Session 16 TS, SOA Experience Study Calculations Educational Tool

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2017 Life & Annuity Symposium

David B. Atkinson

Session 16:
SOA Experience Study Calculations Education Tool
May 8, 2017

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Acknowledgements

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- One member, John McGarry, contributed so much that he became the co-author!
- The project was led and edited by Cynthia MacDonald, FSA, MAAA, of the SOA, with editorial assistance provided by Korrel Rosenberg of the SOA.

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Purpose of Tool

- Standardize terminology and nomenclature used for experience studies
  - Document a shared language for experience study practitioners
- Provide a primer on experience study calculations for those new to the subject
- Document common and traditional practices as well as less well-known or new practices
- Evaluate alternative approaches

Polling Questions

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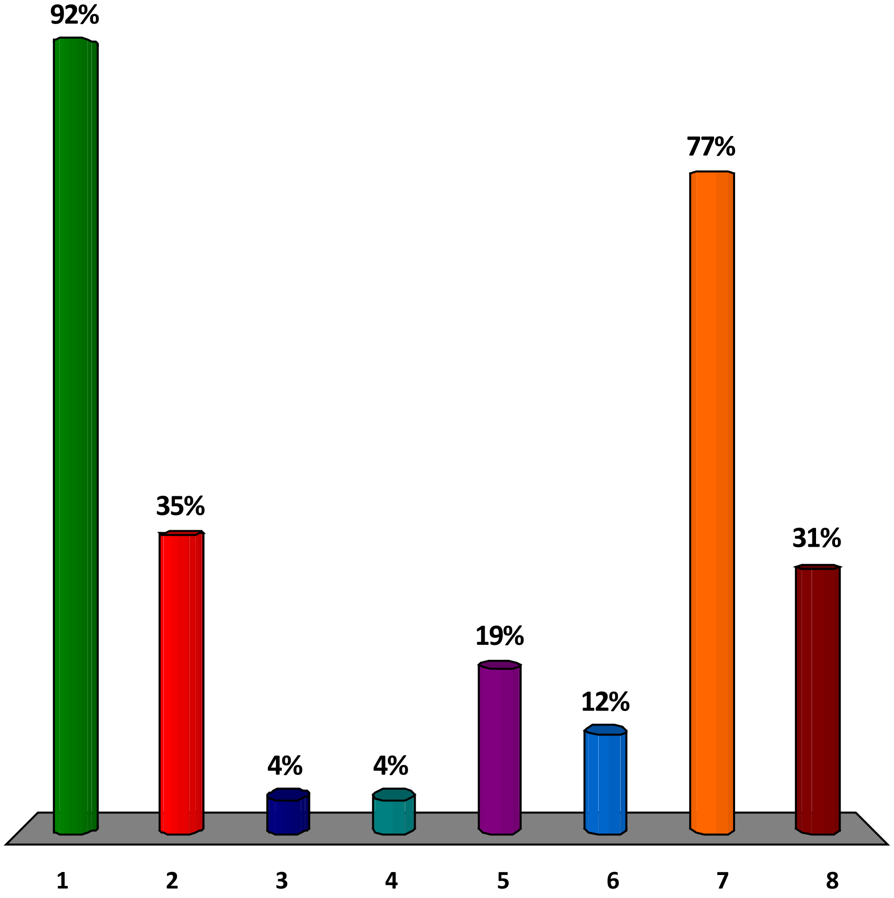
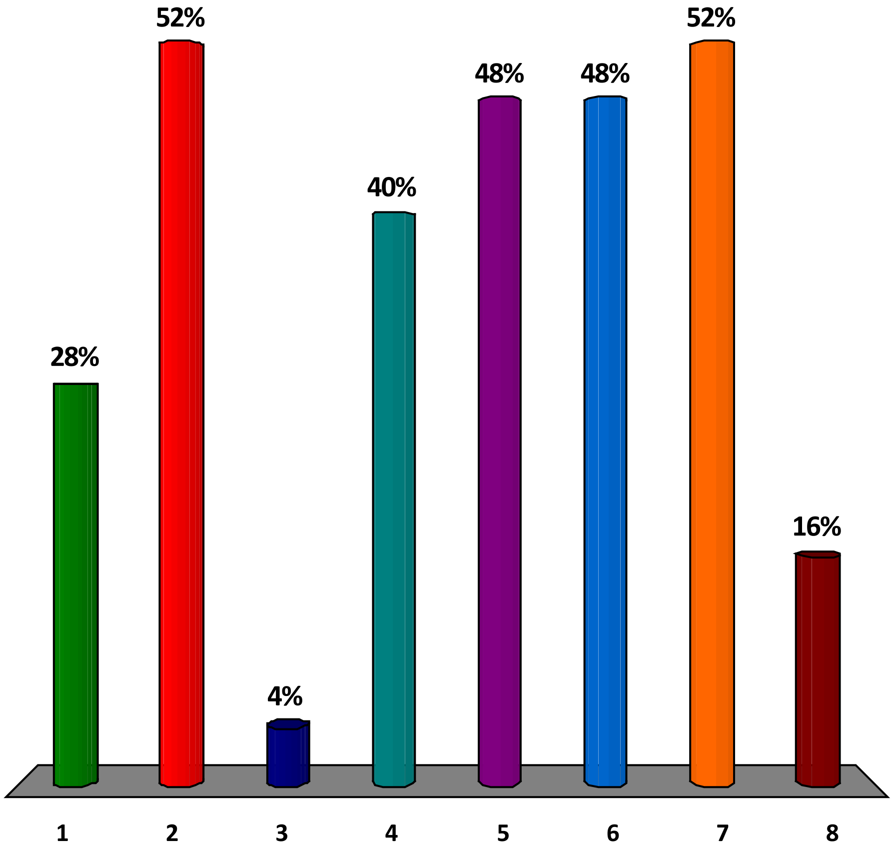
Polling Question #1
Which types of experience studies have you participated in? (Answer all that apply)

1. Life Mortality
2. Annuity Mortality
3. Pension Mortality
4. Critical Illness
5. Disability Income
6. Long Term Care
7. Lapse/Persistency for Traditional Products
8. Lapse/Persistency for Variable Products



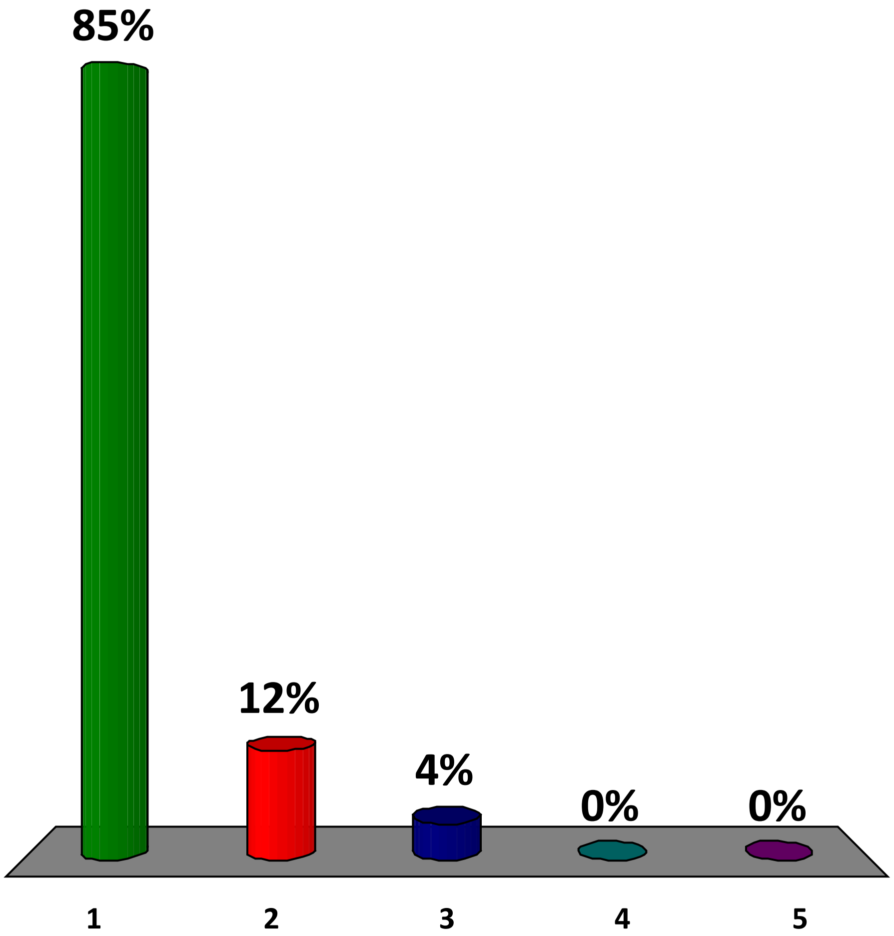
Polling Question #2
Which types of experience studies would you like to know more about?(Answer all that apply)

1. Life Mortality
2. Annuity Mortality
3. Pension Mortality
4. Critical Illness
5. Disability Income
6. Long Term Care
7. Lapse/Persistency for Traditional Products
8. Lapse/Persistency for Variable Products



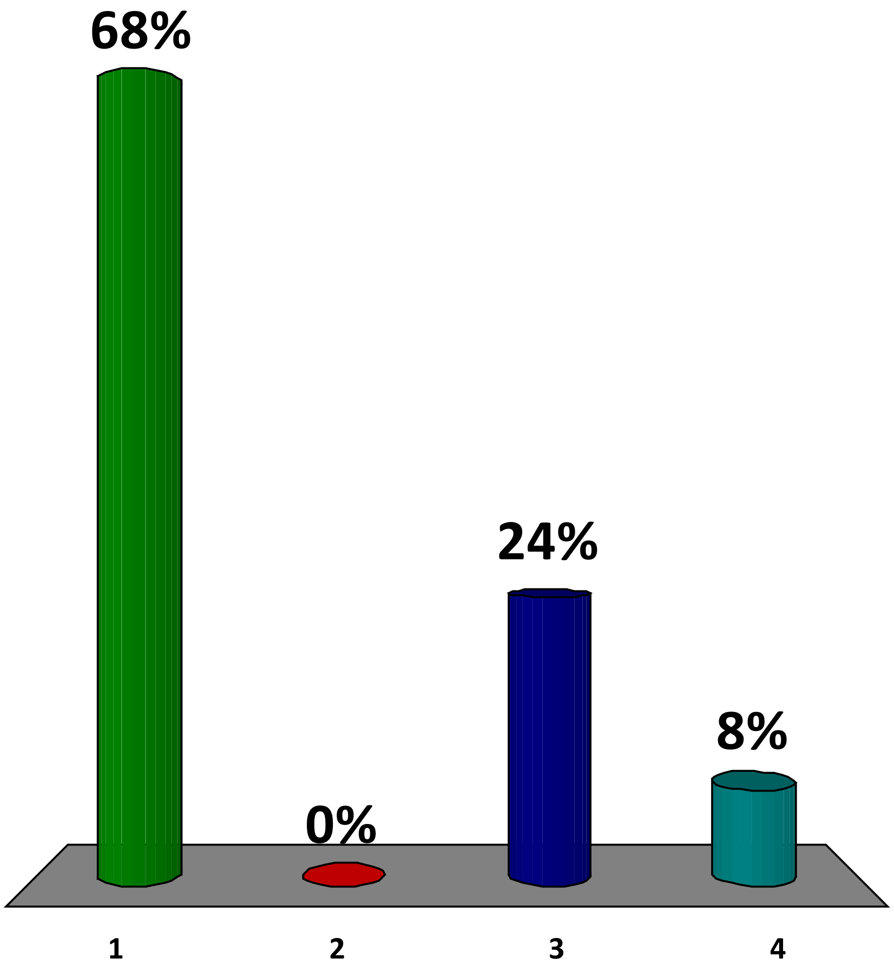
Polling Question #3
Have you reviewed the SOA Experience Study Calculations Educational Tool?
Answer all that apply)

1. No
2. Skimmed it
3. Read basic sections in detail
4. Read advanced sections in detail
5. Read entire document in detail



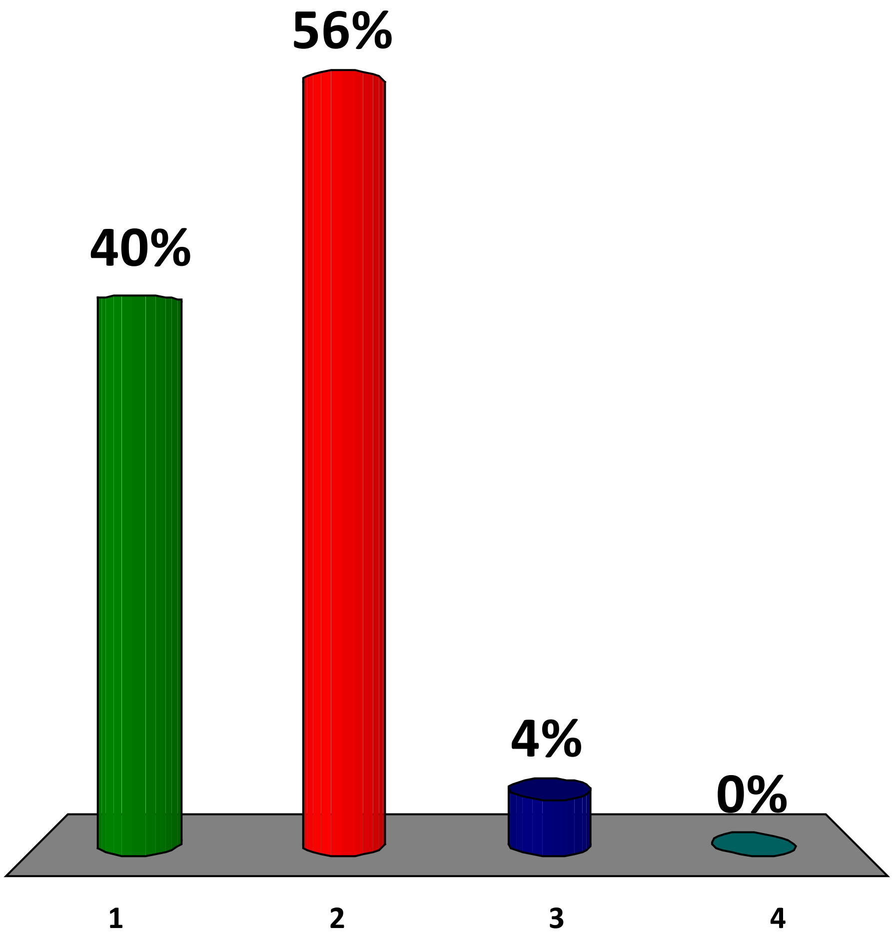
Polling Question #4
Which exposure methods have you used?

