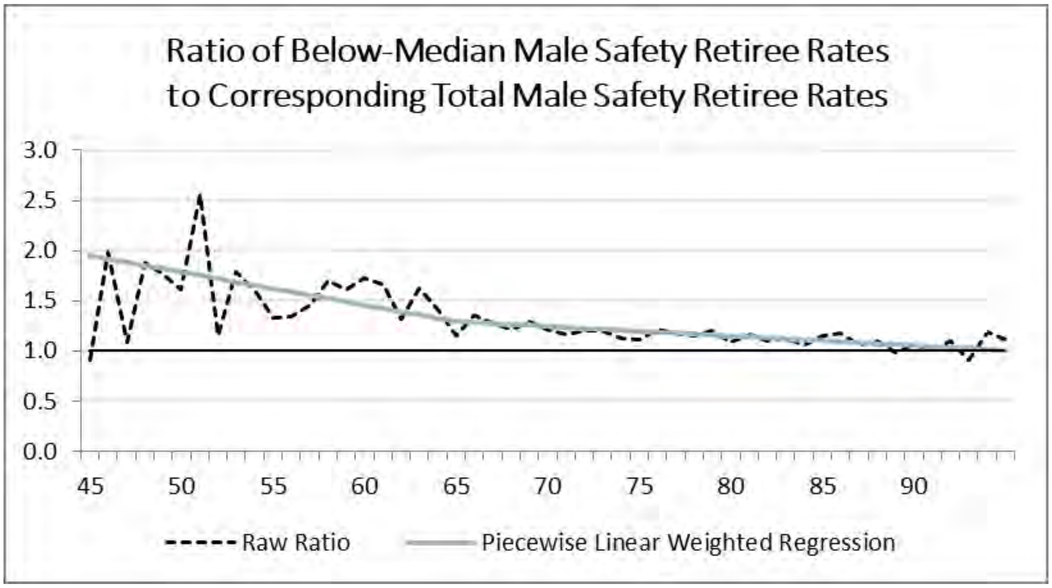

Figure 6.1: Example of Piecewise-Linear Weighted Regression Curve

Finally, given the thinness of the total dataset for female Safety Retirees, the Committee decided to take a more direct approach than PLWR. Specifically, the Above-Median and Below-Median tables for female Safety Retirees were set equal to a constant multiple of the underlying total female Safety Retiree table, based on the ratio of total actual deaths to total expected deaths for all ages through 90. For example, the headcount-weighted rates for Above-Median female Safety Retirees are all 86.0% of the corresponding total female Safety Retiree headcount-weighted rates.

Section 7: Construction of Employee Tables

7.1 Overview

The following steps summarize the process that RPEC utilized to construct the Employee mortality tables:

1. The raw gender-specific rates for each job category and weighting combination were graduated using the GAM methodology described in Section 5.

2. Each of these sets of smoothed rates was extended (when necessary) beyond the end of the oldest graduation age to age 80 by determining the constant annual increase factor that would converge exactly to the corresponding Retiree age 100 rate.

3. Each of these sets of smoothed rates was extended backwards from age 25 (the age at which all of the Employee rate graduations commenced) to age 18 by fitting a cubic polynomial to the following four mortality rates: the Juvenile rates at ages 16 and 17 (see Section 9) and the corresponding graduated Employee rates at ages 25 and 26.

4. For each of the resulting 12 separate mortality tables (two genders, two weightings and three job categories), two additional tables were developed. These additional tables reflect subpopulations of the corresponding dataset, bifurcated based on whether the Employee's salary amount was above- or below-median for that population.

Overall, a total of 36 Employee tables were created.

7.2 Extension to Age 80 for Teachers and Public Safety

Since there was sufficient reliable data within the General population to graduate Employee rates through age 80, RPEC needed to develop extended Employee rates only for Teachers and Safety. In each of those cases, rates were extended beyond the oldest graduation age (age 75 for Teachers and age 65 for Safety) by solving for the constant exponential rate that, if applied to the oldest graduated rate, would equal the corresponding age 100 Retiree rate.

Considering the amount-weighted Safety table for males as an example, the graduated age-65 Employee rate is 0.00410, the corresponding age-100 Retiree rate is 0.32609, and the resulting exponential factor is 1.133186. Hence the mortality rate for each of the ages 66 through 80 was calculated as 1.133186 times the rate at the preceding age.

7.3 Extension Down to Age 18

Gender-specific mortality rates for Employees ages 18 through 24 were interpolated using a cubic polynomial matching the following four rates: Juvenile rates at ages 16 and 17 (described in Section 9) and the Employee rates at ages 25 and 26.

7.4 Development of Above-Median and Below-Median Employee Tables

The Committee considered the use of both the PLWR methodology described in Subsection 6.5 and a constant scaling factor that could be applied to each of the respective job category-specific Employee tables to obtain Above-Median and Below-Median versions of those tables. After reviewing the results of the PLWR methodology, the Committee concluded that there was not enough variation at ages over 40 (the piecewise linear bend point) to warrant the use of the more complicated PLWR methodology for the Employee tables.

Therefore, the Above-Median and Below-Median Employee rates at ages 25 and older were each set equal to a constant multiple of the corresponding total job category-specific Employee table, based on the ratio of total actual deaths to total expected deaths for ages 25 through 70 (25 through 65 for Safety). For example, the amount-weighted rates for Below-Median male Teachers starting at age 25 are all 135.0% of the corresponding (total subpopulation) male Teacher Employee amount-weighted rates. The rates for ages 18 through 24 were developed using cubic polynomial interpolation, as described in Subsection 7.3.

Section 8: Construction of Disabled Retiree Tables

8.1 Overview

Early in the process of constructing tables for Disabled Retirees, RPEC observed that the overall pattern of raw mortality rates for Safety Disabled Retirees was significantly different than that for the Disabled Retirees in both the other two job categories. This result was not unexpected, because of (1) the nature of the work performed by people in these professions and (2) the fact that Disabled Retirement provisions in plans covering Safety workers are typically quite different from those covering other (Non-Safety) occupations.

Based on that observation, along with the fact that the overall mortality patterns for Disabled Retirees in the Teachers and General job categories were relatively similar at ages 60 and older, the Committee decided to combine these two Disabled Retiree datasets for table construction purposes. Hence, two sets of gender-/weighting-specific Disabled Retiree mortality tables were created: one based on Safety experience and another based on the combined experience of Teachers and General. This latter subgroup is referred to as "Non-Safety" throughout the remainder of this section.

8.2 Process

The following describes the process that RPEC utilized to construct the Disabled Retiree mortality tables:

1. The raw gender-/weighting-specific rates for the Safety and Non-Safety subgroups were graduated using the GAM methodology described in Section 5. Except for the female Safety subpopulation, graduated rates were used between ages 50 and 95 for all Disabled Retiree subgroups. The graduated rates for female Safety Disabled Retirees covered only ages 50 through 75, because of the limited size of this dataset.

2. Each of these sets of smoothed rates was extended to the corresponding age 100 (nondisabled) Retiree rate using quintic polynomial interpolation. The quintic interpolation process started at ages 83, 84 and 85 and ended at ages 100, 101 and 102 for all subgroups except female Safety, for which the interpolation process started at ages 73, 74 and 75. For ages 100 and older, the Disabled Retiree rates were assumed to equal the corresponding Retiree rates.

3. Each of these sets of smoothed rates was extended backwards starting at age 49 down to age 18 by applying a constant scaling factor to the corresponding Employee rates.[28] Each of these scaling factors was calculated as the ratio of the age-50 rate for a given Disabled Retiree subgroup to the age-50 rate for the corresponding Employee subgroup. Table 8.1 summarizes those scaling factors.[29]

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[28] Note that a combined Teachers and General Employee table was constructed solely for the purpose of extending the non-Safety Disabled Retiree to ages 18 through 49.

[29] The scaling factors used to develop the headcount-weighted Disabled Retiree rates for a small number of ages (below age 50) were adjusted to ensure those rates were never lower than their amount-weighted counterparts.

Gender/Job Category	Amount-Weighted	Headcount-Weighted
Female Safety	3.34	3.06
Male Safety	2.95	3.42
Female Non-Safety	18.53	15.88
Male Non-Safety	11.32	10.14

Table 8.1: Scaling Factors Used to Develop Disabled Retiree Rates below Age 50

As can be seen in Table 8.1, the factors used to develop the Disabled Retiree rates below age 50 were much smaller for Safety members than those for Non-Safety members. The fact that the Disabled Retiree mortality rates for Safety members are generally lower than the corresponding Disabled Retiree rates for Non-Safety members is not surprising, given that the plan provisions for disability retirement benefits (eligibility and amount) for those in Safety professions are typically considerably less restrictive than those for members in other public-sector jobs. The difference between Safety and Non-Safety Disabled Retiree mortality rates can also be seen by comparing the displays in Subsection 10.4.

Because of the limited predictive value of benefit amount for Disabled Retirees and the thinness of the dataset for female Safety Disabled Retirees, the Committee decided not to develop separate Above- and Below-Median versions of these tables.

Section 9: Construction of Juvenile Tables

For completeness, RPEC has also included a set of gender-specific Juvenile mortality rates for ages 0 through 17. The rates of ages 0 through 12 were set equal to the adjusted[30] "unismoke" 2015 Valuation Basic Table (VBT) age nearest birthday rates [SOA 2015]. Both of the gender-specific Juvenile rates for ages 16 and 17 were based on the adjusted 2015 VBT rates multiplied by the ratio of (1) the actual number of deaths from all three job categories for Employees between ages 25 and 34 to (2) the total expected number of deaths between ages 25 and 34 based on the adjusted 2015 VBT rates. The Juvenile rates for ages 13, 14 and 15 were calculated using cubic polynomial interpolation.

Note that the gender-specific Juvenile rates are the same for both the amount and headcount weightings.

[30] The 2015 VBT rates were adjusted back to July 1, 2010, using mortality improvement Scale MP-2017.

Section 10: Comparison of Rates

RPEC produced job category-specific comparisons of amount-weighted[31] mortality rates to those in three previously published SOA tables. Two of those previously published SOA tables, RP-2000 and RP-2006, were selected by the Committee because they formed the basis for the mortality assumptions currently used by many public-sector plans; see [SOA 2018]. Comparisons were also made to mortality rates from the White Collar (WC) version of RP-2006 because some of the Pub-2010 tables developed in this report tended to track more closely with the RP-2006 WC rates than they did with either the RP-2000 or the (aggregate) RP-2006 rates.

All the graphs presented in Section 10 are based on mortality rates projected to July 1, 2018,[32] as follows:

- RP-2000 rates are projected with Scale BB from 2000
- RP-2006 rates and RP-2006 White Collar rates are projected with Scale MP-2017 from 2006
- Pub-2010 rates are projected with Scale MP-2017 from 2010.

A ratio less than 1.0 means that the projected Pub-2010 mortality rate is smaller than the corresponding projected RP-2000 or projected RP-2006 mortality rate.

Finally, it should be noted that the RP-2000 and RP-2006 tables combined data for Retirees and Contingent Survivors into a "Healthy Annuitant" table. This complicates the direct comparisons of those historical rates to the Pub-2010 Retiree rates and the Pub-2010 Contingent Survivor rates presented in Subsections 10.1 and 10.3, respectively.

[31] Corresponding graphs constructed using the headcount-weighted mortality rates are very similar to the amount-weighted versions shown in this section and therefore are not included in this report.
[32] See Subsection 13.6 for a discussion of projecting mortality rates to July 1, 2018.

10.1 Comparison of Retiree Rates

Both Figure 10.1(F) and Figure 10.1(M) show that the projected PubT-2010 rates are lower than and closest to the projected RP-2006 White Collar rates. They are considerably lower than the projected RP-2000 or projected RP-2006 rates except at ages beyond 95 for females, when the PubT-2010 rates are higher than the projected RP-2000 rates.

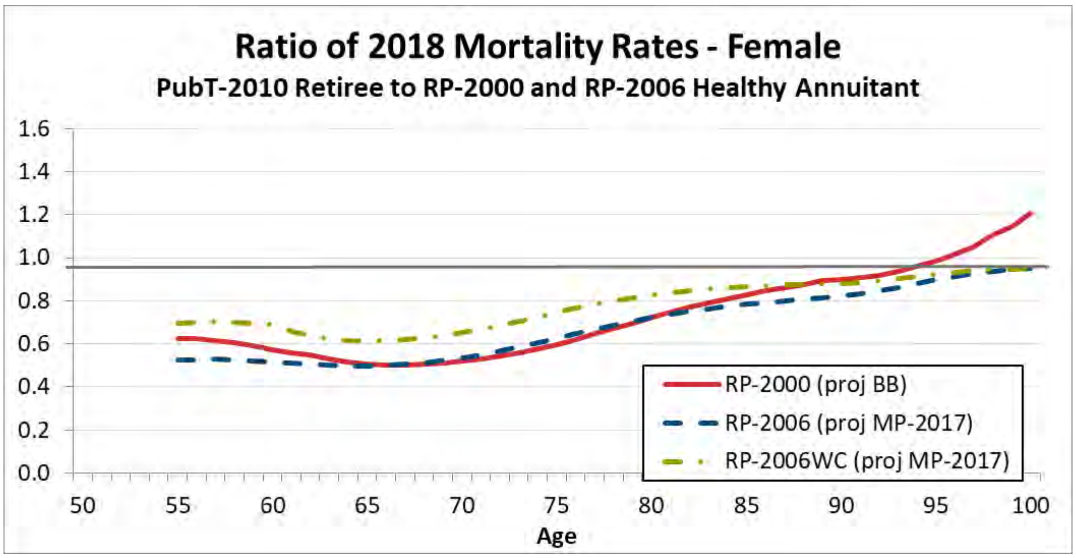

Figure 10.1(F): Female Retiree Teachers

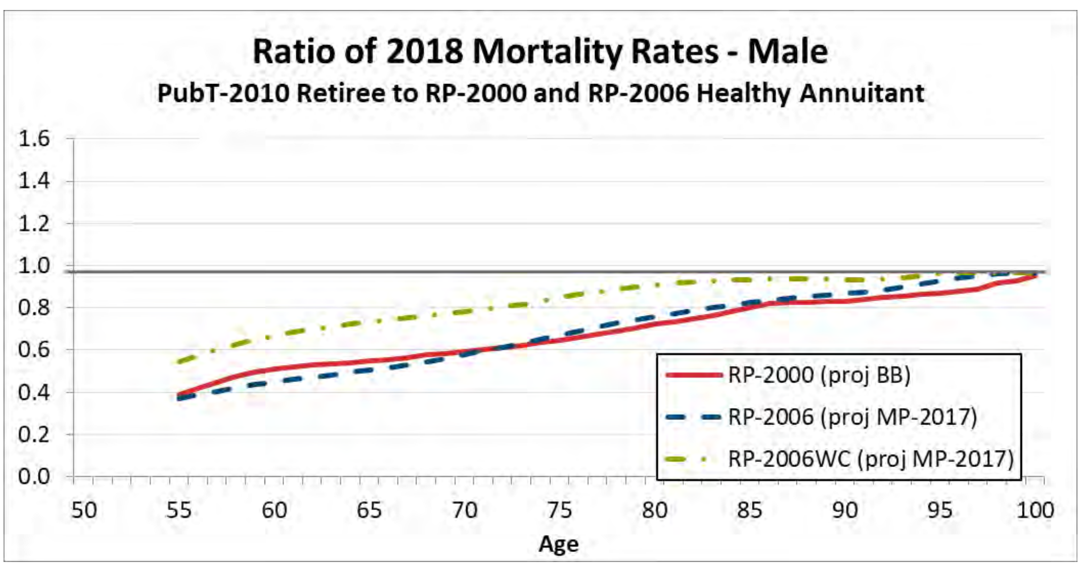

Figure 10.1(M): Male Retiree Teachers

Figure 10.2(F) shows that the projected female PubS-2010 rates are lower than the projected RP-2006 White Collar rates at the younger ages and gradually become very close to the projected RP-2006 beyond around age 75. Figure 10.2(M) shows that the projected male PubS-2010 rates exhibits a similar pattern as the female PubS-2010 rates shown in Figure 10.2(F).

