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Aging and Retirement

Liability-Driven Investment

Benchmark Model

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Executive Summary

The prevalence of defined benefit (DB) pension plans has been shrinking in the past two decades, due in part to decreasing interest rates, longevity risk and high market volatility. Plan sponsors moved from DB plans to DC plans or a hybrid of both to transfer the DB risks to plan participants or the insurance market. For the remaining DB plans, liability-driven investment (LDI) strategies are getting more popular to reduce the risks associated with pension liability. The philosophy of LDI for pension funds is similar to asset liability management in the banking and insurance industry. The goal of LDI asset investment is not to maximize the return of the asset portfolio but to maximize asset performance relative to plan liability. It is a systematic approach to balancing pension liability hedging and pension asset growth. It considers a wide range of risks, including interest rate risk, inflation risk, longevity risk, credit risk and foreign-exchange risk.

A few challenges exist in LDI modeling and implementation for pension plans. In a low-interest-rate environment, alternative investments such as real estate, private equity, infrastructure and commodities are used to support the high expected asset return. These asset classes are less liquid or largely driven by specific factors in addition to the general market trend. Asset allocation and selection can be done at a more granular level than market indices. For example, high-dividend large-cap stocks and long-term bonds may be preferred. This requires modeling at the asset subclass level. The relationships between alternative assets/subclasses and liabilities need to be studied, as the general relationship between the asset market values and pension liabilities may not be appropriate for them. The interdependency of asset subclasses in a pension asset portfolio also needs careful analysis. LDI cares about not only normal scenarios but also stress scenarios. The interdependency is usually stronger in stress scenarios. Linear relationships assumed in correlation matrix approaches may need to be adjusted to better handle nonlinear relationships. Because of the long-term nature of DB pension plans, economic cyclical patterns also need to be embedded in economic scenarios. LDI analysis and decision making need to be based on holistic, consistent and realistic scenarios.

To understand these challenges, we have developed an LDI benchmark model. It starts from an economic scenario generator, which includes fundamental economic factors, asset returns and other factors. Macroeconomic models are used to represent the general economy and systemic risk. Fundamental economic factors such as GDP growth rates, interest rates, credit defaults, credit spread, unemployment rates and inflation rates are generated based on the macroeconomic models to maintain both contemporary and intertemporal consistency.

With the fundamental economic factors, the return of each asset class is assumed to be the sum of two factors: a systemic factor and an idiosyncratic factor. The systemic factor is determined based on the relationship between the asset return and the general economy governed by fundamental economic factors. The idiosyncratic factor is determined by the unique features of each asset class. On average, the idiosyncratic factor is independent from the systemic factor but could have higher volatility and nonzero correlation with the systemic factor during economic recessions.

The economic scenario generator provides a bridge between assets and liabilities in the LDI benchmark model. The exposure of assets and liabilities to common factors embedded in the scenarios can be assessed. With the LDI benchmark model, the financial outcome of LDI strategies can be predicted under different scenarios. This model allows users to test different LDI strategies for asset allocation purposes. The LDI benchmark model can be used to predict the benchmark funding status, given a chosen asset allocation strategy and an economic scenario. Based on that, it is possible to evaluate the performance of LDI in the short or medium term. Running different asset allocation plans through the model enables users to choose the most appropriate liability-driven asset allocation plan, considering the return, risk and plan sponsor's risk appetite. The model is also helpful for measuring, optimizing and managing the risks arising from pension asset-liability mismatch.

This report contains three examples of applying the LDI benchmark model to LDI analysis. First, an example of asset class analysis uses the LDI benchmark model to analyze the appropriateness of each asset class in DB plan investment. Both historical performance and future predicted performance are used to understand the trade-off between risk and return for each asset class. Next is an investment decision example, which shows how the LDI benchmark model can be used to determine the optimal asset allocation plans, given the plan liability and plan sponsor's risk appetite. Finally, a

risk capital example uses a simplified approach to assess the economic capital requirements for different asset allocation plans. An Excel tool accompanying this report also is developed to illustrate the LDI benchmark model with numerical examples.

Section 1: Introduction

A defined benefit (DB) pension plan has the obligation to provide retirement income to its participants. For a DB plan to be able to meet its commitments, its asset value must grow in line with its liability value. During the latter part of the 20th century, high interest rates meant fund insufficiency was less of a concern. Plan sponsors had more flexibility in making contributions. The working population was younger with smaller near-term benefit payments. However, the situation has changed. Interest rates and therefore bond returns moved to a much lower level. Equity market volatility appears to be heightened. This can significantly impair the future growth of pension assets, especially for plans with high immediate liquidity needs. With lower expected pension asset returns and lower discount rates for liability valuation, the funding level of DB plans has dropped, and required contributions have increased materially.

Mortality improvement is another challenge DB plans face. Funding levels dropped further with increased liability caused by increased longevity. Pension legislation, including the Employee Retirement Income Security Act (ERISA) in 1974 and the Pension Protection Act (PPA) in 2006, requires plan sponsors to improve the funding level and enhance the protection for plan participants. The financial impact of the difficulties faced by the DB plans is reflected in plan sponsors' financial statements as well.

DB plan sponsors have responded to these challenges in various ways. Some companies adopted defined contribution (DC) plans that transfer the investment risk and longevity risk to plan participants. Some companies transfer the risks to plan participants at the time of their retirement with lump sum payments. Some companies transfer the risks to a third party such as an insurance company by buying annuity products. Some companies make additional contributions or adjust retirement income to improve the funding status. On the investment side, focus has been moving from pure asset growth to asset performance relative to pension liability. The gradual shift to liability-driven investment (LDI) makes pension asset performance less sensitive to liability risks. Like asset and liability management (ALM), which is popular in the banking and insurance industry, LDI strategies are designed so that the sensitivities of assets and liabilities offset each other to a great extent. These strategies focus on the performance of the asset portfolio relative to the liability portfolio and seek to minimize (1) the probability that the asset value will drop below the liability value and (2) the chance of liquidity insufficiency. Long-term return is more important for liability-driven investment than short-term return. Day-to-day market gains and losses are unlikely to be fully realized for a defined benefit pension fund with a long investment horizon.

Given that pension liability is largely affected by interest rates and inflation rates, existing designs of LDI use fixed-income securities and financial derivatives to reduce the mismatch between DB plan assets and liabilities. In addition to hedging the liability risk, LDI strategies may also seek high returns by investing in riskier assets, including equity and alternative investments. According to the OECD (2014) in a report on pension funds' long-term investments, large pension funds have a growing investment in alternative assets such as real estate, private equity, infrastructure, natural resources and hedge funds. The uniqueness of LDI and the changing economic environment make it necessary to study the assumptions used for asset return projections and interdependencies in economic scenario generation.

This report explores various LDI strategies and their impact on asset allocation in DB plans. By studying various assets employed in LDI and their interdependencies, it aims to provide an LDI benchmark model that can project the performance of LDI strategies in different economic scenarios. Liability hedging strategies, detailed asset allocation choices at the subclass level (including large-cap stocks and commercial real estate), current funding level, plan sponsor's risk appetite, plan participants' demographic information, and the time horizon of performance measurement also are considered in the LDI benchmark model. The model can be used to assess the performance of different LDI strategies under deterministic and stochastic scenarios. It can be used for investment strategy design, financial projection, risk capital modeling and risk assessment. While the benchmark model presented in this report is a good starting point to incorporate empirical macroeconomic modeling into LDI modeling, we focus on the structure rather than the specific models or assumptions that should be used. Even though the model assumptions have been internally calibrated, the benchmark model should not be used as given. Model risk is high if users do not understand what the model is doing. Practitioners will have different opinions on modeling details.

The report proceeds as follows:

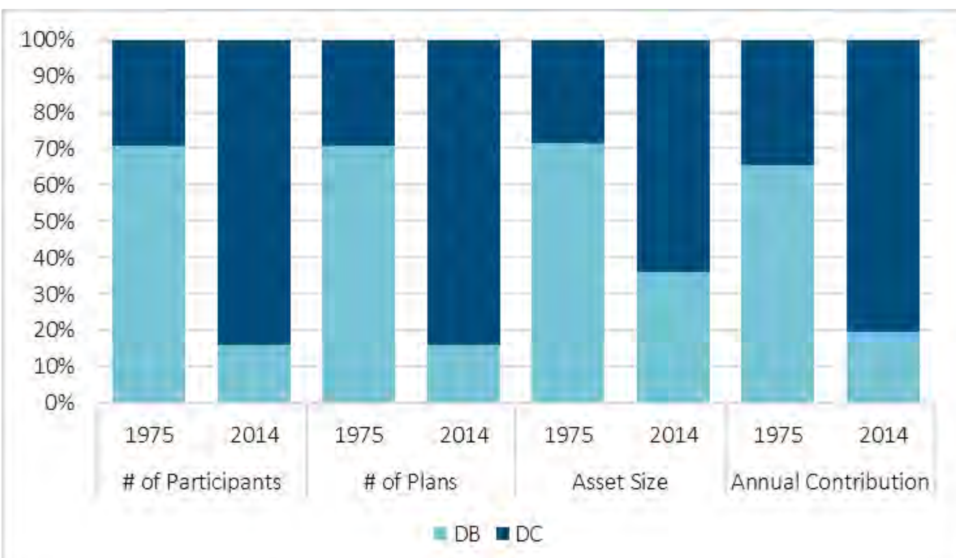
- Section 2, "Liability-Driven Investment Strategy," discusses common liability-driven investment strategies, the asset allocation of employer-sponsored defined benefit pension plans, and special asset selection considerations.
- Section 3, "Literature Review," summarizes some existing research and empirical analysis of the investment performance and dependency of a variety of asset classes, including bonds, equities and alternative investments. It lays out the background for the benchmark model explained in Section 4. This section may be skipped, as it is independent from the introduction of LDI benchmark model in Section 4.
- Section 4, "LDI Benchmark Model," describes a framework for constructing appropriate benchmarks for liability-driven investment, considering specific investment strategies, asset class dependency, pension liabilities, investment horizon and economic cycle.
- Section 5, "Application," discusses how the new benchmarking method can be used to help investment decision making and risk analysis.
- Section 6, "Conclusion," summarizes the key points of this research and concludes the main body of the report.

Section 2: Liability-Driven Investment Strategy

DB plans have become less popular in the past three decades but are still an important component of employer-sponsored pension plans. According to the U.S. Department of Labor (2016), the number of participants grew from 33 million to 38 million. Plan assets increased from a total value of $186 billion to $2,985 billion. DB pension contributions increased from an annual value of $24 million to $98 million. The number of DB plans dropped from 103,346 in 1975 to 44,869 in 2014. But as Figure 1 shows, even though DB plans grew in terms of assets and participants, DC plans grew much faster, capturing a much larger share of all these measures.

Figure 1

US Private Pension Plan Composition, 1975 vs. 2014



Sources: Employee Benefits Security Administration (EBSA). 2016. Private Pension Plan Bulletin. U.S. Department of Labor.

However, even with the drastic loss of popularity, private pension DB plans' assets are still sizable. Furthermore, most pension plans in the public sector are DB plans. Federal employee DB plans include the Civil Service Retirement System

(CSRS) and Federal Employees' Retirement System (FERS). According to Issacs (2015), the federal plans had a total asset value of $844.6 billion at the end of 2013. State and local government DB plan pension assets had a total value of $3,596 billion at the end of 2013.

This section explains the details of major LDI strategies. These strategies are the foundation of this report's proposed LDI benchmark model for DB plans.

2.1 Investment Philosophy

The philosophy of LDI is similar to asset liability management in the banking and insurance industry. The goal of LDI asset investment is not to maximize the return of the asset portfolio but to maximize the performance of assets relative to liabilities, such as maximizing the pension surplus or funding ratio. In this report, a synthetic approach is used in LDI to look at the aggregate performance of a diversified asset portfolio relative to a liability portfolio. It is different from some traditional approaches such as cash flow matching or duration matching.

2.1.1 Pension Liability Risk

Pension liabilities represent the actuarial present value of pension benefits paid, either as a lump sum or a series of payments, to plan participants after they retire. The amount is usually linked to the plan participant's expected retirement age, salary level, years of service, and accrual rate. The benefit amount may be further indexed by the inflation rate to maintain the purchasing power of pension benefits. The liability is simply the present value of future benefit payments. Two types of liability are normally measured in pension plans: settlement obligation and projected benefit obligation (PBO). Settlement only considers the expected benefit payments based on current accrued benefits. They are the benefits that plan participants will get if they terminate employment and plan membership at the valuation date. In contrast, PBO projects the payments considering factors such as salary changes and early-retirement subsidies that will arise only if employment and plan membership continue. An actuarial cost method is used to attribute a portion of the projected benefits to past, current and future service. In this report, PBO is used to represent pension liability.

Pension liability may be affected by a variety of risk factors:

1. **Wage inflation.** Future increases in salary lead to higher pension benefits. For young plan participants, because of the long-term horizon and the power of compounding, the impact of wage inflation can be material. Wage inflation can be predicted based on industry prospects, inflation rate, economic environment and the impact of union power.
2. **Discount rate.** The value of all future pension benefits is discounted back to the valuation date. The present value of a benefit payment at the end of the 40th year can be reduced by 32% if the discount rate changes from 4% to 5%. Different discount rates are used for different purposes in liability valuation, as listed in Table 1. The liability value can be affected by interest rates, corporate bond spread or risky asset return, depending on the type of discount rate used in valuation.