1. Traditional/Actuarial exposure
2. Distributed exposure
3. Fractional exposure (e.g., monthly or quarterly exposures for an annual rate study)
4. Daily or exact exposure (using annualized daily rates or force of decrement)



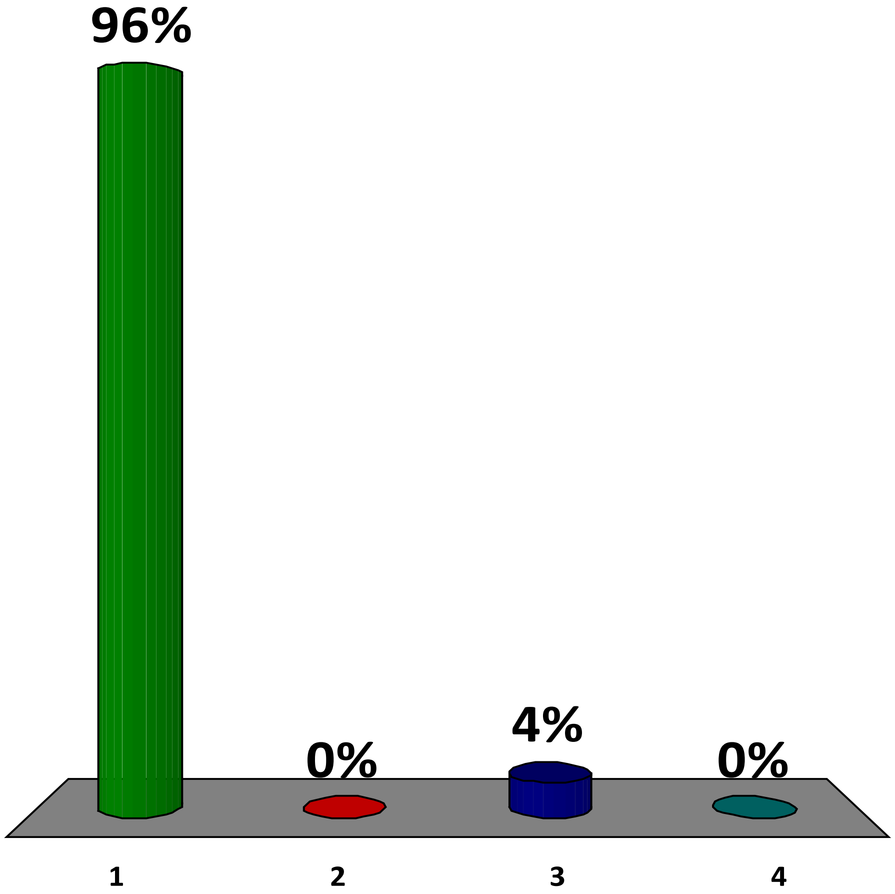
Polling Question #5
How comfortable are you in using the force of mortality (or, more generally, the force of decrement)?

1. Not at all comfortable with it
2. A little comfortable, but I don’t use it
3. I use it, but am not really comfortable with it
4. Very comfortable with it



Polling Question #6
Have you used dependent rates in an experience study? (Answer all that apply)

1. No
2. Yes, for a Pension study
3. Yes, for a Critical Illness study
4. Yes, for another type of study



Brief Overview of the Experience Study Calculations Educational Tool

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Exposure Methods

- Traditional/Actuarial Exposure
  - Focus of early chapters
  - Starts with simple population/cohort mortality study
  - Gradually expands, covering many variations
  - Simple, instructive examples
- Distributed Exposure – used by some companies for studies split by policy and calendar year – more to come on this from John McGarry
- Daily Exposure or, more generally, fractional(e.g., monthly, quarterly) exposure: Used to calculate an average fractional rate that can be annualized

Distribution of Deaths

- Compares and “debunks” common and uncommon methods used for distribution of deaths within a one-year period:
  - Balducci Hypothesis
  - Uniform Distribution of Deaths
  - Constant Fractional (e.g., monthly or daily) Rate
  - Constant Force of Mortality
  - Linear Force of Mortality
- More to come from John McGarry…

Types of Rates

- **Independent rates**
  - Commonly used for life insurance: exposure is calculated separately for lapse and mortality studies.
- **Dependent rates**
  - Commonly used when a multiplicity of rates must be calculated, such as mortality rates by cause of death. One combined exposure calculation is used for all rates.
- **Central rates**
  - Used in population studies. Similar to dependent rates but based on “central exposure,” which is the mean or mid-year population.

Utilization Studies

- Frequency
- Severity
- Utilization with maximum limits
- Loss ratios
- Utilization rate for variable annuity withdrawals

Practical Considerations

- Amount-weighted distortions
- Multi-year studies
- Homogeneity of data
- Reporting lags
- Events that are not uniformly distributed
- Distortions from use of partial policy years

Product-Related Considerations

- Individual Life
  - Grace period, contested claims, reinsurance, backdated new business and more
- Group Life
- Morbidity Products
  - Claim incidence, severity and termination studies
- Disability Income
  - Elimination period, partial disability, recovery followed by relapse, claim settlements and more

Product-Related Considerations (continued)

- Long-Term Care
  - Elimination period, inflation protection, benefit utilization rate and more
- Deferred Annuities
  - Utilization rates and contract-year challenges
- Payout Annuities
- Retirement Pensions
- Credit Life and Disability

Your Questions…

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2017 Life & Annuity Symposium

John McGarry

Session 16: SOA Experience Study Calculations Education Tool

May 9th, 2017

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Experience Study Methods, Distributions and Errors

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Agenda

A. Main Study Methods

B. Generalized Distribution

C. Testing Methods

D. Study Errors

E. Conclusions

A. Main Study Methods

- Traditional Exposure – Annual Rate
- Distributed Exposure – Annual Rate
- Daily Exposure – Average Annual Force

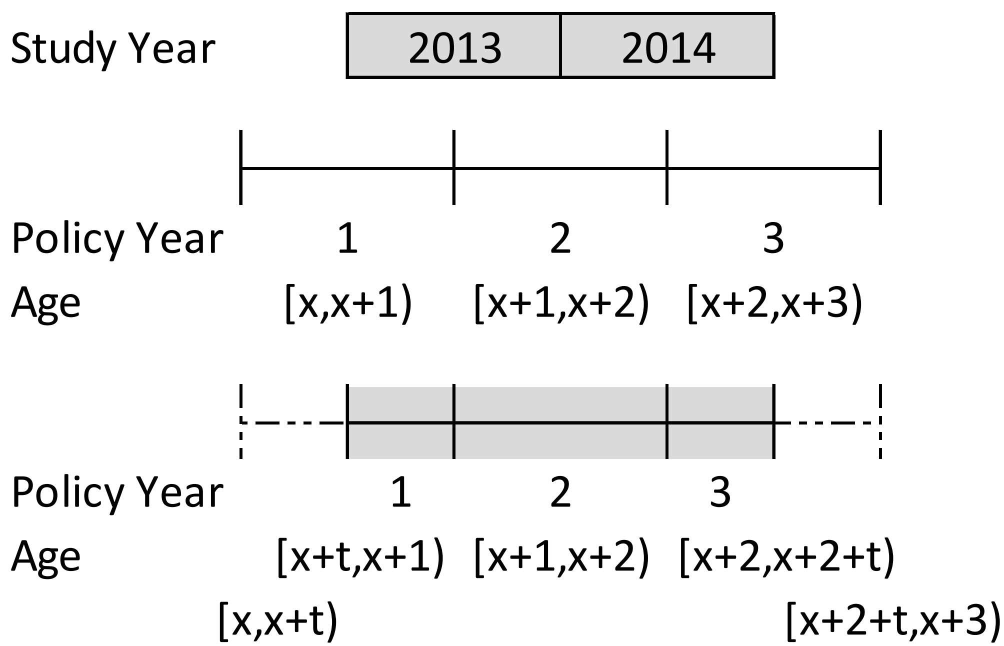
- A Policy Year rate interval is used here to illustrate the studies by age, as is generally the case for insurance, although a Life Year rate interval would be more natural for pensions.

Traditional and Distributed Exposure

- Full Policy Year: [x,x+1)
  - Lives active throughout are assigned 1 year.
  - Deaths are assigned 1 year.
- Assumption
  - There is no assumption about distribution of decrements (e.g. deaths), which could be continuous or discrete.
- Ignoring other decrements, e.g. withdrawals

Partial Policy Years

- At the start and end of a calendar year study, the Policy and Study Years intersect to give partial Policy Years.
  - 2 year study 2013-2014,
  - Policy Year = sequential count of policy years in study,
  - t = the period from the policy anniversary to the year end as a fraction of year.



Traditional Exposure – Partial Years

- First Partial Year: [x,x+t)
  - Lives active throughout are assigned t years.
  - Deaths are assigned 1 year.
- Second Partial Year: [x+t,x+1)
  - Lives active throughout are assigned 1-t years.
  - Deaths are assigned 1-t years.
- Assumption
  - The rate for the Second Partial Year is proportional to annual rate (Balducci).
    - [x+t,x+1): ${}_{1-t}q_{x+t} = (1-t)q_x$.
  - The rate is decreasing over the year.

Distributed Exposure – Partial Years

- First Partial Year: [x,x+t)
  - Lives active throughout are assigned t years.
  - Deaths are assigned t years in current period and 1-t years in following period.
  - Exposure is independent of the timing of deaths.
- Second Partial Year: [x+t,x+1)
  - Lives active throughout are assigned 1-t years.
  - Deaths are assigned 1-t years.
- Assumption
  - The rate for the First Partial Year is proportional to the annual rate.
    - [x,x+t): ${}_tq_x = tq_x$
  - The rate is increasing over year.
  - The deaths are distributed uniformly over the year.

Daily Exposure

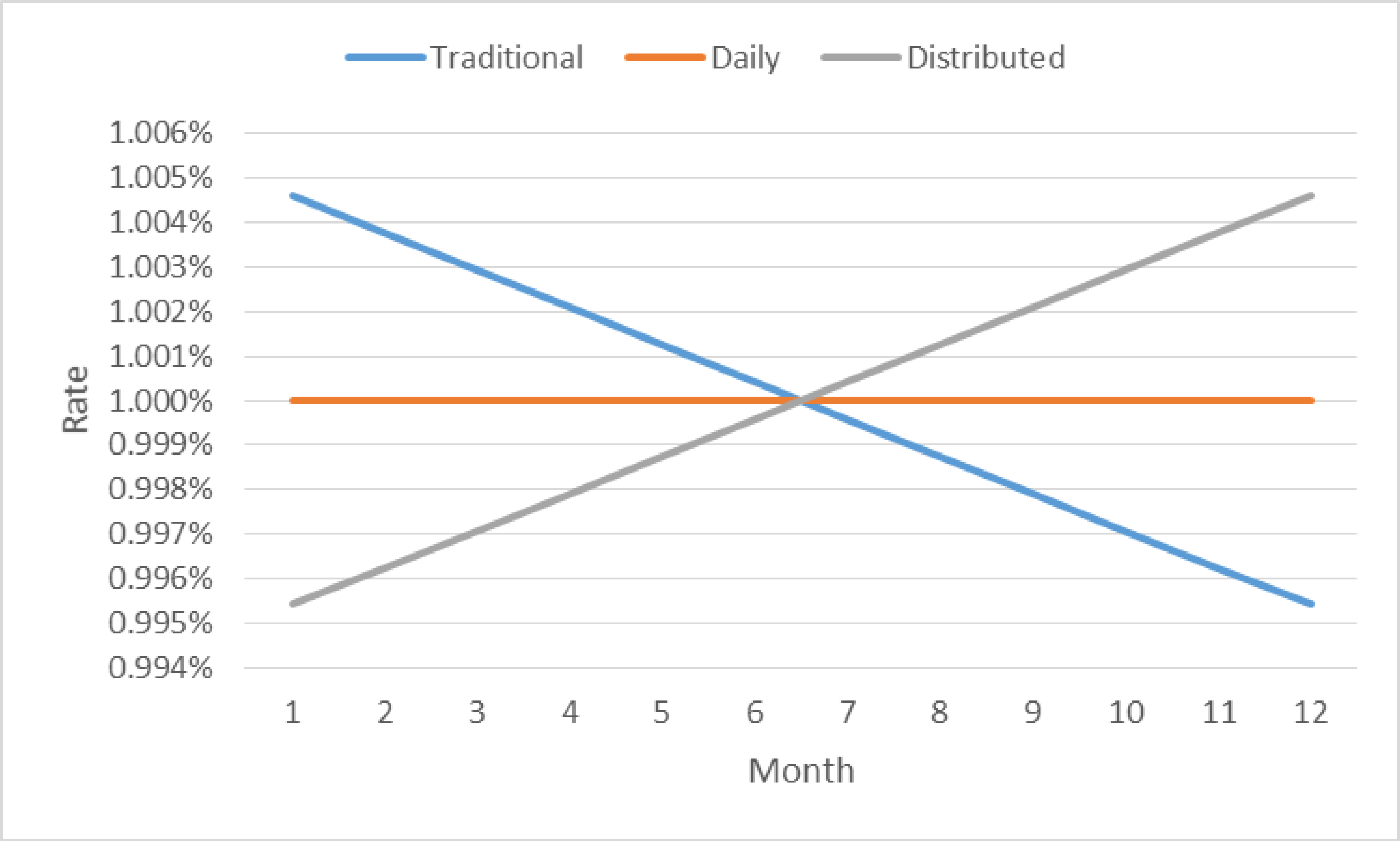
- Exposure for Force – Full Policy Year
  - Lives active throughout are assigned 1 year.
  - Deaths are assigned exposure in years to date of death.
- Rate
  - Calculate average force of mortality,
    - $\bar{\mu}_x \approx d_x / E_x^F$.
  - Calculate the annual rate,
    - $q_x = 1 - e^{-\bar{\mu}_x}$.
- Assumption
  - The force of mortality is constant throughout year.

Method Assumptions

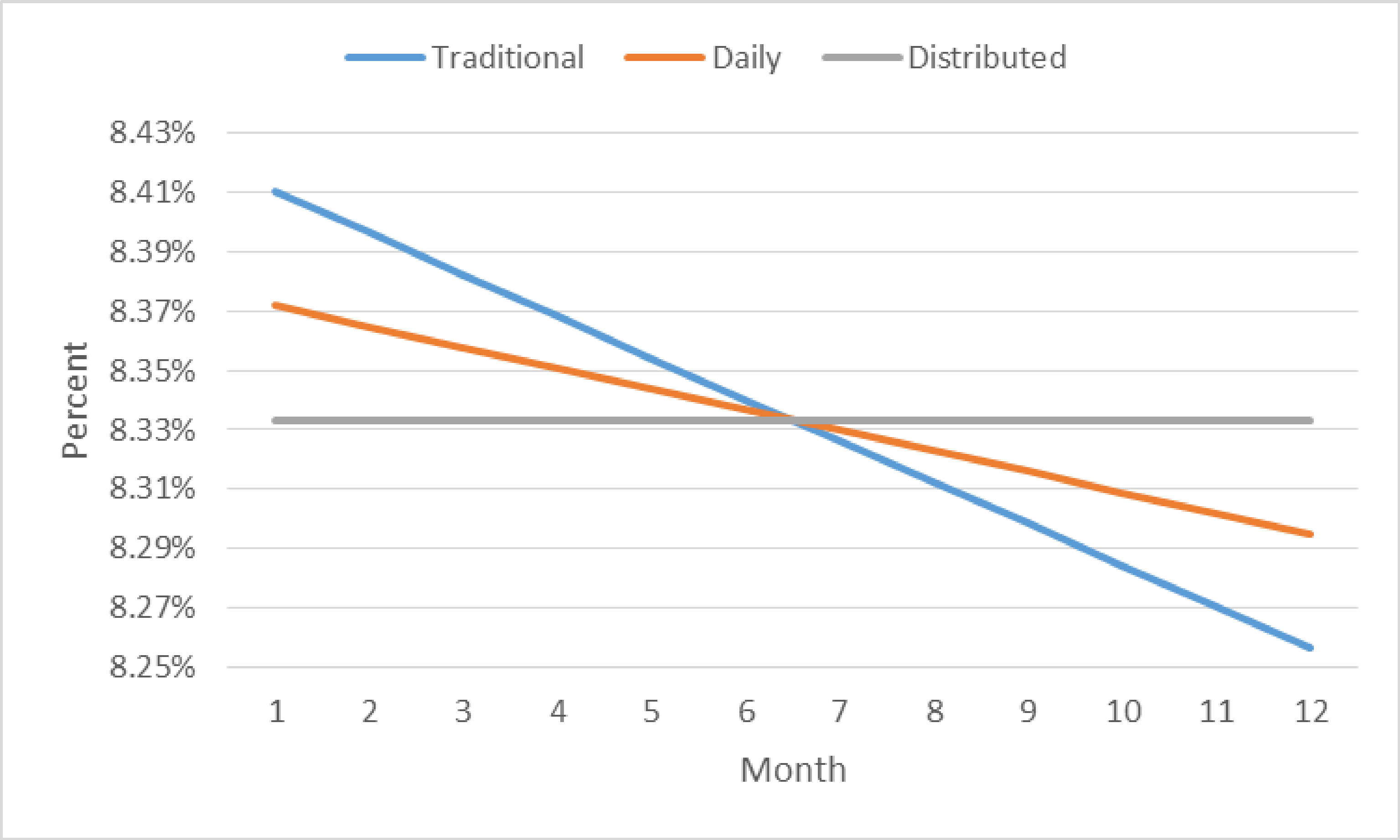
- The method assumptions will be investigated by:
  - projecting the lives and deaths for each method using its underlying assumption, and
  - then calculating the exposures and annual rates for each month in the year.
- Sample Mortality Rate:
  - 1%.