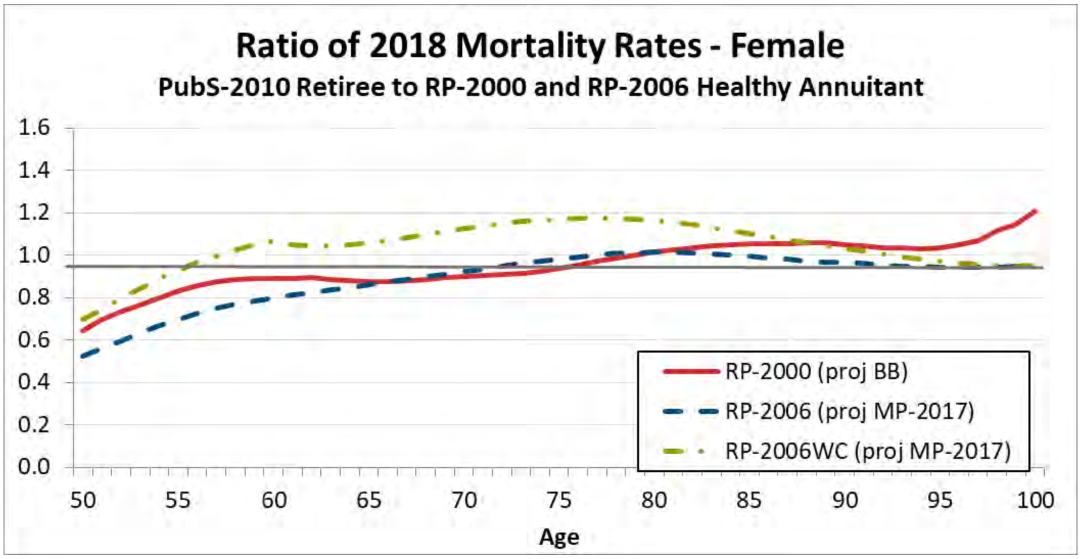

Figure 10.2(F): Female Retiree Safety

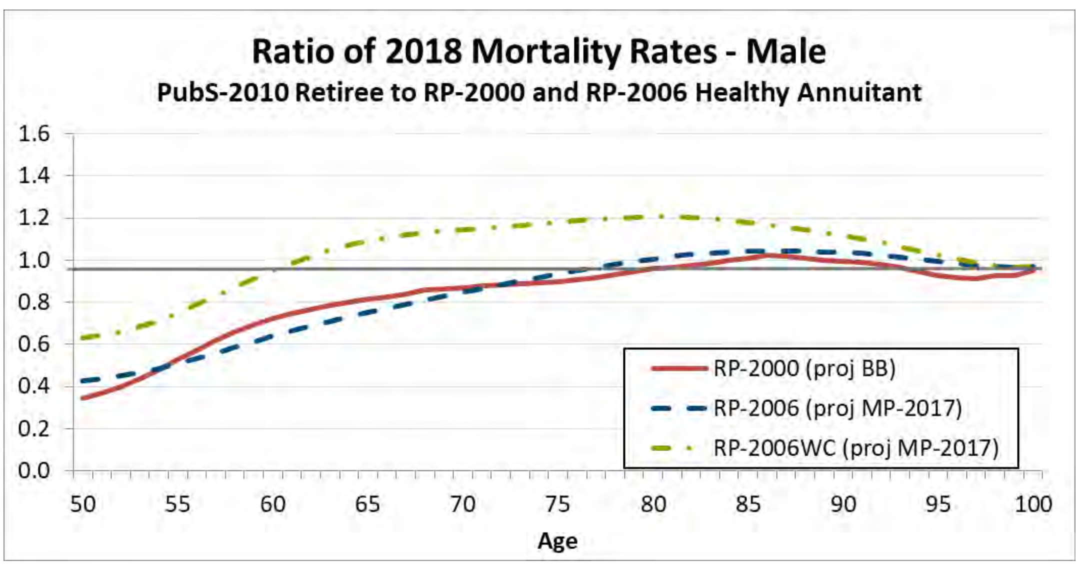

Figure 10.2(M): Male Retiree Safety

Figure 10.3(F) shows that the projected female PubG-2010 rates are closest to the projected RP-2006 White Collar rates for the age range shown. Figure 10.3(M) shows that the projected male PubG-2010 are generally higher than the projected RP-2006 White Collar rates but lower than the projected RP-2006 rates.

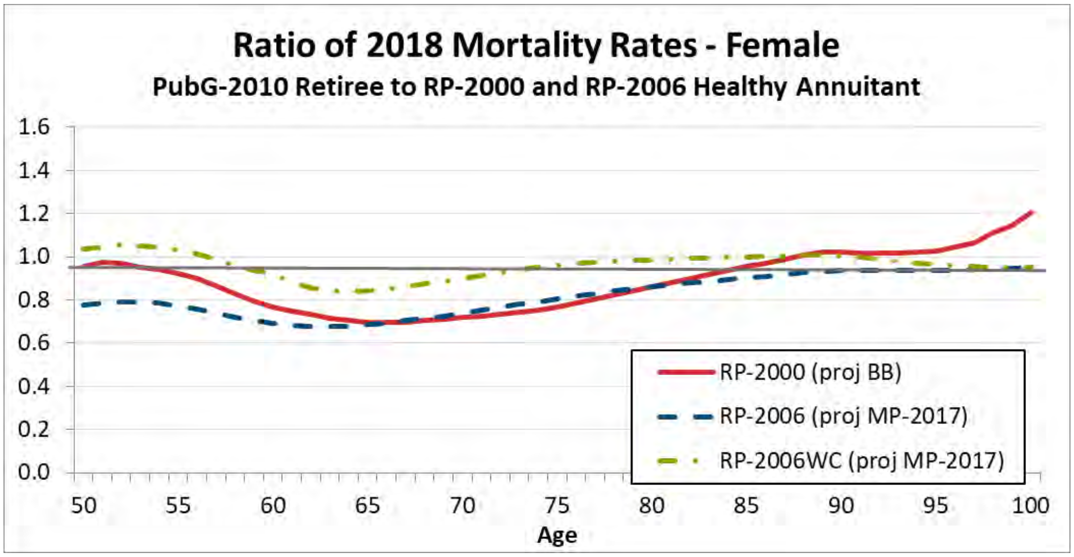

Figure 10.3(F): Female Retiree General

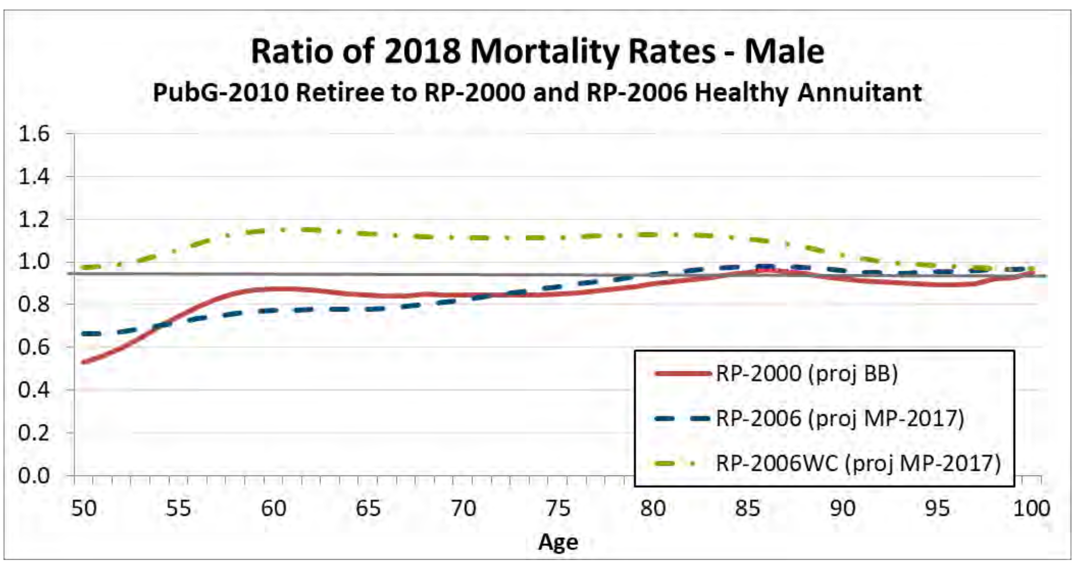

Figure 10.3(M): Male Retiree General

10.2 Comparison of Employee Rates

Figure 10.4(F) shows that the projected female PubT-2010 rates are lower than those in all three of the projected comparator tables. They are comparatively closest to the projected RP-2006 White Collar rates. Although Figure 10.4(M) shows a similar pattern, the ratio to the projected RP-2000 rates is higher than the ratio to RP-2006 White Collar rates starting at around age 65. This is in part because of the higher mortality improvement under Scale BB that is used to project RP-2000.

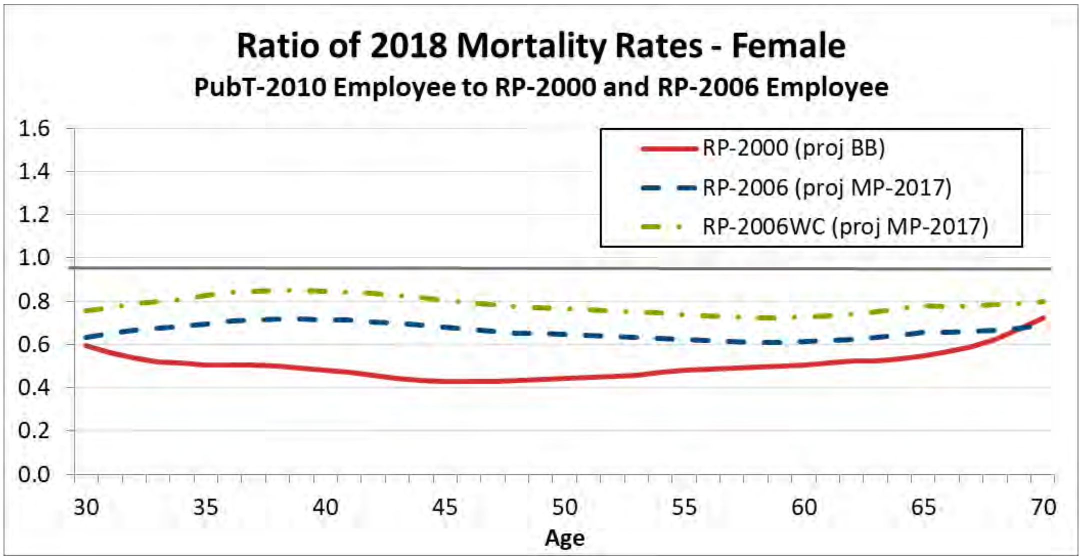

Figure 10.4(F): Female Employee Teachers

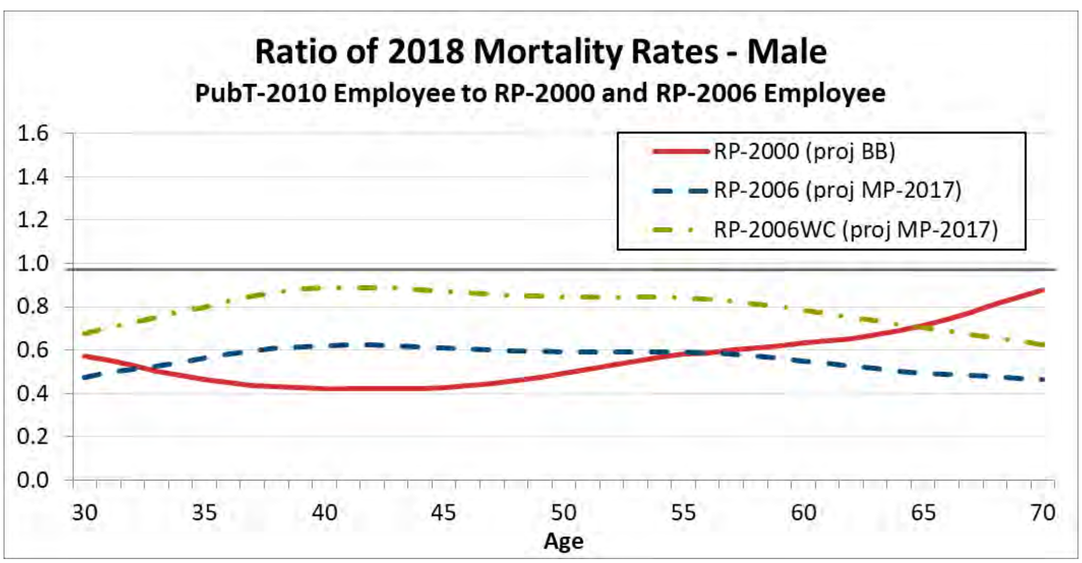

Figure 10.4(M): Male Employee Teachers

Figure 10.5(F) shows that the projected female PubS-2010 rates are higher than the projected RP-2006 rates at the younger Employee ages but gradually become lower starting at approximately age 45. Figure 10.5(M) shows the projected male PubS-2010 rates are in between the projected RP-2006 White Collar rates and the projected RP-2006 rates up to about age 45, after which point the PubS-2010 rates become lower than the RP-2006 White Collar rates.

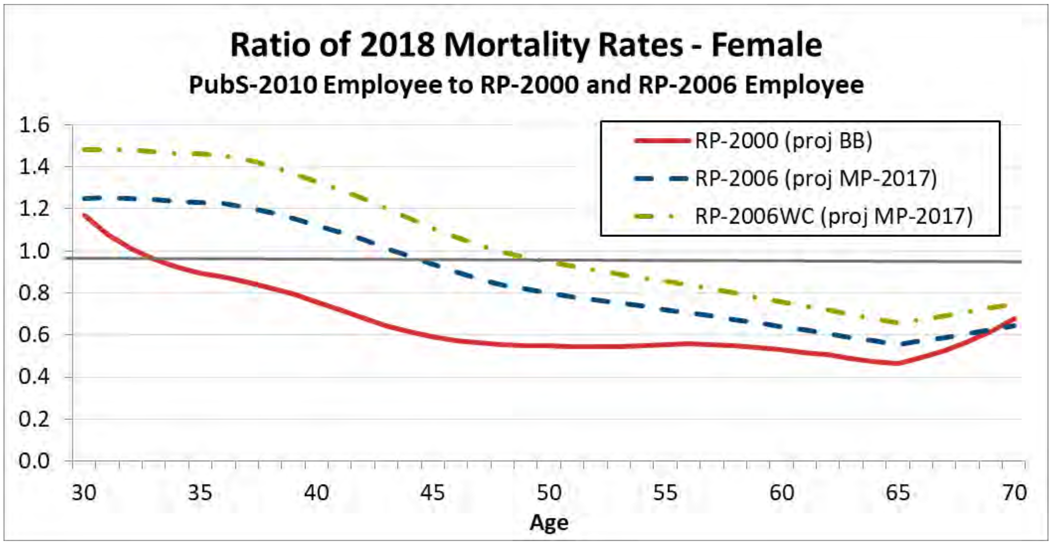

Figure 10.5(F): Female Employee Safety

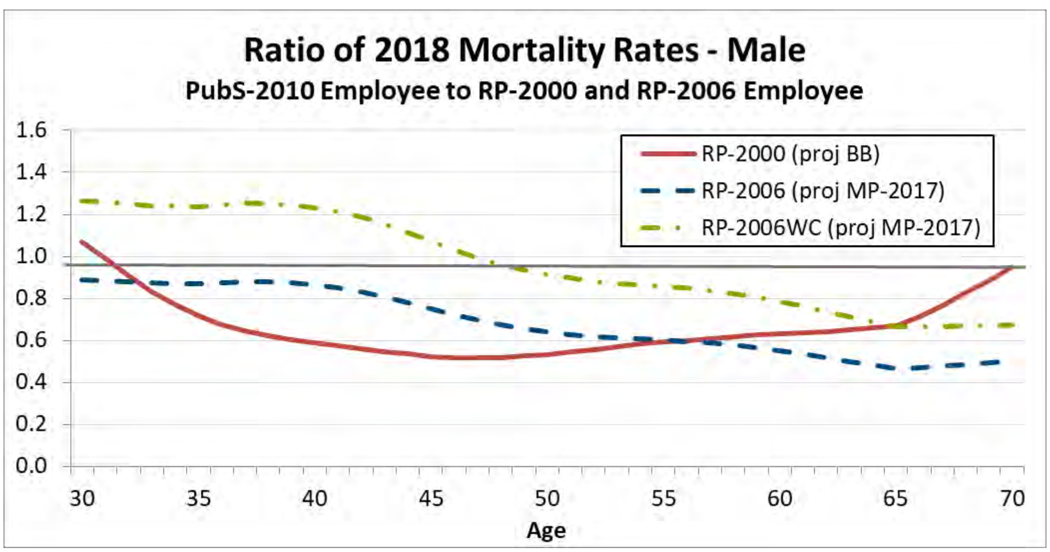

Figure 10.5(M): Male Employee Safety

Figure 10.6(F) shows that the projected female PubG-2010 rates are lower than (but closest to) the projected RP-2006 White Collar rates. Figure 10.6(M) shows that the projected male PubG-2010 rates are generally closest to the projected RP-2006 rates before around age 50.

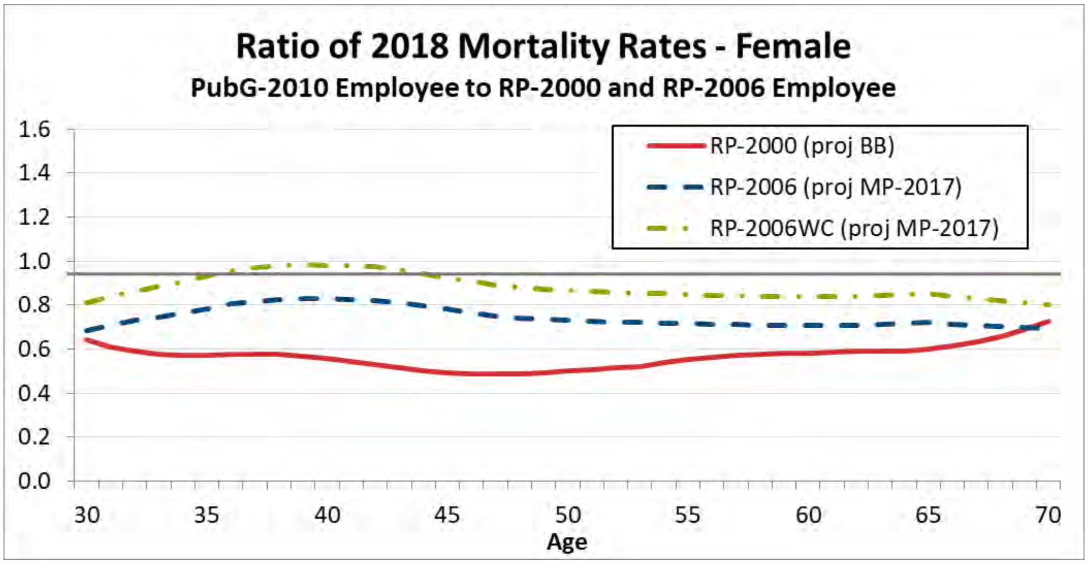

Figure 10.6(F): Female Employee General

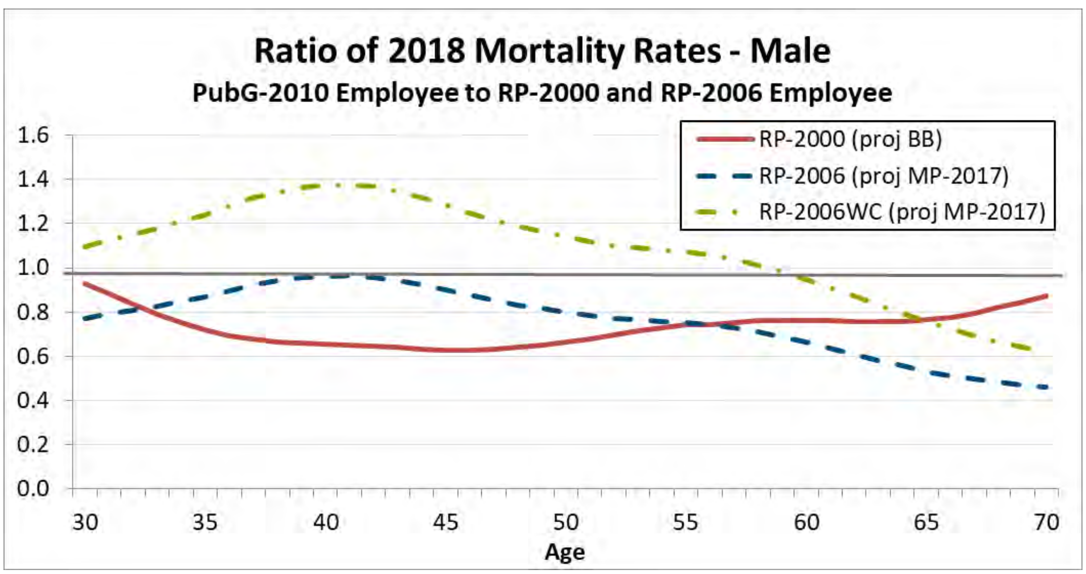

Figure 10.6(M): Male Employee General

10.3 Comparison of Contingent Survivor Rates

Figure 10.7(F) shows that the projected female Pub-2010 Contingent Survivor rates are closer to the projected RP-2006 Healthy Annuitant rates. Figure 10.7(M) shows that the projected male Pub-2010 Contingent Survivor rates are quite a bit higher than the projected RP-2000 and the projected RP-2006 Healthy Annuitant rates for almost all ages before age 85.