Table 1

Pension Liability Discount Rate

Purpose	Discount Rate
Fair valuation (exit price)	Rates prescribed for lump sum settlements or implied from annuity prices
Actuarial valuation (ongoing concern)	Expected asset return
PPA and Pension Benefit Guaranty Corporation (PBGC) contributions	24-month average of high-quality corporate bond yields
ERISA minimum funding (MAP-21)[a]	25-year average of high-quality corporate bond yields
Pension accounting (U.S. GAAP)	High-quality corporate bond yields or yields implied from annuity prices
GASB for public-sector DB plans	Expected asset return
Capital budgeting	Weighted average cost of capital (WACC) or required return on capital (hurdle rate)

[a] The Moving Ahead for Progress in the 21st Century Act (MAP-21) allows plan sponsors to incorporate the 25-year average of high-grade corporate bond yields when valuing ERISA pension liabilities. Compared with the original 24-month average of bond yields, it reduces the required pension contribution and sensitivity to current market conditions. It was further extended by the Highway and Transportation Funding Act (HATFA) in 2014.

In this report, current AA-rated corporate bond yields are used as pension liability discount rates. They are low-risk rates that immediately reflect new market conditions. Assuming that the plan sponsor still wants to manage the plan on an ongoing basis, exit price is too conservative and not appropriate. Normally, the plan sponsor still wants to gain investment return higher than the risk-free rate and invests in risky assets. Expected asset return could be too aggressive and may lead to concerns of a large probability of underfunding. A moving average of bond yields does not reflect the real market return in a timely manner. Although current AA-rated corporate bond yields are used for LDI analysis in this report, other discount rates may be used as well, because the debate on discount rates continues.

3. **Inflation rate.** The inflation rate could affect pension liability through wage inflation for active participants. Pension benefits may also be indexed to the consumer price index (CPI) rate to preserve purchasing power. The inflation adjustment may be floored and capped as well.
4. **Foreign-exchange (FX) rate.** For multinational companies, pension benefits may be denominated in foreign currencies for foreign employees. FX rates could have a significant impact on the value of pension liabilities in reporting currency.
5. **Mortality rate.** Unexpected future mortality improvements may lead to a longer benefit period, more benefit payments and a higher liability value.
6. **Plan participants' options.** Choices such as early or deferred retirement, lump sum or life annuity, and early withdrawal can change the duration and amount of pension liability as well.

Discount rate risk can be significant when discount rates are linked to market bond yields. For example, in a normal pension plan, liability duration is usually longer than asset duration, especially for a new plan or a plan with a large portion of active participants. If the liability duration is 25 years and the asset duration is 10 years, a bond yield reduction of 0.25% will increase the asset value by 2.5% and increase the liability by 6.25%. Even though the asset return is positive because of the interest rate change, the funding level will decrease due to the impact on the liability side.

For an aging or frozen DB plan, liquidity risk caused by pension liability needs to be considered as well. Available positive cash flows include pension contributions, asset income (coupon, dividend income, rent, etc.) and redemption. When available cash flows cannot cover pension benefit payments, existing assets need to be sold to meet liquidity requirements.

2.1.2 LDI Objectives

Switching from traditional pension investment strategies to LDI usually involves de-risking. In general, LDI tries to mitigate liability risks while maintaining a sufficient and sustainable expected long-term asset return.

1. **Liability hedging.** Investment strategies are designed so that the impact of major liability risks is offset, either fully or partially, by asset movement. For example, increasing the asset duration can reduce the negative impact of interest rate decreases. Hedging liability risks causes pension funding levels and required pension contributions to be less volatile. The financial impact of a pension plan on the plan sponsor's balance sheet and income statement will be more stable as well.
2. **Pension asset growth.** As with traditional pension investment strategies, high asset return is an important LDI goal. Given that the liability risk is largely hedged, high asset return can improve the funding status of the plan and may eventually reduce the required contributions to the plan.

LDI is a process of finding the optimal risk return trade-off for DB plans. The balance of the two objectives is driven by many factors:

1. **Market condition.** Current market conditions determine the cost of hedging liability risks and therefore the hedging ratio. A low-interest-rate environment, an illiquid longevity risk transfer market, and a low expected inflation rate could reduce the desirability of long-term risk hedging.
2. **Regulation.** Stringent pension regulation would require higher pension contributions in adverse situations. Less volatile funding status reduces the possibility of extra pension contributions to meet the required funding ratio. A higher level of risk hedging is preferred, given stringent pension regulation. On the contrary, countercyclical regulation that provides temporary relief of pension contributions would discourage pension liability hedging to a certain degree.
3. **Current funding level.** The funding level's impact on liability hedging is twofold. A high funding level gives the plan sponsor more flexibility for risky asset investment. A low funding level permits less tolerance of liability risks. However, a fund with a very low funding level may never have a chance to improve its funding status with full liability hedging. In that situation, achieving high asset return could dominate liability hedging in LDI.
4. **Target funding level.** The target funding level and planned pension contributions determine the required return on assets and implicitly affect LDI strategies to achieve the target funding level. A narrow gap between current and target levels means little change in the hedging level. A wide gap means material de-risking when the target level is higher than the current level, and risk taking when the target level is lower.
5. **Plan sponsor's financial strength.** Great financial strength gives the DB plan more risk-taking capability. The plan sponsor can always make extra contributions to the pension fund if necessary.
6. **Plan sponsor's risk appetite.** In addition to plan sponsor's risk tolerance driven by its financial strength, the plan sponsor's willingness to take risk also has an impact on the level of risk hedging. A low level of risk aversion means more risk taking for high returns and a lower level of hedging.

These factors could change from time to time. Therefore, LDI objectives need to be reviewed regularly, and the asset portfolio needs to be adjusted dynamically.

2.1.3 LDI Strategies

To achieve the objectives of LDI, one can use many investment strategies. To facilitate the discussion, we define the following terms:

$\mathrm{AR}_t =$ continuous pension asset return for period *t*.
$B_t =$ pension benefit payment in period *t*. Benefit payments are assumed to be evenly distributed in the period and are modeled as one single payment at the midpoint of the period.
$C_t =$ pension contribution in period *t*. Pension distributions are assumed to be evenly distributed in the period and are modeled as one single contribution at the midpoint of the period.
$A_t = A_{t-1}e^{\mathrm{AR}_t} + (C_t - B_t)e^{\frac{\mathrm{AR}_t}{2}} =$ pension asset value at time *t*.
$\mathrm{DR}_{t,i} =$ continuous discount rate at time *t* for cash flow in period *t* + *i* for pension liability valuation.
$I_t =$ investment earnings for period *t*.
$L_t = \sum_{i=1}^{\infty} e^{-\mathrm{DR}_{t,i} \cdot i} \cdot B_{t+i} =$ pension liability value at time *t*.
$S_t = A_t - L_t =$ pension surplus at time *t*.
$\mathrm{FR}_t = \frac{L_t}{A_t} =$ pension plan funding ratio at time *t*.
$H_t =$ target liability hedging ratio at time *t*.

The biggest pension liability risk is interest rate. Pension liability has a long duration, which makes it very sensitive to interest rate changes. DB plan interest rate risk can be mitigated with interest rate risk immunization methods, such as the following methods widely used in life insurance companies.

1. **Duration matching.** Duration measures the sensitivity of an asset or liability to a small parallel shift of the discount yield curve:

$$\text{Asset duration} = \mathrm{AD}_t = \frac{A_t^- - A_t^+}{2A_t\Delta y}$$

$$\text{Liability duration} = \mathrm{LD}_t = \frac{L_t^- - L_t^+}{2L_t\Delta y}$$

where

$\frac{A_t^-}{L_t^-}$ = asset/liability value with a yield decrease;

$\frac{A_t^+}{L_t^+}$ = asset/liability value with a yield increase; and

Δy = increase or decrease in the yield.

The target asset duration AD_t can be solved as

$$\mathrm{AD}_t \cdot A_t \cdot \Delta y = \mathrm{LD}_t \cdot L_t \cdot H_t \cdot \Delta y \Rightarrow \mathrm{AD}_t = \frac{\mathrm{LD}_t \cdot L_t \cdot H_t}{A_t}$$

For pension liability, a shift of the yield curve changes not only the discount rates, but also the expected benefit payments in the future through wage inflation and inflation adjustment to pension benefits. Theoretically, an interest rate cut would increase aggregate demand in the economic system and usually increase nominal wage rate and inflation rate. For pension assets, embedded options in callable bonds and puttable bonds may be exercised with the yield changes and need to be considered in duration calculation as well.

2. **Duration and convexity matching.** Duration matching is valid only for small parallel changes. To incorporate the second-order impact of yield curve changes on asset or liability value, we can calculate and match convexity as well:

$$\text{Asset convexity} = \mathrm{AC}_t = \frac{A_t^- + A_t^+ - 2A_t}{2A_t(\Delta y)^2}$$

$$\text{Liability convexity} = \mathrm{LC}_t = \frac{L_t^- + L_t^+ - 2L_t}{2L_t(\Delta y)^2}$$

$$\text{Asset value change} = \Delta A_t = -\mathrm{AD_t} \cdot A_t \cdot \Delta y + \mathrm{AC}_t \cdot A_t \cdot (\Delta y)^2$$

$$\text{Liability value change} = \Delta L_t = -\mathrm{LD_t} \cdot L_t \cdot \Delta y + \mathrm{LC}_t \cdot L_t \cdot (\Delta y)^2$$

Given the duration matching, the target asset convexity can be solved as $\mathrm{AC}_t = \mathrm{LC}_t \cdot H_t$.

3. **Key-rate duration matching.** Even with duration and convexity matching, nonparallel yield curve change can cause different changes between assets and liabilities. Key-rate duration measures the sensitivity to the interest rate at a specified maturity. A series of key-rate durations can capture the key terms on the yield curve. The target asset key-rate durations can be calculated the same way as target duration except replacing duration with a series of key rate durations.
4. **Cash flow matching.** Duration, convexity and key-rate duration matching try to match the value changes. When a benefit payment is due, some assets may have to be sold to meet the liquidity requirement. A more conservative approach is to match the target liability cash flows with asset cash flows. Both the interest rate risk and liquidity risk are addressed in this approach. However, given the long time horizon of pension benefits, uncertainty about mortality experience, and the relatively high cost of cash flow matching, usually only short-term cash flow matching is used together with interest rate sensitivity matching strategies to mitigate interest rate risk.
5. **Liability replicating portfolio.** Except for cash flow matching, which is immune to material interest rate changes, the previous matching approaches work only for small changes. However, fixed-income securities may not always be available for matching long-dated pension benefits. A liability replicating portfolio intends to capture both small and big changes in interest rate level and yield curve shape. The replicating portfolio is composed of asset instruments selected so the portfolio mimics the value and sensitivity to the interest rate

curve as much as possible. The effectiveness of replication needs to be assessed under best-estimate scenarios, stochastic scenarios and stress scenarios:

$$\min_{\mathrm{RP}} \sum_{i,t} w_i \left(V_{\mathrm{RP}}^i(t) - V_L^i(t) \right)^2$$

s.t. $V_{RP}(0) = H \cdot V_L(0)$

where

RP = replicating portfolio;
w_i = weight assigned to scenario *i*;
$V_{\mathrm{RP}}^i(t)$ = value of replicating portfolio at time *t* under scenario *i*; and
$V_L^i(t)$ = value of liability at time *t* under scenario *i*.

Pension asset portfolios can be built based on the liability replication portfolio and the target hedging ratio. Any surplus can be invested in risky assets for a higher return.

In many cases, plan sponsors do not want to hedge all the liability risk when constructing investment strategy. The target hedging ratio, which is the portion of risk to be hedged, usually depends on the plan sponsor's risk appetite and current funding ratio. A lower hedging ratio implies higher risk but also higher potential of upside gain.

Traditionally, fixed-income securities such as government bonds and high-grade corporate bonds are used to construct the hedging portfolio. Interest rate and credit spread movements change the value of hedging assets and hedged liability. During economic recessions, interest rates and credit spreads tend to move in different directions, which reduces the aggregate impact of discount rate changes.

However, if most assets are invested in safe fixed-income securities, the expected return will be too low to meet pension obligations. Actual returns are much more volatile than corporate bond yields, which are the basis of private-sector DB plan discount rates. Actual asset portfolios take more risk than a liability hedging portfolio. Expected asset returns used for pension funding have declined but are still well above the high-grade corporate bond yields used for accounting. According to a corporate pension funding study by Milliman (2016),[1] the expected rate of return on assets dropped from 9.4% in 2000 to 7.2% in 2015. A public pension plan funding survey conducted by the National Association of State Retirement Administrators (NASRA 2016) indicates that the median return assumption dropped from 8% in 2001 to 7.6% in 2015. Even with lower investment return assumptions and a less risky asset portfolio, hedging the entire liability risk with high-grade fixed-income securities is still not a realistic choice. Long-term outperformance of risky assets over high-grade bonds is desired and necessary to control the expected amount of pension contribution.

An alternative way of hedging interest rate risk is to use financial derivatives. Two instruments are widely used by pension managers to increase the hedging ratio without physically investing in the bond markets:

1. **Interest rate swaps.** An interest rate swap is a financial agreement between two parties to exchange cash flows based on a notional amount and interest rates. One party makes fixed payments based on a fixed interest rate. The other party makes floating payments based on floating rates such as the Libor rates. Pension funds usually make floating payments and receive fixed payments to reduce their interest rate exposure. When the interest rate goes down, the pension liability value increases with offsetting reduced floating payments. Entering an interest rate swap usually does not require any initial payments. The fixed rate is determined so that the initial cost of the swap is zero. However, risks exist when using interest rate swaps for interest rate risk hedging. The term of a swap contract may not be long enough to hedge long-dated liabilities. Counterparty risk may cause termination of payment exchanges in the future. Basis risk could be material if the discount rate curve used for liability valuation deviates from the swap rate curve.