Annualized Effective Rates by Month

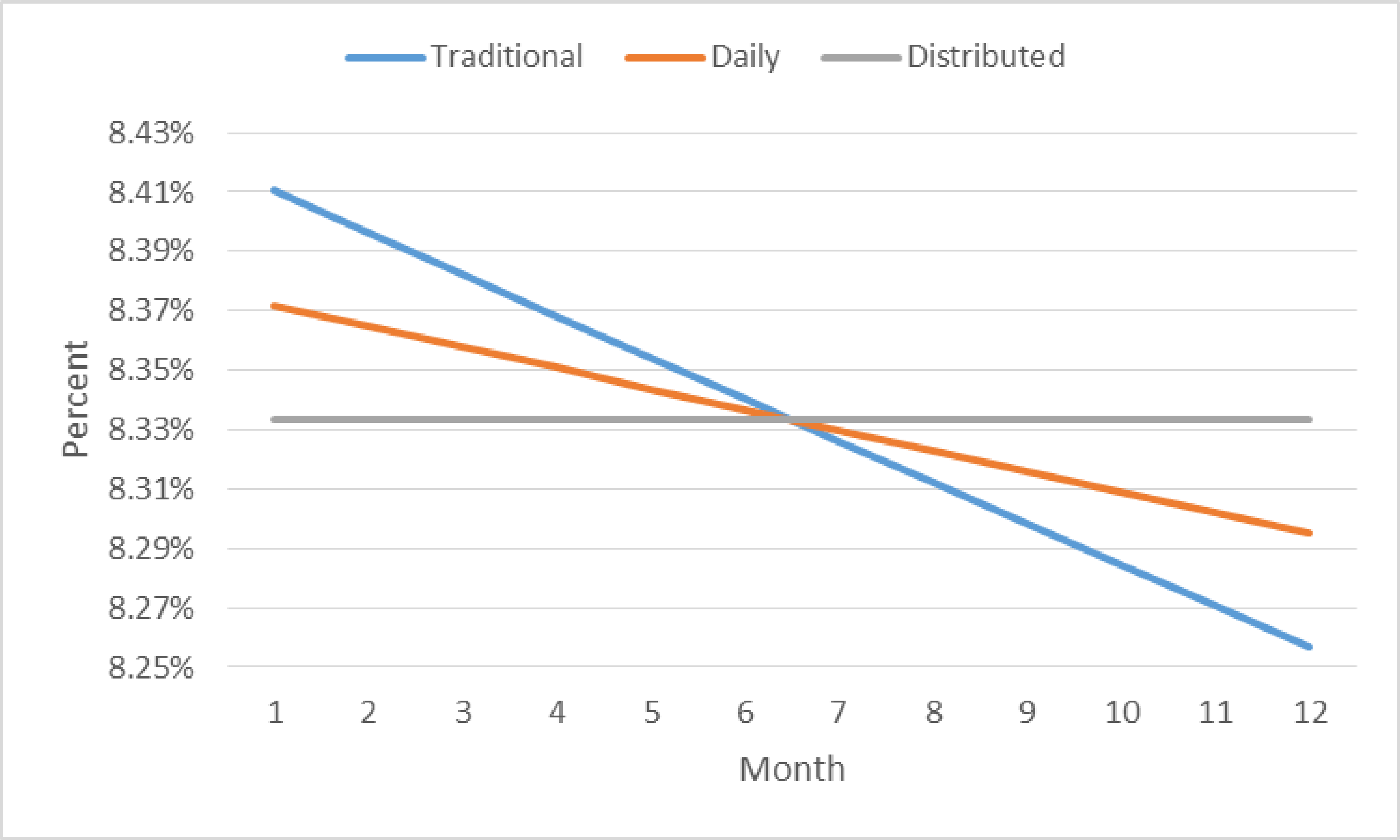
Assumed Distributions by Method



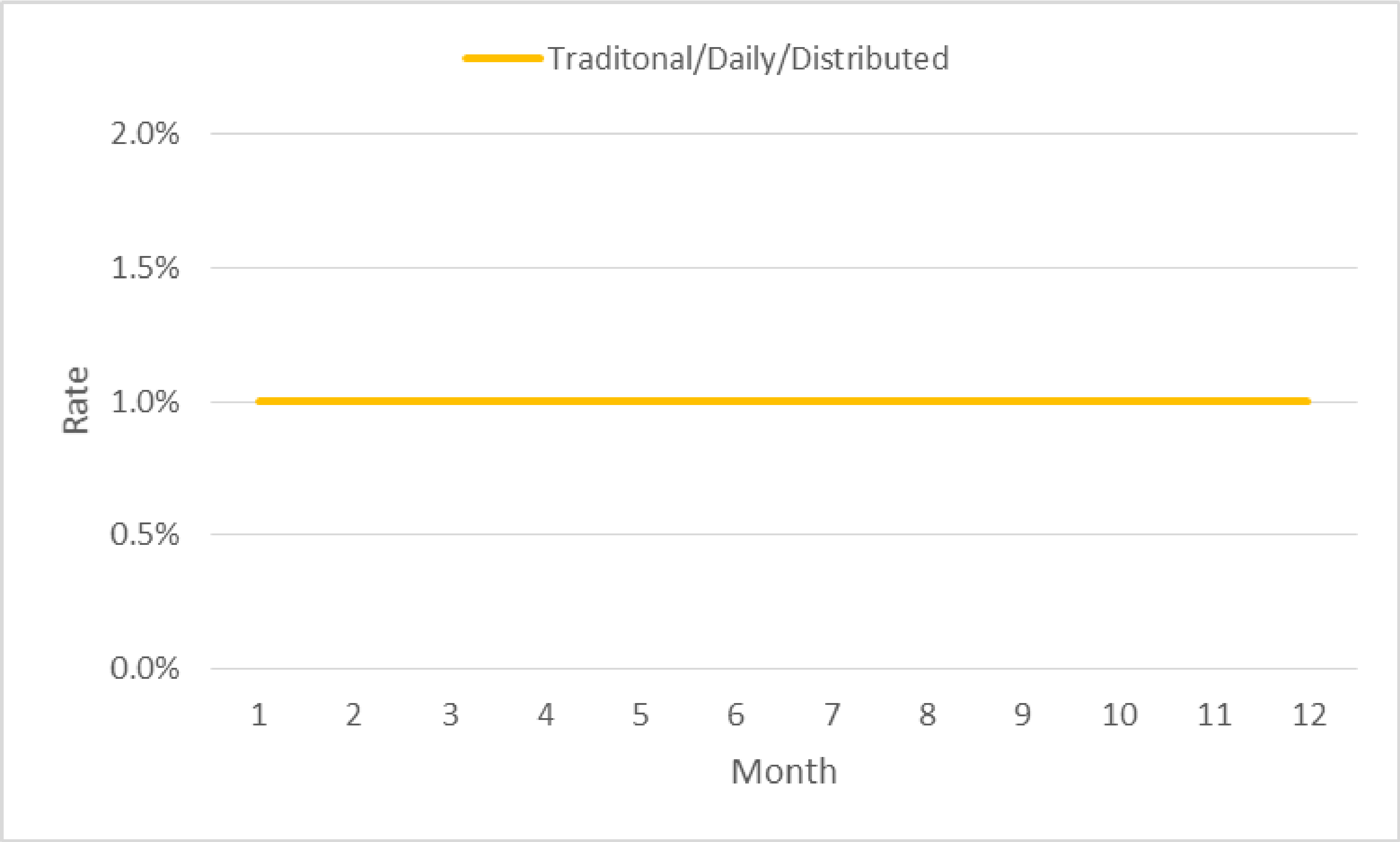
Distribution of Exposures By Month



Distribution of Deaths by Month



Calculated Rates by Month

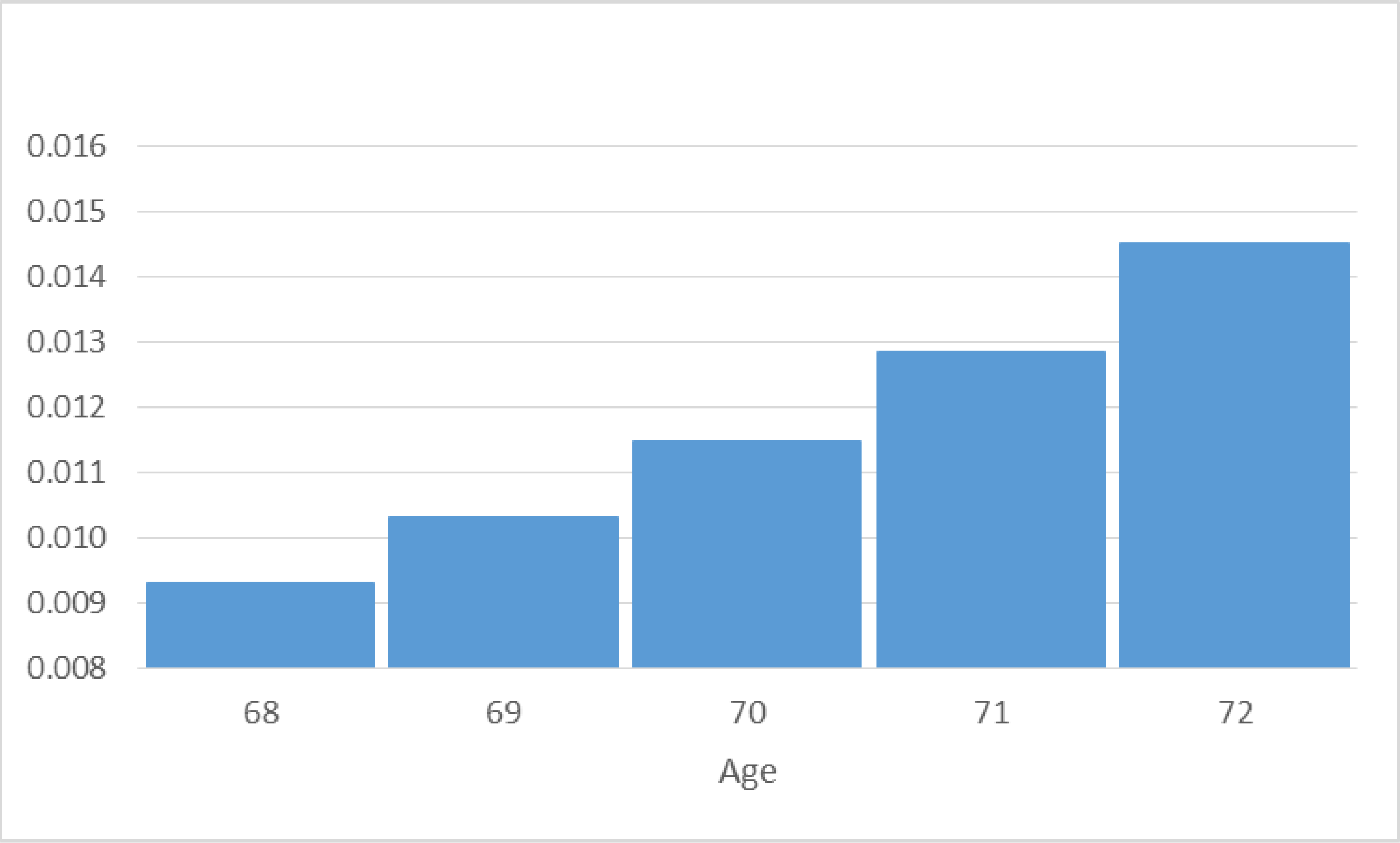


C. Generalized Distribution

- The implicit distribution of each method is applied to all ages in a study.
- But in a study, the actual distributions will vary by age (or duration), and by the type of study.
- A generalized decrement distribution:
  - will allow the methods to be tested using more realistic distributions.
  - can be defined by assuming that the force changes linearly over the year, using the “force gradient” as the distribution parameter.