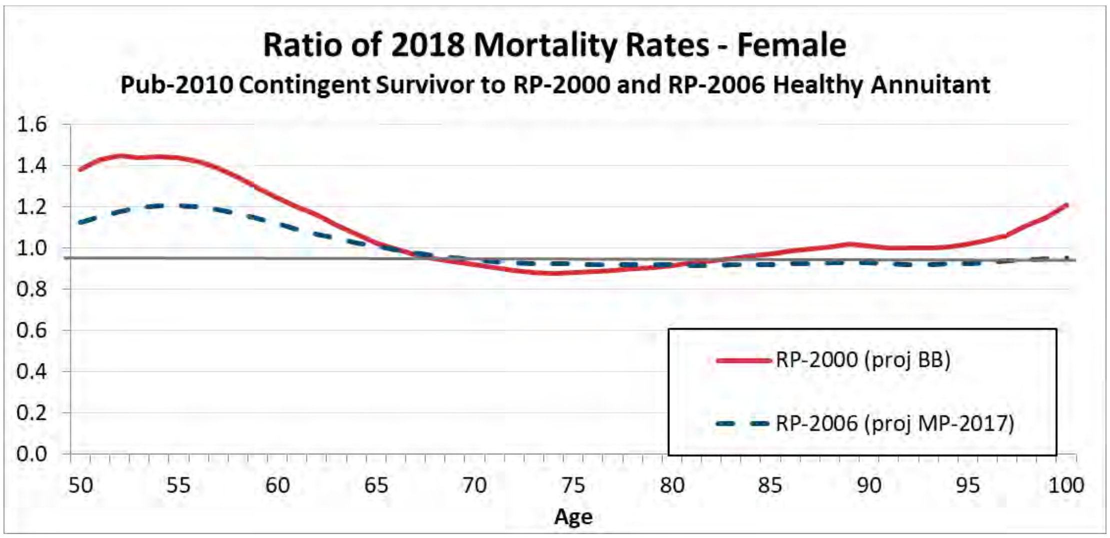

Figure 10.7(F): Female Contingent Survivor

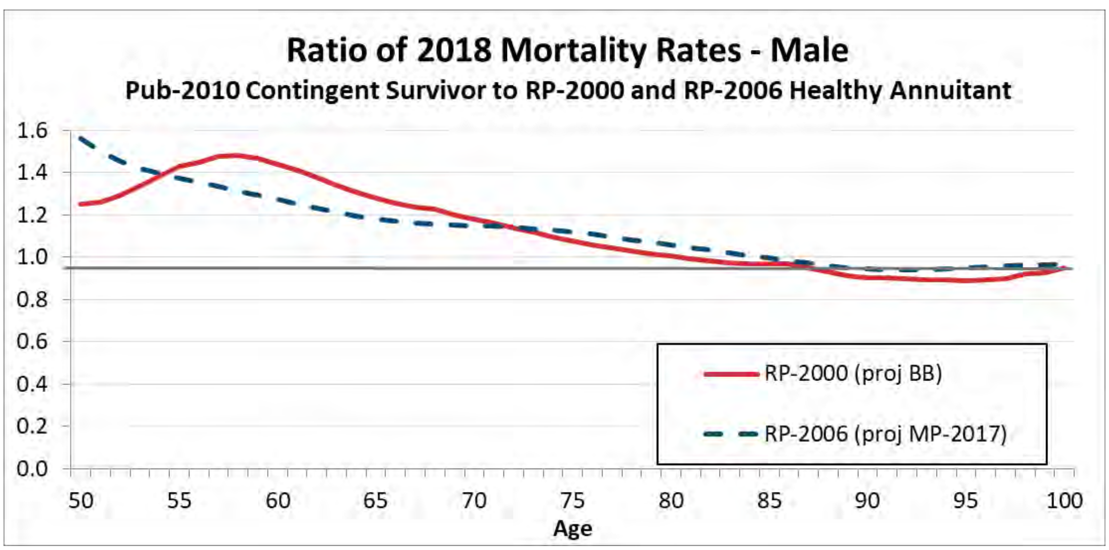

Figure 10.7(M): Male Contingent Survivor

10.4 Comparison of Disabled Rates

In addition to comparisons to the RP-2000 and RP-2006 Disabled Retiree rates, the projected PubS-2010 Disabled Retiree rates are also compared to the projected RP-2006 Healthy Annuitant rates (the yellow dashed line). Figures 10.8(F) and 10.8(M) show that the projected PubS-2010 Disabled Retiree rates are generally much lower than either of the corresponding projected RP-2000 or projected RP-2006 rates. Interestingly, they are relatively close to the projected RP-2006 Healthy Annuitant rates. This seems consistent with the criteria for disability retirement associated with the employee's capacity to perform their job duties, which are much more physically demanding for most Safety employees.

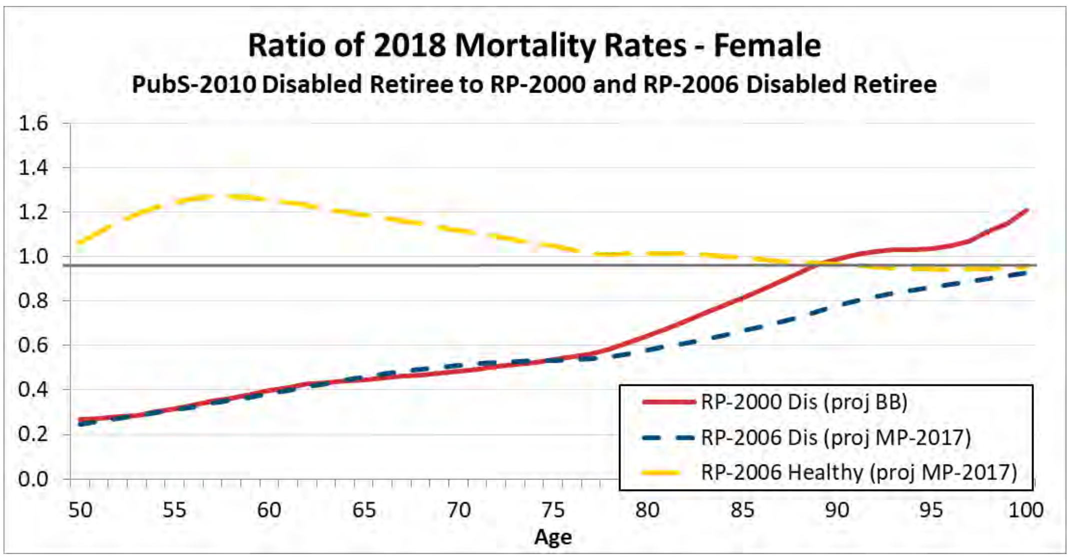

Figure 10.8(F) Female Disabled Retiree Safety

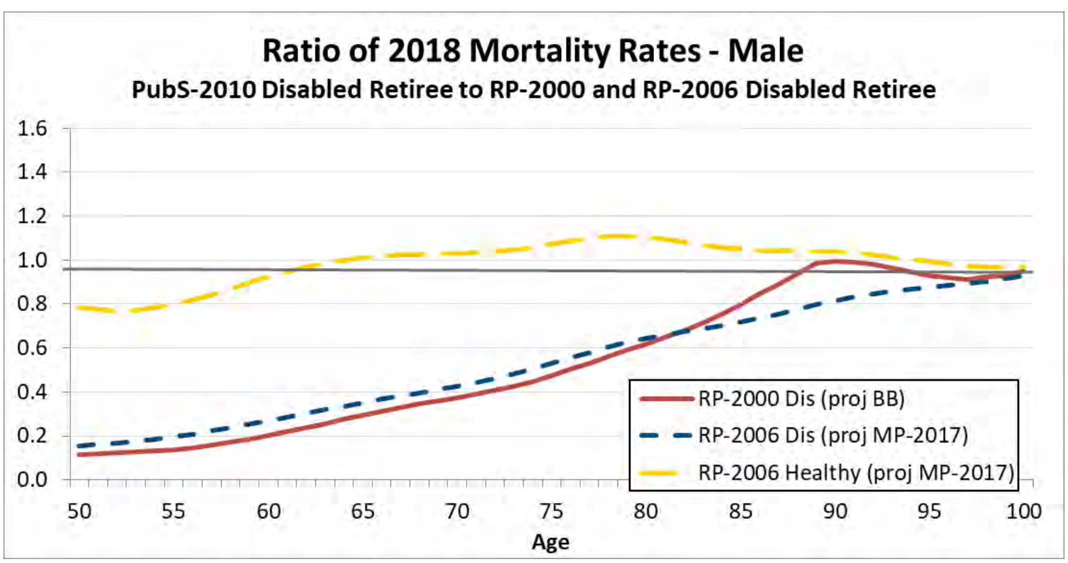

Figure 10.8(M): Male Disabled Retiree Safety

Figure 10.9(F) shows that the projected female Non-Safety Disabled Retiree rates (denoted here as PubNS-2010) are higher than the projected RP-2006 Disabled Retiree rates before approximately age 65 and are slightly lower than the projected RP-2006 rates beyond age 65. Figure 10.9(M) shows that the projected male PubNS-2010 rates are lower than the projected RP-2000 and projected RP-2006 Disabled Retiree rates at almost all ages shown.

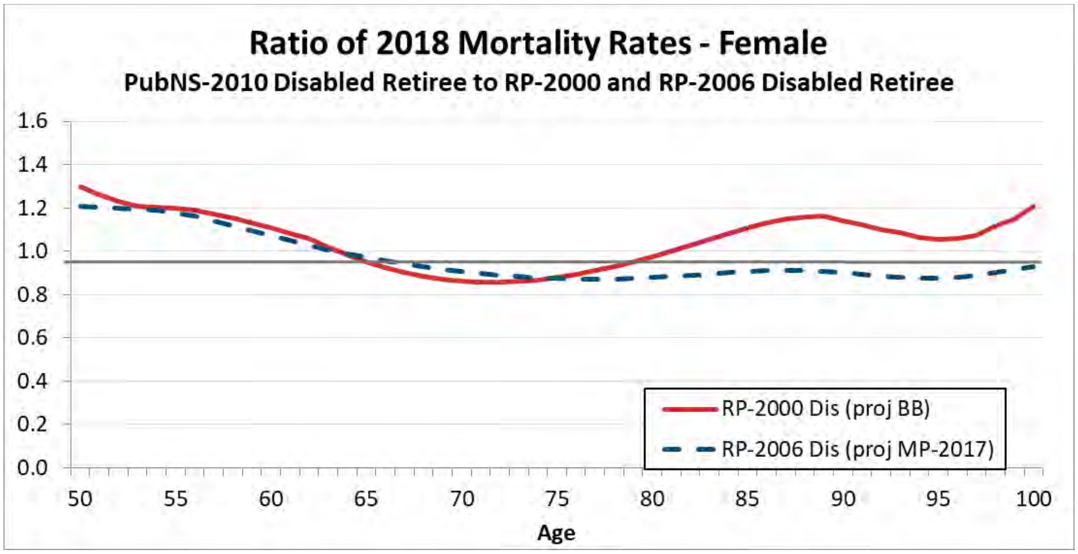

Figure 10.9(F): Female Disabled Retiree Non-Safety

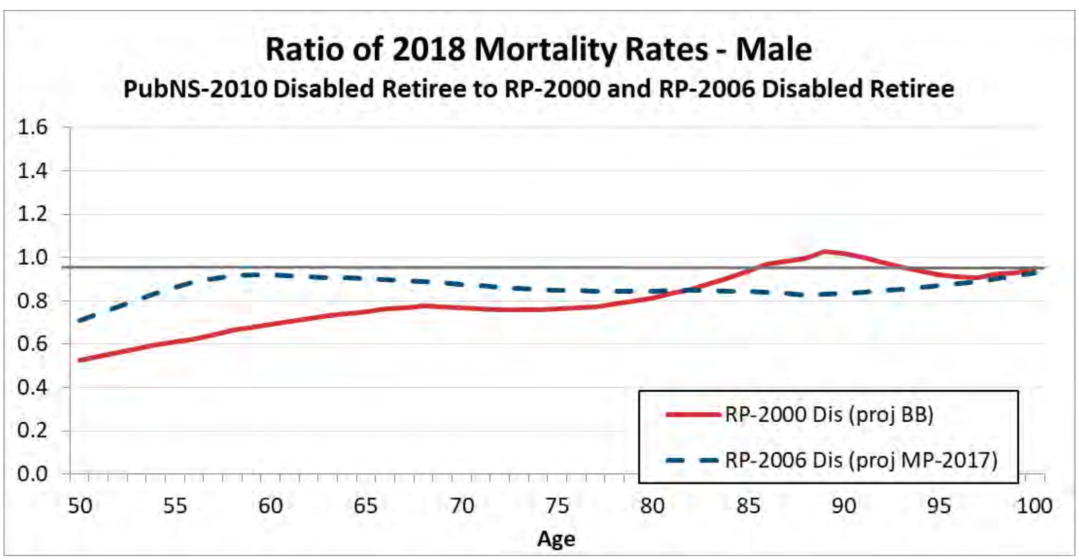

Figure 10.9(M): Male Disabled Retiree Non-Safety

Section 11: Annuity Comparisons

11.1 Comparison of Annuity Values to Other Published SOA Tables

11.1.1 Basis of Annuity Calculations

All annuity values in this section were calculated as of July 1, 2018,[33] using a pre-retirement discount rate of 7.0% and a post-retirement discount rate of 5.0%. The 7.0% rate was chosen to be broadly representative of discount rates recently used in the funding valuations of public-sector retirement plans, and the “spread” of 2.0% broadly representative of recent post-retirement cost-of-living adjustments. Annuity comparisons based on a flat 7.0% discount rate (pre- and post-retirement) are presented in Appendix D.1.

For all deferred-to-age-62 annuity calculations shown in this report, RPEC used Employee rates for ages less than 62 and Retiree (or Healthy Annuitant)[34] rates for ages 62 and older. All monthly annuity values were calculated using the standard approximation to Woolhouse’s formula:

$$ {}_{n|}\ddot{a}^{(12)} \approx {}_{n|}\ddot{a}_x - (11/24)\ {}_{n|}E_x $$

11.1.2 Comparisons of Amount-Weighted Deferred Annuities for Nondisabled Members

The Committee developed deferred-to-age-62 annuity-due values for nondisabled members in each of the three job categories and compared those to deferred annuity values developed using the previously published SOA tables (and associated mortality improvement scales) described in Section 10.

Table 11.1 shows that the amount-weighted deferred annuity values developed using the new tables for Teachers are significantly higher than those developed using any of the three previously released SOA tables. In fact, the deferred annuities values produced using the new PubT-2010 tables are generally more than 4% higher for females and approximately 3% higher for males relative to those developed using the White Collar version of the RP-2006 table.