2. **Repos.** A repo allows a pension fund to sell government bonds with an agreement to buy them back in the future at a pre-specified price. With the repo, the sold government bonds can still be used to hedge interest rate risk. In addition, the cash received from the counterparty can be used to buy more bonds and increase the hedging ratio. The term of a repo could range from overnight to few months. It needs to be rolled over after expiry to maintain the increased hedging ratio.

Interest rate swaps and repos can be used together with fixed-income securities to meet a hedging target. Their duration, convexity, key-rate duration and cash flows can be calculated for matching strategies. Swaps and repos can also be included in liability replicating portfolios.

Wage inflation and inflation adjustments can change pension benefit amounts. Inflation-indexed bonds such as Treasury Inflation-Protected Securities (TIPS) can be used to hedge inflation risk. Pension funds can also use Inflation swaps to make fixed payments and receive floating payments linked to the Consumer Price Index (CPI). The liability cost of high inflation rates can be offset by the floating payments.

Credit spread risk also is an important factor to consider in LDI strategies. A lower credit spread could lead to a lower discount rate for pension liability valuation. High-grade corporate bonds can be invested to offset the risk, as their yields are the benchmark used for pension funding and accounting. Depending on the market liquidity, credit spread options may be used to offset the impact of credit spread change on pension liability as well.

Given the volatility of FX rates and the extent of economic globalization, FX risk also may be considered in LDI strategies to smooth the impact of currency value changes on pension funding. The ideal way is to back all foreign pension liability fully with foreign pension assets. Financial derivatives such as FX futures and forwards can be used for FX risk hedging as well.

Besides economic risks, longevity risk also has an impact on pension liabilities. The benefit payment period and number of payments are uncertain. Pension liabilities are thereby sensitive to mortality rates and future mortality improvement assumptions. Plan sponsors have several choices to reduce longevity risk:

1. Longevity swaps entail pension funds paying fixed premiums and receiving floating payments linked to actual mortality experience of a pension scheme. If people live longer than expected, the floating payments are higher to offset the impact of increased pension benefit payments.
2. Plan sponsors can sell longevity bonds to mitigate longevity risk as well. The principal is available to bond issuers if the mortality experience is lower than expected.
3. Plan sponsors can also transfer DB plan liabilities to a third party such as an insurance company. By buying annuity products for retirees, plan sponsors arrange to have future longevity risk and economic risk borne by the third party. However, this will require liquid assets and extra cost embedded in the annuity price.

Most economic factors are correlated with each other, and their dependency needs to be reflected in liability risk hedging as well. For example, during a financial crisis, lower interest rates are likely to be accompanied by higher credit spreads, lower inflation rates and more volatile FX rate changes. An effective hedging portfolio needs to mitigate the aggregate impact on pension liabilities.

In addition to risks affecting both pension liabilities and assets, some risks mainly affect the asset side. Market risk caused by equity, real estate and commodity investments needs to be considered when determining asset allocation for LDI. The allocation to these risky assets can be adjusted to align the risk exposure with the plan sponsor's risk tolerance. Financial derivatives such as equity index futures and options can be used to hedge public equity market risk as well. Counterparty risk and default risk also need to be considered for assets. They may be mitigated by requesting high-quality collateral or entering credit default swaps.

Given different possible LDI strategies and the variety of financial instruments that can be used to achieve LDI objectives, it is important but difficult to understand how these strategies determine the asset allocation plan and how the assets interact with each other and with the liabilities. The LDI benchmark model discussed in Section 4 incorporates all these considerations to provide a holistic view of LDI strategies.

2.1.4 LDI Optimization

Many optimization goals can be used to choose an appropriate hedging ratio and an asset allocation plan. Plan sponsors may want to minimize liability shortfall probability, surplus at risk or surplus return volatility, given a desired return. Or they may want to maximize the funding ratio, given a specified risk tolerance.

Sharpe and Tint (1990) studied the surplus optimization based on surplus return. The risk measure is the volatility of the surplus return. Surplus is $S_t = A_t - H \cdot L_t$, where *H* is the target hedging ratio. A normal surplus definition would require *H* = 1. Surplus return is defined as

$$z_t = R_t^A - H\frac{L_t}{A_t}R_t^L$$

where

$R_t^A =$ asset return in period t; and

$R_t^L =$ liability return in period t. This is the increase in liabilities during the period due to discounting and inflation. It can be estimated as the return that could have been realized on the assets if they had been invested in a liability-replicating portfolio.

The goal is to find the weight on each asset class that minimizes the following utility function.

$$\max_w E(z_t) - \frac{\mathrm{Var}(z_t)}{\lambda}$$

where

λ = the degree of risk aversion to return volatility (a higher λ means a lower level of risk aversion); and

w = a vector that contains the allocation percentage for each asset class.

Sharpe and Tint (1990) show that the maximization problem is equivalent to

$$\max_w E(R_t^A) - \frac{\mathrm{Var}(R_t^A)}{\lambda} + 2\frac{H}{\lambda}\frac{L_t}{A_t}\mathrm{Cov}(R_t^A,\ R_t^L)$$

The third term that considers the interaction between asset and liability is called liability hedging credit.

Ang et al. (2013) change the risk measure from variance to downside risk only. The maximization problem becomes

$$\max_w E(R_t^A) - \frac{\mathrm{Var}(R_t^A)}{\lambda} - \frac{c}{A_t}P(w,\ L_t,\ A_t)$$

where

c = the penalty cost for downside risk, which indicates the degree of risk aversion to downside shortfall (a higher *c* means more risk averse); and

$P(w,\ L_t,\ A_t)$ = the price of a European put option of pension asset A_t with exercise price L_{t+1} and expiry date *t* + 1.

Maximizing the first two terms generates the traditional mean-variance efficient portfolio. Maximizing the third term generates the best liability hedging portfolio (LHP). The optimal solution to the aggregate maximization is a weighted portfolio including both mean-variance efficient portfolio and the LHP, in addition to risk-free assets. The optimal hedging ratio can be determined as the investment in the LHP divided by L_t.

Martellini (2006) uses the utility function of the funding ratio at a specified future point as the target for maximization. If the constant relative risk aversion (CRRA) utility function is used, the problem is

$$\max_w E\left(\frac{{\mathrm{FR}_T}^{1-\gamma}}{1-\gamma}\right)$$

where

$\mathrm{FR}_T =$ funding ratio at time *T*; and

$\gamma =$ degree of relative risk aversion.

As in Ang et al. (2013), the optimal three-fund asset allocation consists of a mean-variance efficient performance portfolio, an LHP and a risk-free asset.

However, some assumptions used in these optimization models may not be meaningful in the real world. Correlation between asset and liability is not constant and could be higher in extreme scenarios. Nonlinear relationships among economic variables also require portfolio optimization methods different from the mean-variance method. Closed-form solutions are not available if alternative assets and financial derivatives are included in the asset portfolio. To address these challenges, the LDI benchmark model described in Section 4 uses a different approach to optimize LDI strategies. Stochastic analysis is adopted to evaluate the hedging effectiveness and return potential. External factors such as regulation, plan sponsor's financial condition and risk appetite discussed in Section 2.1.2 also may be included in the analysis.

2.1.5 LDI Time Horizon

The duration of a pension liability is long. The last payment for current plan participants could be 70 or more years away. LDI needs to consider these long-dated benefit payments, but its time horizon is much shorter. LDI strategies are expected to be reviewed and revised every one to five years, given the following considerations:

1. Market conditions change from time to time. The confidence interval of long-term prediction could be too wide to be useful. Revisiting LDI strategies makes sure that the current asset allocation plan is still relevant in a changing economic environment.
2. The plan sponsor's financial condition and risk appetite may change. Plan participants' demographic composition also may change. LDI strategies could be materially affected by these changes.
3. The success of LDI strategies needs to be measured in a timely manner. Having a short or medium time horizon facilitates the performance measurement of pension fund investment managers.
4. Some financial instruments used in LDI strategies have a liquid market only for short- or medium-term contracts. For rollovers after maturity, reassessment of investment strategies is necessary.

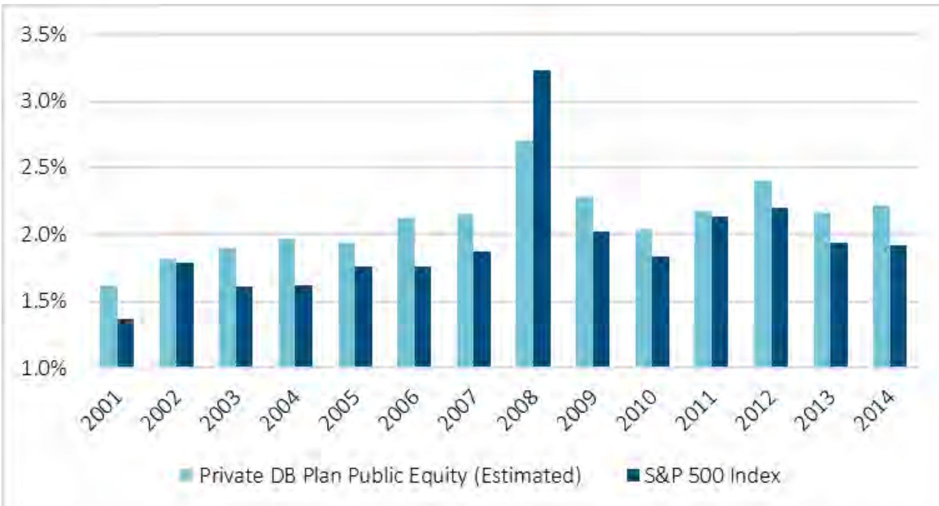
2.2 Asset Allocation

DB plan asset allocations experienced material changes in the past decade. According to Aguirre and McFarland (2016), the average equity allocation dropped from 45% in 2009 to 36% in 2015 for a sample of 274 Fortune 1000 companies. Debt investments increased from 36% to 41%. Clearly, the liability hedging ratio increased in the past seven years. Real estate investments increased from 3.2% to 4.3%, which may be largely attributed to real estate market recovery. Hedge fund allocations, including the use of financial derivatives such as interest rate swaps, increased from near zero to 4.5%. With more and more pension funds adopting LDI strategies, higher allocation to debt instruments and more use of financial derivatives are expected.

Individual asset selection also needs to be considered in LDI strategies. Diversification of individual asset holdings is necessary for limiting credit risk and exposure to individual sectors. The correlation between plan assets and a plan sponsor's financial conditions may need to be controlled as well. High positive correlation means that the plan sponsor may have little financial resources available for making pension contributions when they are most needed. Some assets may be selected over others with the same level of risk for yield pickup. For example, when swap rates are lower than government bond yields, repos are more cost-effective for hedging pension liability for a higher return.

High-yield dividend stocks are also preferred by DB plans considering the additional available liquidity from dividend payments. A long holding period makes short-term stock market volatility less important for DB plans. Figure 2 shows the average dividend yield of U.S. private-sector DB plans' stock investments compared with the S&P 500 Index dividend yield. Except in 2008, when the stock market crashed, DB plans' dividend yield is higher than that of large-cap equity.

Figure 2
US Private-Sector DB Plan Dividend Yields, 2001–2014



Sources: Private-sector DB plan dividend yields are dividend yields of common stock investments derived from the data available in EBSA (2000–2014). DB plans with no fewer than 100 participants are included. Assuming a quarterly dividend payment frequency, the dividend yields are calculated as follows:

$$\frac{\text{Total Dividends}}{\frac{3}{8}(\text{Starting Common Stock Value}) + \frac{5}{8}(\text{Ending Common Stock Value})}$$

S&P 500 Index dividend yield is estimated based on SPDR S&P 500 ETF data available at https://finance.yahoo.com/quote/SPY?p=SPY.

These asset selection preferences could have a material impact on asset performance compared to a general market index. They need to be considered when designing and evaluating LDI strategies.

Section 3: Literature Review

The relationship between pension assets and liabilities is complicated and driven by many economic and insurance factors. To evaluate a variety of LDI strategies against pension liabilities consistently, the relationship can be modeled in the following steps:

1. Pension assets are mapped into fundamental factors such as GDP growth rate, interest rate, credit spread, credit default, equity index, inflation rate, unemployment rate, consumption and investment. Assets can be modeled at sub-class level to reflect their specific characteristics.
2. Pension liabilities are mapped into the same set of key factors. The mapping can be done directly by using liability information or by modeling liabilities as negative assets using a liability replicating portfolio.
3. The dependency among key risk factors can be modeled based on empirical studies or economic theories. In the economic system, these key factors are linked together. With knowledge of the dependency among risk factors, plan sponsors can model the relationship between assets and liabilities based on their exposure to these risk factors.

Some studies exist for both risk factor mapping and dependency among risk factors. They are usually very generic topics not confined to DB pension investment. This section summarizes some of the literature to provide some theoretical support for the LDI benchmark model. The focus is on asset correlation, alternative investments (such as real estate, commodities, infrastructure and hedge funds), public-equity subclasses, equity duration and macroeconomic models. For most of the issues, different conclusions exist. The LDI benchmark model in Section 4 LDI Benchmark Model does not choose a specific conclusion but allows different inputs. To be as objective as possible, we used empirical analysis when illustrating the model for LDI analysis in Section 5 Application.

3.1 Asset Return and Credit Default Correlation

Asset return correlation and credit default correlation are important for determining the co-movements of assets, especially for investment strategies without sufficient portfolio diversification. Frye (2008) suggested different treatments for default correlation and asset return correlation. Asset return correlation is close to default correlation only when the default is defined as a shortfall (asset < liability) and the issuer's liability value stays fixed. These assumptions are hardly true in reality. Calibration based on default data instead of asset value data is suggested to model correlation regarding credit defaults.