Annual Rates for Discrete Ages 68-72

- VBT 2015 Ult M SM ANB

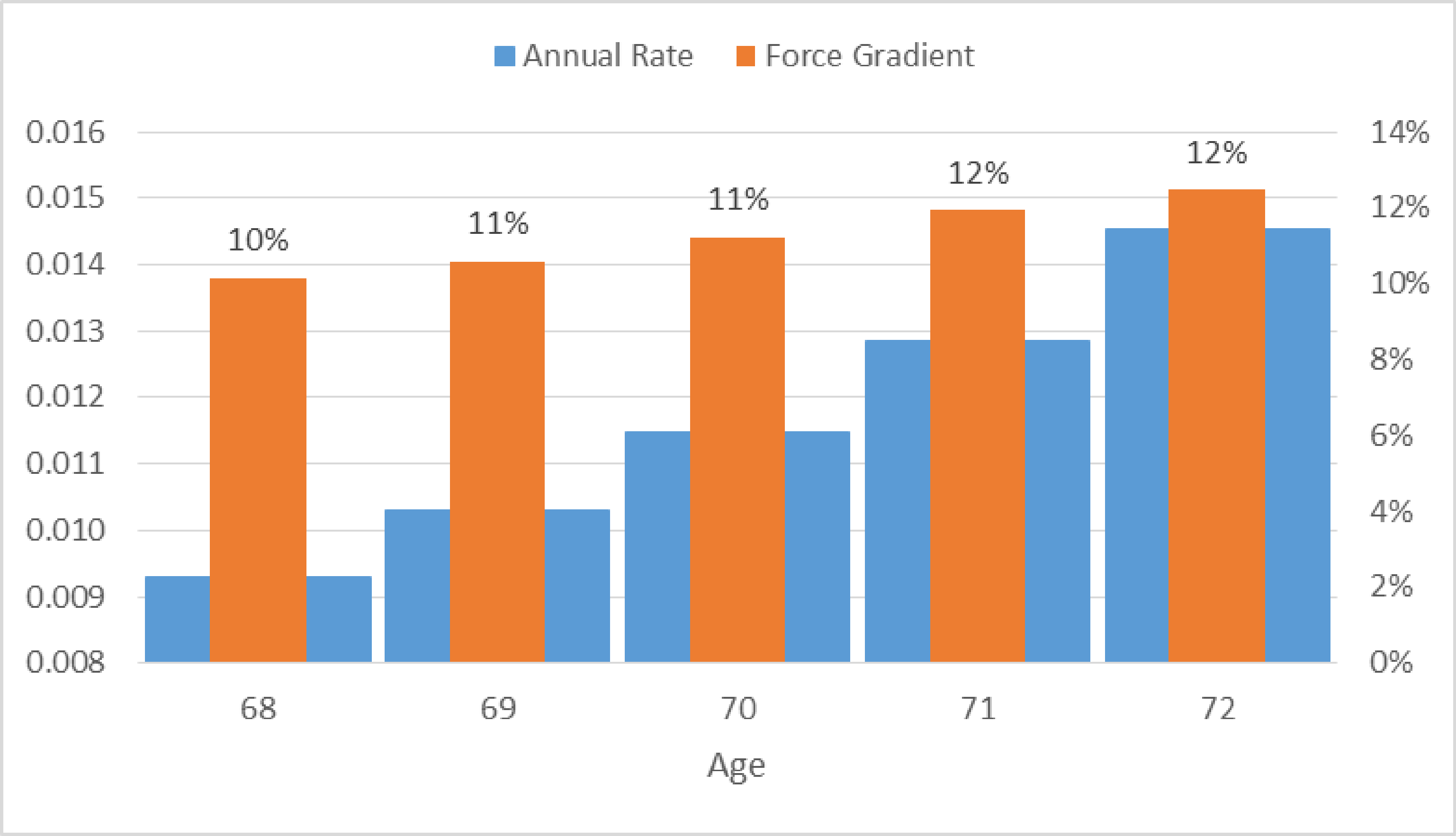


Linear Force Model

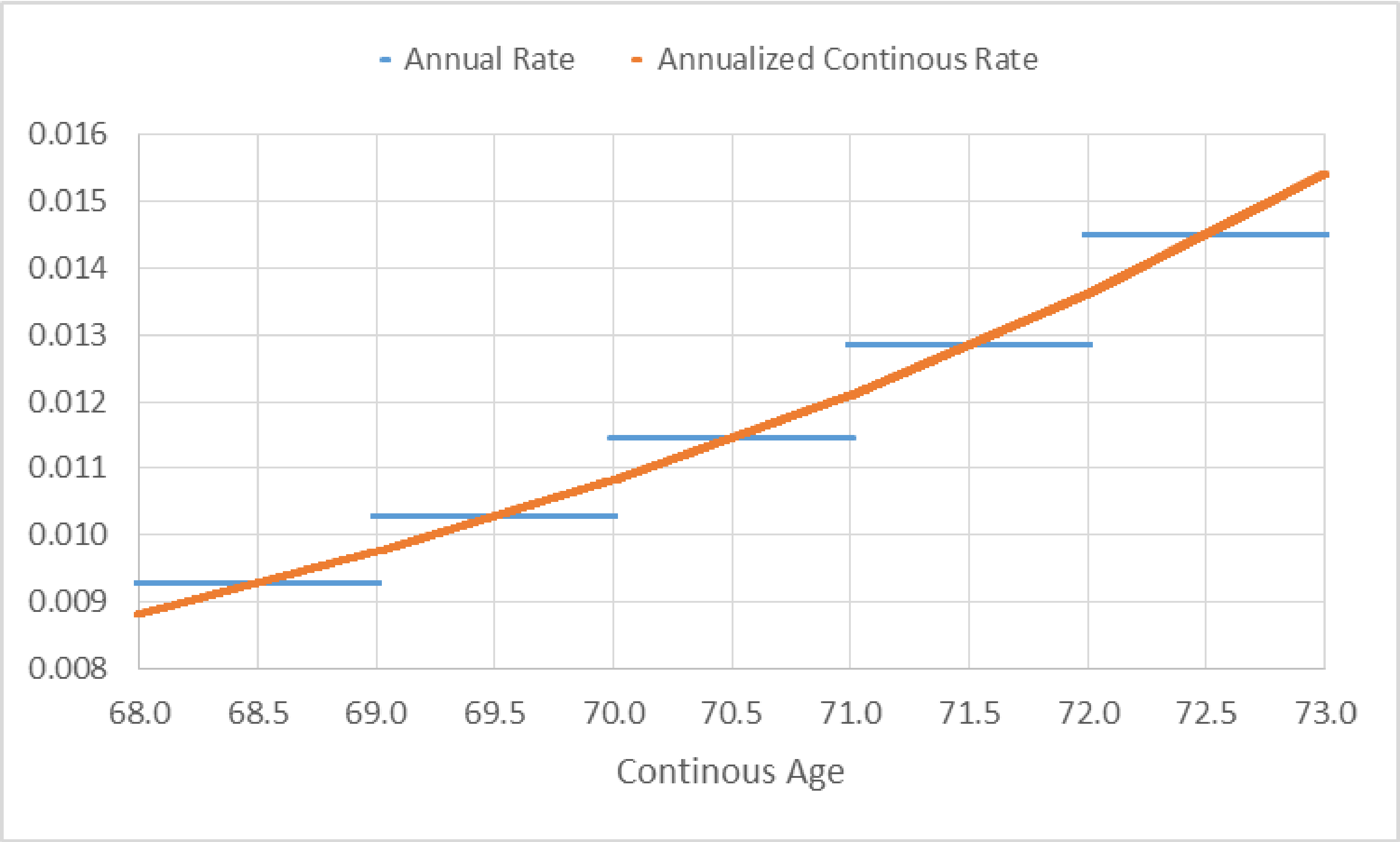
- The rates are interpolated within the year by assuming that the force of mortality changes linearly.
- The force at an exact age is the average force plus the time weighted proportion of the change in force,
  - $\mu_{x+s} = \bar{\mu}_x + (s - ½)\Delta\mu_x$.
- The change in force is the difference between the exact force at the start and end of the year:
  - $\Delta\mu_x = \mu_{x+1} - \mu_x \approx ½(\bar{\mu}_{x+1} - \bar{\mu}_{x-1})$.
- The Force Gradient is the generalized distribution parameter, defined as the change in force divided by the average force:
  - $\Delta_x = \Delta\mu_x / \bar{\mu}_x$.

Force Gradient for Discrete Ages

- Ages 68-70 VBT 2015 Ult M SM ANB



Annualized Rates for Continuous Ages

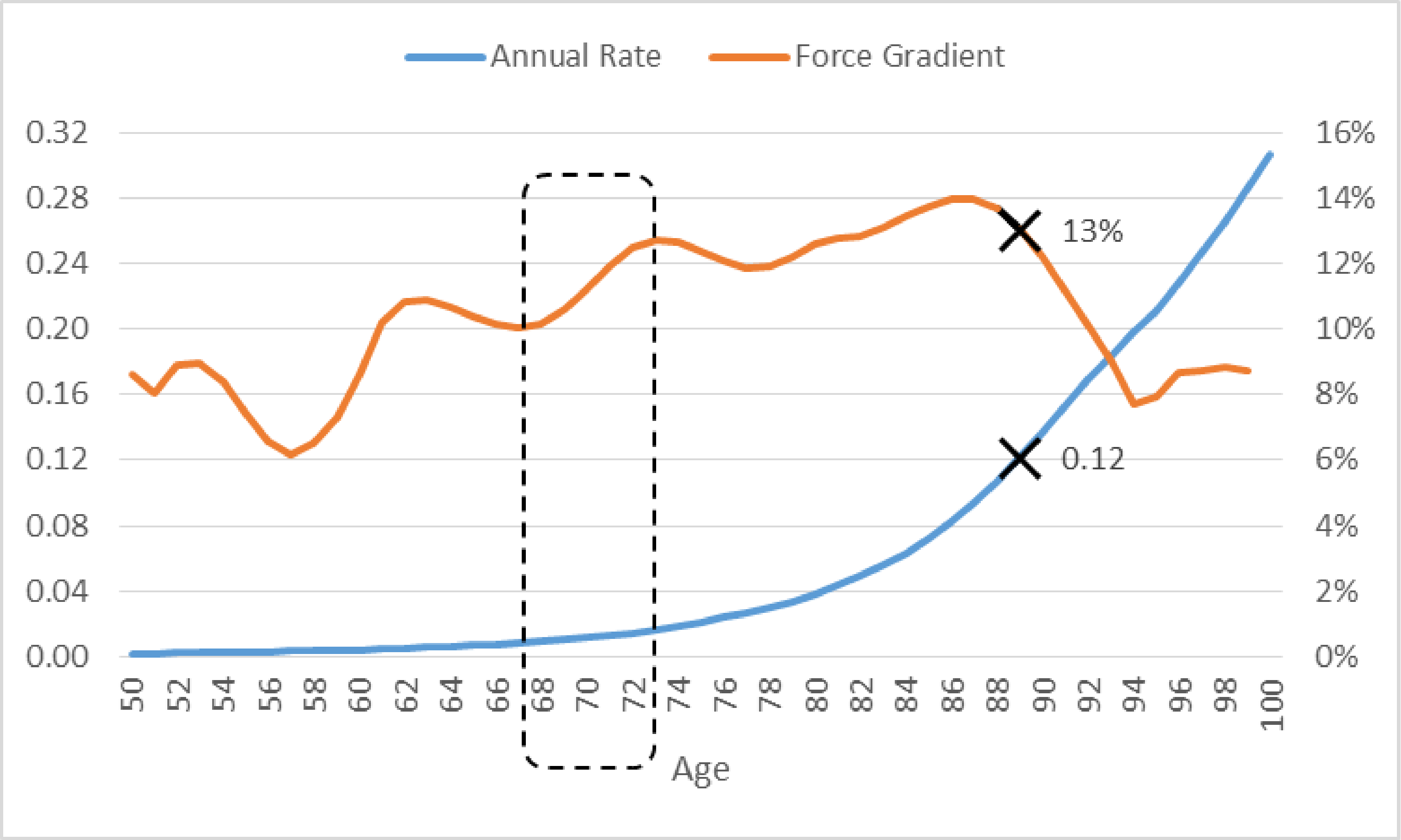


Standard Table Distributions

- The range of distributions can be found by calculating the (force) gradients for the standard tables of different types of study.
- Standard tables:
  - Mortality – VBT 2015 Ult M SM ANB,
  - Lapse - ILPS 2009 Whole Life Lives FA<5k,
  - Lapse - ILPS 2009 Term 10 Lives Age30-39.