[33] See Subsection 13.6 for a more detailed discussion of the calculation of annuity values as of July 1, 2018.
[34] Healthy Annuitant rates were used in the annuity calculations based on the RP-2000 and RP-2006 tables, since those studies did not develop separate Retiree and Contingent Survivor rates.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubT-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubT-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.2406**	*5.7%*	*6.3%*	*3.7%*
	Age 35	2.2720	2.2594	2.3191	**2.4113**	*6.1%*	*6.7%*	*4.0%*
	Age 45	4.4101	4.3812	4.5028	**4.6932**	*6.4%*	*7.1%*	*4.2%*
	Age 55	8.6263	8.5443	8.7849	**9.1655**	*6.3%*	*7.3%*	*4.3%*
	Age 65	13.0772	12.9595	13.3331	**13.9245**	*6.5%*	*7.4%*	*4.4%*
	Age 75	9.8517	9.6858	10.0300	**10.5286**	*6.9%*	*8.7%*	*5.0%*
	Age 85	6.3586	6.0423	6.2543	**6.6215**	*4.1%*	*9.6%*	*5.9%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1867**	*5.8%*	*7.9%*	*2.8%*
	Age 35	2.1668	2.1251	2.2369	**2.3018**	*6.2%*	*8.3%*	*2.9%*
	Age 45	4.1995	4.1143	4.3391	**4.4721**	*6.5%*	*8.7%*	*3.1%*
	Age 55	8.2051	8.0345	8.4670	**8.7317**	*6.4%*	*8.7%*	*3.1%*
	Age 65	12.3695	12.2340	12.8373	**13.2171**	*6.9%*	*8.0%*	*3.0%*
	Age 75	8.9093	8.9690	9.4431	**9.7232**	*9.1%*	*8.4%*	*3.0%*
	Age 85	5.3409	5.4378	5.6904	**5.8822**	*10.1%*	*8.2%*	*3.4%*

Table 11.1: Amount-Weighted Teachers

The comparisons for Safety in Table 11.2 are quite different from those of Teachers. The amount-weighted deferred annuity values for PubS-2010 turn out to be quite close to those produced using RP-2000 rates projected generationally with Scale BB. Except at the oldest male ages, the deferred annuity values under the new tables for Safety fell between those developed using the RP-2006 and RP-2006 WC tables.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubS-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubS-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.1820**	*0.7%*	*1.3%*	*-1.2%*
	Age 35	2.2720	2.2594	2.3191	**2.2919**	*0.9%*	*1.4%*	*-1.2%*
	Age 45	4.4101	4.3812	4.5028	**4.4517**	*0.9%*	*1.6%*	*-1.1%*
	Age 55	8.6263	8.5443	8.7849	**8.6740**	*0.6%*	*1.5%*	*-1.3%*
	Age 65	13.0772	12.9595	13.3331	**13.0713**	*0.0%*	*0.9%*	*-2.0%*
	Age 75	9.8517	9.6858	10.0300	**9.7245**	*-1.3%*	*0.4%*	*-3.0%*
	Age 85	6.3586	6.0423	6.2543	**6.1480**	*-3.3%*	*1.8%*	*-1.7%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1330**	*1.0%*	*3.1%*	*-1.8%*
	Age 35	2.1668	2.1251	2.2369	**2.1949**	*1.3%*	*3.3%*	*-1.9%*
	Age 45	4.1995	4.1143	4.3391	**4.2582**	*1.4%*	*3.5%*	*-1.9%*
	Age 55	8.2051	8.0345	8.4670	**8.2939**	*1.1%*	*3.2%*	*-2.0%*
	Age 65	12.3695	12.2340	12.8373	**12.4434**	*0.6%*	*1.7%*	*-3.1%*
	Age 75	8.9093	8.9690	9.4431	**8.9533**	*0.5%*	*-0.2%*	*-5.2%*
	Age 85	5.3409	5.4378	5.6904	**5.3471**	*0.1%*	*-1.7%*	*-6.0%*

Table 11.2: Amount-Weighted Safety

In Table 11.3, the amount-weighted deferred annuity values developed using PubG-2010 for female General members are greater than virtually all corresponding values developed using the previously published SOA tables. The closest match for female General members was the RP-2006 WC table, which produced deferred annuity values that were generally about 0.5% to 1.0% lower. The amount-weighted deferred annuity values developed using PubG-2010 for male General members are somewhat greater than those developed using RP-2000 and RP-2006, and somewhat lower than those developed using RP-2006 WC.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubG-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubG-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	1.1735	1.1671	1.1960	**1.2085**	*3.0%*	*3.5%*	*1.0%*
	Age 35	2.2720	2.2594	2.3191	**2.3444**	*3.2%*	*3.8%*	*1.1%*
	Age 45	4.4101	4.3812	4.5028	**4.5554**	*3.3%*	*4.0%*	*1.2%*
	Age 55	8.6263	8.5443	8.7849	**8.8853**	*3.0%*	*4.0%*	*1.1%*
	Age 65	13.0772	12.9595	13.3331	**13.4541**	*2.9%*	*3.8%*	*0.9%*
	Age 75	9.8517	9.6858	10.0300	**10.0760**	*2.3%*	*4.0%*	*0.5%*
	Age 85	6.3586	6.0423	6.2543	**6.2831**	*-1.2%*	*4.0%*	*0.5%*
Males	Age 25	1.1220	1.0994	1.1543	**1.1344**	*1.1%*	*3.2%*	*-1.7%*
	Age 35	2.1668	2.1251	2.2369	**2.1955**	*1.3%*	*3.3%*	*-1.9%*
	Age 45	4.1995	4.1143	4.3391	**4.2605**	*1.5%*	*3.6%*	*-1.8%*
	Age 55	8.2051	8.0345	8.4670	**8.3168**	*1.4%*	*3.5%*	*-1.8%*
	Age 65	12.3695	12.2340	12.8373	**12.5732**	*1.6%*	*2.8%*	*-2.1%*
	Age 75	8.9093	8.9690	9.4431	**9.1604**	*2.8%*	*2.1%*	*-3.0%*
	Age 85	5.3409	5.4378	5.6904	**5.5437**	*3.8%*	*1.9%*	*-2.6%*

Table 11.3: Amount-Weighted General

11.1.3 Comparisons of Headcount-Weighted Deferred Annuities for Nondisabled Members

Since the RP-2000 tables were developed on an amount-weighted basis only, RPEC decided to compare headcount-weighted annuity values to RPH-2006 and RPH-2006 WC (the headcount-weighted versions of RP-2006 and RP-2006 WC) and to the corresponding job category-specific amount-weighted annuity values.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubT.H-2010 (with MP-2017) from:*		
	Base Rate →	RPH-2006	RPH-2006 WC	PubT-2010	**PubT.H-2010**	RPH-2006	RPH-2006 WC	PubT-2010
	Proj. Scale →	MP-2017	MP-2017	MP-2017	**MP-2017**	MP-2017	MP-2017	MP-2017
Females	Age 25	1.1521	1.1878	1.2406	**1.2338**	*7.1%*	*3.9%*	*-0.5%*
	Age 35	2.2291	2.3025	2.4113	**2.3977**	*7.6%*	*4.1%*	*-0.6%*
	Age 45	4.3225	4.4706	4.6932	**4.6669**	*8.0%*	*4.4%*	*-0.6%*
	Age 55	8.4442	8.7294	9.1655	**9.1174**	*8.0%*	*4.4%*	*-0.5%*
	Age 65	12.8513	13.2668	13.9245	**13.8554**	*7.8%*	*4.4%*	*-0.5%*
	Age 75	9.5814	9.9470	10.5286	**10.4440**	*9.0%*	*5.0%*	*-0.8%*
	Age 85	5.9948	6.2230	6.6215	**6.5202**	*8.8%*	*4.8%*	*-1.5%*
Males	Age 25	1.0685	1.1334	1.1867	**1.1724**	*9.7%*	*3.4%*	*-1.2%*
	Age 35	2.0638	2.1951	2.3018	**2.2734**	*10.2%*	*3.6%*	*-1.2%*
	Age 45	3.9944	4.2567	4.4721	**4.4157**	*10.5%*	*3.7%*	*-1.3%*
	Age 55	7.8134	8.3133	8.7317	**8.6257**	*10.4%*	*3.8%*	*-1.2%*
	Age 65	11.9457	12.6387	13.2171	**13.0623**	*9.3%*	*3.4%*	*-1.2%*
	Age 75	8.7317	9.2887	9.7232	**9.5631**	*9.5%*	*3.0%*	*-1.6%*
	Age 85	5.3145	5.6203	5.8822	**5.7641**	*8.5%*	*2.6%*	*-2.0%*

Table 11.4: Headcount-Weighted Teachers

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubS.H-2010 (with MP-2017) from:*		
	Base Rate →	RPH-2006	RPH-2006 WC	PubS-2010	**PubS.H-2010**	RPH-2006	RPH-2006 WC	PubS-2010
	Proj. Scale →	MP-2017	MP-2017	MP-2017	**MP-2017**	MP-2017	MP-2017	MP-2017
Females	Age 25	1.1521	1.1878	1.1820	**1.1697**	*1.5%*	*-1.5%*	*-1.0%*
	Age 35	2.2291	2.3025	2.2919	**2.2664**	*1.7%*	*-1.6%*	*-1.1%*
	Age 45	4.3225	4.4706	4.4517	**4.4005**	*1.8%*	*-1.6%*	*-1.1%*
	Age 55	8.4442	8.7294	8.6740	**8.5766**	*1.6%*	*-1.7%*	*-1.1%*
	Age 65	12.8513	13.2668	13.0713	**12.9476**	*0.7%*	*-2.4%*	*-0.9%*
	Age 75	9.5814	9.9470	9.7245	**9.6649**	*0.9%*	*-2.8%*	*-0.6%*
	Age 85	5.9948	6.2230	6.1480	**6.1330**	*2.3%*	*-1.4%*	*-0.2%*
Males	Age 25	1.0685	1.1334	1.1330	**1.1074**	*3.6%*	*-2.3%*	*-2.3%*
	Age 35	2.0638	2.1951	2.1949	**2.1434**	*3.9%*	*-2.4%*	*-2.3%*
	Age 45	3.9944	4.2567	4.2582	**4.1554**	*4.0%*	*-2.4%*	*-2.4%*
	Age 55	7.8134	8.3133	8.2939	**8.0909**	*3.6%*	*-2.7%*	*-2.4%*
	Age 65	11.9457	12.6387	12.4434	**12.1368**	*1.6%*	*-4.0%*	*-2.5%*
	Age 75	8.7317	9.2887	8.9533	**8.6186**	*-1.3%*	*-7.2%*	*-3.7%*
	Age 85	5.3145	5.6203	5.3471	**5.0917**	*-4.2%*	*-9.4%*	*-4.8%*

Table 11.5: Headcount-Weighted Safety

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubG.H-2010 (with MP-2017) from:*		
	Base Rate →	RPH-2006	RPH-2006 WC	PubG-2010	**PubG.H-2010**	RPH-2006	RPH-2006 WC	PubG-2010
	Proj. Scale →	MP-2017	MP-2017	MP-2017	**MP-2017**	MP-2017	MP-2017	MP-2017
Females	Age 25	1.1521	1.1878	1.2085	**1.1932**	*3.6%*	*0.5%*	*-1.3%*
	Age 35	2.2291	2.3025	2.3444	**2.3137**	*3.8%*	*0.5%*	*-1.3%*
	Age 45	4.3225	4.4706	4.5554	**4.4949**	*4.0%*	*0.5%*	*-1.3%*
	Age 55	8.4442	8.7294	8.8853	**8.7713**	*3.9%*	*0.5%*	*-1.3%*
	Age 65	12.8513	13.2668	13.4541	**13.2898**	*3.4%*	*0.2%*	*-1.2%*
	Age 75	9.5814	9.9470	10.0760	**9.9106**	*3.4%*	*-0.4%*	*-1.6%*
	Age 85	5.9948	6.2230	6.2831	**6.1242**	*2.2%*	*-1.6%*	*-2.5%*
Males	Age 25	1.0685	1.1334	1.1344	**1.1000**	*2.9%*	*-2.9%*	*-3.0%*
	Age 35	2.0638	2.1951	2.1955	**2.1262**	*3.0%*	*-3.1%*	*-3.2%*
	Age 45	3.9944	4.2567	4.2605	**4.1230**	*3.2%*	*-3.1%*	*-3.2%*
	Age 55	7.8134	8.3133	8.3168	**8.0498**	*3.0%*	*-3.2%*	*-3.2%*
	Age 65	11.9457	12.6387	12.5732	**12.1776**	*1.9%*	*-3.6%*	*-3.1%*
	Age 75	8.7317	9.2887	9.1604	**8.8102**	*0.9%*	*-5.2%*	*-3.8%*
	Age 85	5.3145	5.6203	5.5437	**5.3131**	*0.0%*	*-5.5%*	*-4.2%*

Table 11.6: Headcount-Weighted General

As can be seen in Tables 11.4, 11.5 and 11.6, the overall impact of moving from the headcount-weighted RPH-2006 and RPH-2006 WC tables to the job-specific headcount-weighted Pub-2010 tables is similar to the patterns observed with the corresponding amount-weighted tables. Considering female Safety members, for example, switching to PubS.H-2010 from RPH-2006 produces deferred annuity values that range from 0.7% to 2.3% higher, compared to a range of 0.4% to 1.8% higher for the corresponding amount-weighted values in Table 11.2.

Broadly speaking, the difference between headcount-weighted and corresponding amount-weighted annuity values represents a measure of the dispersion of individual amounts within the population being studied and the sensitivity of mortality to differences in income. More specifically, the larger the dispersion in underlying amounts, the greater (positive) differential between the amount-weighted annuity values relative to their headcount-weighted counterparts. Therefore, the final columns in the three tables above indicate that

- The salary/benefit amount dispersion within the Teacher population (especially females) was considerably less than that for Safety and General and
- The salary/benefit amount dispersion for males is greater than that for females within all three job categories.

11.2 Above- and Below-Median Annuity Values

As discussed in Section 4, the multivariate analysis performed on the Employee, Retiree and Contingent Survivor subpopulations revealed clear evidence for variations in mortality experience based on income quartile. As a result, RPEC decided to produce additional sets of mortality rates (both amount- and headcount-weighted) based on whether the member fell into one of the two higher income quartiles[35] (above-median) or one of the two lower income quartiles (below-median).

Tables 11.7, 11.8 and 11.9 show comparisons of the amount-weighted deferred annuity values computed using the Above- and Below-Median tables to those computed using the corresponding "total dataset" table for each job category.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018			*Percentage Change from: PubT-2010 (with MP-2017):*	
	Base Rate →	**PubT-2010**	PubT-2010(A)	PubT-2010(B)	PubT-2010(A)	PubT-2010(B)
	Proj. Scale →	**MP-2017**	MP-2017	MP-2017	MP-2017	MP-2017
Females	Age 25	**1.2406**	1.2446	1.2308	*0.3%*	*-0.8%*
	Age 35	**2.4113**	2.4194	2.3915	*0.3%*	*-0.8%*
	Age 45	**4.6932**	4.7089	4.6559	*0.3%*	*-0.8%*
	Age 55	**9.1655**	9.1932	9.1046	*0.3%*	*-0.7%*
	Age 65	**13.9245**	13.9648	13.8683	*0.3%*	*-0.4%*
	Age 75	**10.5286**	10.5951	10.4552	*0.6%*	*-0.7%*
	Age 85	**6.6215**	6.7021	6.5497	*1.2%*	*-1.1%*
Males	Age 25	**1.1867**	1.1994	1.1605	*1.1%*	*-2.2%*
	Age 35	**2.3018**	2.3276	2.2485	*1.1%*	*-2.3%*
	Age 45	**4.4721**	4.5235	4.3669	*1.2%*	*-2.4%*
	Age 55	**8.7317**	8.8302	8.5395	*1.1%*	*-2.2%*
	Age 65	**13.2171**	13.3754	12.9797	*1.2%*	*-1.8%*
	Age 75	**9.7232**	9.9124	9.5137	*1.9%*	*-2.2%*
	Age 85	**5.8822**	6.0521	5.7739	*2.9%*	*-1.8%*

Table 11.7: Amount-Weighted Teachers: Above- and Below-Median

[35] See Subsection 3.8 for a description of how income quartiles were calculated.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018			*Percentage Change from: PubS-2010 (with MP-2017):*	
	Base Rate →	**PubS-2010**	PubS-2010(A)	PubS-2010(B)	PubS-2010(A)	PubS-2010(B)
	Proj. Scale →	**MP-2017**	MP-2017	MP-2017	MP-2017	MP-2017
Females	Age 25	**1.1820**	1.1953	1.1614	*1.1%*	*-1.7%*
	Age 35	**2.2919**	2.3187	2.2512	*1.2%*	*-1.8%*
	Age 45	**4.4517**	4.5042	4.3752	*1.2%*	*-1.7%*
	Age 55	**8.6740**	8.7728	8.5415	*1.1%*	*-1.5%*
	Age 65	**13.0713**	13.2262	12.8928	*1.2%*	*-1.4%*
	Age 75	**9.7245**	9.9062	9.5168	*1.9%*	*-2.1%*
	Age 85	**6.1480**	6.3092	5.9636	*2.6%*	*-3.0%*
Males	Age 25	**1.1330**	1.1651	1.0908	*2.8%*	*-3.7%*
	Age 35	**2.1949**	2.2597	2.1099	*3.0%*	*-3.9%*
	Age 45	**4.2582**	4.3878	4.0890	*3.0%*	*-4.0%*
	Age 55	**8.2939**	8.5484	7.9681	*3.1%*	*-3.9%*
	Age 65	**12.4434**	12.8692	11.9777	*3.4%*	*-3.7%*
	Age 75	**8.9533**	9.4598	8.5673	*5.7%*	*-4.3%*
	Age 85	**5.3471**	5.8116	5.1526	*8.7%*	*-3.6%*

Table 11.8: Amount-Weighted Safety: Above- and Below-Median

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018			*Percentage Change from: PubG-2010 (with MP-2017):*	
	Base Rate →	**PubG-2010**	PubG-2010(A)	PubG-2010(B)	PubG-2010(A)	PubG-2010(B)
	Proj. Scale →	**MP-2017**	MP-2017	MP-2017	MP-2017	MP-2017
Females	Age 25	**1.2085**	1.2151	1.1871	*0.5%*	*-1.8%*
	Age 35	**2.3444**	2.3578	2.3004	*0.6%*	*-1.9%*
	Age 45	**4.5554**	4.5818	4.4683	*0.6%*	*-1.9%*
	Age 55	**8.8853**	8.9339	8.7226	*0.5%*	*-1.8%*
	Age 65	**13.4541**	13.5250	13.2388	*0.5%*	*-1.6%*
	Age 75	**10.0760**	10.1620	9.8548	*0.9%*	*-2.2%*
	Age 85	**6.2831**	6.3632	6.1196	*1.3%*	*-2.6%*
Males	Age 25	**1.1344**	1.1549	1.0689	*1.8%*	*-5.8%*
	Age 35	**2.1955**	2.2370	2.0624	*1.9%*	*-6.1%*
	Age 45	**4.2605**	4.3427	3.9953	*1.9%*	*-6.2%*
	Age 55	**8.3168**	8.4732	7.8093	*1.9%*	*-6.1%*
	Age 65	**12.5732**	12.8090	11.8695	*1.9%*	*-5.6%*
	Age 75	**9.1604**	9.4047	8.5582	*2.7%*	*-6.6%*
	Age 85	**5.5437**	5.7352	5.1997	*3.5%*	*-6.2%*

Table 11.9: Amount-Weighted General: Above- and Below-Median

Consistent with the concept of "amount dispersion" discussed in Subsection 11.1.3, the impact of income (i.e., moving from the total dataset table to either the Above- or Below-Median tables) is considerably smaller for Teachers (especially females) than for Safety or General members. And, once again, the impact for males in each of the three job categories is considerably larger than that for females, most dramatically for General members.