Many reports from investment banks suggested a trend of increasing correlation among asset classes and therefore risk factors. J.P. Morgan (2011) attributes the increase of asset correlation to macro volatility, risk management and less availability of alpha return. CRISIL (2013) conducted a survey on asset correlation and found that high cross-asset correlation is considered a key risk factor that affected fund performance negatively. Alpha-generating probabilities also declined with rising correlation.

Basel III allows banks to use an internal-model based approach to determine required capital. An asymptotic single-factor model (ASFM) is used to reflect the asset correlation for credit risk:

$$\text{Credit Risk Capital} = \left(\text{LGD} \cdot \Phi\left(\frac{\Phi^{-1}(\text{PD}) + \sqrt{\rho}\Phi^{-1}(99.99\%)}{\sqrt{1-\rho}}\right) - \text{LGD} \cdot \text{PD}\right) \cdot \text{MA} \cdot \text{SF} \cdot \text{MCR} \cdot \text{EAD}$$

where

LGD = loss given default;
$\Phi(x)$ = cumulative distribution function of normal distribution;
PD = probability of default;
ρ = single asset correlation factor;
MA = maturity adjustment;
SF = scaling factor;
MCR = minimum capital requirement as percentage of asset; and
EAD = exposure at default.

Lopez (2002) studied the relationship between the single correlation factor and the default probability and firm size. It was found that the correlation factor increases when the default probability decreases and firm size increases. It is consistent with common understanding, as default is more of an individual event than a common event. A bigger firm tends to have more exposure to systemic risk than idiosyncratic risk.

Using default data of Japanese companies, Hashimoto (2009) found that a single correlation factor may not be adequate to estimate asset correlation. It varies by industry, firm size, credit rating and region. The correlation is high for firms with a high or low credit rating and low for firms with a medium credit rating.

Hamerle et al. (2003) also studied the single correlation factor using default data and found that it is lower than implied by asset data. Hamerle and Roesch (2015) also compared the Gaussian copula and *t* copula in credit risk correlation modeling. Though the *t* copula provides a fatter tail than Gaussian copula, the impact is small and the Gaussian copula is still an acceptable choice, even though the estimation may be biased.

Geidosch (2013) compared different estimators for asset allocations and studied the correlation of the RMBS portfolio that experienced the 2008 subprime crisis. The correlation is lower than expected.

To estimate the single asset correlation factor, Martin (2013) examined several methods including conditional value at risk (VaR) based on a fitted beta distribution and the Vasicek model with PD as exogenous variables and the firm value depending on both systemic risk and idiosyncratic risk.

The LDI benchmark model proposed in Section 4 treats the correlation at the asset class or subclass level by assuming that a pension plan normally invests in a highly diversified portfolio for each asset class or subclass. For asset classes other than bonds, asset returns implicitly include the impact of defaults, as the default loss will be translated into a reduction of the returns. Asset correlation and default correlation are separately modeled for corporate bonds based on empirical analysis. The correlation between corporate bond asset return and corporate bond default rate is also maintained.

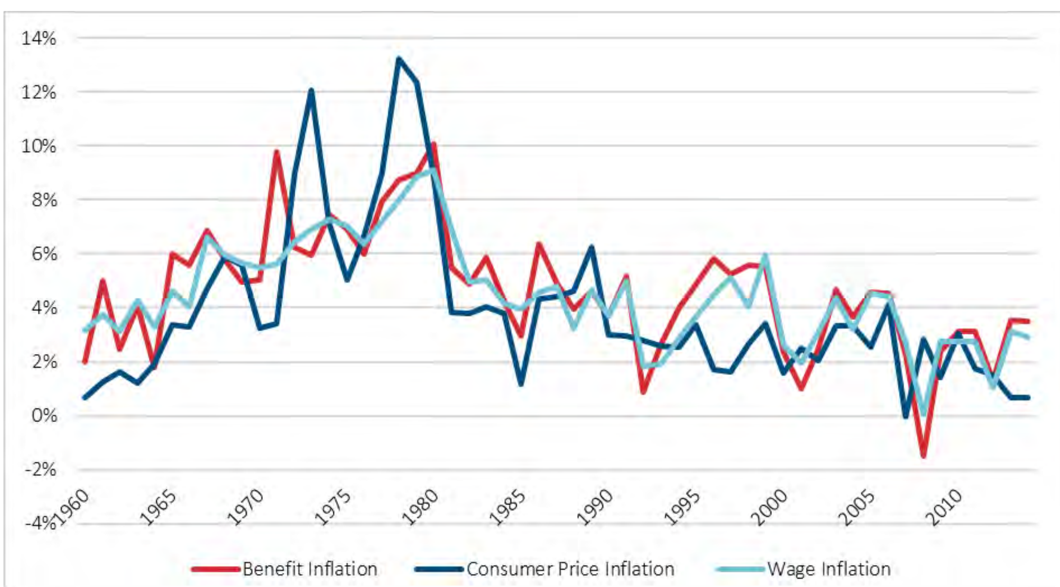
3.2 Real Estate and Inflation

One of the key issues in LDI is to hedge the inflation of pension liability. Real estate investment is used by some pension sponsors for inflation protection and high expected return. But there is some doubt about the effectiveness of real estate's inflation hedging. In addition, a higher inflation rate is usually accompanied by a higher interest rate. Their impacts offset each other. Another complication is that typical pension liability is only partially and imperfectly linked to the inflation rate. The benefit amount is indexed to pre-retirement wage inflation, which can be quite different from the inflation rate derived from the Consumer Price Index (CPI). Wage inflation varies widely by industry and employer, while CPI is a general index of the entire economy. Wage inflation data also needs to be used carefully to appropriately estimate the "real" wage inflation reflected in pension benefits. Demographic changes in the population are included in the average wage data but not in the wage progression for individual plan participants, which determines pension liability.

Figure 3 shows the benefit inflation, consumer price inflation, and wage inflation from 1960 to 2014. Benefit inflation is based on the national average wage index published by the Social Security Administration. It is used to adjust a person's retirement benefit to reflect the rise of living standard. The consumer price inflation rate is calculated based on the CPI for all urban consumers published by the Bureau of Labor Statistics (BLS). Wage inflation is calculated as the annual percentage change of total personal income divided by the number of nonfarm employees.

Figure 3

US Benefit, Consumer Price and Wage Inflation Rates, 1960–2014



Sources: Benefit inflation is from the national average wage index, published by the Social Security Administration, https://www.ssa.gov/oact/cola/AWI.html.

Consumer price inflation is the CPI for all urban consumers, published by the Bureau of Labor Statistics.

Wage inflation is equal to total personal income divided by the number of nonfarm employees. Total personal income is from Wage and Salary Disbursements, published by the Bureau of Economic Analysis. The number of nonfarm employees is the total of nonfarm payrolls, published by the Bureau of Labor Statistics.

Although the general trends are similar, the magnitude and sometimes direction of the consumer price inflation rate is different from those of wage and benefit inflation. Table 2 lists the mean and standard deviation for each of three inflation series and their correlation matrix. The correlation between wage inflation and consumer price inflation is 66%, which indicates that hedging CPI risk is not enough for hedging wage inflation risk—even in the absence of employer-specific variations.

Table 2

US Inflation Descriptive Statistics, 1960–2014

	Mean	*Volatility*	*Correlation*		
			Benefit Inflation	*Consumer Price Inflation*	*Wage Inflation*
Benefit Inflation	4.6%	2.3%	1.00	0.66	0.90
Consumer Price Inflation	3.9%	2.9%	0.66	1.00	0.76
Wage Inflation	4.5%	1.9%	0.90	0.76	1.00

Sources: Benefit inflation is from the national average wage index, published by the Social Security Administration, https://www.ssa.gov/oact/cola/AWI.html.

Consumer price inflation is the CPI for all urban consumers, published by the Bureau of Labor Statistics.

Wage inflation is equal to total personal income divided by the number of nonfarm employees. Total personal income is from Wage and Salary Disbursements, published by the Bureau of Economic Analysis. The number of nonfarm employees is the total of nonfarm payrolls, published by the Bureau of Labor Statistics.

Most of the existing studies focus on the relationship between real estate and consumer price inflation. Fama and Schwert (1977) studied the relationship between asset returns and inflation rates. Private residential real estate hedged both the expected and unexpected inflation rate effectively. However, labor income and therefore wage inflation had little relationship with both the expected and unexpected inflation rates. Wurtzebach et al. (1991) studied the commercial real estate market from 1978 to 1990 and found that commercial real estate provided a good hedge of inflation risk during both a high-inflation period and a low inflation period. This research found that vacancy rate also is an important determinant of real estate return (for office and industrial property) and may distract from its hedging effectiveness against inflation.

The Investment Property Forum (2011) studied the relationship between real estate investment and inflation rate in the U.K. market. The researchers found that U.K. real estate is not a perfect hedge of inflation rates, though inflation rate is an important driver of U.K. real estate return. Instead, U.K. real estate moved more closely with GDP growth and therefore wage inflation.

Case and Wachter (2011) compared equity real estate investment trusts (REITs) with other assets in terms of the effectiveness of inflation hedging. Investment alternatives include commodities, stocks, Treasury Inflation Protection Securities (TIPS) and gold. The effectiveness is driven by high/low-inflation environment and time horizon. In the high-inflation environment, commodities outperformed equity REITs. In the low-inflation environment, stocks outperformed equity REITs. The researchers suggest using a mix of all these asset types, instead of a single asset type, for hedging inflation risk.

In addition to empirical analysis, there is literature that investigates whether the inflation rate affects real estate prices or the other way around. However, different studies drew opposite conclusions.

3.3 Hedge Funds

Hedge funds are used by pension plans for higher expected return and diversification. Given the heterogeneity and lack of transparency of the investment strategies of hedge funds, analysis based on the average performance of hedge funds is not useful unless a pension plan has a diversified hedge fund portfolio. One solution is to map the invested hedge funds into risk factors using historical performance data. By doing this, we can evaluate hedge funds consistently with liabilities and other asset classes. However, hedge funds may employ short strategies and financial derivatives to increase the leverage ratio. Standard risk factors such as bond yield and equity return may not be sufficient to explain fund returns. Researchers have made efforts to identify the common driving factors of hedge funds.

Lhabitant (2003) mapped hedge fund returns to pure investment-style indices, including convertible arbitrage, distressed security, emerging market, event-driven, fixed-income arbitrage, global macro, long/short equity, merger arbitrage, relative value and short-selling index.

Bussière et al. (2014) mapped hedge fund returns to 12 risk factors, including equity index returns of different markets, Treasury bond and corporate bond yields, options on bonds, currency and commodities, return on variance swap, and liquidity. They found that the commonality of hedge fund returns doubled between 2003 and 2006.

These new risk factors, called hedge fund betas by Berger et al. (2008), may be added into LDI analysis to better capture the return and risk of hedge fund investment. Other investment alternatives such as private equity may be less accessible than hedge funds. Like hedge funds, they can be mapped to risk factors to assess their correlation with other asset classes and pension liabilities.

3.4 Commodities

Some pension plans also invest in commodity markets such as precious metals and energy. Commodities are usually considered good hedges for long-term inflation, especially unexpected inflation. However, other asset classes such as bonds and equities are usually considered good hedges for expected inflation. Commodities are also considered an important source of inflation. These features make commodity investment a potential inflation-hedging instrument for pension plans.

Greer (2000) studied the return of an unleveraged commodity index, the Chase Physical Commodity Index. Based on the data from 1970 to 1999, the author concludes that on average, the index return is comparable to equity returns and volatility. The index return is positively correlated with inflation and more correlated with the changes in inflation. It is negatively correlated with the equity and bond markets. Breaking it down to the commodity level, commodity price changes differed greatly.

However, commodity markets are generally more volatile than equity markets. This could cause problems for pension plans that need at least annual valuation. Commodity markets are also materially affected by idiosyncratic factors. For example, the crude oil price and futures risk premium are largely affected by the crude oil inventory. Gorton et al. (2012) studied 31 commodity futures using inventory data from 1971 to 2010 to verify the negative relationship between risk premium and inventory. The study also found that positions in futures markets have no predicting power for the futures risk premium.

3.5 Infrastructure

Infrastructure investment also is used in pension plans' diversified asset portfolio. It is usually preferred for its low correlation with other asset classes, inflation protection, high cash yield, and lesser sensitivity to economic cycles. Unlike infrastructure debt investment, infrastructure equity investment is believed to provide a significant protection against inflation risk. Kohn et al. (2009) provide an overview of infrastructure investment regarding benefits and risks. They divided the investment into four categories: regulated assets, transportation assets, communication assets and social infrastructure assets. Different infrastructure assets have different risk and return expectations. Infrastructure portfolios also provided stable returns higher than the inflation rates.

Some researchers expressed doubts about the inflation-hedging effectiveness of infrastructure investment. Rodel and Rothballer (2012) used data mostly for the years 1990 to 2009 to analyze investment returns based on inflation. They found that infrastructure investment is not better than equity investment in terms of inflation hedging.

Another purported benefit of infrastructure investment for pension plans is the long duration of the projects, which could match the long duration of pension liabilities. However, many other risks, such as liquidity risk and political risk, may be associated with infrastructure equity investment. These need to be considered when making infrastructure investment decisions for pension plans.