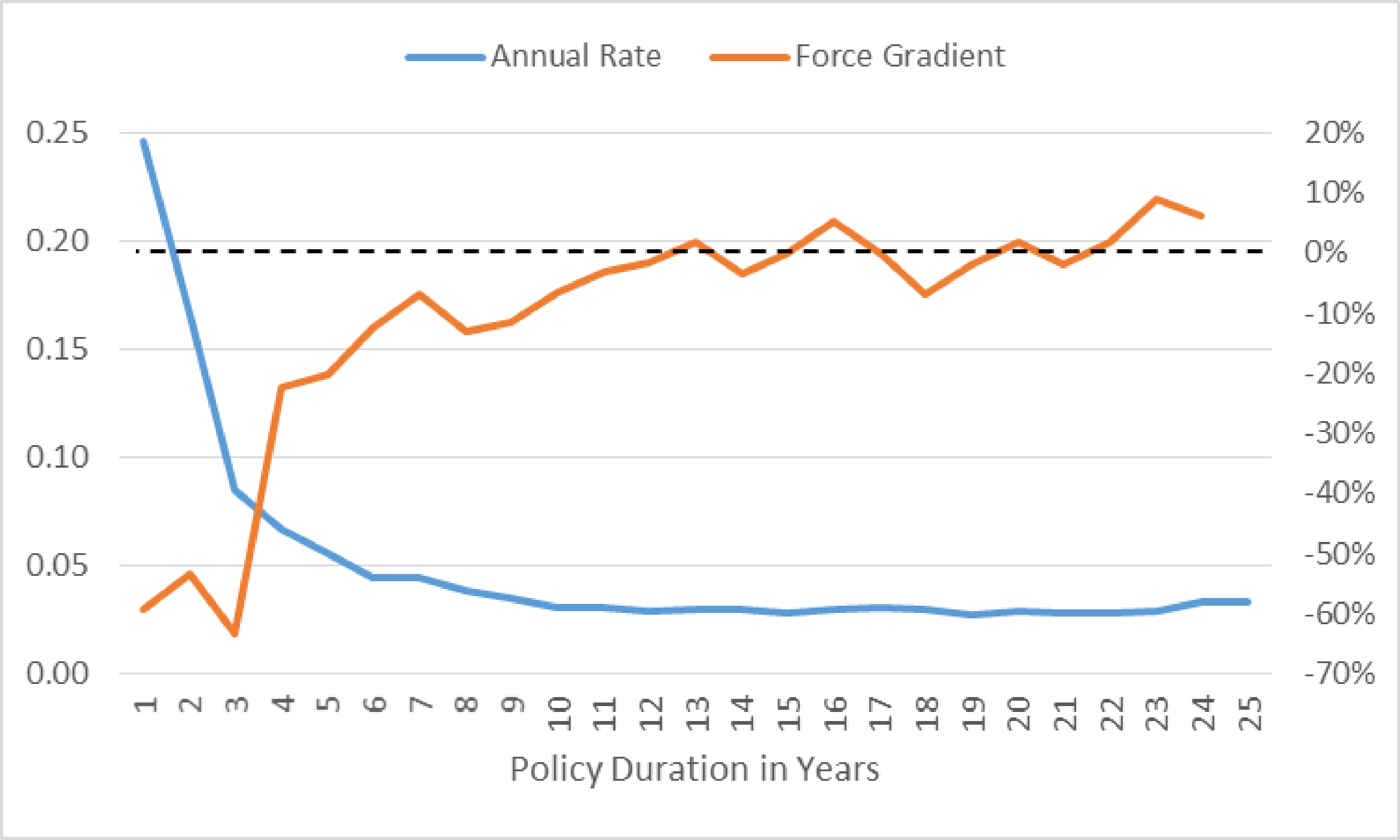
Mortality – VBT 2015

- Ages 50-100



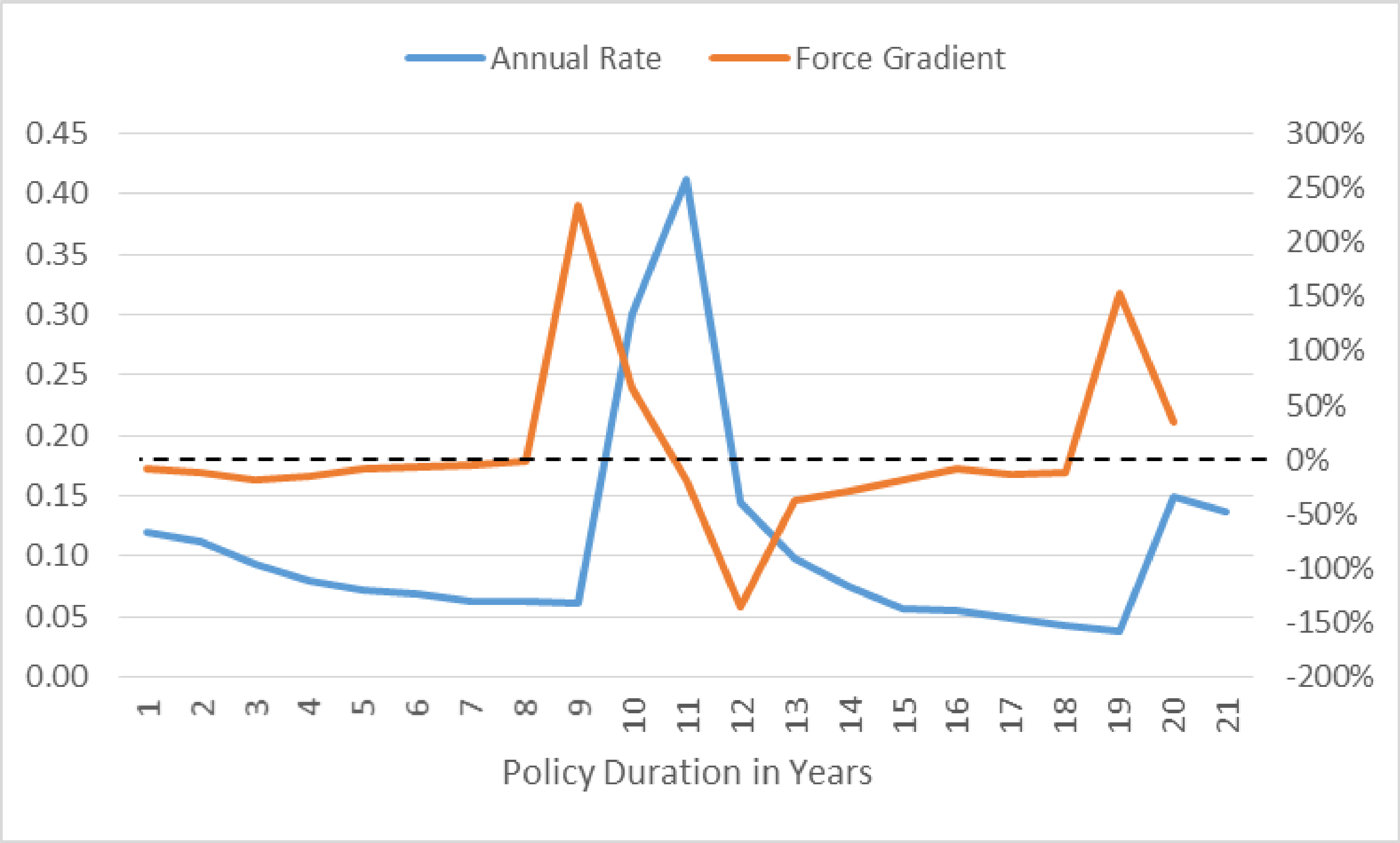
Lapse – IPS 2009 Whole Life

- Years 1-25



Lapse – IPS 2009 Term 10

- Years 1-21

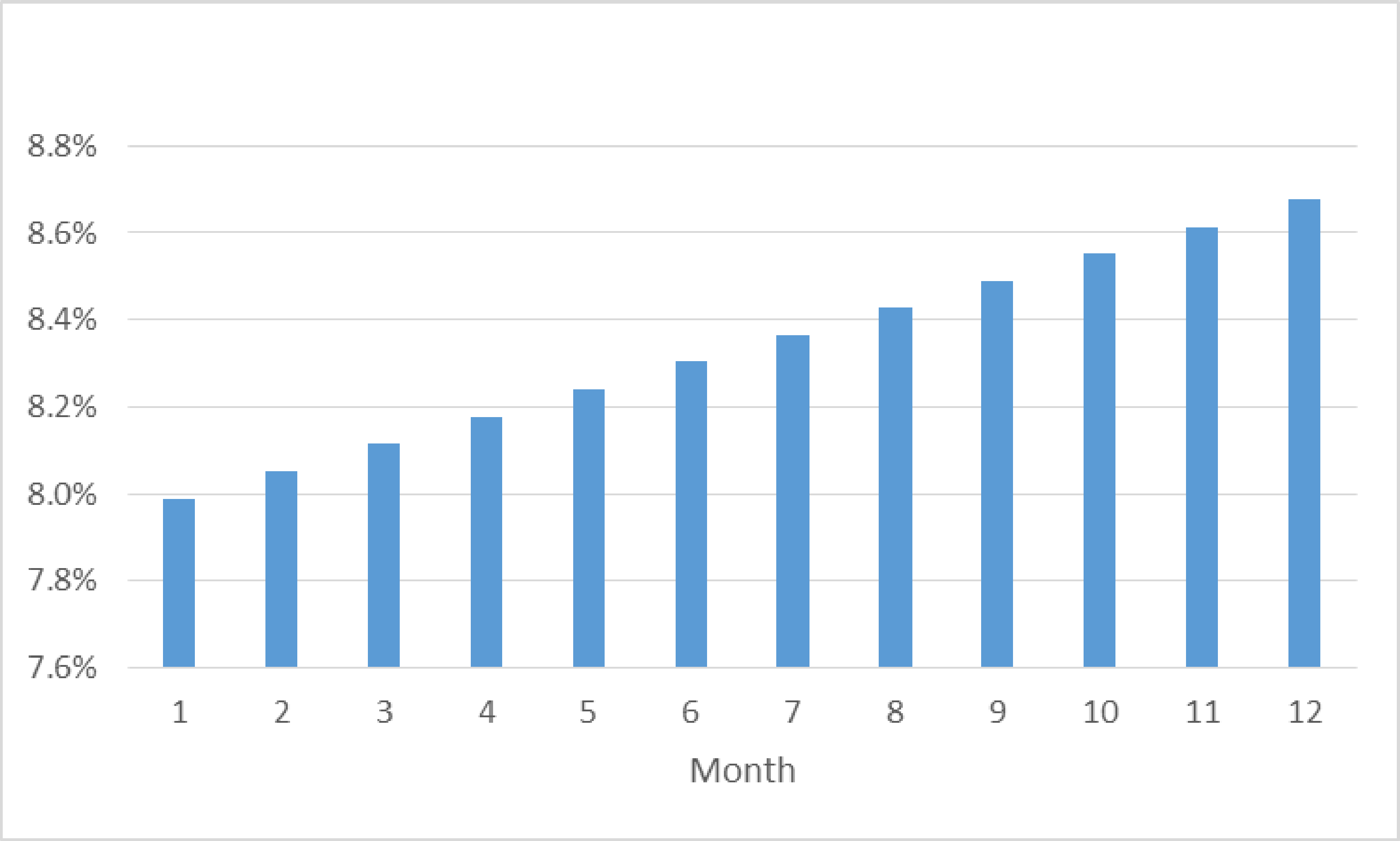


E. Testing Methods

- Example 1
  - Whole Life, Mortality, Age 69.
  - Mortality Rate 1%.
  - Force Gradient 10%, increasing.
- Example 2
  - Whole Life, Lapse, Year 4.
  - Lapse Rate 10%.
  - Force Gradient -40%, decreasing.
- Example 3
  - 10 Year Renewable Term, Renewal Lapse, Year 10.
  - Lapse Rate 50%.
  - Lapses At Year End (no lapses over the year).

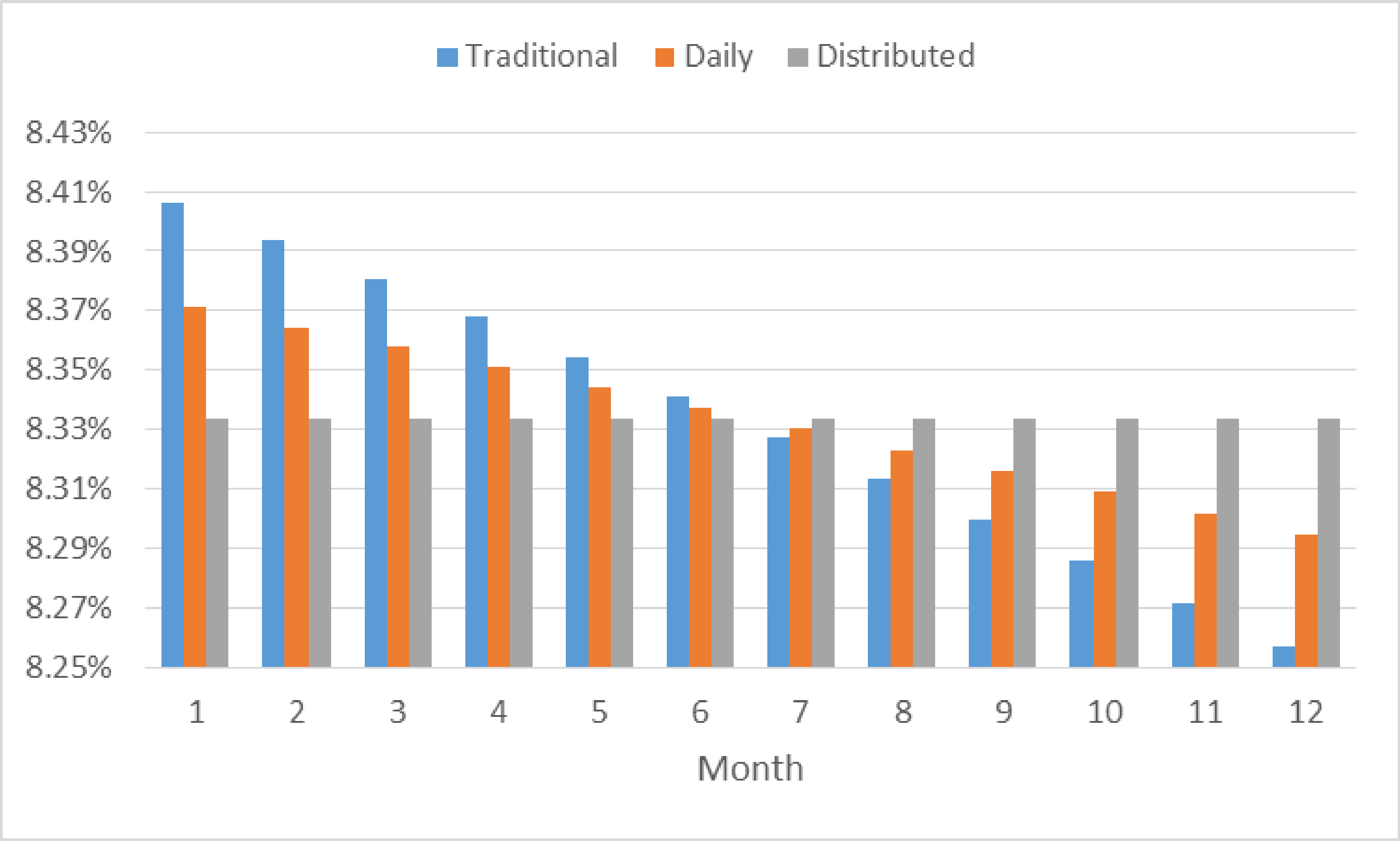
Distribution of Deaths by Month

- Mortality Rate 1%, Force Gradient 10%.



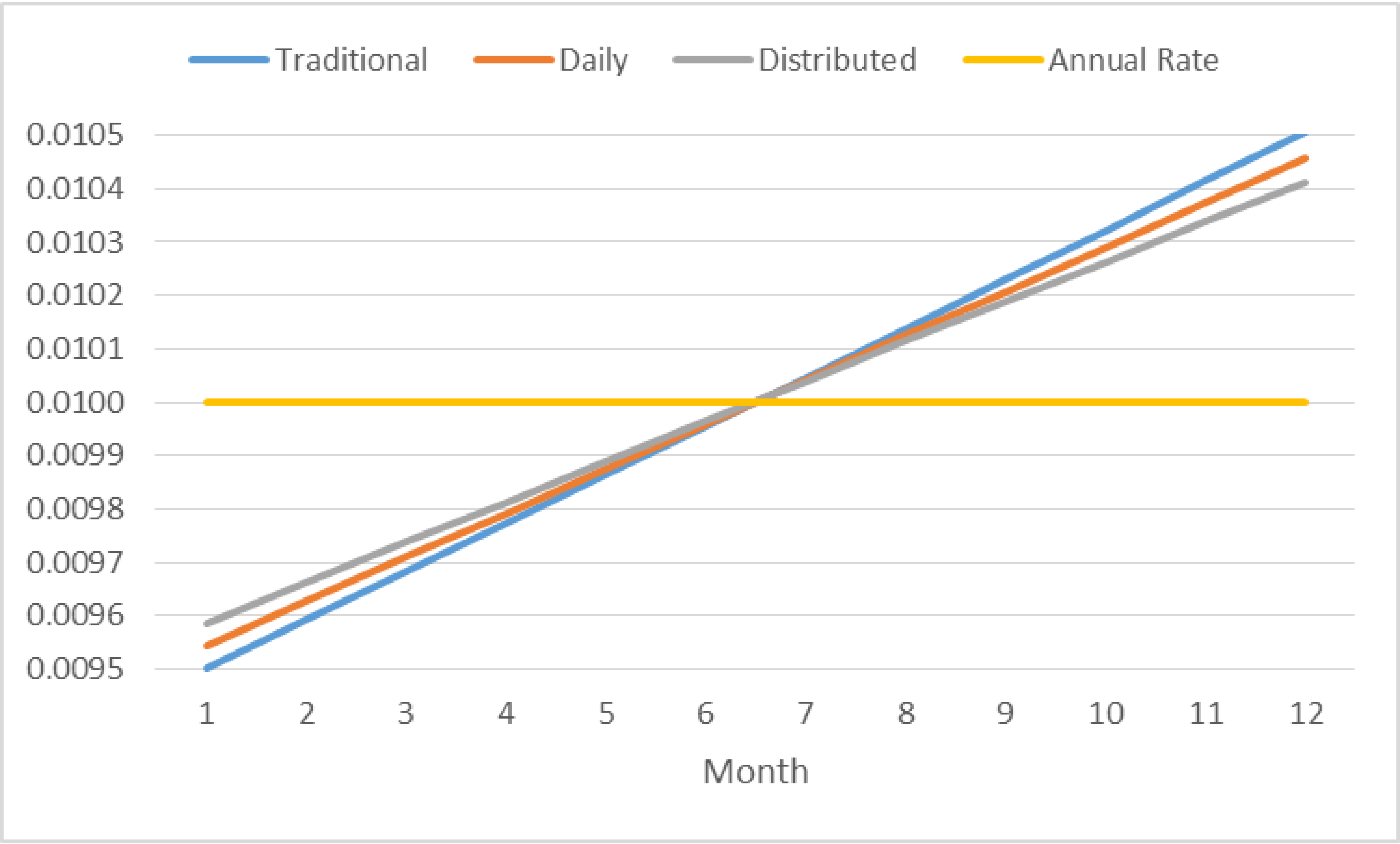
Distribution of Exposures by Month

- Mortality Rate 1%, Force Gradient 10%.