11.3 Disabled Retiree Annuity Values

The following tables show comparisons of immediate annuity factors at 5.0% interest developed using Disabled Retiree rates for the Safety (Table 11.10) and Non-Safety (Table 11.11)[36] job categories to those computed using previously published mortality tables.

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving To PubS-2010 Disabled (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006	PubS-2010	RP-2000	RP-2006	RP-2006
	Proj. Scale →	BB	MP-2017	MP-2017	MP-2017	BB	MP-2017	MP-2017
	Health Status →	Disabled	Disabled	Healthy	Disabled	Disabled	Disabled	Healthy
Females	Age 55	13.3968	13.1372	15.4328	15.2247	*13.6%*	*15.9%*	*-1.3%*
Females	Age 65	11.2428	11.0030	12.9595	12.8394	*14.2%*	*16.7%*	*-0.9%*
Females	Age 75	8.6290	8.1069	9.6858	9.7013	*12.4%*	*19.7%*	*0.2%*
Females	Age 85	6.0040	5.2344	6.0423	6.1480	*2.4%*	*17.5%*	*1.8%*
Males	Age 55	11.1623	11.8324	14.7912	14.7858	*32.5%*	*25.0%*	*0.0%*
Males	Age 65	9.6464	9.8049	12.2340	12.0854	*25.3%*	*23.3%*	*-1.2%*
Males	Age 75	7.3500	7.2871	8.9690	8.7503	*19.1%*	*20.1%*	*-2.4%*
Males	Age 85	5.0599	4.5914	5.4378	5.3433	*5.6%*	*16.4%*	*-1.7%*

Table 11.10: Amount-Weighted Safety Disabled Retirees

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018			*Percentage Change of Moving To PubNS-2010 (with MP-2017) from:*	
	Base Rate →	RP-2000	RP-2006	PubNS-2010	RP-2000	RP-2006
	Proj. Scale →	BB	MP-2017	MP-2017	BB	MP-2017
	Health Status →	Disabled	Disabled	Disabled	Disabled	Disabled
Females	Age 55	13.3968	13.1372	13.1663	*-1.7%*	*0.2%*
Females	Age 65	11.2428	11.0030	11.3616	*1.1%*	*3.3%*
Females	Age 75	8.6290	8.1069	8.5479	*-0.9%*	*5.4%*
Females	Age 85	6.0040	5.2344	5.5744	*-7.2%*	*6.5%*
Males	Age 55	11.1623	11.8324	12.3043	*10.2%*	*4.0%*
Males	Age 65	9.6464	9.8049	10.3605	*7.4%*	*5.7%*
Males	Age 75	7.3500	7.2871	7.9033	*7.5%*	*8.5%*
Males	Age 85	5.0599	4.5914	5.1327	*1.4%*	*11.8%*

Table 11.11: Amount-Weighted Non-Safety Disabled Retirees

As explained in Sections 4 and 8, generally less restrictive definitions of disability for Safety workers lead to abnormally lower rates of mortality in that job category for Disabled Retirees than what is typically expected in most other lines of work. This explains the much higher annuity factors produced by the Safety Disabled Retiree table compared to those computed with previous SOA disability tables. As illustrated in Table 11.10, the annuity factors for the PubS-2010 Disabled Retiree table are much closer to (but lower than) those for the PubS-2010 Retiree table than those for previous disabled mortality tables.

[36] The Pub-2010 table for Non-Safety Disabled Retirees is denoted PubNS-2010 in Table 11.11.

For Non-Safety Disabled Retirees, the annuity factors for females range from a 1.1% decrease relative to RP-2006 at age 55 to a 6.5% increase at age 85. The male annuity factors show a greater increase over RP-2006 than the females, ranging from 2.4% at age 55 to 11.8% at age 85.

11.4 Contingent Survivor Annuity Values

As discussed previously, the annuity factors above for the Pub-2010 tables were developed using Employee mortality up until age 62 and Retiree mortality at ages 62 and above. The RP-2000 tables and RP-2006 tables combined Retiree and Contingent Survivor experience into a "Healthy Annuitant" table. This means that for ages 62 and up, the above nondisabled exhibits effectively compare Retiree-only mortality for the Pub-2010 tables to a combination of Retiree and Contingent Survivor mortality for the older tables.

Table 11.12 shows a comparison at a 5.0% (post-retirement) interest rate of immediate annuity factors using the Contingent Survivor mortality rates (denoted PubCS-2010) to those generated by prior tables. A discussion of the application of the Contingent Survivor tables can be found in Subsection 12.4.

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubCS-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubCS-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 55	15.6172	15.4328	15.7820	**15.4698**	*-0.9%*	*0.2%*	*-2.0%*
	Age 65	13.0772	12.9595	13.3331	**13.1534**	*0.6%*	*1.5%*	*-1.3%*
	Age 75	9.8517	9.6858	10.0300	**9.9546**	*1.0%*	*2.8%*	*-0.8%*
	Age 85	6.3586	6.0423	6.2543	**6.2942**	*-1.0%*	*4.2%*	*0.6%*
Males	Age 55	15.0143	14.7912	15.3887	**14.4426**	*-3.8%*	*-2.4%*	*-6.1%*
	Age 65	12.3695	12.2340	12.8373	**11.9953**	*-3.0%*	*-2.0%*	*-6.6%*
	Age 75	8.9093	8.9690	9.4431	**8.8766**	*-0.4%*	*-1.0%*	*-6.0%*
	Age 85	5.3409	5.4378	5.6904	**5.5602**	*4.1%*	*2.3%*	*-2.3%*

Table 11.12: Amount-Weighted Contingent Survivors

Section 12: Application of Tables

12.1 "As-of Date" of the Pub-2010 Tables

After accounting for data exclusions, RPEC determined that the central year of the study's observation period was approximately the 12-month period from July 1, 2010, through June 30, 2011. Therefore, the mortality rates for each age, *x*, in the Pub-2010 tables should be interpreted as one-year probabilities of death for a person exactly age *x* on July 1, 2010.

The study's "as-of date" of July 1, 2010, notwithstanding, RPEC believes that the Pub-2010 tables could represent reasonable benchmarks for mortality rates for any date within calendar year 2010. However, some users might believe that an adjustment of the Pub-2010 rates would be more appropriate for calculations with as-of dates other than July 1. In that case, practitioners could consider using the appropriate fraction of the calendar 2010 mortality improvement rates from the same mortality improvement scale assumed for the projection of future mortality rates. For example, if the assumed age-65 mortality improvement rate in 2010 was 1.2%, then an actuary performing a measurement as of April 1 could consider first increasing the age-65 Pub-2010 rate by 0.3% before projecting that rate into the future.

12.2 Selecting Appropriate Benchmark Mortality Tables

The potential uses of the Pub-2010 tables as published depend to a certain extent on the size and credibility of the underlying population to which the tables would be applied. For example, very large public retirement systems that have performed recent mortality experience studies might compare those results to one or more of the Pub-2010 tables, or blends/adjustments thereof, taking job categories and pay/benefit amount into consideration. Covered populations that are not large enough to support fully credible mortality results might use suitably selected Pub-2010 tables as "benchmark" starting points; i.e., tables that in conjunction with a recent mortality experience study could be used with appropriate adjustments, or as reference tables for credibility-weighted blended mortality rates.

As detailed in Section 4, the multivariate analyses performed on nondisabled members revealed that job category and income quartile were both statistically significant covariates for virtually all status groups. Therefore, consistent with the principles of ASOP 35, actuaries are encouraged to take such characteristics of the covered population into consideration when selecting appropriate benchmark mortality tables. Given the high statistical significance of income as a predictor of mortality, as well as the absence of separate tables reflecting geographic region,[37] the Above-Median or Below-Median tables developed in this report should be considered as an alternative to the corresponding "total population" table, whenever appropriate.

In those situations where the covered population includes more than one job category and/or income level,[38] to the extent practical, the actuary could consider using a number of separate tables for

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[37] See Subsection 13.2.

[38] It should also be noted that even in public-sector retirement plans that cover members in a single job category, different member subgroups can often have disparate income profiles and possibly be covered by a separate set of plan provisions. It would not be unusual, for example, for a plan covering General members to include both judges and low-paid administrative staff.

subgroups stratified into appropriate job/income subpopulations or develop custom tables using appropriately weighted combinations of those tables.[39]

12.3 Comments on Disabled Retiree Tables

Developing reliable mortality rates for Disabled Retirees has always presented special challenges to those tasked with the construction of mortality tables for retirement plans. These challenges include the following:

- Issues with the accurate tracking of those who initially retire under a disability retirement provision but are automatically reclassified as a Retiree upon attainment of some fixed age. See the discussion of "Unclear Disability Status" in Subsection 3.4, for example.

- The more subjective nature of disability retirement eligibility criteria compared to other (nondisabled) retirement provisions. This issue was particularly significant in the current study due to the disparity between Safety and non-Safety members addressed in Subsections 4.6 and 8.2.

In accordance with ASOP 35, actuaries should use professional judgment when applying any of the Disabled Retiree mortality tables developed in this report, especially when the plan's disability retirement provisions—particularly the eligibility criteria—are known to be particularly strict or broad.

12.4 Comments on Contingent Survivor Tables

12.4.1 Contingent Survivor Mortality Rates

The plan-level data questionnaire included an item allowing contributors to indicate whether beneficiary mortality experience was reliably tracked prior to the death of the primary member. The vast majority of contributors indicated that such information was not maintained in their database and could not reliably measure deaths for beneficiaries who predecease the primary member. As a result, a minimal number of deaths were collected for beneficiaries that had yet to receive annuity payments, and the Committee elected not to study mortality experience for this group. Therefore, the Contingent Survivor database for this study only contains life-years of exposure for Contingent Survivors after the death of the primary member.

Because the Pub-2010 Contingent Survivor tables were constructed using experience specifically from designated beneficiaries who had survived deceased plan members, these rates could be appropriate for application to current Contingent Survivors within plan populations. Above- and Below-Median versions can help practitioners tailor their mortality assumption to the applicable populations.

[39] See Subsection 13.1 for RPEC's rationale for not developing "combined" public plan tables.

12.4.2 Joint-and-Survivor Annuities

The availability of distinct tables for Retirees and Contingent Survivors raises the question of which set of mortality rates might be used for designated beneficiaries in the calculation of joint-and-survivor annuities. There are several possibilities including, but not limited to, the three discussed below:

1. One approach ("Approach 1") would be to assume the same mortality basis as the Retiree, except using the rates applicable to the beneficiary's gender. It should be noted that Pub-2010 Retiree experience includes many members with joint-and-survivor annuities, and presumably additional members with spouses/partners not designated under joint-and-survivor options. Over the years, a percentage of these Retirees will lose a spouse/partner, and any grieving widow(er) effect would be reflected in the Retiree experience. On average, the Retiree rates may contain a reasonable provision for this impact. In addition, the socioeconomic status correlated to job category, and above- or below-median designation within job category, could well be relevant in predicting beneficiary mortality.

2. Another approach ("Approach 2") would use the Retiree basis (with beneficiary gender, as in Approach 1 above) while the Retiree is alive, but utilize Contingent Survivor mortality rates after the death of the Retiree. The rationale would be that portions of the present value calculation that specifically address the beneficiary's mortality while a Contingent Survivor of the deceased member might be appropriately modeled by the Contingent Survivor rates. This approach, in which the applicable beneficiary mortality rates (Retiree or Contingent Survivor) depend on whether or not the primary retiree is alive, may not be easy to implement in the typical valuation software in use today.

3. A third approach ("Approach 3") would be to assume Contingent Survivor mortality rates for the beneficiary both before and after the death of the original member. It is possible that the Contingent Survivor mortality experience in Pub-2010 shows higher mortality due to a number of factors correlated with beneficiary status, apart from a grieving widow(er) effect. In that case, Contingent Survivor mortality might be appropriate both before and after the death of the original member.

Comparisons of joint-and-100%-survivor annuity values calculated using Approach 2[40] to those developed using previously published SOA tables can be found in Appendix D.2. Comparisons of the Approach 2 joint-and-100%-survivor annuity values to those calculated using Approaches 1 and 3 are shown in Appendix D.3. Although the Approach 2 values generally fall between the corresponding Approach 1 and Approach 3 values, the magnitude and direction of the differences among the three approaches will vary by job category and the ages of the joint annuitants.

Per ASOP 35, the selection or development of beneficiary mortality rates should reflect the actuary's judgement, consider the intended purpose and incorporate actual plan experience to the extent it is

[40] The Committee would like to make it clear that using Approach 2 for the basis for these comparisons does not represent a recommendation or preference.

deemed credible (per Actuarial Standard of Practice No. 25, *Credibility Procedures* [ASB 2013]) and predictive.

12.5 Amount-Weighted and Headcount-Weighted Tables

The reason for using a weighted version of a mortality table—either amount-weighted or headcount-weighted—is to obtain the most appropriate result for the particular application at hand. For the measurement of most pension obligations, tables weighted by amount (salary for active employees and benefit amount for those in payment status) generally produce the most appropriate results. On the other hand, headcount-weighted tables might be more appropriate for applications such as the measurement of obligations for retirement programs with benefit structures uncorrelated with income, such as many retiree medical or retiree life insurance programs.

As a consequence, this report includes both amount-weighted and headcount-weighted versions of each of the public plan mortality tables. Per ASOP 35, the actuary should select a mortality assumption that is appropriate for the purpose of the measurement. Therefore, it would not necessarily be inappropriate—or inconsistent—to use amount-weighted tables to measure pension obligations and the corresponding headcount-weighted tables to measure most postretirement medical obligations, even when the two covered populations are identical.

12.6 Projection of Mortality Rates beyond 2010

The Committee believes that for most pension-related actuarial applications, the Pub-2010 mortality rates (including those for Disabled Retirees) should be projected with an appropriate mortality improvement scale, and that generational projection should be considered as an approach to projecting future mortality rates. In all cases, the selection of a mortality improvement assumption must satisfy the applicable requirements of ASOP 35.

Section 13: Observations and Other Considerations

13.1 Rationale for No "Combined" Public Table

The Committee did consider publishing a "combined" public plans table that included all of the data received for the study from each of the three job categories. Ultimately, it was decided that this would not be done. In addition to the statistically significant differences in mortality by job category discussed in Section 4, it was determined that "combined" rates at various ages were often more reflective of the relative concentrations of the component subpopulations (Teachers, Safety, General) than of underlying mortality characteristics. The covered populations in many public retirement plans have demographic characteristics (including job category) that are quite different than that of the population that would have been used to develop any combined Pub-2010 table. Therefore, it would be better for the actuary with knowledge of the specific member demographics either to segregate the populations and use appropriate tables for each, or to construct a custom combined table using appropriate weighted averages of the job category and Above- or Below-Median rates from this study.

Last, many public-sector retirement programs specifically cover Teachers, Safety or General employee populations. Even those that cover multiple populations often provide different benefit features and track census data separately by job categories.

13.2 Comments on Geographic Region

Data were collected from public pension systems from across the country, which allowed the Committee an opportunity to investigate whether geographic region is an effective predictor of relative mortality experience. As explained in Section 4, for some subsets of the data, it did appear as though there were significant variations by geography. However, the predictive value of geographic region was limited, and it was not used in the development of mortality tables because of two main factors:

1. Results from the multivariate analysis showed that the explanatory power of geography was considerably lower than that of job category or amount-based quartile. For Employees, the explanatory power was typically one-third to one-tenth that of salary quartile. For Nondisabled Annuitants, the explanatory power was less than one-half that of job category, which in turn was less than one-half that of benefit quartile. Therefore, any attempt to construct mortality tables from broad geographic regions would have resulted in mortality rates that were more reflective of the differences in distribution of job categories and amount-based quartiles than between regions in the study dataset.[41]

2. The data submitted for the study were not uniform across broad geographies. Some observed differences between geographic regions are potentially more a result of the locations of specific submitters than any systematic longevity differences by region. In three of the four regions, for

[41] RPEC considered constructing additional mortality tables by geographic region within each job category. However, this would have resulted in a number of "gender/region/job" datasets with very low credibility. Furthermore, several of the subpopulations would have been heavily concentrated in one or two plans. Therefore, no such tables were developed.

example, approximately two-thirds of the Retiree data came from public pension systems located within the same state.

13.3 Comments on "Bump" in Young Employee Rates

Most Employee tables show a decline in mortality rates with advancing age at some point between the ages of 19 and 25. As described in Section 7, mortality rates in this age range were developed by fitting a cubic polynomial to the juvenile rates at ages 16 and 17 and the graduated rates at ages 25 and 26. Per Section 9, the mortality rates at ages 16 and 17 were computed using the mortality improvement-adjusted 2015 VBT rates at those ages, adjusted using a ratio of the mortality predicted by the graduated public plan rates from ages 25–34 to that predicted by the corresponding 2015 VBT rates.

The 2015 VBT also exhibited a period of declining mortality at young employment ages, so this pattern is not inconsistent with that found in other studies [SOA 2015].

13.4 Rationale for Above- and Below-Median Tables

Income quartile was found to be a significant variable in the RP-2014 study, and tables were created for top-quartile and bottom-quartile groups (within gender). Practitioners have reported difficulty in using quartile-based tables, except in limited situations such as supplemental plans that provide benefits in excess of qualified plan limits and are clearly high-paid, top-quartile members, or alternatively for low-paid groups that are easily identifiable as bottom-quartile.

The multivariate analysis completed as part of this study seemed to indicate a significant difference between below-median (quartiles Q1 and Q2) and above-median (Q3 and Q4) mortality. Differences between each of the bottom two quartiles (Q1 and Q2) were less stable in models that contained more explanatory variables, but differences between above- and below-median remained strong. Given these observations and the potentially greater ease of application, Above-Median and Below-Median tables were constructed for this study rather than top-quartile and bottom-quartile tables.

13.5 Tabulation of Exposures and Deaths

Exposures and deaths were tabulated using age-nearest-birthday as of January 1 of each calendar year in the study's observation period. For example, a retiree exact-age 69.75 on January 1, 2010, who remained alive throughout 2010, would have been credited with a full year of age-70 exposure for 2010. If that retiree had died at any time during 2010, he or she would have been treated as an age-70 death, even if the retiree was older than age 70.5 at the time of death (i.e., the death was after October 1, 2010).