3.6 Public Equity Subclasses

As LDI strategies may favor certain subclasses of public equity, it is helpful for studying the characteristics of these subclasses. Eun et al. (2008) suggest that large-cap stocks and global market indices tend to move together, which mitigates the benefit of international diversification. Small-cap stocks offer better diversification even after considering market friction cost. A report from FTSE Russell (2015) also showed that small-cap stocks have larger movements, either up or down, than large-cap stocks. The correlation between the FTSE Global Small Cap Index and FTSE All-World Index (large-cap stocks) rose above 95% during the 2008 financial crisis but gradually reduced to around 85% in 2015. Small-cap stocks are still useful for better diversification.

Switzer (2010) studied the behavior of small-cap and large-cap stocks of the U.S. and Canadian market. His results suggest that small-cap stocks outperform over the year after an economic trough and tend to lag one year before an economic peak. The U.S. small-cap premium can be explained by the default risk, but the average premium did not materially change during recessions. The difference between Canadian and U.S. equity returns can be explained by the differences in default risk and inflation risk.

Global X (2016) studied the top 10% of U.S. stocks in terms of dividend yield in the U.S. market from 1960 to 2015. High-yield dividend stocks outperformed the S&P 500 index by 3.2% on average. In the 10 rising-interest-rate periods, high-yield dividend stocks outperformed the market index in seven periods. They underperformed in the other three periods, when interest rates increased rapidly. The correlation between high-yield dividend stocks and the market index is 56% from 1985 to 2015, which shows potential for better diversification. It is an ex-post analysis that may not be helpful for asset selection because it is uncertain whether a current high-yield dividend stock will stay the same way in the long run. However, the lower correlation with the general market still implies it is a good addition to the asset portfolio.

3.7 Equity Duration

The sensitivity of equities to interest rate movements is an important yet complicated topic in the investment area. Firm value and therefore equity value are surely affected by interest rates. Intuitively, an interest rate increase would attract more money into the bond market and may negatively affect the stock market and therefore equity valuation. Interest rates are also an indicator of the economic cycle. Financial-industry companies have large exposure to interest rate risk. Therefore, equity duration is expected to vary widely by company and economic conditions.

Like bond duration, equity duration is a concise way to measure the impact of interest rates on equity value. Many different definitions of equity duration exist. For example, Leibowitz (1986) defines the duration of equity portfolio as

$$\beta_{\mathrm{EP}} \cdot D_E = \beta_{\mathrm{EP}} \cdot \frac{\sigma_E}{\sigma_B} \rho_{E,B} D_B$$

where

β_{EP} = beta value of the equity portfolio, which represents its correlation with the general equity market;
D_E = duration of the equity market;
σ_E = volatility of the equity market;
σ_B = volatility of the bond market;
$\rho_{E,B}$ = correlation between the equity and bond markets; and
D_B = duration of the bond market.

This definition of equity duration relies entirely on the bond market to derive the interest rate sensitivity. It incorporates the general impact of the interest rate but does not reflect its specific impact on each firm.

Dechow et al. (2002) defined implied equity duration as being like bond Macaulay duration. The coupons and redemption value are replaced by future cash distributions defined as the difference between earnings and changes in book value. The bond yield is replaced by equity return. The research found that implied equity duration is highly correlated with book-to-market ratio.

Blitzer and Dash (2004) applied the concept of equity duration to pension asset allocation. Equity duration was calculated based on the dividend discount model:

$$P = \frac{D}{(k-g)}$$

where

P = stock price
D = dividend
k = discount rate
g = dividend growth rate

Equity duration is calculated as the sensitivity to the discount rate k:

$$\frac{1}{P}\frac{\delta P}{\delta k} = -\frac{1}{k-g}\left(1 - \frac{\delta g}{\delta k}\right)$$

Based on a 30-year history of the U.S. equity market (1973–2003), the equity duration had a downward trend reaching 15.2 years in 2003. Blitzer et al. (2010) updated the analysis to include data that covers the 2008 financial crisis. Equity duration climbed to 42.6 years in 2008 and declined to 21 years in 2010, mainly caused by the vast fluctuation of quarter-over-quarter dividend growth rates. Clearly, the equity duration is much more sensitive than bond duration to the economic environment. Equity duration is high for high-growth stocks or stocks with low discount rates.

For LDI analysis, the goal is to evaluate the sensitivity of equity to interest rates. Equity duration is more useful when adopting the format of effective duration. Using the simple dividend discount valuation model, changes in the interest rate could lead to changes in both dividend timing/amount and the discount rate. The discount rate reflects the financial structure of the company. When interest rates are lower, the required return (discount rate) is likely to change as well, because equity return is usually seen as the sum of the risk-free rate and the equity risk premium. At the same time, earnings and dividends are expected to be changed based on the new interest rates. Depending on the interpretation of interest rate movements, the impact could be a short-term fluctuation because of economic cycle or a long-term change because of economic structural change. Pension equity portfolios could differ from the general equity market. The impact of interest rates on individual stocks needs to be assessed and reflected in the equity duration as well. These considerations make equity duration calculation for LDI complicated and somewhat subjective. In the LDI benchmark model proposed in Section 4, equity duration is not directly used to assess the impact of interest rate on equity portfolio. Rather equity scenarios are directly linked to interest rate scenarios to reflect the interest rate sensitivity of the equity portfolio.

3.8 Macroeconomic Model

All the asset classes are linked to economic growth to a certain degree. Macroeconomic models can link together all the economic variables and therefore asset classes. They are able to maintain the consistency among asset scenarios. Two types of macroeconomic models can be used for maintaining the consistency. The first type relies on empirical analysis of the economic variables. Historical dependencies are then assumed to continue in the future. For example, Cornell (2009) analyzed the relationship between GDP growth and equity returns of the U.S. market from 1947 to 2008. As the long-term real GDP growth rate is unlikely to exceed 3% by much, it was concluded that long-term real equity return is likely to be capped around 4% to 5%. A report by MSCI Barra (2010) also explored the relationship between GDP growth and equity returns. The researchers tested the supply model empirically, based on global stock markets. It turned out that GDP growth is not a good predictor for equity returns because of globalization and growth from new enterprises rather than growth of existing enterprises. Real GDP growth is viewed as a cap on the long-term real stock returns rather than an important determinant.

The second type of macroeconomic model builds the foundation of the economy, considering aggregate demand, aggregate supply, labor market, monetary policies, fiscal policies and economic turbulence. These models include factors such as interest rate, GDP growth, inflation and unemployment rate. Examples include the dynamic stochastic general equilibrium (DSGE) models used by central banks for monetary-policy making. Sbordone et al. (2010) provide an introduction of the Federal Reserve Bank of New York DSGE to the public, using a small-scale model. The model was used to explain the unexpected inflation increase in the first half of 2004. Gust and Lopez-Salido (2009) incorporated a limitation on household financial-portfolio rebalancing and found this can account for excess equity returns following a monetary shock. The model can help explain the sensitivity of stock prices to monetary policies by generating countercyclical movements in the equity premium. Benes et al. (2014) built a DSGE model to explain the occurrence of financial crisis by incorporating bank loans in the economic system.

Challenges exist to using DSGE models for real-world investment analysis. For example, it is difficult to calibrate variables in a DSGE model such as the equity risk premium to the asset market. Asset markets are not always affected by the economy in a rational way. Simplified assumptions required for solving a DSGE model could also leave behind many details that are important for the asset market. One needs to address these issues before using DSGE models to evaluate investment strategies.

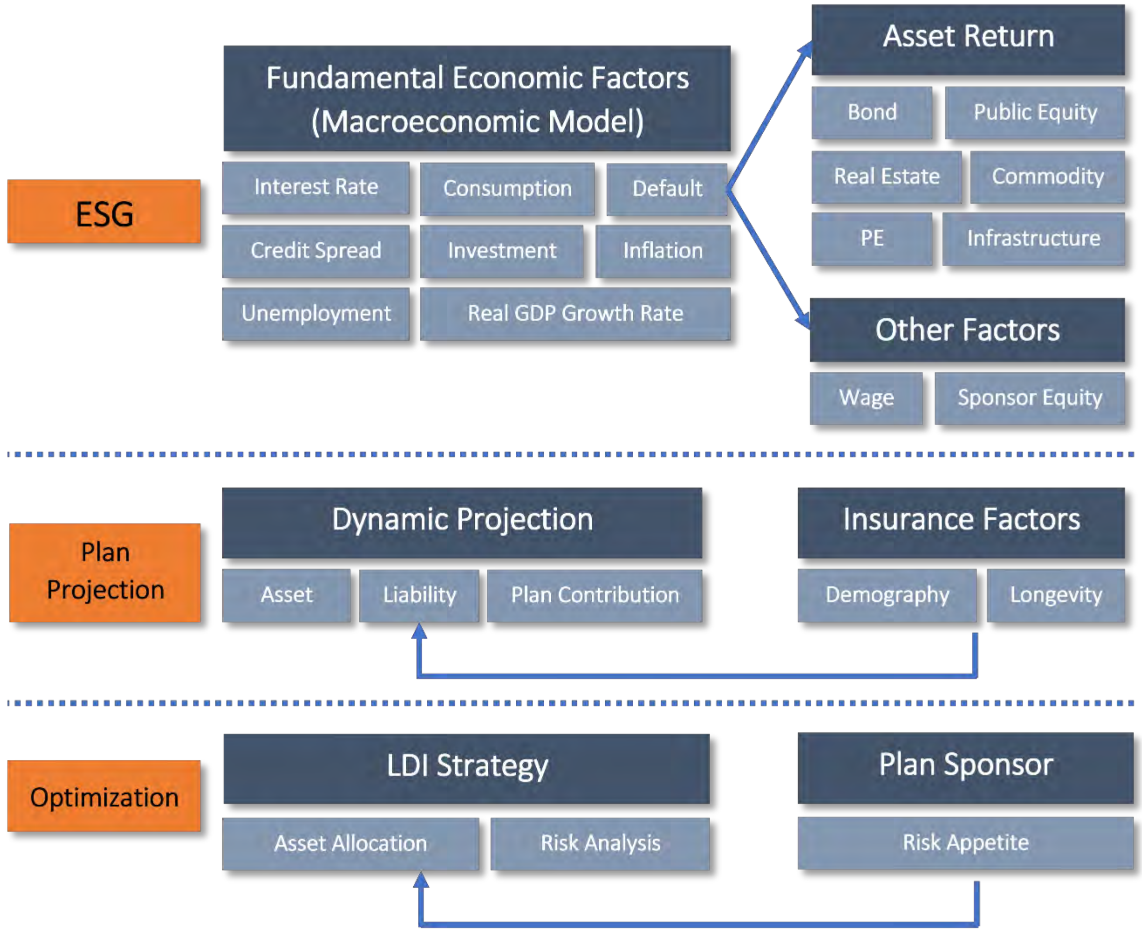
This report uses a hybrid approach. Starting from the fundamental economic factors, we used a vector autoregression model based on empirical analysis to govern the macroeconomic projection. Asset returns and other relevant factors are then built upon the fundamental economic factors to maintain consistency but also allow idiosyncratic shocks.

Section 4: LDI Benchmark Model

4.1 Overview

The LDI benchmark model is composed of three elements: economic scenario generation (ESG), plan projection and investment strategy optimization. It can project the funding status of a pension plan given an asset allocation plan and an economic scenario. It provides a benchmark for evaluating LDI strategies and actual LDI performance on a consistent basis. Figure 4 shows the structure of the model.

Figure 4
LDI Benchmark Model Structure



4.1.1 Economic Scenario Generation (ESG)

The LDI benchmark model starts from an economic scenario generator, which generates fundamental economic factors, asset returns and other factors. Unlike scenario generators driven by financial market data, the economic scenario generator relies on macroeconomic models to represent the economic system. Fundamental economic factors such as GDP growth rates, interest rates, credit defaults, credit spread, unemployment rate and inflation rate can be chosen to govern the systemic risk in the economic scenario generator. These fundamental economic factors can be considered the roots of our economic and financial system. They define the current economic status and predict the future status, whether it is in expansion or recession. Macroeconomic models study both the contemporary and intertemporal relationships among fundamental economic factors to predict the future. They ensure the consistency and reasonableness of generated economic scenarios.

With the fundamental economic factors, asset returns can be generated through the process of fund mapping. The return of each asset class is generated as the sum of two factors: a systemic factor and an idiosyncratic factor. The systemic factor is determined based on the relationship between the asset return and the general economy governed by fundamental economic factors. The idiosyncratic factor is determined by the unique features of each asset class. The systemic factor can be determined based on historical data or adjusted with forward-looking views of the future economy. The idiosyncratic factor is independent of the systemic factor on average but usually has higher volatility and nonzero correlation with the systemic factor during economic recessions.

In addition to asset returns, some other factors, such as wage inflation and plan sponsor's equity return, may be important for LDI analysis. Wage inflation is important for projecting plan liability and benefit payments, which are usually linked to the wage level in the future. The plan sponsor's equity return is a useful indictor of whether sufficient plan contributions will be available in the future. These factors can be generated using the same fund-mapping process as is used for asset returns.

4.1.2 Plan Projection

With the economic scenario generator ready, the next part is plan projection. The economic scenario generator builds the connections between and within assets and liabilities in the LDI benchmark model:

1. Plan assets are linked to stochastic scenarios via asset returns.
2. Pension liability discount rates are linked to economic scenarios. For example, AA-rated corporate bond yields may be used for liability valuation.
3. Future benefit amounts may be linked to wage inflation before retirement and price inflation after retirement. Both consumer price inflation and wage inflation are part of the economic scenario generator.
4. Future plan contributions are determined by discounting additional plan benefits with expected future investment returns. The expected rate of return is determined by the economic scenario generator as well. Determination of plan contributions is key to the dynamic projection of plan assets and liabilities.

Insurance factors such as demographic information of plan participants and longevity risk also are included in the framework to measure liability risk. These factors affect the timing and amount of future benefit payments and therefore the asset allocation plan.