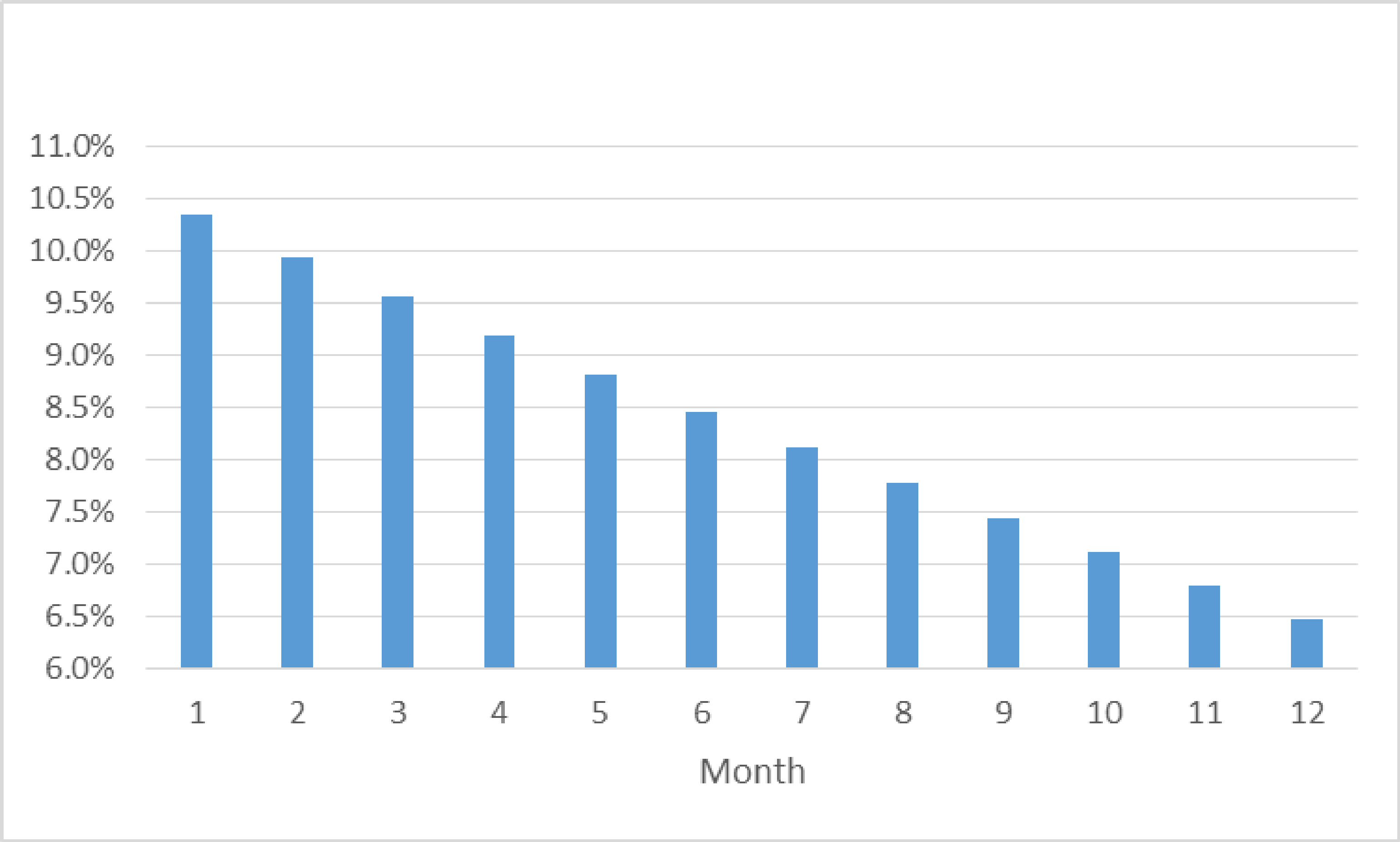
Calculated Rates by Month

- Mortality Rate 1%, Force Gradient 10%.



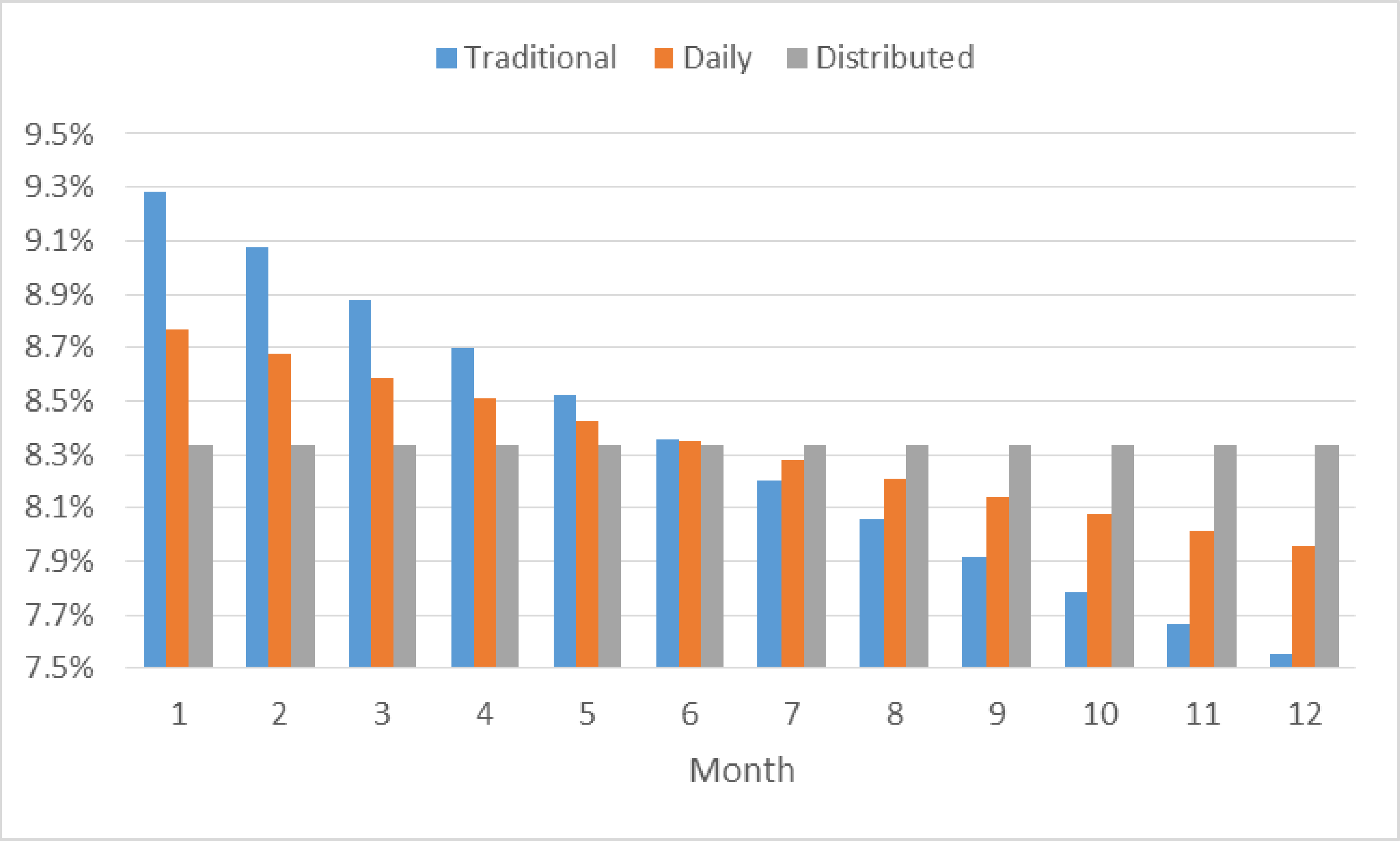
Distribution of Lapses by Month

- Lapse Rate 10%, Force Gradient -40%.



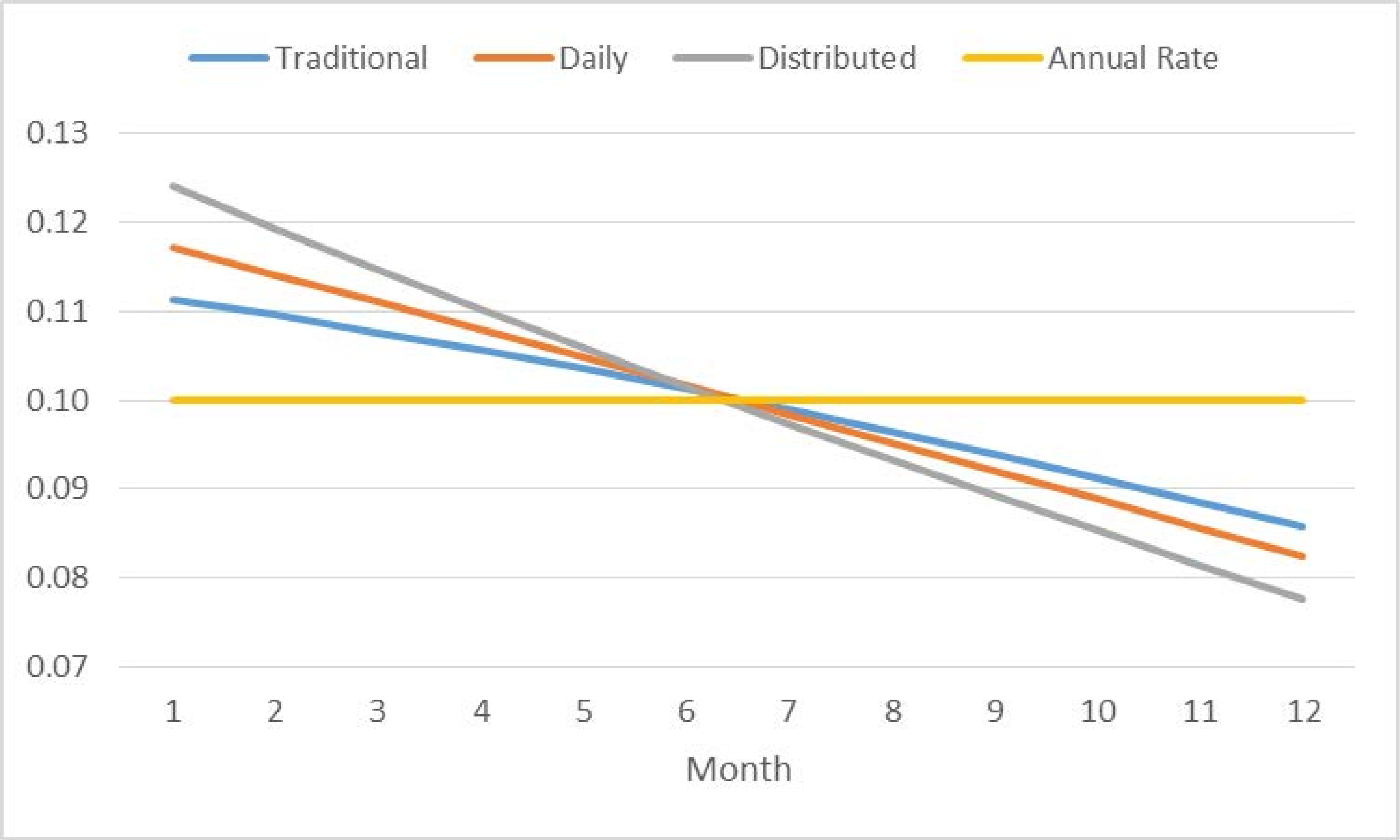
Distribution of Exposures by Month

- Lapse Rate 10%, Force Gradient -40%.



Calculated Rates by Month

- Lapse Rate 10%, Force Gradient -40%.



Error in Methods - Monthly

- The Monthly Error Estimate From the Annual Rate
  = Time at Mid Point from Mid Year
  * (Gradient * Rate + Flag * Rate Squared)
  $= T(\Delta_x q_x + F(q_x)^2) = T(\Delta_x + F q_x) q_x.$
- Where
  - Time $T = (M - 6.5)/12$,
  - Method Flag F = Traditional 1,
    Daily 0,
    Distributed -1.
- Method Assumptions expressed as Linear Force:
  - Daily: $\Delta_x = 0$
  - Distributed: $\Delta_x = q_x$
  - Traditional: $\Delta_x = -q_x$

Errors in each Month M

- Mortality Rate 1%, Force Gradient 10%.
  - Time $T = (M - 6.5)/12$.

Month	Time	Traditional	Daily	Distributed
1	-0.46	-0.00050	-0.00046	-0.00041
2	-0.38	-0.00041	-0.00038	-0.00034
3	-0.29	-0.00032	-0.00029	-0.00026
4	-0.21	-0.00023	-0.00021	-0.00019
5	-0.13	-0.00014	-0.00013	-0.00011
6	-0.04	-0.00005	-0.00004	-0.00004
7	0.04	0.00005	0.00004	0.00004
8	0.13	0.00014	0.00013	0.00011
9	0.21	0.00023	0.00021	0.00019
10	0.29	0.00032	0.00029	0.00026
11	0.38	0.00041	0.00038	0.00034
12	0.46	0.00050	0.00046	0.00041

Errors in each Month M

- Lapse Rate 10%, Force Gradient -40%.
  - The error estimates more approximate with large parameters.

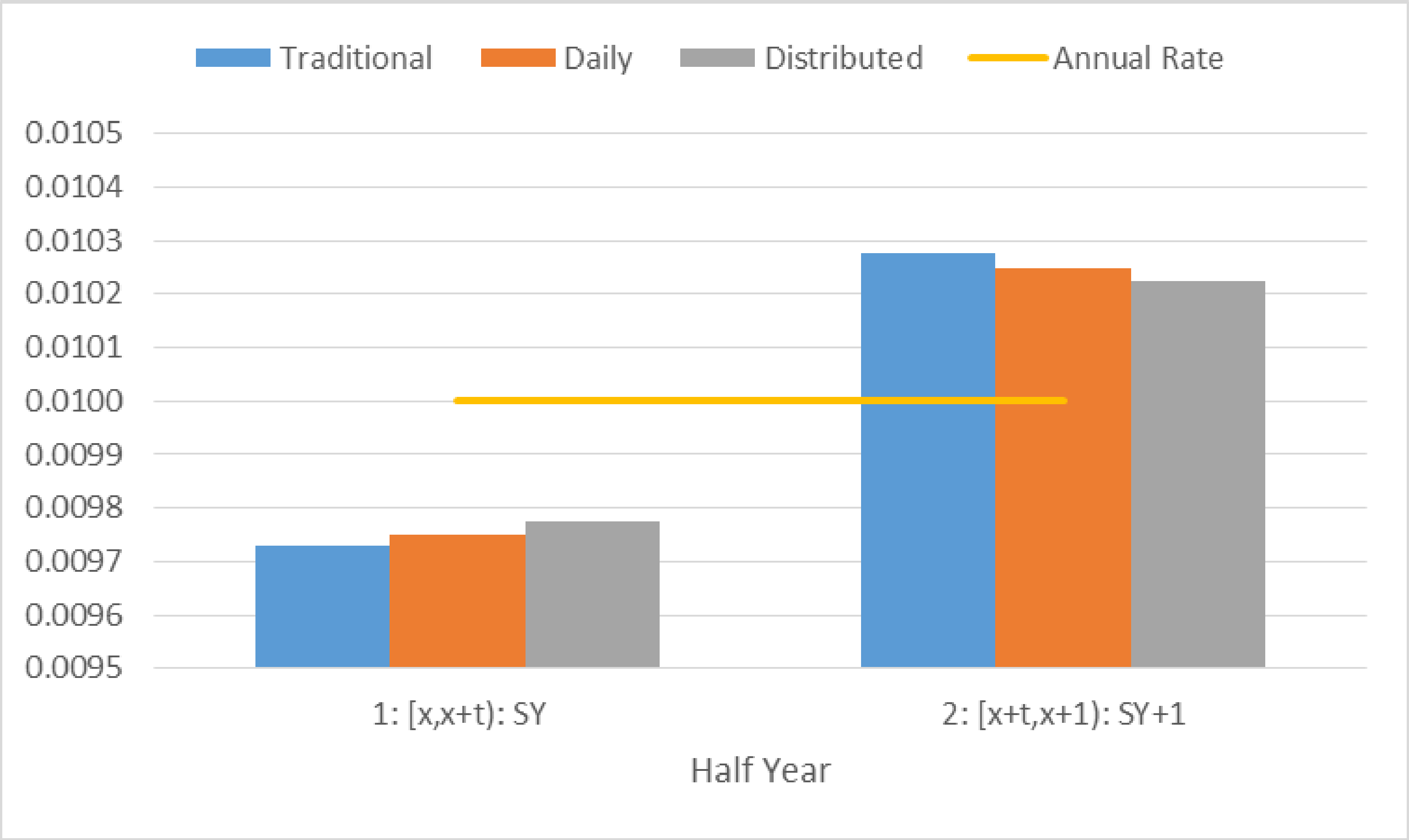
Month	Time	Traditional	Daily	Distributed
1	-0.46	0.01375	0.01833	0.02292
2	-0.38	0.01125	0.01500	0.01875
3	-0.29	0.00875	0.01167	0.01458
4	-0.21	0.00625	0.00833	0.01042
5	-0.13	0.00375	0.00500	0.00625
6	-0.04	0.00125	0.00167	0.00208
7	0.04	-0.00125	-0.00167	-0.00208
8	0.13	-0.00375	-0.00500	-0.00625
9	0.21	-0.00625	-0.00833	-0.01042
10	0.29	-0.00875	-0.01167	-0.01458
11	0.38	-0.01125	-0.01500	-0.01875
12	0.46	-0.01375	-0.01833	-0.02292

Partial Years – Half Yearly

- The monthly breakdown illustrates the method behavior, but it is partial years that contribute to a study.
- Partial years always occur at the start and end of a study, and by Study Year, every full policy year is partitioned into partial policy years.
- The split into partial years is determined by the policy anniversary for each life, and the distribution of policy anniversaries over the year will determine the aggregate partial years in the study.
- The calculated rates by half year illustrate the aggregate differences in a study where, on average, policy anniversaries occur at mid year.