These tabulation rules, in conjunction with the study's central year, produced raw one-year mortality probabilities as of July 1, 2010.

13.6 Application of Mortality Improvement Rates

A central study year consisting of portions of two consecutive calendar years raises the question of how a mortality projection scale with improvement factors described in terms of age and calendar year (such as MP-2017) might be applied to the PubT-2010 base rates. The base mortality rates and annuity factors displayed in this report are shown as of July 1, 2018. To compute the mortality rates underpinning these amounts, RPEC applied full years of mortality improvement prospectively from 2010 rather than a fractional blend of calendar-year improvement rates. For example, the age-65 male Teachers rate applicable in calendar year 2018 was calculated by applying the cumulative 2011–2018 age-65 male mortality improvement rates from MP-2017 to the base mortality rate for an age-65 male Teacher from the PubT-2010 table. The Committee believes that this is the most common approach among pension practitioners, but that other approaches might also be reasonable.

For purposes of comparing mortality rates and annuity values in this report, the RP-2000 and RP-2006 rates were projected using an integral number of calendar years of mortality improvement from 2000 and 2006, respectively.

13.7 Age-65 Life Expectancy Comparison

Table 13.1 presents a comparison of 2018 complete cohort life expectancy values at age 65. These values are based on the headcount-weighted Pub-2010 tables and the headcount-weighted RPH-2006 tables.

	Age-65 Cohort Life Expectancies (Complete) Generational @ July 1, 2018				*Percentage Change of Moving from RPH-2006 (with MP-2017) to:*		
Base Rate →	**RPH-2006**	PubT.H-2010	PubS.H-2010	PubG.H-2010	PubT.H-2010	PubS.H-2010	PubG.H-2010
Proj. Scale →	**MP-2017**	MP-2017	MP-2017	MP-2017	MP-2017	MP-2017	MP-2017
Females	**22.40**	25.03	22.68	23.48	*11.7%*	*1.3%*	*4.8%*
Males	**20.01**	22.70	20.27	20.49	*13.4%*	*1.3%*	*2.4%*

Table 13.1: Age-65 Complete Cohort Life Expectancies

Table 13.1 indicates that of the three job categories, Teachers have the longest age-65 life expectancy by a substantial margin, followed by General and then Public Safety. Appendix D.4 contains additional life expectancy comparisons for ages other than 65.

Section 14: Reliance and Limitations

The Pub-2010 Public Retirement Plans Mortality Tables released in conjunction with this report have been developed from public pension mortality experience in the United States and are intended for use in connection with actuarial applications related to public-sector retirement programs. No assessment has been made concerning the applicability of these tables to other purposes.

Appendixes

Appendix A: Reconciliation of Excluded Data

Table A.1 summarizes the amount of data received for the study and the amount of data that was excluded from the final dataset. Approximately 90% of all the data submitted was included in the development of Pub-2010 Mortality Tables, with the largest component of excluded data attributable to Employee groups with low actual-to-expected death ratios that could not be confirmed by the contributor.

		Life-Years of Exposure (in thousands)				
		Employees	Healthy Retirees	Contingent Survivors	Disabled Retirees	Total
(a)	**Total Beginning Exposures**	**30,375**	**17,745**	**1,898**	**1,451**	**51,469**
(b)	Estimated exposures for plans with missing critical information or data provided in an unusable format	403	191	20	9	623
(c)	Estimated exposures for months with anamolous death counts that could not be confirmed by the contributor	540	240	33	60	873
(d)	**Exposures before in-depth A/E analysis**	**29,432**	**17,314**	**1,845**	**1,382**	**49,973**
(e)	Exposures for data subgroups with suspiciously low A/E ratios that could not be confirmed by the contributor	2,063	4	-	8	2,075
(f)	2013 Employees data deletion	154	-	-	-	154
(g)	Exposures with ages outside of age ranges used in final graduation	1,058	278	130	74	1,540
(h)	**Exposures in Final Dataset**	**26,157**	**17,032**	**1,715**	**1,300**	**46,204**

Table A.1: Reconciliation of Excluded Data

Below is a more detailed description of the intermediate line items in Table A.1:

(b) Estimated exposures for plans with missing critical information or data provided in an unusable format

This includes estimated life-years for the unusable submissions described in Subsection 3.3. As the data was not processed, the life-year counts were estimated from the plans' publicly available valuation reports.

(c) Estimated exposures for months with anomalous death counts that could not be confirmed by the contributor

These counts represent the estimated amount of data excluded for time periods for which a plan had an unusual pattern of month-by-month death counts that was not confirmed by the contributor, as detailed in Subsection 3.5. As these exclusions took place before exposure was calculated, these were estimated based on a given plan's total included exposure and the fraction of the study period that was excluded for that plan.

(e) Exposures for data subgroups with suspiciously low A/E ratios that could not be confirmed by the contributor

As discussed in Subsection 3.6, some subgroups within certain plans exhibited abnormally low mortality experience that could not be confirmed by the contributor, indicative of issues with the reliability of death tracking.

(f) 2013 Employees Data Deletion

The data for a small number of plans showed an unreasonably large drop in mortality for Employees in 2013, the final year of the study. These observations were anomalous far beyond that which could be explained by simple year-to-year variance and seemed to reflect a potential reporting lag in categorizing the most recent employee terminations as deaths.

(g) Exposures with ages outside of age ranges used in final graduation

As described in Sections 6 through 8, graduations of the raw data were performed for specific age ranges within each status/job category/gender subset of the total database. Those ranges were determined based on the ages for which a sufficiently robust amount of data was provided. This effectively excluded a small amount of life-years of data for relatively very old or young members within each subset, which are reflected in this row of the table.

Appendix B: Summary of Final Datasets

The tables in this appendix summarize the exposures, deaths and resulting raw death rates upon which the Pub-2010 Mortality Tables were constructed. The data in these tables (and in Table 3.1) reflect the data ultimately used in the graduations described in Sections 6–8; additional life-years of data were processed that fell outside of the graduation age ranges. The data reconciliation in Appendix A shows the small amount of data that was processed but not included in the graduations due to age.

Gender-specific tables are shown separately for each member subgroup: Employee, Healthy Retiree, Disabled Retiree and Contingent Survivor. The exposure sums (by age band, job category or income grouping) might not match the total because of rounding.

Table Key

	Table Number	
	Females	Males
Employees		
• Teachers	1	2
• Safety	3	4
• General	5	6
Retirees		
• Teachers	7	8
• Safety	9	10
• General	11	12
Disabled Retirees		
• Safety	13	14
• Non-Safety	15	16
Contingent Survivors	17	18

Summary of Teachers Female Employee Dataset

	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25-29	664,018	72	662,252	69	28,207,108	2,687	0.000108	0.000104	0.000095
30-34	771,877	144	766,947	142	40,334,353	6,646	0.000187	0.000185	0.000165
35-39	737,075	197	733,819	197	43,063,103	10,996	0.000267	0.000268	0.000255
40-44	718,187	280	715,987	276	43,247,181	14,800	0.000390	0.000385	0.000342
45-49	674,479	447	672,385	438	41,369,711	24,338	0.000663	0.000651	0.000588
50-54	715,962	686	713,834	679	46,227,254	40,452	0.000958	0.000951	0.000875
55-59	694,639	927	690,157	925	47,644,460	58,223	0.001335	0.001340	0.001222
60-64	359,442	753	355,258	744	24,494,913	46,085	0.002095	0.002094	0.001881
65-69	76,381	257	75,325	253	4,749,139	15,165	0.003365	0.003359	0.003193
70-75	16,920	116	16,746	115	881,551	5,958	0.006856	0.006867	0.006759
TOTAL	5,428,981	3,879	5,402,712	3,838	320,218,773	225,350			
Above Median			2,865,761	2,130	224,815,459	169,170			
Below Median			2,536,951	1,708	95,403,313	56,181			

Table B.1

Summary of Teachers Male Employee Dataset

	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25-29	205,169	55	204,547	54	8,484,708	1,492	0.000268	0.000264	0.000176
30-34	276,331	85	274,655	82	14,977,653	3,997	0.000308	0.000299	0.000267
35-39	296,443	117	295,405	114	18,941,734	6,276	0.000395	0.000386	0.000331
40-44	285,005	158	284,315	154	19,646,716	9,367	0.000554	0.000542	0.000477
45-49	245,844	228	245,235	227	17,380,208	13,948	0.000927	0.000926	0.000803
50-54	244,791	388	244,135	383	17,846,081	25,007	0.001585	0.001569	0.001401
55-59	239,505	532	237,993	530	17,856,735	35,604	0.002221	0.002227	0.001994
60-64	141,567	487	140,426	481	10,187,843	31,447	0.003440	0.003425	0.003087
65-69	40,774	234	40,394	232	2,601,546	13,298	0.005739	0.005743	0.005112
70-75	12,424	120	12,309	118	654,528	5,583	0.009659	0.009587	0.008530
TOTAL	1,987,854	2,404	1,979,413	2,375	128,577,752	146,018			
Above Median			1,048,395	1,229	90,391,872	106,852			
Below Median			931,019	1,146	38,185,879	39,167			

Table B.2

Summary of Public Safety Female Employee Dataset

Age Band	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25-29	41,553	6	41,454	6	2,161,355	289	0.000144	0.000145	0.000134
30-34	55,911	19	55,847	19	3,420,355	1,149	0.000340	0.000340	0.000336
35-39	62,839	38	62,804	38	4,232,386	2,305	0.000605	0.000605	0.000544
40-44	66,965	41	66,925	41	4,843,951	2,652	0.000612	0.000613	0.000548
45-49	61,113	49	61,076	49	4,527,943	3,294	0.000802	0.000802	0.000728
50-54	42,551	44	42,505	44	3,042,832	2,655	0.001034	0.001035	0.000872
55-59	22,732	39	22,685	39	1,588,303	2,191	0.001716	0.001719	0.001379
60-65	10,390	23	10,381	23	729,296	1,529	0.002214	0.002215	0.002097
TOTAL	364,054	259	363,678	259	24,546,421	16,064			

	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths
Above Median	191,095	115	16,996,704	10,068
Below Median	172,582	144	7,549,717	5,996

Table B.3

Summary of Public Safety Male Employee Dataset

Age Band	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25-29	188,476	89	188,266	89	11,010,888	4,570	0.000472	0.000473	0.000415
30-34	243,497	124	243,363	124	17,011,639	7,921	0.000509	0.000510	0.000466
35-39	289,259	161	289,159	160	22,343,259	10,070	0.000557	0.000553	0.000451
40-44	314,400	239	314,275	239	26,118,278	17,877	0.000760	0.000760	0.000684
45-49	267,577	280	267,370	280	23,142,426	21,724	0.001046	0.001047	0.000939
50-54	170,966	280	170,794	280	14,657,743	21,364	0.001638	0.001639	0.001458
55-59	84,147	176	84,059	176	6,938,531	13,271	0.002092	0.002094	0.001913
60-65	33,896	123	33,846	123	2,631,410	8,086	0.003629	0.003634	0.003073
TOTAL	1,592,218	1,472	1,591,131	1,471	123,854,173	104,884			

	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths
Above Median	830,593	641	83,581,632	63,634
Below Median	760,538	830	40,272,542	41,250

Table B.4

Summary of General Female Employee Dataset

Age Band	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25–29	724,354	115	715,247	114	22,635,231	2,704	0.000159	0.000159	0.000119
30–34	892,070	198	885,171	197	33,142,920	5,878	0.000222	0.000223	0.000177
35–39	1,046,821	340	1,041,420	339	40,846,684	10,926	0.000325	0.000326	0.000267
40–44	1,341,288	685	1,336,574	681	52,914,975	23,701	0.000511	0.000510	0.000448
45–49	1,690,287	1,305	1,685,928	1,300	67,843,497	45,298	0.000772	0.000771	0.000668
50–54	1,879,310	2,198	1,874,691	2,184	78,112,346	77,353	0.001170	0.001165	0.000990
55–59	1,574,159	2,573	1,570,037	2,561	66,519,874	94,115	0.001635	0.001631	0.001415
60–64	871,032	2,081	866,972	2,067	35,528,819	75,262	0.002389	0.002384	0.002118
65–69	242,736	955	241,280	945	8,770,899	31,709	0.003934	0.003917	0.003615
70–74	70,102	420	69,469	415	2,026,362	11,087	0.005991	0.005974	0.005471
75–80	25,630	272	25,254	260	601,241	5,777	0.010612	0.010296	0.009609
TOTAL	10,357,789	11,142	10,312,043	11,063	408,942,848	383,809			
Above Median			5,493,829	4,869	310,210,083	265,985			
Below Median			4,818,214	6,194	98,732,766	117,824			

Table B.5

Summary of General Male Employee Dataset

Age Band	Number		Number with Amount		Annual Salary Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
25–29	462,944	192	455,839	192	15,941,949	5,679	0.000415	0.000421	0.000356
30–34	558,433	288	553,701	284	24,469,182	9,897	0.000516	0.000513	0.000404
35–39	641,125	412	637,961	406	32,043,432	15,964	0.000643	0.000636	0.000498
40–44	803,464	730	800,825	728	43,259,142	32,145	0.000909	0.000909	0.000743
45–49	997,523	1,426	995,297	1,421	55,985,091	67,523	0.001430	0.001428	0.001206
50–54	1,115,416	2,370	1,112,995	2,364	63,960,873	118,050	0.002125	0.002124	0.001846
55–59	973,906	2,915	971,368	2,897	55,547,556	137,931	0.002993	0.002982	0.002483
60–64	585,326	2,500	582,896	2,482	32,128,465	117,468	0.004271	0.004258	0.003656
65–69	193,252	1,160	192,089	1,148	9,236,413	47,344	0.006003	0.005976	0.005126
70–74	67,503	650	66,835	636	2,490,779	20,905	0.009629	0.009516	0.008393
75–80	27,476	424	27,072	415	798,715	10,155	0.015432	0.015330	0.012714
TOTAL	6,426,369	13,067	6,396,878	12,973	335,861,596	583,063			
Above Median			3,474,003	5,496	250,498,987	386,581			
Below Median			2,922,875	7,477	85,362,609	196,481			

Table B.6

Summary of Teachers Female Healthy Retiree Dataset

Age Band	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
55-59	225,147	658	209,176	615	7,957,092	19,754	0.002923	0.002940	0.002483
60-64	680,745	2,480	644,169	2,369	25,598,038	89,968	0.003643	0.003678	0.003515
65-69	659,317	3,645	634,986	3,527	22,704,530	120,922	0.005528	0.005554	0.005326
70-74	442,637	4,591	426,469	4,442	13,347,668	132,756	0.010372	0.010416	0.009946
75-79	320,931	6,333	309,949	6,135	8,403,754	160,491	0.019733	0.019794	0.019097
80-84	250,762	9,292	241,851	8,969	5,671,941	205,057	0.037055	0.037085	0.036153
85-89	162,111	11,466	156,690	11,089	3,037,180	207,479	0.070729	0.070770	0.068313
90-95	96,387	12,955	93,153	12,508	1,486,637	192,274	0.134406	0.134273	0.129335
TOTAL	2,838,037	51,420	2,716,443	49,654	88,206,840	1,128,701			
Above Median			1,369,899	15,003	69,079,752	673,050			
Below Median			1,346,544	34,651	19,127,088	455,651			

Table B.7

Summary of Teachers Male Healthy Retiree Dataset

Age Band	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
55-59	95,473	350	88,486	334	3,915,047	11,674	0.003666	0.003775	0.002982
60-64	332,313	1,686	313,365	1,617	15,070,437	68,849	0.005074	0.005160	0.004568
65-69	358,155	2,878	343,361	2,753	15,285,102	111,775	0.008036	0.008018	0.007313
70-74	264,393	3,966	254,051	3,808	10,183,967	140,201	0.015000	0.014989	0.013767
75-79	215,128	6,103	207,427	5,858	7,486,845	196,013	0.028369	0.028241	0.026181
80-84	153,989	7,890	148,401	7,564	4,786,863	228,681	0.051237	0.050970	0.047773
85-89	82,357	7,776	79,414	7,489	2,170,603	194,565	0.094418	0.094303	0.089636
90-95	28,644	4,817	27,723	4,669	631,740	102,604	0.168168	0.168414	0.162414
TOTAL	1,530,452	35,466	1,462,228	34,092	59,530,604	1,054,362			
Above Median			734,417	10,741	44,207,569	596,769			
Below Median			727,811	23,351	15,323,035	457,593			

Table B.8

Summary of Public Safety Female Healthy Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
45-49	10,954	18	10,906	18	419,329	562	0.001643	0.001650	0.001341
50-54	20,961	58	20,888	56	852,793	1,662	0.002767	0.002681	0.001949
55-59	26,783	108	26,701	108	953,274	3,266	0.004032	0.004045	0.003426
60-64	23,292	129	23,188	127	695,609	3,396	0.005538	0.005477	0.004882
65-69	16,432	172	16,349	171	403,720	3,673	0.010468	0.010459	0.009098
70-74	8,684	145	8,625	143	181,303	2,699	0.016697	0.016580	0.014888
75-79	4,303	135	4,273	135	79,733	2,459	0.031377	0.031595	0.030837
80-84	2,173	121	2,164	121	36,181	2,124	0.055694	0.055906	0.058713
85-89	991	65	991	65	16,613	1,104	0.065564	0.065564	0.066458
90-95	343	50	339	50	5,322	813	0.145857	0.147363	0.152856
TOTAL	114,915	1,001	114,425	994	3,643,877	21,758			
Above Median			56,556	267	2,817,018	12,385			
Below Median			57,869	727	826,859	9,374			