4.1.3 Optimization

With the ability to project plan assets and liability, the next part is to choose the most appropriate LDI strategy. Each candidate asset allocation plan can be run through the projection model under different scenarios. Risk and return trade-offs of asset allocation plans can be compared. Given a plan sponsor's risk appetite and other constraints, the model permits determination of the optimal asset allocation plan.

4.1.4 What are the Objectives of the LDI Benchmark Model?

The LDI benchmark model has three primary objectives:

1. The model maintains a high level of consistency between asset and liability projection using macroeconomic models. Asset returns, liability discount rates, wage inflation and future plan contributions are all governed by economic conditions.
2. The model follows basic economic patterns in the real world. Unlike most economic scenario generators used for risk analysis, economic cycles are embedded in each scenario in the LDI benchmark model, no matter how extreme the scenario is. Lower interest rates, higher unemployment rates and credit spreads, and bear equity markets are observed during economic recessions in each scenario. Higher correlation is preserved during economic recessions. Investment strategy analysis based on these reasonable and explainable scenarios is more meaningful and easier to communicate.
3. The LDI benchmark model is a conditional model reflecting the current status of the economy. This makes it more relevant to real-world investment decision making.

The economic scenario generator, projection model and optimization process in the LDI benchmark model can vary between pension plans. In practice, assumptions and models are chosen based on funding policy and views of the future. The future economy is uncertain, and it is difficult to have a uniform view. Even economists have a wide range of predictions of the economy in the short term. In fact, different views drive buy and sell decisions and are necessary for a liquid market. Instead of trying to unify the views of the future economy, this report shows one possible example of an LDI benchmark model based on empirical analysis. It is a relatively objective approach but not necessarily the most appropriate approach. The remainder of this section explains the specific application of the LDI benchmark model adopted in this report. Numerical examples are presented in Section 5. Technical details are documented in the appendixes.

4.2 Economic Scenario Generation

Many economic scenario generators can be used to generate real-world economic scenarios for LDI analysis. This report uses an economic scenario generator based on empirical analysis. The economic scenario generator is composed of three parts: fundamental economic factors, asset returns and other relevant factors.

A vector autoregression (VAR) model is used to describe the intertemporal relationships among eight fundamental economic factors for the U.S. economy: real GDP growth rate, inflation rate, unemployment rate, short-term interest rate, long-term interest rate, credit spread, consumption and investment. These fundamental economic factors govern the period-to-period changes in status of the economy and indirectly affect the asset returns and other model outcomes in a consistent way:

$$\mathbf{F}_t = \mathbf{c} + \mathbb{A}_1 \mathbf{F}_{t-1} + \mathbf{e}_t$$

where

$\mathbf{F}_t$ is a column vector with fundamental economic factors at time t or during period t;
$\mathbf{c}$ is a column vector representing the constant terms of the fundamental economic factors;
$\mathbb{A}_1$ is a square matrix containing the model parameters describing the linear dependence of fundamental economic factors; and
$\mathbf{e}_t$ is a column vector to store the error terms of fundamental economic factors that cannot explained by linear models.

After the scenarios of fundamental economic factors are generated, the model generates asset returns and other economic factors, including wage inflation and plan sponsor's equity return, based on linear models with fundamental economic factors as the explanatory variables. Both contemporary and intertemporal relationships are represented by the linear functions

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\phi}_1 y_{t-1} + \boldsymbol{\phi}_2 \mathbf{y}_{t-2} + \mathbb{B}_0 \mathbf{F}_t + \mathbb{B}_1 \mathbf{F}_{t-1} + \mathbb{B}_2 \mathbf{F}_{t-2} + \mathbf{e}_t$$

where

$\mathbf{y}_t$ is a column vector containing the returns of all asset classes and other factors during period t;
$\boldsymbol{\alpha}$ is a column vector containing the constant terms of all asset classes;
$\boldsymbol{\phi}_i$ is a column vector containing parameters to govern the relationship between current return and return for all asset classes during period $t - i$;
$\mathbb{B}_i$ is a matrix that contains all asset classes' model parameters for the fundamental economic factors during period $t - i$; and
$\mathbf{F}_t$ is a column vector including all the fundamental economic factors at time t or during period t.

Model outcomes used in this report include the U.S. Treasury bond yield curve (for one-, two-, three-, five-, seven-, 10-, 20- and 30-year yields); the AAA-, AA-, A- and BBB-rated corporate bond credit spread and default rate; large-, mid- and small-cap equity index dividend yields and capital returns; high-dividend equity index dividend yields and capital returns; equity, mortgage and REIT cap rates and capital returns; private-equity and infrastructure index dividend yields and capital returns; oil, gold and commodity index total returns; wage inflation and plan sponsor equity return.

The last step of the economic scenario generation process is to project bond fund returns based on bond fund investment strategy, term mix and reinvestment strategy. Simulated yield curve movements, defaults and credit rating migration need to be reflected when projecting the bond fund returns.

Both the VAR(1) and linear models are calibrated using historical data from 1991 to 2016, where available. The generated scenarios have the following patterns that are beneficial for LDI analysis:

1. Economic cyclical patterns are built into each individual scenario. No scenario is extremely bad for all the future years.
2. Under each scenario, economic factors and asset returns move in a consistent way. Both good and bad scenarios involve the following conditions during an economic recession as determined by fundamental economic factors:
   - Lower GDP growth rates, interest rates, equity returns and oil prices; and
   - Higher unemployment rates, credit spreads, default rates and gold prices

Based on these scenarios, an LDI strategy can be formulated using reasonable and realistic scenarios. More details of economic scenario model calibration, simulation and validation are documented in Appendix A: Economic Scenario Generation for LDI.

4.3 Liability Model

Future pension benefit payments can be projected based on demographic details of plan participants (age, gender, occupation, length of service, expected retirement age, current salary, etc.), mortality assumptions, wage inflation, benefit rate, cost of living adjustments (COLA), COLA limit, lump sum payment option and so on. Except for wage inflation and COLA, which are linked to the model inflation rate, all other assumptions are deterministic. With the projected benefit payments, the liability value (PBO) can be calculated as

$$L_t = \sum_{i=1}^{\infty} \frac{B_{t+i}}{(1 + \mathrm{DR}_{t,i})^i}$$

where $\mathrm{DR}_{t,i}$ is the discount rate applicable to benefit payments due i time periods after the liability value is determined, at time t. In this report, it is the AA-rated i-year corporate bond spot rate at time t.

The liability value is projected dynamically. At the end of each period, the accrued benefit payments will increase because of extra service accrued during the period for active plan participants. Accordingly, contributions will be made to the plan to fund the extra benefit payments as follows:

$$C_t = \sum_{i=1}^{\infty} \frac{\Delta B_{t,i}}{\left(1 + \mathrm{CR}_{t,i}\right)^i} [1 + (\mathrm{SER}_t - \overline{\mathrm{SER}}) \times \mathrm{adj}]$$

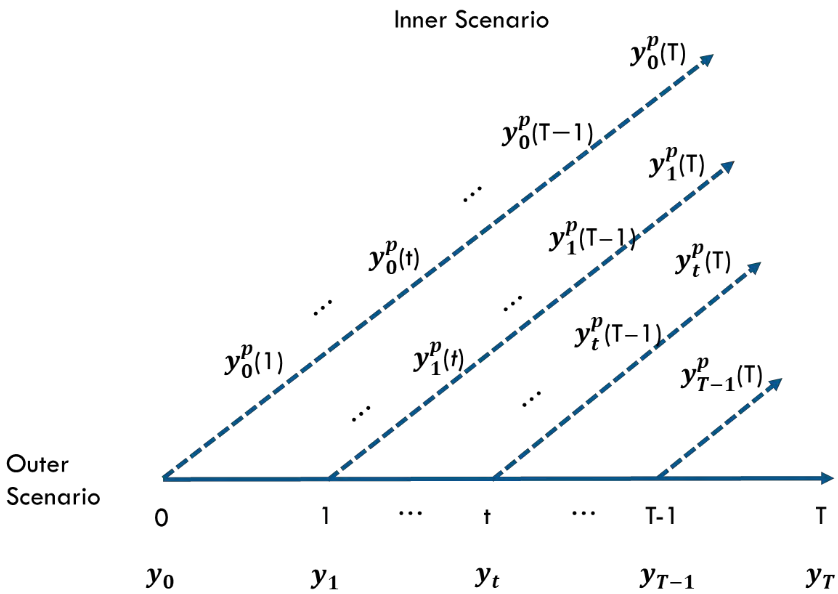
where

$\Delta B_{t,i}$ = extra benefit payments at time $t + i$ caused by the extra length of service during period t;
$\mathrm{CR}_{t,i}$ = discount rate of term i at time t used to determine the amount of plan contribution (could be the expected investment return based on actual asset allocation or the liability discount rate);
SER_t = plan sponsor's equity return during period t;
$\overline{\mathrm{SER}}$ = expected plan sponsor's equity return; and
adj = adjustment factor to reflect the impact of sponsor's equity return on plan contributions (adjusted to mitigate the impact of financial stress experienced by the plan sponsor).

In practice, the discount rates are often set to be the expected rate of investment return for the purpose of determining the plan contribution. The scenarios generated by the economic scenario generator include volatilities of all periods till the end of the projection. During the dynamic projection, the future volatilities are unknown. Expected return needs to reflect the current economic state but exclude the unknown future volatilities. As shown in Figure 5, the outer scenario is the entire scenario, including the volatility of all periods. The inner scenarios are the expected scenarios at each time point. At time *t*, the then-current plan assets and plan liabilities are determined based on the outer scenario till time *t*.

The inner scenario at time t is constructed with the expected future economic fundamental factors and asset returns till the end of projection at time T.

Figure 5
Dynamic Projection



Discount rate $\mathrm{CR}_{t',i}$ can be derived using the following steps:

Step 1. Generate the fundamental economic factors with the random term $\mathbf{e}_t$ till time t' (simulated economic factors) and without the random term thereafter (expected economic factors):

$$\mathbf{F}_{t'}^{p}(t) = \begin{matrix} c + \mathbb{A}_1\mathbf{F}_{t-1} + \mathbf{e}_t & \quad t \le t' \\ c + \mathbb{A}_1\mathbf{F}_{t-1} & \quad t > t' \end{matrix}$$

where $\mathbf{F}_{t'}^{p}(t)$ is a column vector that contains expected fundamental economic factors, given the history till time t'.

Step 2. Generate the asset returns with the random term $\mathbf{e}_t$ till time t' (simulated asset returns) and without the random term thereafter (expected asset returns):

$$\mathbf{y}_{t'}^{p}(t) = \begin{matrix} \alpha + \phi_1\mathbf{y}_{t-1} + \phi_2\mathbf{y}_{t-2} + \mathbb{B}_0\mathbf{F}_t + \mathbb{B}_1\mathbf{F}_{t-1} + \mathbb{B}_2\mathbf{F}_{t-2} + \mathbf{e}_t & \quad t \le t' \\ \alpha + \phi_1\mathbf{y}_{t-1} + \phi_2\mathbf{y}_{t-2} + \mathbb{B}_0\mathbf{F}_t + \mathbb{B}_1\mathbf{F}_{t-1} + \mathbb{B}_2\mathbf{F}_{t-2} & \quad t > t' \end{matrix}$$

Where $\mathbf{y}_{t'}^{p}(t)$ is a column vector that contains expected asset returns and other factors, given the history till time t'.

Step 3. Calculate the expected aggregated asset return based on the asset mix:

$$\mathrm{CR}_{t',i} = \mathbf{w}_{t'+i}\mathbf{y}_{t'+i}^{p}$$

where $\mathbf{w}_{t'+i}$ is a row vector that contains the asset mix at time $t' + i$. If the plan has a target asset mix and the portfolio is rebalanced at the end of each period, the vector will be constant through time.

By this method, the discount rate used to determine contributions is not fixed but varies according to circumstances. The expected return on a bond portfolio would be sensitive to changes in the average yield to maturity on the bonds, while the expected return on equities and other investments could be sensitive to anticipated real GDP growth rates or other fundamental economic factors.

With the projected benefit payments and plan contributions, asset balances can be dynamically projected under each scenario, considering the net cash flow from the liability side.

4.4 Asset Model

With the economic scenario generator and liability cash flow projection, asset projection is straightforward. A simplified version is used here, assuming that the benefit payment B_t and plan contribution C_t occur at the end of each period:

$$A_t = A_{t-1}(1 + c_t + r_t) - B_t + C_t$$

where

$A_t =$ total asset value at time t;
$c_t =$ aggregate income rate (coupon, cap rate or dividend yield) of the asset portfolio during period t, equal to the weighted average income rate of all asset classes with weight equal to the investment amount at the beginning of period t; and
$r_t =$ aggregate capital return (price return) of the asset portfolio during period t, or the weighted average capital return of all asset classes with weight equal to the investment amount at the beginning of period t.

Based on the asset allocation plan, the asset portfolio may be rebalanced, which will affect the calculation of future c_t and r_t. This is where LDI strategy plays its important role in matching liabilities with assets.