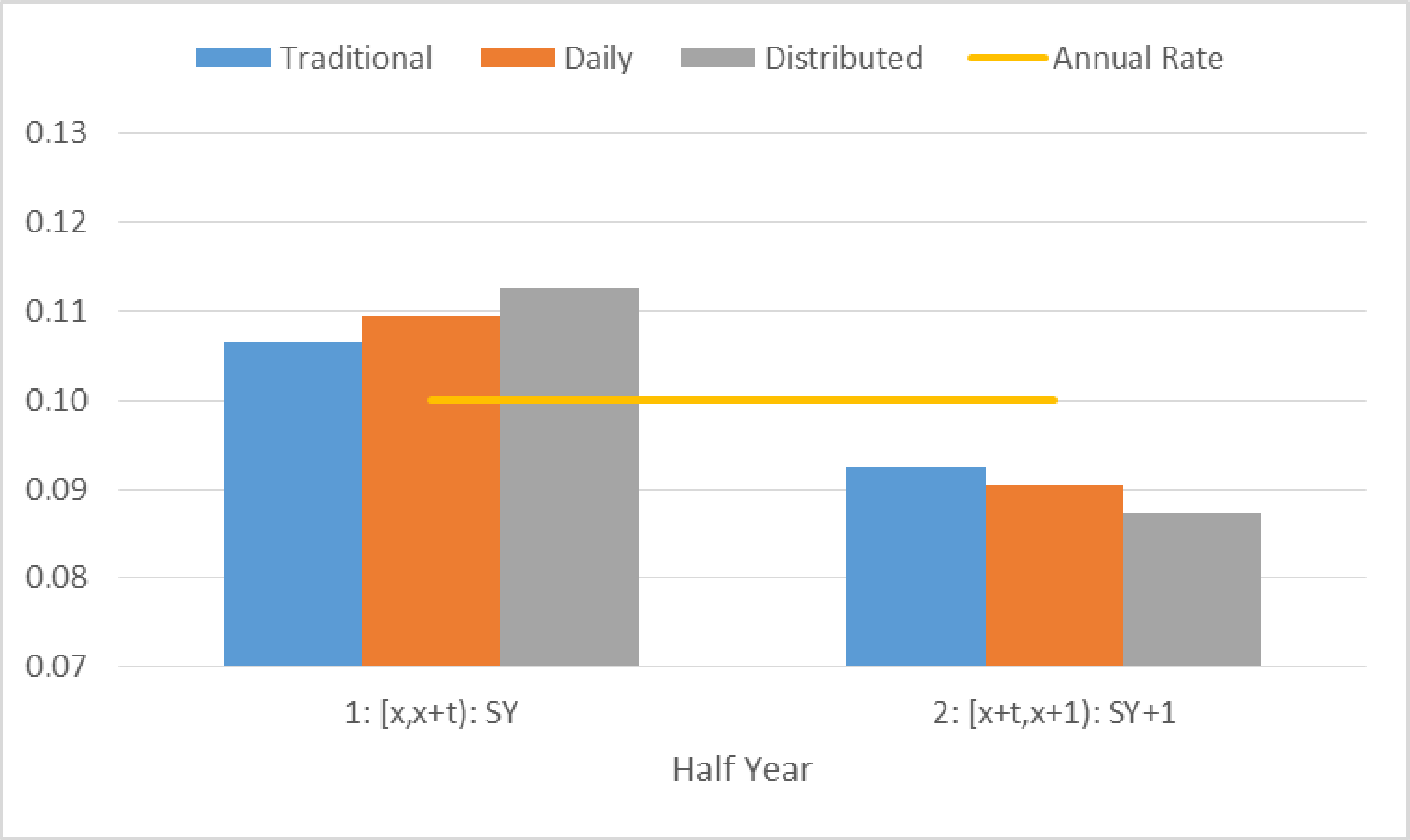
Calculated Rates by Half Year

- Mortality Rate 1%, Force Gradient 10%.



Calculated Rates by Half Year

- Lapse Rate 10%, Force Gradient -40%.



Errors in each Half Year H

- Time $T = (H - 1.5)/2$.
- Mortality Rate 1%, Force Gradient 10%.

Half Year	Time	Traditional	Daily	Distributed
1: [x,x+t): SY	-0.25	-0.00028	-0.00025	-0.00023
2: [x+t,x+1): SY+1	0.25	0.00028	0.00025	0.00023

- Lapse Rate 10%, Force Gradient -40%.

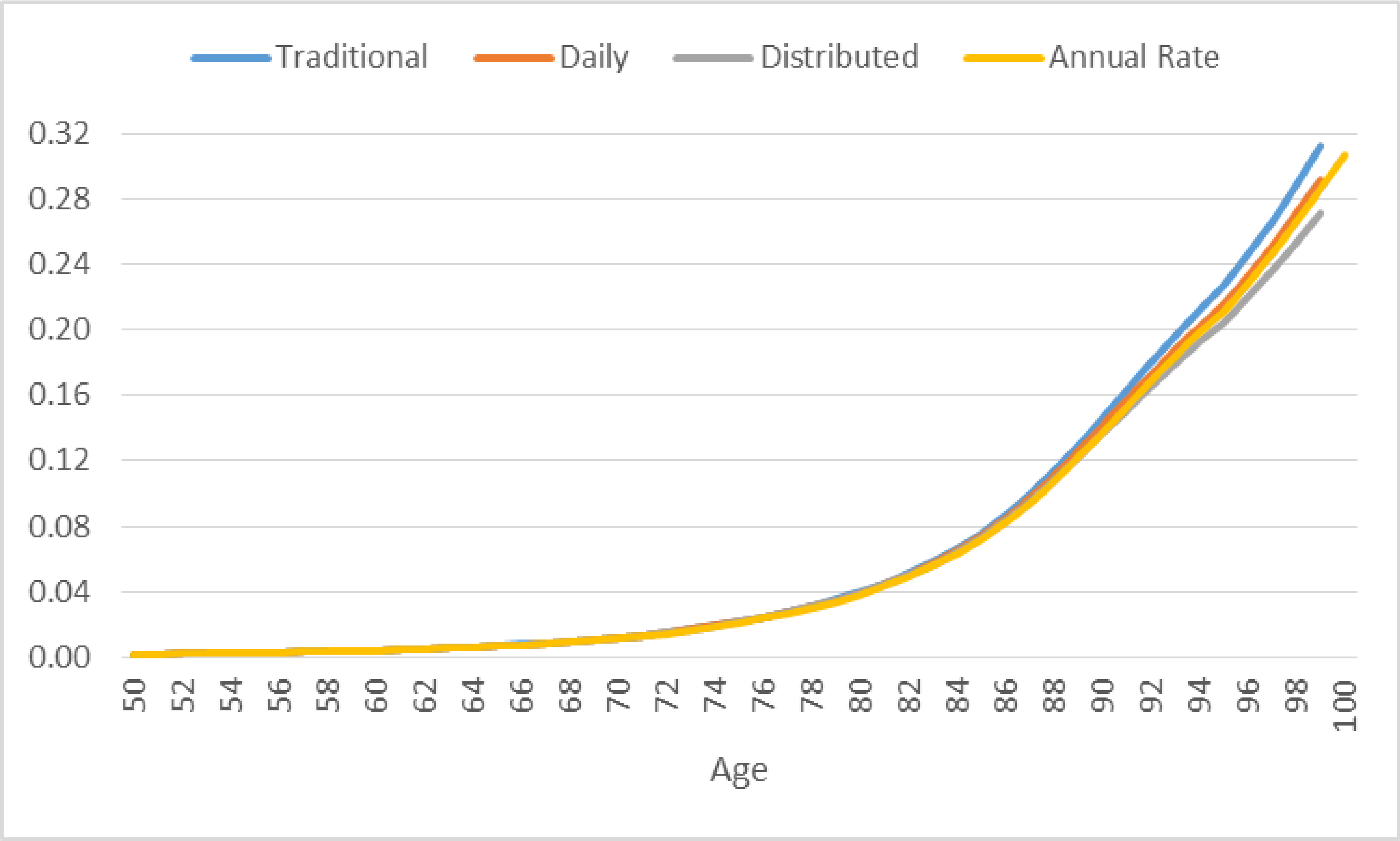
Half Year	Time	Traditional	Daily	Distributed
1: [x,x+t): SY	-0.25	0.00750	0.01000	0.01250
2: [x+t,x+1): SY+1	0.25	-0.00750	-0.01000	-0.01250

Partial Year Rate Estimates

- The partial year rates that arise in a study can be examined using the standard tables.
- For example, the Second Partial Year rate can be estimated as the annual rate plus the error estimate:
  - $q_x^{PY2} = q_x + ¼(\Delta_x + Fq_x)q_x$.
- Standard tables:
  - Mortality – VBT 2015 Ult M SM ANB,
  - Lapse - ILPS 2009 Whole Life Lives FA<5k,