Table B.9

Summary of Public Safety Male Healthy Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
45-49	50,098	107	49,788	103	2,198,539	3,475	0.002136	0.002069	0.001580
50-54	111,630	368	111,211	367	5,532,612	14,018	0.003297	0.003300	0.002534
55-59	167,304	764	166,359	756	7,994,930	28,849	0.004567	0.004544	0.003608
60-64	196,051	1,546	194,857	1,538	8,556,098	55,120	0.007886	0.007893	0.006442
65-69	165,039	2,047	163,874	2,038	6,334,174	69,405	0.012403	0.012436	0.010957
70-74	109,953	2,411	108,883	2,386	3,716,761	73,060	0.021928	0.021913	0.019657
75-79	66,734	2,638	65,966	2,605	2,030,657	71,177	0.039530	0.039490	0.035051
80-84	41,591	2,959	41,071	2,917	1,134,887	70,533	0.071145	0.071023	0.062150
85-89	20,540	2,536	20,256	2,502	521,400	59,205	0.123468	0.123519	0.113549
90-95	6,999	1,411	6,890	1,383	164,105	30,835	0.201602	0.200717	0.187897
TOTAL	935,939	16,787	929,155	16,595	38,184,162	475,676			
Above Median			458,448	4,241	27,783,448	228,479			
Below Median			470,707	12,354	10,400,714	247,197			

Table B.10

Summary of General Female Healthy Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
50-54	115,286	486	114,133	481	2,350,981	6,395	0.004216	0.004214	0.002720
55-59	591,571	2,477	588,911	2,469	13,831,796	45,285	0.004187	0.004192	0.003274
60-64	1,319,332	7,135	1,314,498	7,112	27,970,845	129,280	0.005408	0.005410	0.004622
65-69	1,455,182	12,292	1,454,097	12,277	26,395,111	202,379	0.008447	0.008443	0.007667
70-74	1,129,962	16,303	1,129,691	16,289	17,709,992	232,888	0.014428	0.014419	0.013150
75-79	876,516	22,172	876,383	22,147	11,967,554	283,613	0.025296	0.025271	0.023698
80-84	700,543	31,448	700,438	31,427	8,504,998	361,267	0.044891	0.044868	0.042477
85-89	477,463	39,486	477,360	39,461	5,079,023	399,043	0.082700	0.082665	0.078567
90-95	243,748	36,820	243,689	36,795	2,330,940	331,316	0.151058	0.150992	0.142138
TOTAL	6,909,604	168,619	6,899,200	168,458	116,141,240	1,991,467			
Above Median			3,487,984	60,468	96,931,114	1,408,421			
Below Median			3,411,215	107,990	19,210,126	583,046			

Table B.11

Summary of General Male Healthy Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
50-54	79,455	452	78,902	447	2,057,992	7,184	0.005689	0.005665	0.003491
55-59	406,235	2,986	404,851	2,973	12,607,163	67,869	0.007350	0.007343	0.005383
60-64	916,487	8,383	914,202	8,356	27,552,331	196,909	0.009147	0.009140	0.007147
65-69	1,004,698	13,617	1,004,107	13,605	27,368,614	304,451	0.013553	0.013549	0.011124
70-74	796,902	18,048	796,682	18,021	19,608,126	374,093	0.022648	0.022620	0.019078
75-79	631,385	24,303	631,218	24,280	14,307,243	479,736	0.038492	0.038465	0.033531
80-84	475,343	31,973	475,200	31,951	10,032,179	600,764	0.067263	0.067237	0.059884
85-89	279,919	31,834	279,758	31,813	5,366,448	562,493	0.113726	0.113716	0.104817
90-95	112,134	21,126	112,033	21,112	1,851,581	321,507	0.188400	0.188444	0.173639
TOTAL	4,702,559	152,722	4,696,954	152,558	120,751,677	2,915,006			
Above Median			2,376,859	52,941	97,675,068	1,958,279			
Below Median			2,320,095	99,617	23,076,609	956,727			

Table B.12

Summary of Public Safety Female Disabled Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
50-54	10,086	39	10,080	39	324,296	1,127	0.003867	0.003869	0.003476
55-59	8,797	61	8,795	61	271,446	1,586	0.006934	0.006936	0.005843
60-64	6,290	63	6,287	62	181,831	1,527	0.010016	0.009861	0.008400
65-69	3,593	50	3,593	50	99,325	1,071	0.013917	0.013917	0.010788
70-75	1,905	46	1,905	46	50,407	991	0.024143	0.024143	0.019660
TOTAL	30,671	259	30,661	258	927,304	6,303			

Table B.13

Summary of Public Safety Male Disabled Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
45-49	33,104	127	33,098	127	1,535,373	4,962	0.003836	0.003837	0.003232
50-54	41,809	233	41,793	232	2,000,708	7,793	0.005573	0.005551	0.003895
55-59	48,087	351	48,078	350	2,228,772	12,376	0.007299	0.007280	0.005553
60-64	61,555	678	61,538	677	2,616,830	23,758	0.011014	0.011001	0.009079
65-69	57,136	972	57,100	971	2,289,304	32,920	0.017012	0.017005	0.014380
70-74	37,652	958	37,623	957	1,450,376	32,375	0.025444	0.025436	0.022322
75-79	23,735	1,088	23,727	1,088	885,391	36,530	0.045840	0.045854	0.041259
80-84	14,997	1,096	14,990	1,095	551,716	36,913	0.073083	0.073049	0.066905
85-89	7,192	855	7,192	855	255,911	28,890	0.118881	0.118881	0.112891
90-95	2,256	429	2,255	429	78,143	14,566	0.190124	0.190243	0.186402
TOTAL	327,523	6,787	327,394	6,781	13,892,525	231,084			

Table B.14

Summary of Non-Safety Female Disabled Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
45-49	28,590	379	28,352	375	481,100	6,107	0.013256	0.013227	0.012694
50-54	62,102	995	61,627	977	1,039,649	16,596	0.016022	0.015854	0.015963
55-59	104,825	1,992	103,729	1,967	1,794,617	33,717	0.019003	0.018963	0.018788
60-64	119,509	2,613	118,065	2,582	1,949,716	39,483	0.021864	0.021869	0.020251
65-69	89,817	2,382	88,944	2,365	1,309,536	31,709	0.026521	0.026590	0.024214
70-74	58,753	2,086	58,185	2,071	762,667	25,189	0.035504	0.035593	0.033028
75-79	36,236	1,798	35,848	1,776	421,755	19,010	0.049620	0.049542	0.045074
80-84	23,268	1,739	22,962	1,719	242,255	17,210	0.074737	0.074864	0.071040
85-89	13,931	1,668	13,749	1,645	136,281	15,120	0.119734	0.119644	0.110950
90-95	6,204	1,109	6,118	1,096	61,268	10,474	0.178763	0.179142	0.170956
TOTAL	543,235	16,761	537,578	16,573	8,198,844	214,617			

Table B.15

Summary of Non-Safety Male Disabled Retiree Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
45-49	21,790	322	21,695	321	417,909	5,643	0.014778	0.014796	0.013502
50-54	45,882	890	45,707	881	879,573	16,223	0.019397	0.019275	0.018445
55-59	73,780	1,786	73,341	1,770	1,382,378	31,215	0.024207	0.024134	0.022580
60-64	85,949	2,544	85,248	2,532	1,576,979	42,552	0.029599	0.029702	0.026984
65-69	66,765	2,516	66,230	2,505	1,179,351	40,048	0.037684	0.037823	0.033958
70-74	43,742	2,057	43,394	2,042	729,576	31,258	0.047025	0.047057	0.042844
75-79	29,140	1,932	28,839	1,913	463,603	27,546	0.066300	0.066334	0.059417
80-84	18,421	1,665	18,226	1,647	287,415	24,005	0.090388	0.090364	0.083520
85-89	9,631	1,296	9,549	1,283	144,474	17,572	0.134561	0.134364	0.121626
90-95	3,325	557	3,295	555	45,571	7,343	0.167527	0.168457	0.161143
TOTAL	398,425	15,565	395,523	15,449	7,106,827	243,405			

Table B.16

Summary of Female Contingent Survivor Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
50–54	41,120	212	40,853	208	633,023	2,421	0.005156	0.005091	0.003824
55–59	70,133	432	69,659	430	1,204,842	6,300	0.006160	0.006173	0.005229
60–64	113,621	996	112,754	985	2,082,091	14,726	0.008766	0.008736	0.007073
65–69	150,445	1,991	149,262	1,973	2,772,463	30,695	0.013234	0.013218	0.011072
70–74	185,160	3,519	183,686	3,483	3,218,087	52,473	0.019005	0.018962	0.016306
75–79	224,164	6,695	222,251	6,637	3,627,580	96,093	0.029867	0.029863	0.026489
80–84	258,694	12,768	256,124	12,604	3,745,084	166,517	0.049356	0.049211	0.044463
85–89	226,307	19,250	224,153	19,029	2,857,488	227,039	0.085061	0.084893	0.079454
90–95	132,883	19,226	131,677	19,041	1,420,939	195,827	0.144683	0.144604	0.137815
TOTAL	1,402,527	65,089	1,390,420	64,390	21,561,598	792,092			
Above Median			699,867	26,613	17,503,861	585,395			
Below Median			690,553	37,777	4,057,737	206,696			

Table B.17

Summary of Male Contingent Survivor Dataset

	Number		Number with Amount		Annual Benefit Amount ($ thousands)		Raw Death Rates Based on		
Age Band	Exposed Life Years	Deaths	Exposed Life Years	Deaths	Exposed $ Years	$ Weighted Deaths	Number	Number with Amount	Amount
50–54	12,941	119	12,730	117	125,411	1,104	0.009196	0.009191	0.008803
55–59	20,854	210	20,505	204	241,225	2,134	0.010070	0.009949	0.008845
60–64	34,485	425	33,923	417	449,653	4,882	0.012324	0.012293	0.010856
65–69	41,272	753	40,655	746	547,586	8,994	0.018245	0.018350	0.016424
70–74	43,744	1,273	43,134	1,259	519,482	13,792	0.029101	0.029188	0.026550
75–79	48,328	2,084	47,640	2,053	516,999	21,209	0.043122	0.043094	0.041023
80–84	49,068	3,412	48,184	3,347	471,082	30,363	0.069537	0.069463	0.064454
85–89	39,990	4,450	39,282	4,368	332,932	35,039	0.111277	0.111195	0.105244
90–95	21,844	3,829	21,438	3,744	165,068	27,954	0.175289	0.174642	0.169349
TOTAL	312,526	16,555	307,491	16,255	3,369,438	145,471			
Above Median			153,491	6,882	2,783,193	111,577			
Below Median			154,001	9,373	586,245	33,893			

Table B.18

Appendix C: Discussion of GAM Graduation

The splines that underlie the GAM models have a long history. Before the advent of computers, engineers and drafting technicians used "splines" to draw curves. Such splines were thin flexible strips of wood or metal. The technician would place wooden or metal dowels vertically on the drawing surface and position the spline strip such that the strip passed through the dowels and then was flexed to the desired curve. By both arranging the positions and orientations of the dowels and setting the length of the strip between each dowel, a technician could obtain a wide range of smooth curves.

With the advent of computer-aided drafting and design, mathematical representations of splines were developed by practitioners with the desired features that were needed to solve problems specific to their fields. The Renault engineer Pierre Bézier is remembered for his introduction of Bézier splines, which can be thought of as a weighted average of *n* control points, with the weighting determined by the binomial formula. Since then, computer representations of splines have found uses throughout engineering, statistics and visual effects.

There is a large and growing diversity of spline types of one and higher dimensions. The GAM framework is agnostic with respect to spline type, but the most natural and easiest to understand type for the purposes of one-dimensional mortality modeling is the class of cubic regression splines. A cubic spline is a type of spline constructed using piecewise cubic polynomials that pass through a certain set of points called knots. The cubic splines used in the GAM models are set up in a way similar to traditional drafting. In R's mgcv package, 10 knots are placed evenly by default over the attained age range of the data, with one knot reserved for each end. For example, if the attained ages run from 50 to 95, then knots are placed at ages 50 and 95, and eight other knots are placed evenly, in this case at quinquennial ages. Then a model matrix is set up in R. The model matrix is configured such that, when combined with the model coefficients, (a) the function is a cubic polynomial between knots, expressed relative to some basis, and (b) each of the zeroth, first and second derivatives of the cubic polynomials agree at the knots. Model coefficients are then determined using optimization routines. If the GAM model equation is specified as

$$\ln q_x = f(x) + \varepsilon_x$$

where $f(x)$ is the function of cubic splines and ε_x is the error term (as is encountered for the RPEC graduations using a binomial likelihood with log link), then subject to appropriate weighting w_x, and other considerations related to the fitting algorithm, the goal of the optimization is to find the function *f*(*x*) such that the following is minimized:

$$\sum_x w_x\left(\ln q_x - f(x)\right)^2 + \lambda \int_{x_{min}}^{x_{max}} [f''(x)]^2 dx$$

The formula represents a trade-off between rewarding a tight fit of the data (the summation on the left) and rewarding a curve with low curvature (the integral on the right). The parameter λ is the smoothness penalty, with higher values increasing the penalty for curvature.

Compare this with the objective function of Whittaker-Henderson (Type B) graduation. For every raw mortality rate q_x and weighting factor w_x, find $\hat{q}_x$ that minimizes

$$\sum_x w_x(q_x - \hat{q}_x)^2 + h\sum_x(\Delta^n \hat{q}_x)^2$$

where h is the smoothing penalty, and $\Delta^n \hat{q}_x$ is the *n*th difference of the fitted rates.

Any Whittaker-Henderson graduation can be recast as a regression using penalized splines, or p-splines.[42] To translate, set a knot at every age, use a p-spline basis dimension of zero (hence step functions at every age), and set the order of difference penalty equal to n.[43] The smoothness parameter can be either specified or made part of the minimization.

Technical discussion

A generalized additive model (GAM) extends the generalized linear model (GLM) by including specifications for a smooth function of one or more predictors (e.g., a smooth function of age) and a penalty term to penalize the "wigglyness" of that function. The main advantage to this approach is that the modeler is freed of the chore of hunting for an appropriate polynomial or other smooth function that both fits the data and permits stable predictions from the model. Since GAMs extend GLMs, many of the intuitions from fitting GLMs carry over to GAMs.

In turn, GLMs extend linear models to broader types of data. In linear models, a response is regressed linearly onto a collection of predictor variables using least squares minimization, and it is assumed that the response data are independent normally distributed random variables with mean equal to a linear combination of the predictors. Least-squares minimization is equivalent to maximizing the likelihood of the independent normally distributed data. Replacing the likelihood function with exponential families (e.g., binomial, Poisson etc.) leads to GLMs.

The remainder of Appendix C is meant to provide a high-level overview of statistical techniques underpinning GAMs. Readers interested in learning more about GAM's mathematical underpinnings are advised to consult *Generalized Additive Models: An Introduction in R,* 2nd edition, by Simon Wood [Wood 2017] from which much of Appendix C is adapted.

The methodology for fitting GAMs tracks these extensions. First, convert the GAM problem to a penalized GLM problem by setting up model and penalty matrices that reflect the specified spline structure. Second, convert the GLM problem into an iteratively reweighted least squares problem, and finally iterate the fit by alternating between optimizing the regression parameters for fixed smoothing parameter, and optimizing the smoothing penalty for fixed regression parameters. In addition, the R package mgcv automates this procedure in the function "gam."

[42] P-splines are B-splines that add a difference penalty on the regression coefficients for the spline. The penalties are analogous to the penalties in Whittaker-Henderson graduation.
[43] Iain Currie (n.d.), "Back to the Future with Whittaker Smoothing," in *Longevitas*. Retrieved July 10, 2018, from https://www.longevitas.co.uk/site/informationmatrix/whittaker.html.

The algorithm in gam does the following:

1. Set up the matrices and other parameters for the problem, along with any computational optimizations.
2. Minimize generalized cross-validation (GCV) with respect to λ using a version of Newton's method for a fixed vector of regression parameters (the so-called "outer loop").
    a. For fixed λ, fit GLM using penalized iteratively reweighted least squares (the so-called "inner loop").
    b. Compute derivative information to enable the minimization in the outer loop.
3. Generate statistics for final model.

When the algorithm is done, the modeler receives a model that both fits the data optimally (up to the limitations of its specifications) *and* has the optimal smoothness, all without the need for hand-tuning the smoothness parameter. This is an improvement on the process that is commonly carried out in Whittaker-Henderson graduation, in which the modeler uses trial-and-error and visual inspection to get to an acceptable smoothing parameter.

Appendix D: Additional Annuity Comparisons

D.1: Annuity Comparisons at 7% Interest

Tables D.1, D.2 and D.3 show annuity factor comparisons for Teachers, Safety and General, respectively, at a flat 7.0% interest rate, in contrast to the 7.0% pre-retirement/5.0% post-retirement interest rate structure shown in Tables 1.1–1.3 and Tables 11.1–11.3.