With projected assets and liabilities, economic funding ratio FR_t and economic funding surplus/shortfall FS_t can be calculated accordingly:

$$\mathrm{FR}_t = \frac{A_t}{L_t}$$

$$\mathrm{FS}_t = A_t - L_t$$

4.5 Dependency

The dependency in the LDI benchmark model is governed by the economic scenario generator. The economic scenario generator generates scenarios of fundamental economic factors that determine the status of the economy. Based on the fundamental economic factors, it generates asset returns and other factors such as wage inflation and sponsor's equity returns. Pension plan assets and liabilities are then linked together through the scenarios. It is important that the economic scenario generator can maintain the dependency among all the economic variables observed in the historical data. This is achieved in five places:

1. Intertemporal dependency among fundamental economic factors is built in the economic scenario generator with the VAR(1) model. The fundamental economic factors jointly determine the status of the economy: recession or expansion.
2. The error terms (nonsystemic component) of the fundamental economic factors in the VAR(1) model are not independent from one another. Their contemporary correlations are reflected when generating the random part of the fundamental economic factors.
3. Both contemporary and intertemporal dependency between asset returns and fundamental economic factors are governed by the linear models used to generate asset return scenarios.
4. The error terms (idiosyncratic factors) of asset return variables or other factors are not independent from one another. Their contemporary correlations are reflected when generating the random part of the asset returns.
5. To address the issue of nonlinear relationship, the volatility of idiosyncratic factors, their interdependency and their dependency on fundamental economic factors vary by the status of the simulated economy. Higher volatility and dependency are used when the economy is projected to be in recession.

Each scenario generated by the economic scenario generator will bear the appropriate relationships. Asset and liability projected based on these scenarios will provide a consistent, realistic and holistic view of the pension plan's future. Technical details of modeling dependency can be found in Appendix A: Economic Scenario Generation for LDI.

4.6 Optimization

Regarding LDI optimization, existing literature usually has simplified assumptions about asset return distributions and their relationships, so as to be able to have closed-form solutions. The choice of the optimal strategy may rely on an abstract utility function, which may be too difficult to link to reality in an understandable and verifiable way. Unlike some of the existing literature on pension investment strategy optimization discussed in Section 2.1.4, the optimization of the LDI benchmark model takes a brute-force approach. Closed-form solution is not feasible using the LDI benchmark model because of the way stochastic scenarios are constructed and the dynamic projection of pension plan funding status. With the LDI benchmark model, all possible asset allocation plans are tested and compared to find the best option.

Assessment of the asset allocation plans requires choosing return and risk measures. In this report, the following measures are used:

- **Return measure.** This equals the average funding ratio minus the target funding ratio.
- **Risk measure.** This equals the average funding ratio minus the left-side *x*th percentile of the funding ratio (that is, the lowest [100 – *x*]% of the funding ratio).

The plan contributions under different asset allocation plans may be different. If expected return is used when determining the plan contribution, high-risk plans will have less plan contributions. The difference could be material and needs to be reflected in the return and risk measures. The funding ratio can be modified as below:

$$\text{Modified Funding Ratio} = \text{Funding Ratio} + \frac{\text{DV(Plan Contributions)}^{\text{Low Risk}} - \text{DV(Plan Contributions)}}{\text{Initial Plan Liability}}$$

where

$\text{DV(Plan Contributions)}$ = discounted value of plan contributions at time 0, given the chosen asset allocation plan; and

$\text{DV(Plan Contributions)}^{\text{Low Risk}}$ = discounted value of plan contributions at time 0, given a low-risk allocation plan such as 100% Treasury bond investment.

When the modified funding ratio is used, both plan contributions and fund surplus are considered when assessing asset performance against liability. Alternatively, dollar values of fund surplus can be used to assess investment performance.

The following information may be provided to facilitate the optimization:

1. **Target funding ratio**, which could be 100% or a little higher to avoid both underfunding and overfunding;
2. **Time horizon** at which the return measure and risk measure will be calculated;
3. **Confidence level** *X*%, which defines the percentile at which a risk measure is assessed;
4. **Minimum acceptable funding ratio** that the plan sponsor can tolerate at the confidence level (this is a constraint on asset allocation);
5. **Allocation limit** on specific asset classes (for example, excluding certain asset classes due to regulation or the plan sponsor's own preference, such as not allowing negative allocations, or overlay strategies); and
6. **Other constraints,** such as a limit on annual plan contribution as a percentage of the plan sponsor's equity.

The optimization process follows these steps:

Step 1. Narrow the available asset allocation plans based on the constraints provided. For example, allocation limits may drastically reduce the choice of an asset allocation plan.

Step 2. Evaluate each possible asset allocation plan using stochastic scenarios. Both return and risk are calculated.

Step 3. Remove asset allocation plans that do not meet the constraints. For example, any asset allocation plan that generates a funding ratio lower than the minimum acceptable funding ratio can be removed from the solution set.

Step 4. Select the optimal asset allocation plan with the maximum modified Sharpe ratio:

$$\text{Modified Sharpe Ratio} = \frac{\text{Average Modified Funding Ratio} - 100\%}{\text{Average Funding Ratio} - \text{Left Side } x\text{th Funding Ratio}}$$

The maximum modified Sharpe ratio is one of many possible criteria for choosing the most appropriate asset allocation plan. Qualitative considerations are likely to be involved in the final decision.

In addition to the optimal asset allocation plan, the liability replicating portfolio, which can be considered the minimum variance portfolio against pension liability, can be derived. It is the asset portfolio that best matches the liability. Appendix B: Liability Replicating Portfolio Construction describes the process of deriving a liability replicating portfolio, using an example.

Section 5: Application

In this section, the application of the LDI benchmark model is illustrated, using a sample DB plan. Table 3 describes the DB plan and the liability projection assumptions used for illustration.

Table 3
Sample DB Plan Information

Item	Assumption
Initial plan assets	$10,000,000
Initial plan liabilities	$10,000,000
Valuation date	Dec. 31, 2016
Pension benefit	5-Year Average Salary before Retirement × 1% × Length of Service × COLA
COLA	$\prod_{i=1}^{\text{Current Age} - \text{Retirement Age}} (1 + \min(5\%, \quad 80\% \times \text{Inflation Rate}_i))$
Payment option	1. Retirees can choose the lump sum payment option at retirement. The lump sum amount is equal to the expected future benefit payments discounted by 4%, and 10% of retirees are assumed to use this option. 2. The remaining 90% retirees are assumed to choose life payment option.
Administration, investment and tax expenses	These expenses are not modeled explicitly in this example. It is assumed that administration expenses are included in benefit payments and investment/tax expenses are deducted from gross investment return.
Standard mortality rate	Example ultimate mortality rate till age 110 by gender (life expectancy: 77 years)
Mortality improvement rate	1% per year
Vesting period	3 years

<table>
<tr><td>Vesting period turnover rate</td><td>
<table>
<tr><th>Occupation Type</th><th>Turnover Rate</th></tr>
<tr><td>I</td><td>5%</td></tr>
<tr><td>II</td><td>4%</td></tr>
<tr><td>III</td><td>3%</td></tr>
<tr><td>IV</td><td>2%</td></tr>
<tr><td>V</td><td>1%</td></tr>
</table>
</td></tr>
<tr><td>Salary growth rate</td><td>
Basic salary growth rate is linked to the scenarios of wage inflation. Different occupations have different multiples of the basic growth rate.
<table>
<tr><th>Occupation Type</th><th>Turnover Rate</th></tr>
<tr><td>I</td><td>0.9</td></tr>
<tr><td>II</td><td>1</td></tr>
<tr><td>III</td><td>1.1</td></tr>
<tr><td>IV</td><td>1.2</td></tr>
<tr><td>V</td><td>1.5</td></tr>
</table>
</td></tr>
<tr><td>Plan participant mix</td><td>
<table>
<tr><th>ID</th><th>Retired</th><th>Sex</th><th>Date of Birth</th><th>Date of Hire</th><th>Annual Salary</th><th>Date of Retirement</th><th>Occupation</th><th>Weight</th></tr>
<tr><td>1</td><td>Y</td><td>F</td><td>1950-06-30</td><td>1979-04-30</td><td>50000</td><td>2014-07-31</td><td>II</td><td>15%</td></tr>
<tr><td>2</td><td>Y</td><td>M</td><td>1955-06-30</td><td>1980-05-31</td><td>45000</td><td>2015-06-30</td><td>I</td><td>15%</td></tr>
<tr><td>3</td><td>N</td><td>F</td><td>1981-06-30</td><td>2005-01-31</td><td>80000</td><td>2045-06-30</td><td>IV</td><td>8%</td></tr>
<tr><td>4</td><td>N</td><td>M</td><td>1991-06-30</td><td>2015-01-31</td><td>40000</td><td>2055-06-30</td><td>II</td><td>8%</td></tr>
<tr><td>5</td><td>N</td><td>M</td><td>1971-06-30</td><td>2005-01-31</td><td>120000</td><td>2035-06-30</td><td>V</td><td>8%</td></tr>
<tr><td>6</td><td>N</td><td>F</td><td>1961-06-30</td><td>2000-12-31</td><td>80000</td><td>2020-06-30</td><td>II</td><td>8%</td></tr>
<tr><td>7</td><td>N</td><td>F</td><td>1981-06-30</td><td>2005-01-31</td><td>100000</td><td>2045-06-30</td><td>IV</td><td>8%</td></tr>
<tr><td>8</td><td>N</td><td>M</td><td>1986-06-30</td><td>2005-01-31</td><td>55000</td><td>2050-06-30</td><td>II</td><td>10%</td></tr>
<tr><td>9</td><td>N</td><td>M</td><td>1966-06-30</td><td>1991-01-31</td><td>95000</td><td>2030-06-30</td><td>IV</td><td>10%</td></tr>
<tr><td>10</td><td>N</td><td>F</td><td>1971-06-30</td><td>1995-01-31</td><td>50000</td><td>2035-06-30</td><td>II</td><td>10%</td></tr>
</table>
They are compressed model points to represent the entire portfolio. The weight stands for the portion of the liability portfolio that a model point represents. They are scaled so that the total liability value equals the plan’s initial liability value.
</td></tr>
</table>

Based on the sample DB plan, each asset class is analyzed regarding its fit to pension plan investment, using both the historical experience and the simulated scenarios. The optimal asset allocation plan is also sought to match the plan sponsor’s risk appetite. A simplified risk capital model is discussed to measure the riskiness of each asset allocation plan.

5.1 Asset Class Analysis

Understanding the trade-off between risk and return of each asset class is helpful for designing LDI strategy. It builds the foundation of LDI optimization. Certain asset classes may be found inappropriate for the plan sponsor and excluded from LDI consideration. Analyzing an asset class using the LDI benchmark model is no different from analyzing an asset allocation plan, except that the allocation plan is 100% investment in the asset class. Two approaches are used in this example: empirical analysis and stochastic analysis.

In the empirical analysis, history is assumed to repeat. The funding status of the sample DB plan can then be predicted based on the historical returns. To have an apples-to-apples comparison, future plan contributions are assumed to be the same, irrespective of asset class. Plan contributions are calculated by discounting the extra benefits accrued during the period using an AA-rated corporate bond yield curve. Therefore, for asset classes with high risk and high expected return, a very high average funding ratio would be expected in the empirical analysis. A more reasonable way of determining plan contributions is to use expected return as the discount rate. However, it is difficult to hypothesize the expected return in a deterministic historical scenario, and the impact of different contribution amounts is difficult to normalize.

The historical data of asset returns used for economic scenario calibration described in A.2 Economic Scenario Generation for Asset Return is used for empirical analysis as well. Using exactly the history from 1997 to 2016, the projected funding ratios are listed in Table 4. Clearly, the historical performance of mortgage REITs, private equity, the infrastructure fund index, crude oil and the commodity index is not satisfactory. The liability replicating portfolio derived in Appendix B: Liability Replicating Portfolio Construction also is included in the analysis as a low-risk portfolio. Bonds and the liability replicating portfolio have similar stable performance. Public equities and equity REITs have superior performance.

Table 4
Asset Class Empirical Analysis

Asset Class	Projection Year							
	0	1	2	3	5	10	15	20
Government bond	1	1.21	1.38	1.17	1.33	1.43	1.93	1.78
Corporate bond (AAA to BBB rating)	1	1.18	1.29	1.13	1.38	1.43	1.89	1.95
AAA-rated corporate bond	1	1.17	1.29	1.12	1.39	1.37	1.92	1.69
AA-rated corporate bond	1	1.18	1.30	1.13	1.41	1.41	1.87	1.90
A-rated corporate bond	1	1.18	1.29	1.14	1.34	1.45	1.86	2.02
BBB-rated cororate bond	1	1.19	1.24	1.13	1.29	1.54	1.96	2.23
Public equity, large cap	1	1.41	1.76	2.01	1.44	1.60	1.33	2.20
Public equity, mid cap*	1	1.18	1.29	1.36	1.35	1.85	1.79	3.08
Public equity, small cap*	1	1.24	1.41	1.55	1.64	2.42	2.18	3.99
Public equity, high dividend*	1	1.17	1.25	1.30	1.40	1.99	1.55	2.53
Equity REIT	1	1.27	1.04	0.95	1.20	2.69	2.11	3.13
Mortgage REIT	1	1.10	0.79	0.54	0.93	1.64	0.91	1.17
Private equity*	1	1.09	1.10	1.07	1.00	0.95	0.71	0.79
Infrastructure*	1	1.06	1.04	0.99	0.88	0.74	0.29	0.42
Crude Oil	1	0.73	0.50	1.00	0.69	1.74	2.34	1.06
Gold	1	0.84	0.82	0.79	0.67	1.28	2.56	1.61
Commodity index*	1	0.96	0.73	0.89	0.71	1.38	1.78	0.87
Liability replicating portfolio	1	1.17	1.27	1.11	1.34	1.36	1.83	1.73

*Missing data are reconstructed using the linear models for asset return scenario generation.

In addition to conducting an empirical analysis, it is helpful to understand the correlation between these asset classes and pension liability. A higher correlation indicates that the asset class is a better hedging instrument for pension liability risk. Table 5 shows the correlation coefficients between asset classes and pension liability. Both pension asset value and liability value are projected using the simple DB plan described in Table 3. The assets are assumed to be invested in a single asset class or the liability replicating portfolio.