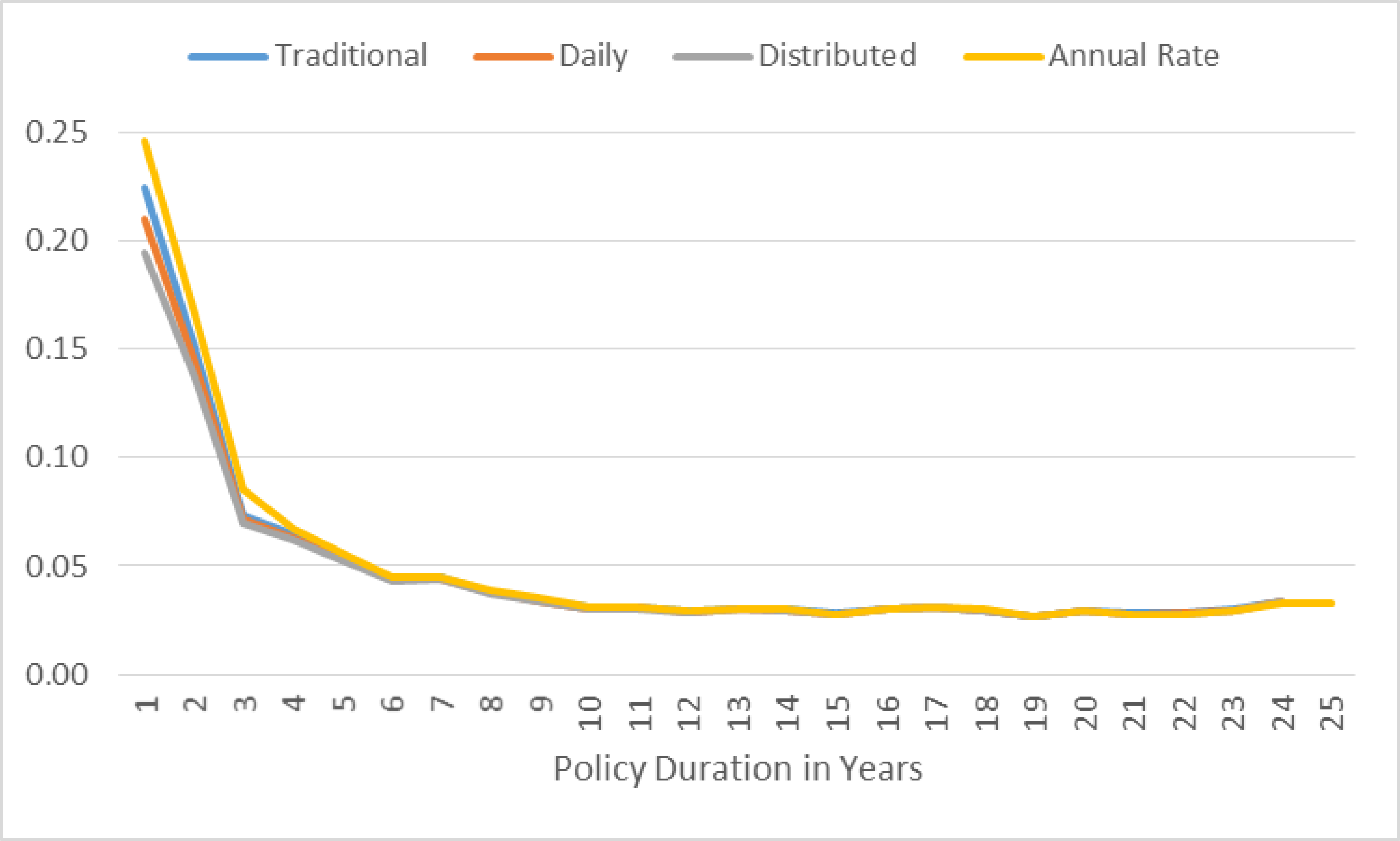
Mortality – VBT 2015

- Ages 50-100



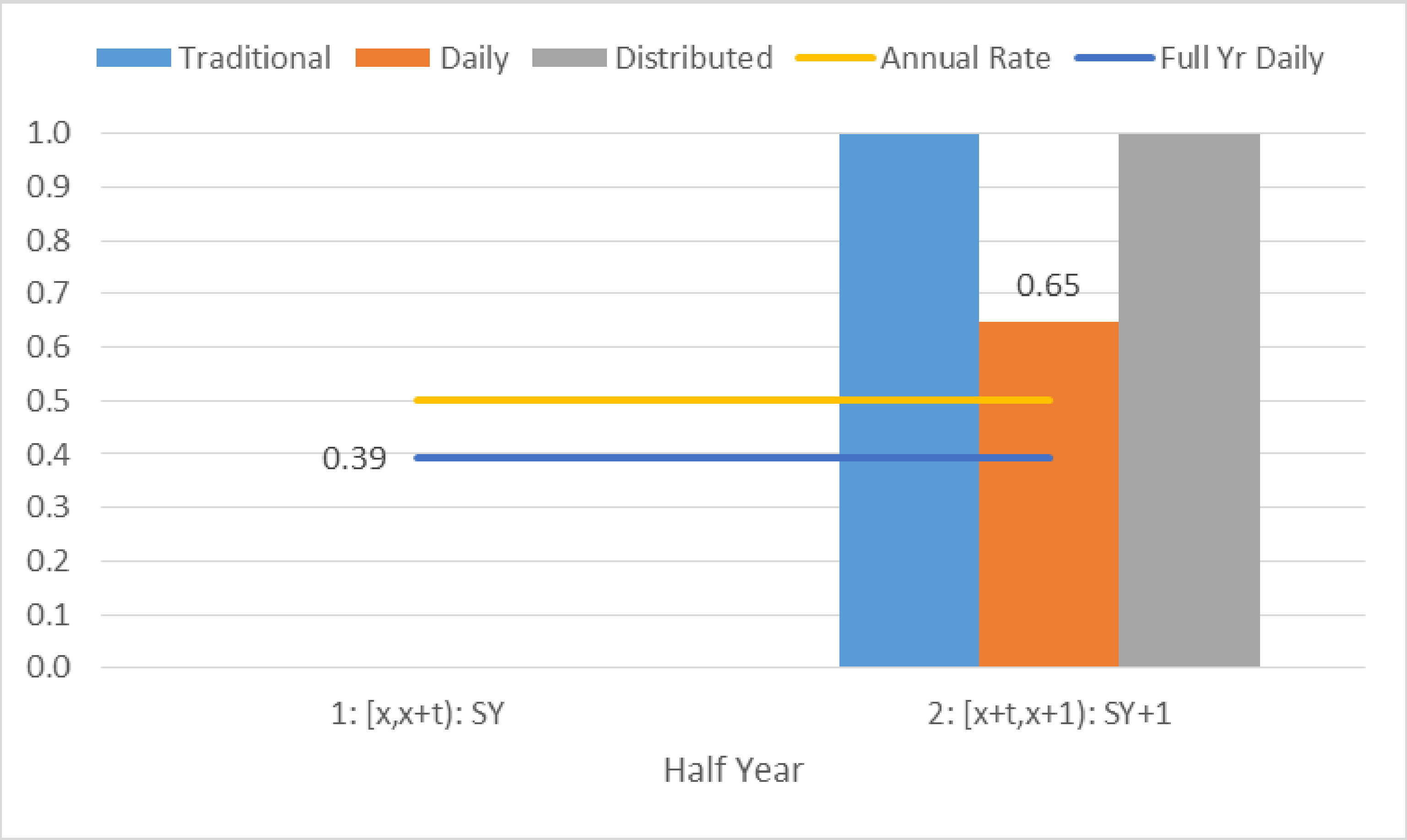
Lapse – IPS 2009 Whole Life

- Years 1-25



Calculated Rates by Half Year

- Lapse Rate 50%, At Year End.



F. Study Errors

- The half year rates just presented give an indication of the partial year errors arising from a single cohort split by Study Year, where a cohort is a set of lives that contributes to a single age for a single policy year.
- Cohorts for rates by:
  - Age – Birth Year (adjusted if using Policy Year rates),
  - Duration – Issue Year.
- The interaction of errors in a study can be illustrated by examining the contributions to all ages from:
  - One cohort in a two year study,
  - Two cohorts in a one year study,
  - Two cohorts in a two year study,
  - Four cohorts in a two year study.

One Cohort, Two Study Years

- 2 year study 2013-2014, Birth Year 1962.
- Ages and Errors by Study Year

	Exact Age			Error		
Age\StdyYr	**All**	2013	2014	**All**	2013	2014
50	**50+t,51**	50+t,51		**+e(50)**	+e(50)	
51	**51,52**	51,51+t	51+t,52	**0**	-e(51)	+e(51)
52	**52,52+t**		52,52+t	**-e(52)**		-e(52)

- e(x) = absolute error in rate (Trad/Dist) or force (Daily) for age x.
- Error signs indicate a increasing distribution to illustrate.
- Error signs would be reversed for a decreasing distribution.
- Age 51: full year, no error; by Study Year, partial year errors.

Two Cohorts, One Study Year

- 1 year study 2013, Birth Years 1961, 1962.
- Ages and Errors by Cohort Year

	Exact Age		Error		
Age\BrthYr	1961	1962	1961	1962	All
50		50+t,51		+e(50)	+e(50)
51	51+t,52	51,51+t	+e(51)	-e(51)	(+e(51)-e(51))/2
52	52,52+t		-e(52)		-e(52)

- The Error All columns shows the study error, i.e. the accumulated errors, weighted by exposure, using equal half years here for simplicity.
- The boundary ages 50 and 52 have the full method error.
- Age 51 has a complete year with errors offset and minimized.
- Independent of study method.

Error for Ages with Complete Years

- For an N year study, each age has N+1 policy years from N+1 cohorts ($n = 0, N$). The Study Error is given by:
  - $SE = \left(+E_{x,0}e_{x,0} - E_{x,N}e_{x,N}\right)/\sum_0^N E_{x,n}$.
- For cohorts, with the same decrement distribution and policy anniversary distribution, that are equal in size,
  - $SE = 0$.
- Simplifying: $SE \approx ½(+e_x - e_x)/N \equiv 0$.
- For cohorts, with the same decrement distribution and policy anniversary distribution, that increase by i% per year simple,
  - $SE = -Ni\left(E_{x,0}e_{x,0}/\sum_0^N E_{x,n}\right) = -Ni\left(E_{x,N}e_{x,N}/\sum_0^N E_{x,n}\right)$
- Simplifying: $SE \approx -½ie_x/(1 + ½Ni)$.
- Maximum Study Error $= -½ie_x$.

Error for Ages with Complete Years

- To illustrate the range of study errors, consider 3 examples with different rates and % method errors.

Example	A	B	C
Rate	1.00%	10%	50%
%Error	1.00%	10%	100%
First Pol Yr	1.01%	11%	100%
Last Pol Yr	0.99%	9%	0%

- For a range of cohort increases, the resulting % maximum study errors and study rates are:

% Max Study Error			
i	A	B	C
0%	0.000%	0.00%	0.0%
1%	-0.005%	-0.05%	-0.5%
5%	-0.025%	-0.25%	-2.5%
10%	-0.050%	-0.50%	-5.0%

Study Rate			
i	A	B	C
0%	1.00%	10.00%	50.00%
1%	1.00%	10.00%	49.75%
5%	1.00%	9.98%	48.75%
10%	1.00%	9.95%	47.50%

Two Cohorts, Two Study Years

- 2 year study 2013-2014, Birth Years 1961, 1962.
- Ages and Errors by Cohort Year

	Exact Age		Error		
Age\BrthYr	1961	1962	1961	1962	All
50		50+t,51		+e(50)	+e(50)
51	51+t,52	51,52	+e(51)	0	+e(51)/3
52	52,53	52,52+t	0	-e(52)	-e(52)/3
53	53,53+t		-e(53)		-e(53)

- The boundary ages 50 and 53 have the full method error.
- Ages 51 and 52 have 1/3rd of the method error.
- There are no ages with complete years of age, i.e. where the sum of the time in each age range is an integer.

Four Cohorts, Two Study Years

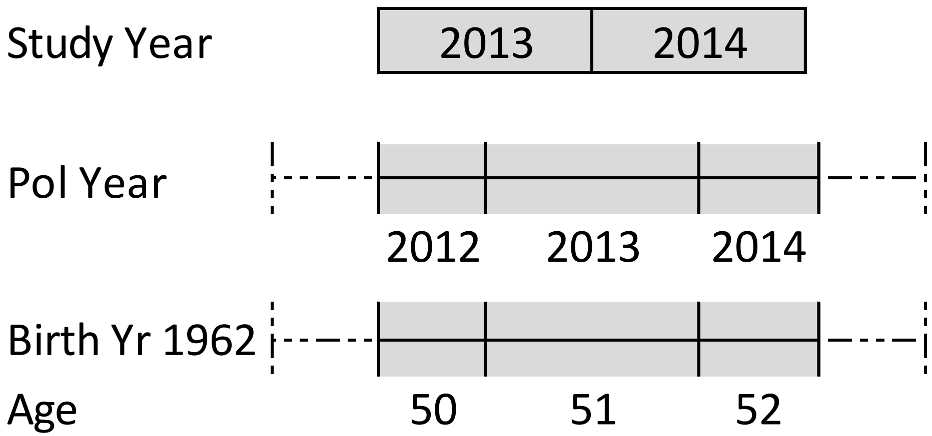
- 2 year study 2013-2014, Birth Years 1959-1962.
- Ages and Errors by Cohort Year

	Exact Age				Error				
Age\BrthYr	1959	1960	1961	1962	1959	1960	1961	1962	All
50				50+t,51				+e(50)	+e(50)
51			51+t,52	51,52			+e(51)	0	+e(51)/3
52		52+t,53	52,53	52,52+t		+e(52)	0	-e(52)	(+e(52)-e(52))/4
53	53+t,54	53,54	53,53+t		+e(53)	0	-e(53)		(+e(53)-e(53))/4
54	54,55	54,54+t			0	-e(54)			-e(54)/3
55	55,55+t				-e(55)				-e(55)

- Ages 52,53: complete years with errors minimized.
- Ages 50,51/54,55: increasing tail of errors to the boundary ages.
- N year Study, M>N cohorts: M+N Ages, M-N complete Ages with minimized error; N tail ages from each boundary Age with errors.

Policy Year Attribute

- Policy Year
  - = the Calendar Year End that falls in the Policy Year.
- This allows the contributions from full and partial policy years to be analyzed separately in a calendar year study, and hence, allowing the study to use the policy-year study period within the calendar-year study period.



One Cohort by Policy Year

- 2 year study 2013-2014, Birth Year 1962.
- Ages and Errors by Policy Year

	Exact Age				Error			
Age\PolYr	**All**	2012	2013	2014	**All**	2012	2013	2014
50	**50+t,51**	50+t,51			**+e(50)**	+e(50)		
51	**51,52**		51,52		**0**		0	
52	**52,52+t**			52,52+t	**-e(52)**			-e(52)

- There are separate contributions by Age and Policy Year.
- The cohort populates cells along the diagonal.

Four Cohorts by Policy Year

- 2 year study 2013-2014, Birth Years 1959-1962.
- Ages and Errors by Policy Year

	Exact Age			Error			
Age\PolYr	2012	2013	2014	2012	2013	2014	All
50	50+t,51			+e(50)			+e(50)
51	51+t,52	51,52		+e(51)	0		+e(51)/3
52	52+t,53	52,53	52,52+t	+e(52)	0	-e(52)	(+e(52)-e(52))/4
53	53+t,54	53,54	53,53+t	+e(53)	0	-e(53)	(+e(53)-e(53))/4
54		54,55	54,54+t		0	-e(54)	-e(54)/3
54			55,55+t			-e(55)	-e(55)

- The Age/Policy Year table is more compact than the Age/Cohort Year table.
- Full and partial years analyzed can be separately.
- Filtering on 2013 gives a policy-year study period.

Test Data by Policy Year

- 2 year study 2013-2014, Birth Years 1959-1962.

Exposure

	Pol Yr			
Age	**2012**	**2013**	**2014**	**Total**
50	996			996
51	977	2,087		3,065
52	882	2,050	1,088	4,020
53	814	1,855	1,073	3,742
54		1,717	973	2,690
55			895	895
Total	**3,669**	**7,709**	**4,029**	**15,408**

Distribution

	Pol Yr		
Age	**2012**	**2013**	**2014**
50	100%		
51	32%	68%	
52	22%	51%	27%
53	22%	50%	29%
54		64%	36%
55			100%
Total	**24%**	**50%**	**26%**

- Contributions for complete Ages, 52,53:
  - Policy Year 2012: second partial year, 22% exposure
  - Policy Year 2013: full year, 50% exposure
  - Policy Year 2014: first partial year, 28% exposure

Test Data by Policy Year and Study Year

- 2 year study 2013-2014, Birth Years 1959-1962.

Exposure

Pol Yr	2012	2013		2014	Total
StdyYr	**2013**	**2013**	**2014**	**2014**	
50	996				996
51	977	1,126	961		3,065
52	882	1,115	935	1,088	4,020
53	814	1,002	853	1,073	3,742
54		937	781	973	2,690
55				895	895
Total	**3,669**	**4,180**	**3,530**	**4,029**	**15,408**

Distribution

Pol Yr	2012	2013		2014
StdyYr	**2013**	**2013**	**2014**	**2014**
50	100%			
51	32%	37%	31%	
52	22%	28%	23%	27%
53	22%	27%	23%	29%
54		35%	29%	36%
55				100%
Total	**24%**	**27%**	**23%**	**26%**

- Similar contributions for:
  - First Partial Years: PY 2013, SY 2013 and PY 2014, SY 2014
  - Second Partial Years: PY 2012, SY 2013 and PY 2013, SY 2014

Duration by Policy Year - Lapse

- 2 year study 2013-2014, Issue Years 2011-2014.
- Durations and Errors by Issue Year.

	Exact Duration			Error			
Dur\PolYr	2012	2013	2014	2012	2013	2014	All
1	1+t,2	1,2	1,1+t	-e(1)	0	+e(1)	(-e(1)+e(1))/4
2	2+t,3	2,3	2,2+t	-e(2)	0	+e(2)	(-e(2)+e(2))/4
3		3,4	3,3+t		0	+e(3)	+e(3)/3
4			4,4+t			+e(4)	+e(4)

- The error signs reversed for a decreasing distribution.
- The latest cohort is in the final study year, resulting in,
  - only four durations, and
  - complete years at lowest durations, with no lower tail of errors.

Eliminating Errors

- Adjust Study Rates
  - Examine the exposure distributions by Age and Policy Year, and identify the Ages with errors.
  - Estimate the annual rate using the partial year rate and an estimate of the gradient.
  - That is, for the Daily method and highest Age,
    - $q_x = q_x^{PY1} / (1 + ¼\Delta_x)$.
- Use the Weighted Exposure method
  - Identify the gradients in the study, and directly weight the daily exposure to reflect the gradients.

Weighted Exposure

- Weighted Exposure
  - Calculate exposure for force.
  - Estimate the gradients, Δ_x, using actual data or a standard table proxy.
  - Calculate weights for partial years or months: $w_x = 1 + T\Delta_x$.
  - Calculate weighted exposure: $E_x^W = w_x E_x^F$.
- Rate
  - Calculate average force of mortality,
    - $\bar{\mu}_x \approx d_x / E_x^W$.
  - Calculate the annual rate,
    - $q_x = 1 - e^{-\bar{\mu}_x}$.
- Assumption
  - The force of mortality changes linearly over the year.

Conclusions - Full Policy Years

- The Traditional and Distributed methods are exact, independent of decrement distribution.
- The Daily method is
  - robust where the linear force assumption holds, with errors up to 1% of the true rate for high gradients,
  - not robust for highly skewed distributions, e.g. at year end, with errors up to 25% of the true rate.

Conclusions - Partial Policy Years

- For small rates with large gradients, e.g. working-age mortality, the methods produce similar errors, around 3% of the true rate.
- For large rates with large positive gradients, e.g. higher-age mortality, the Traditional method has the largest error, up to 10% of the true rate.
- For large rates with large negative gradients, e.g. early-duration lapses, the Distributed method has the largest error, up to 20% of the true rate.

Conclusions – Study Error

- The extent to which errors flow through to the study is determined by the number of Policy Year contributions in a given Age.
- For Ages that have contributions from all Policy Years, the errors will be minimized depending on relative cohort sizes. With cohorts increasing up to 10% pa, errors are less than 1% of the true rate for mortality and lapses and less than 2.5% for renewal lapses.
- There is an increasing tail of errors at the highest Ages, and may be a similar tail at the lowest Ages. For Ages with errors, the annual rate can be estimated from the calculated rate and an estimated gradient.

Questions?