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubT-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubT-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	0.9564	0.9545	0.9748	**1.0059**	*5.2%*	*5.4%*	*3.2%*
	Age 35	1.8580	1.8540	1.8962	**1.9607**	*5.5%*	*5.8%*	*3.4%*
	Age 45	3.6196	3.6075	3.6936	**3.8273**	*5.7%*	*6.1%*	*3.6%*
	Age 55	7.1071	7.0603	7.2300	**7.4969**	*5.5%*	*6.2%*	*3.7%*
	Age 65	10.9394	10.8699	11.1378	**11.5602**	*5.7%*	*6.4%*	*3.8%*
	Age 75	8.6134	8.5002	8.7730	**9.1614**	*6.4%*	*7.8%*	*4.4%*
	Age 85	5.8115	5.5506	5.7343	**6.0466**	*4.0%*	*8.9%*	*5.4%*
Males	Age 25	0.9208	0.9062	0.9466	**0.9699**	*5.3%*	*7.0%*	*2.5%*
	Age 35	1.7854	1.7580	1.8404	**1.8872**	*5.7%*	*7.3%*	*2.5%*
	Age 45	3.4755	3.4163	3.5821	**3.6784**	*5.8%*	*7.7%*	*2.7%*
	Age 55	6.8225	6.6971	7.0143	**7.2060**	*5.6%*	*7.6%*	*2.7%*
	Age 65	10.4576	10.3539	10.7992	**11.0772**	*5.9%*	*7.0%*	*2.6%*
	Age 75	7.8922	7.9390	8.3212	**8.5433**	*8.2%*	*7.6%*	*2.7%*
	Age 85	4.9517	5.0322	5.2534	**5.4180**	*9.4%*	*7.7%*	*3.1%*

Table D.1: Amount-Weighted Teachers at Flat 7.0% Interest

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubS-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	PubS-2010	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	MP-2017	BB	MP-2017	MP-2017
Females	Age 25	0.9564	0.9545	0.9748	**0.9656**	*1.0%*	*1.2%*	*-0.9%*
	Age 35	1.8580	1.8540	1.8962	**1.8785**	*1.1%*	*1.3%*	*-0.9%*
	Age 45	3.6196	3.6075	3.6936	**3.6614**	*1.2%*	*1.5%*	*-0.9%*
	Age 55	7.1071	7.0603	7.2300	**7.1592**	*0.7%*	*1.4%*	*-1.0%*
	Age 65	10.9394	10.8699	11.1378	**10.9527**	*0.1%*	*0.8%*	*-1.7%*
	Age 75	8.6134	8.5002	8.7730	**8.5255**	*-1.0%*	*0.3%*	*-2.8%*
	Age 85	5.8115	5.5506	5.7343	**5.6371**	*-3.0%*	*1.6%*	*-1.7%*
Males	Age 25	0.9208	0.9062	0.9466	**0.9331**	*1.3%*	*3.0%*	*-1.4%*
	Age 35	1.7854	1.7580	1.8404	**1.8141**	*1.6%*	*3.2%*	*-1.4%*
	Age 45	3.4755	3.4163	3.5821	**3.5322**	*1.6%*	*3.4%*	*-1.4%*
	Age 55	6.8225	6.6971	7.0143	**6.9054**	*1.2%*	*3.1%*	*-1.6%*
	Age 65	10.4576	10.3539	10.7992	**10.5243**	*0.6%*	*1.6%*	*-2.5%*
	Age 75	7.8922	7.9390	8.3212	**7.9316**	*0.5%*	*-0.1%*	*-4.7%*
	Age 85	4.9517	5.0322	5.2534	**4.9515**	*0.0%*	*-1.6%*	*-5.7%*

Table D.2: Amount-Weighted Safety at Flat 7.0% Interest

		Monthly Deferred-to-62 Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubG-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	PubG-2010	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	MP-2017	BB	MP-2017	MP-2017
Females	Age 25	0.9564	0.9545	0.9748	**0.9841**	*2.9%*	*3.1%*	*1.0%*
	Age 35	1.8580	1.8540	1.8962	**1.9150**	*3.1%*	*3.3%*	*1.0%*
	Age 45	3.6196	3.6075	3.6936	**3.7330**	*3.1%*	*3.5%*	*1.1%*
	Age 55	7.1071	7.0603	7.2300	**7.3051**	*2.8%*	*3.5%*	*1.0%*
	Age 65	10.9394	10.8699	11.1378	**11.2288**	*2.6%*	*3.3%*	*0.8%*
	Age 75	8.6134	8.5002	8.7730	**8.8082**	*2.3%*	*3.6%*	*0.4%*
	Age 85	5.8115	5.5506	5.7343	**5.7558**	*-1.0%*	*3.7%*	*0.4%*
Males	Age 25	0.9208	0.9062	0.9466	**0.9325**	*1.3%*	*2.9%*	*-1.5%*
	Age 35	1.7854	1.7580	1.8404	**1.8110**	*1.4%*	*3.0%*	*-1.6%*
	Age 45	3.4755	3.4163	3.5821	**3.5270**	*1.5%*	*3.2%*	*-1.5%*
	Age 55	6.8225	6.6971	7.0143	**6.9103**	*1.3%*	*3.2%*	*-1.5%*
	Age 65	10.4576	10.3539	10.7992	**10.6096**	*1.5%*	*2.5%*	*-1.8%*
	Age 75	7.8922	7.9390	8.3212	**8.0934**	*2.5%*	*1.9%*	*-2.7%*
	Age 85	4.9517	5.0322	5.2534	**5.1212**	*3.4%*	*1.8%*	*-2.5%*

Table D.3: Amount-Weighted General at Flat 7.0% Interest

D.2: Joint-and-100%-Survivor Annuities

Tables D.4, D.5 and D.6 show comparisons of Pub-2010 joint-and-100%-survivor annuity values at 5.0% interest to those produced by past SOA tables. As described in Subsection 12.4, the annuity factors for the Pub-2010 tables were developed using Approach 2, in which Retiree mortality tables are used for the beneficiary prior to the death of the primary member, and Contingent Survivor mortality tables are used for the beneficiary after the death of the primary member.

All joint-and-survivor annuity calculations in this Appendix D.2 and the following Appendix D.3 assume that beneficiaries are of the opposite gender of the primary Retiree, and that females are three years younger than their husbands/partners.

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubT-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubT-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 55	16.8167	16.6858	16.9433	**17.1414**	*1.9%*	*2.7%*	*1.2%*
	Age 65	14.4871	14.3801	14.7007	**14.9812**	*3.4%*	*4.2%*	*1.9%*
	Age 75	11.1961	11.0816	11.4104	**11.7222**	*4.7%*	*5.8%*	*2.7%*
	Age 85	7.3428	7.1589	7.3729	**7.6816**	*4.6%*	*7.3%*	*4.2%*
Males	Age 55	17.3375	17.2127	17.4453	**17.5987**	*1.5%*	*2.2%*	*0.9%*
	Age 65	15.2883	15.1688	15.4741	**15.6834**	*2.6%*	*3.4%*	*1.4%*
	Age 75	12.2717	12.1743	12.5112	**12.7416**	*3.8%*	*4.7%*	*1.8%*
	Age 85	8.5048	8.3304	8.5918	**8.8198**	*3.7%*	*5.9%*	*2.7%*

Table D.4: Amount-Weighted Teachers Joint-and-100% Survivor Annuities

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubS-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubS-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 55	16.8167	16.6858	16.9433	**16.7776**	*-0.2%*	*0.5%*	*-1.0%*
	Age 65	14.4871	14.3801	14.7007	**14.4499**	*-0.3%*	*0.5%*	*-1.7%*
	Age 75	11.1961	11.0816	11.4104	**11.1284**	*-0.6%*	*0.4%*	*-2.5%*
	Age 85	7.3428	7.1589	7.3729	**7.2651**	*-1.1%*	*1.5%*	*-1.5%*
Males	Age 55	17.3375	17.2127	17.4453	**17.3392**	*0.0%*	*0.7%*	*-0.6%*
	Age 65	15.2883	15.1688	15.4741	**15.2920**	*0.0%*	*0.8%*	*-1.2%*
	Age 75	12.2717	12.1743	12.5112	**12.2669**	*0.0%*	*0.8%*	*-2.0%*
	Age 85	8.5048	8.3304	8.5918	**8.4251**	*-0.9%*	*1.1%*	*-1.9%*

Table D.5: Amount-Weighted Safety Joint-and-100%-Survivor Annuities

		Monthly Immediate Annuity Due Values Generational @ July 1, 2018				*Percentage Change of Moving to PubG-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RP-2006	RP-2006 WC	**PubG-2010**	RP-2000	RP-2006	RP-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 55	16.8167	16.6858	16.9433	**16.9041**	*0.5%*	*1.3%*	*-0.2%*
	Age 65	14.4871	14.3801	14.7007	**14.6572**	*1.2%*	*1.9%*	*-0.3%*
	Age 75	11.1961	11.0816	11.4104	**11.3589**	*1.5%*	*2.5%*	*-0.5%*
	Age 85	7.3428	7.1589	7.3729	**7.3948**	*0.7%*	*3.3%*	*0.3%*
Males	Age 55	17.3375	17.2127	17.4453	**17.4017**	*0.4%*	*1.1%*	*-0.2%*
	Age 65	15.2883	15.1688	15.4741	**15.4286**	*0.9%*	*1.7%*	*-0.3%*
	Age 75	12.2717	12.1743	12.5112	**12.4501**	*1.5%*	*2.3%*	*-0.5%*
	Age 85	8.5048	8.3304	8.5918	**8.5841**	*0.9%*	*3.0%*	*-0.1%*

Table D.6: Amount-Weighted General Joint-and-100%-Survivor Annuities

D.3: Approaches for Computing Joint-and-Survivor Annuities

As discussed in Subsection 12.4, several possible approaches can be taken to compute a joint-and-survivor annuity value. Approach 2 was used to compute the Pub-2010 joint-and-survivor annuities in Subsection D.2 above. Other possibilities include Approach 1, which uses Retiree mortality for the beneficiary for the entire duration of the annuity, and Approach 3, which uses Contingent Survivor mortality for the beneficiary for the duration of the annuity. Tables D.7, D.8 and D.9 compare the joint-and-100%-survivor annuity values at 5.0% interest for each job category using each of the three methods, using the above Approach 2 values as the baseline.

		Monthly Immediate Annuity Due Values MP-2017 Generational @ July 1, 2018			*Percentage Change of Moving from PubT-2010 Approach 2 to:*	
	Base Rate →	PubT-2010	PubT-2010	PubT-2010	PubT-2010	PubT-2010
	Type →	Approach 1	Approach 2	Approach 3	Approach 1	Approach 3
Females	Age 55	17.2010	17.1414	17.0222	*0.3%*	*-0.7%*
	Age 65	15.0486	14.9812	14.8453	*0.4%*	*-0.9%*
	Age 75	11.7807	11.7222	11.6083	*0.5%*	*-1.0%*
	Age 85	7.7035	7.6816	7.6352	*0.3%*	*-0.6%*
Males	Age 55	17.6791	17.5987	17.4589	*0.5%*	*-0.8%*
	Age 65	15.7997	15.6834	15.5089	*0.7%*	*-1.1%*
	Age 75	12.8828	12.7416	12.5686	*1.1%*	*-1.4%*
	Age 85	8.9420	8.8198	8.6945	*1.4%*	*-1.4%*

Table D.7: Amount-Weighted Teachers Joint-and-100%-Survivor Annuities by Method

		Monthly Immediate Annuity Due Values MP-2017 Generational @ July 1, 2018			*Percentage Change of Moving from PubS-2010 Approx #2 to:*	
	Base Rate →	PubS-2010	PubS-2010	PubS-2010	PubS-2010	PubS-2010
	Type →	Approach 1	Approach 2	Approach 3	Approach 1	Approach 3
Females	Age 55	16.7878	16.7776	16.7068	*0.1%*	*-0.4%*
	Age 65	14.4432	14.4499	14.4017	*0.0%*	*-0.3%*
	Age 75	11.0915	11.1284	11.1350	*-0.3%*	*0.1%*
	Age 85	7.2269	7.2651	7.3218	*-0.5%*	*0.8%*
Males	Age 55	17.3189	17.3392	17.3072	*-0.1%*	*-0.2%*
	Age 65	15.2476	15.2920	15.2953	*-0.3%*	*0.0%*
	Age 75	12.1961	12.2669	12.3222	*-0.6%*	*0.5%*
	Age 85	8.3646	8.4251	8.4910	*-0.7%*	*0.8%*

Table D.8: Amount-Weighted Safety Joint-and-100%-Survivor Annuities by Method

		Monthly Immediate Annuity Due Values MP-2017 Generational @ July 1, 2018			*Percentage Change of Moving from PubG-2010 Approx #2 to:*	
	Base Rate →	PubG-2010	PubG-2010	PubG-2010	PubG-2010	PubG-2010
	Type →	Approach 1	Approach 2	Approach 3	Approach 1	Approach 3
Females	Age 55	16.9303	16.9041	16.8363	*0.2%*	*-0.4%*
	Age 65	14.6779	14.6572	14.5914	*0.1%*	*-0.4%*
	Age 75	11.3621	11.3589	11.3282	*0.0%*	*-0.3%*
	Age 85	7.3900	7.3948	7.4041	*-0.1%*	*0.1%*
Males	Age 55	17.4324	17.4017	17.3243	*0.2%*	*-0.4%*
	Age 65	15.4586	15.4286	15.3437	*0.2%*	*-0.6%*
	Age 75	12.4665	12.4501	12.3953	*0.1%*	*-0.4%*
	Age 85	8.5756	8.5841	8.5714	*-0.1%*	*-0.1%*

Table D.9: Amount-Weighted General Joint-and-100%-Survivor Annuities by Method

D.4: Complete Cohort Life Expectancies

Tables D.10, D.11 and D.12 display comparisons of complete cohort life expectancies as of July 1, 2018, at a variety of ages for Teachers, Safety and General members, respectively.

		Cohort Life Expectancies (Complete) Generational @ July 1, 2018				*Percentage Change of Moving to PubT.H-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RPH-2006	RPH-2006 WC	**PubT.H-2010**	RP-2000	RPH-2006	RPH-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	64.77	63.87	65.17	**66.87**	*3.2%*	*4.7%*	*2.6%*
	Age 35	54.03	53.11	54.43	**56.18**	*4.0%*	*5.8%*	*3.2%*
	Age 45	43.42	42.47	43.79	**45.58**	*5.0%*	*7.3%*	*4.1%*
	Age 55	33.04	32.05	33.33	**35.10**	*6.2%*	*9.5%*	*5.3%*
	Age 65	23.17	22.40	23.48	**25.03**	*8.0%*	*11.7%*	*6.6%*
	Age 75	14.79	14.14	14.85	**15.82**	*6.9%*	*11.9%*	*6.6%*
	Age 85	8.23	7.61	7.95	**8.40**	*2.1%*	*10.4%*	*5.7%*
Males	Age 25	62.48	60.71	62.95	**64.28**	*2.9%*	*5.9%*	*2.1%*
	Age 35	51.73	50.07	52.29	**53.59**	*3.6%*	*7.0%*	*2.5%*
	Age 45	41.15	39.50	41.68	**42.99**	*4.5%*	*8.8%*	*3.1%*
	Age 55	30.77	29.22	31.29	**32.56**	*5.8%*	*11.4%*	*4.1%*
	Age 65	20.98	20.01	21.69	**22.70**	*8.2%*	*13.4%*	*4.6%*
	Age 75	12.76	12.46	13.49	**13.99**	*9.6%*	*12.3%*	*3.8%*
	Age 85	6.59	6.58	7.02	**7.23**	*9.7%*	*9.9%*	*3.0%*

Table D.10: Teachers Complete Cohort Life Expectancies

		Cohort Life Expectancies (Complete) Generational @ July 1, 2018				*Percentage Change of Moving to PubS.H-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RPH-2006	RPH-2006 WC	**PubS.H-2010**	RP-2000	RPH-2006	RPH-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	64.77	63.87	65.17	**64.47**	*-0.5%*	*0.9%*	*-1.1%*
	Age 35	54.03	53.11	54.43	**53.74**	*-0.5%*	*1.2%*	*-1.3%*
	Age 45	43.42	42.47	43.79	**43.12**	*-0.7%*	*1.5%*	*-1.5%*
	Age 55	33.04	32.05	33.33	**32.60**	*-1.3%*	*1.7%*	*-2.2%*
	Age 65	23.17	22.40	23.48	**22.68**	*-2.1%*	*1.3%*	*-3.4%*
	Age 75	14.79	14.14	14.85	**14.34**	*-3.1%*	*1.4%*	*-3.4%*
	Age 85	8.23	7.61	7.95	**7.83**	*-4.8%*	*2.9%*	*-1.5%*
Males	Age 25	62.48	60.71	62.95	**61.73**	*-1.2%*	*1.7%*	*-1.9%*
	Age 35	51.73	50.07	52.29	**51.09**	*-1.2%*	*2.0%*	*-2.3%*
	Age 45	41.15	39.50	41.68	**40.50**	*-1.6%*	*2.5%*	*-2.8%*
	Age 55	30.77	29.22	31.29	**30.02**	*-2.4%*	*2.8%*	*-4.0%*
	Age 65	20.98	20.01	21.69	**20.27**	*-3.4%*	*1.3%*	*-6.6%*
	Age 75	12.76	12.46	13.49	**12.20**	*-4.4%*	*-2.1%*	*-9.6%*
	Age 85	6.59	6.58	7.02	**6.26**	*-5.0%*	*-4.8%*	*-10.8%*

Table D.11: Safety Complete Cohort Life Expectancies

		Cohort Life Expectancies (Complete) Generational @ July 1, 2018				*Percentage Change of Moving to PubG.H-2010 (with MP-2017) from:*		
	Base Rate →	RP-2000	RPH-2006	RPH-2006 WC	**PubG.H-2010**	RP-2000	RPH-2006	RPH-2006 WC
	Proj. Scale →	BB	MP-2017	MP-2017	**MP-2017**	BB	MP-2017	MP-2017
Females	Age 25	64.77	63.87	65.17	**65.28**	*0.8%*	*2.2%*	*0.2%*
	Age 35	54.03	53.11	54.43	**54.54**	*0.9%*	*2.7%*	*0.2%*
	Age 45	43.42	42.47	43.79	**43.91**	*1.1%*	*3.4%*	*0.3%*
	Age 55	33.04	32.05	33.33	**33.42**	*1.1%*	*4.3%*	*0.3%*
	Age 65	23.17	22.40	23.48	**23.48**	*1.3%*	*4.8%*	*0.0%*
	Age 75	14.79	14.14	14.85	**14.75**	*-0.3%*	*4.3%*	*-0.7%*
	Age 85	8.23	7.61	7.95	**7.80**	*-5.2%*	*2.5%*	*-1.9%*
Males	Age 25	62.48	60.71	62.95	**61.68**	*-1.3%*	*1.6%*	*-2.0%*
	Age 35	51.73	50.07	52.29	**50.98**	*-1.5%*	*1.8%*	*-2.5%*
	Age 45	41.15	39.50	41.68	**40.43**	*-1.8%*	*2.3%*	*-3.0%*
	Age 55	30.77	29.22	31.29	**30.05**	*-2.3%*	*2.8%*	*-4.0%*
	Age 65	20.98	20.01	21.69	**20.49**	*-2.3%*	*2.4%*	*-5.5%*
	Age 75	12.76	12.46	13.49	**12.59**	*-1.4%*	*1.0%*	*-6.7%*
	Age 85	6.59	6.58	7.02	**6.58**	*-0.2%*	*0.0%*	*-6.3%*

Table D.12: General Complete Cohort Life Expectancies

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