Table 5

Annual Correlation: Asset Class and Pension Liability

Asset Class	Historical Experience	Good Scenario	Bad Scenario
Government bond	98%	92%	85%
Corporate bond (AAA to BBB rating)	98%	98%	98%
AAA-rated corporate bond	98%	95%	92%
AA-rated corporate bond	98%	98%	98%
A-rated corporate bond	97%	98%	96%
BBB-rated corporate bond	97%	97%	98%
Public equity, large cap	86%	92%	77%
Public equity, mid cap	93%	94%	70%
Public equity, small cap	93%	96%	74%
Public equity, high dividend	91%	94%	80%
Equity REIT	94%	88%	−11%
Mortgage REIT	66%	96%	97%
Private equity	83%	35%	−40%
Infrastructure	−54%	−52%	−43%
Oil	67%	79%	57%
Gold	88%	66%	−44%
Commodity index	78%	94%	−68%
Liability replicating portfolio	98%	97%	98%

Annual correlation is used here to incorporate both contemporary and intertemporal dependence. The impact of an economic event is likely to spread to all asset classes and to pension liability within one year. The correlation is assessed over a 20-year time horizon for each scenario, rather than a set of stochastic scenarios. This is to recognize the nonlinear relationship and the importance of path dependence in pension investment, like other long-term investment activities. The correlation is assessed using the historical experience from 1997 to 2016, a good scenario and a bad scenario. The hypothetical good scenario and bad scenario are selected from the stochastic scenarios generated by the economic scenario generator described in Appendix A: Economic Scenario Generation for LDI. Using a balanced asset allocation plan, the projected funding ratio after 20 years is 146% under the good scenario and 80% under the bad scenario. Figures A.9 and A.34 to A.39 in the appendix show the economic fundamental factors and asset returns of the good and bad scenario.

Based on the results in Table 5, the correlation under the good scenario is mostly higher than that under the bad scenario, where high hedging effectiveness is desired. Corporate bonds are the best hedging candidates, followed by government bonds, public equity, mortgage REITs and oil, considering all three scenarios. Other investigated asset classes have a negative correlation with pension liability, which indicates their value in the pension asset portfolio is not direct risk mitigation but higher return and diversification.

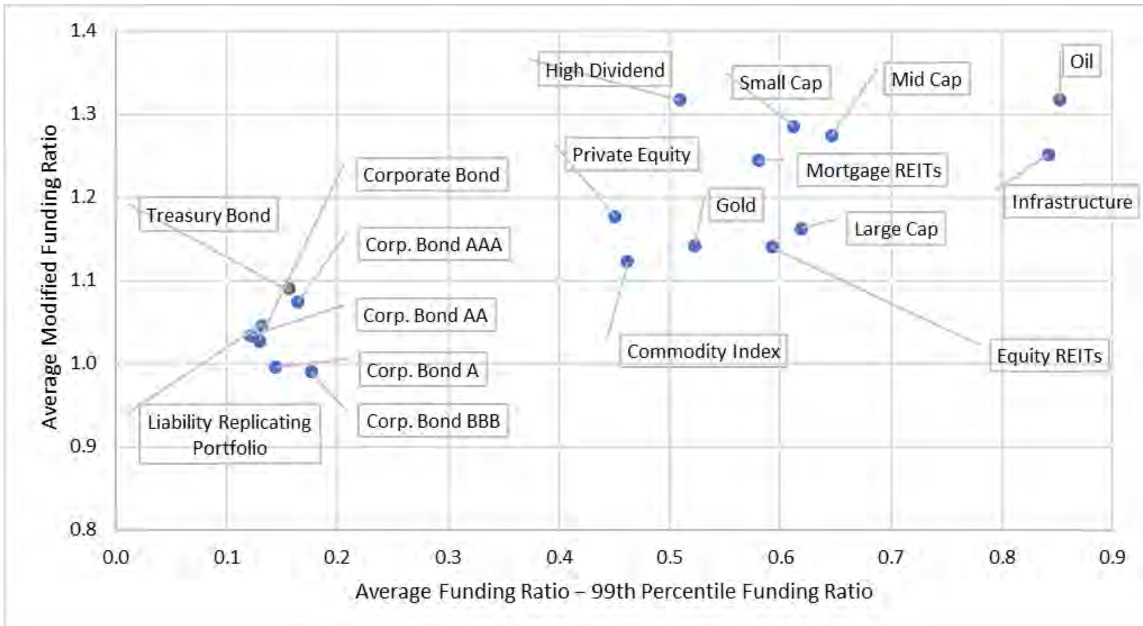
The stochastic analysis approach uses stochastic scenarios generated by the economic scenario generator to predict the future performance of each asset class. Unlike empirical analysis that has only one deterministic scenario, stochastic analysis has multiple scenarios that can predict both return and risk. Plan contributions are determined using expected asset return and are different by asset class. The modified funding ratio described in Section 4.5 Optimization is used to offset the difference in the comparison. Table 6 lists the risk, return, minimum funding ratio and modified Sharpe ratio after three years, using 100 scenarios. High-dividend public equity, small-cap public equity and government bonds are the top three asset classes with a high modified Sharpe ratio. Mortgage REITs, private equity, infrastructure funds, and crude oil, which performed badly in empirical analysis, are not that bad in stochastic analysis. The economic scenario generator maintains a reasonable expected relationship between risk and return which may not be fully observed in a period of history.

Table 6
Asset Class Stochastic Analysis

Asset Allocation Plan	Average—99th Percentile Funding Ratio (Risk)	Modified Average Funding Ratio (Return)	Minimum Funding Ratio	Modified Sharpe Ratio
Government bond	0.16	1.09	0.93	0.58
Corporate bond (AAA to BBB rating)	0.13	1.03	0.90	0.21
AAA-rated corporate bond	0.18	1.07	0.91	0.42
AA-rated corporate bond	0.12	1.03	0.91	0.28
A-rated corporate bond	0.14	1.00	0.85	-0.03
BBB-rated corporate bond	0.16	0.99	0.81	-0.06
Public equity, large cap	0.57	1.16	0.54	0.28
Public equity, mid cap	0.59	1.27	0.63	0.46
Public equity, small cap	0.56	1.28	0.67	0.51
Public equity, high dividend	0.46	1.32	0.81	0.69
Equity REIT	0.54	1.14	0.55	0.26
Mortgage REIT	0.54	1.24	0.66	0.45
Private equity	0.52	1.18	0.73	0.34
Infrastructure fund	0.86	1.25	0.41	0.29
Crude oil	0.81	1.32	0.46	0.39
Gold	0.50	1.14	0.62	0.28
Commodity index	0.50	1.12	0.66	0.25
Liability replicating portfolio	0.12	1.05	0.91	0.37

Figure 6 shows the risk and return of each asset class and the liability replicating portfolio. A- or BBB-rated corporate bonds, the commodity index, equity REITs, large-cap public equity and the infrastructure fund seem to have lower efficiency than other asset classes, which are closer to the efficient frontier.

Figure 6
Asset Class Risk-Return Analysis



The analysis above is based on a simple open DB plan. Many factors, such as the plan status (open or closed), age of the workforce, and benefit indexing method, could affect the liability risk and the best matching asset portfolio. The conclusions may be different for other DB plans.

5.2 Investment Decision

Theoretically, the best asset allocation plan for LDI can be sought by running all possible plans with the LDI benchmark model. Following the optimization process listed in Section 4.5, the asset allocation plan that meets all the constraints and has the highest modified Sharpe ratio is the optimal asset allocation plan.

In this example, the plan sponsor has a target funding ratio equal to 100%. At a confidence level of 99%, the sponsor does not want the funding ratio to fall below 70% at the end of three years. Short selling is not allowed in pension plan investment.

Given the specified constraints, millions of asset allocation plans are still possible. In practice, it is unlikely that all these plans will be run through the benchmark model. Several methods may be used to reduce the number of asset allocation plans:

1. Each asset class can be evaluated separately. For asset classes with similar risk and return characteristics and high correlation, certain asset classes with relatively low modified Sharpe ratios may be removed from the asset allocation plans.
2. A subset of possible asset allocation plans can be sampled from the entire set. A suboptimal asset allocation plan can be found from the subset.

In addition to the 18 allocation plans (individual asset classes and the liability replicating portfolio) shown in Table 6, 40 extra balanced plans are analyzed, with the results listed in Table 7. Some allocation plans do not fit the plan sponsor's risk appetite, as they have minimum funding ratios less than 70%.

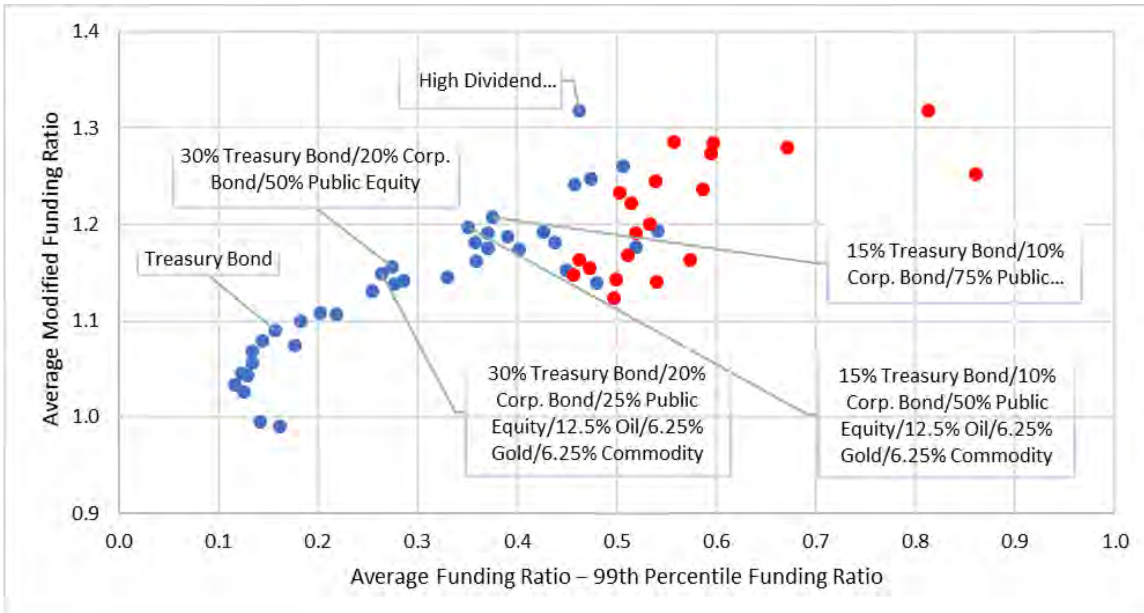
Table 7
Asset Allocation Plan Analysis

Asset Allocation Plan	Average – 99th Percentile Funding Ratio	Modified Average Funding Ratio	Minimum Funding Ratio	Modified Sharpe Ratio
80% Treasury bonds + 20% corporate bonds	0.14	1.08	0.93	0.55
60% Treasury bonds + 40% corporate bonds	0.13	1.07	0.93	0.51
40% Treasury bonds + 60% corporate bonds	0.13	1.06	0.92	0.42
20% Treasury bonds + 80% corporate bonds	0.13	1.04	0.91	0.34
Public equity: 25% each for large cap, mid cap, small cap, high dividend	0.51	1.26	0.70	0.51
Public equity: 50% large cap, 25% small cap, 25% high dividend	0.50	1.23	0.68	0.46
Public equity: 75% large cap, 25% high dividend	0.53	1.20	0.62	0.37
Public equity: 50% large cap, 25% mid cap, 25% small cap	0.51	1.22	0.66	0.43
Public equity: 75% large cap, 25% mid cap	0.52	1.19	0.62	0.37
Public equity: 75% large cap, 25% small cap	0.54	1.19	0.60	0.36
25% each for equity REITs, mortgage REITs, private equity, infrastructure	0.43	1.19	0.74	0.45
50% equity REITs, 25% private equity, 25% infrastructure	0.46	1.16	0.67	0.35
75% equity REITs, 25% infrastructure	0.47	1.15	0.64	0.32
50% equity REITs, 25% mortgage REITs, 25% private equity	0.40	1.17	0.73	0.43
75% equity REITs, 25% mortgage REITs	0.51	1.17	0.61	0.33
75% equity REITs, 25% private equity	0.46	1.15	0.65	0.32
Commodities: 75% crude oil, 25% gold	0.67	1.28	0.57	0.41
Commodities: 50% crude oil, 50% gold	0.59	1.24	0.62	0.40
Commodities: 50% crude oil, 25% gold, 25% commodity index	0.60	1.28	0.65	0.48
Commodities: 25% crude oil, 50% gold, 25% commodity index	0.45	1.15	0.72	0.34
Commodities: 25% crude oil, 25% gold, 50% commodity index	0.48	1.14	0.69	0.29
45% Treasury bonds, 30% corporate bonds, 25% public equity	0.20	1.11	0.88	0.53
45% Treasury bonds, 30% corporate bonds, 12.5% real estate, 6.25% private equity, 6.25% infrastructure	0.18	1.10	0.91	0.54
45% Treasury bonds, 30% corporate bonds, 12.5% oil, 6.25% gold, 6.25% commodities	0.22	1.11	0.88	0.49
30% Treasury bonds, 20% corporate bonds, 50% public equity	0.27	1.16	0.85	0.57
30% Treasury bonds, 20% corporate bonds, 25% real estate, 12.5% private equity, 12.5% infrastructure	0.25	1.13	0.86	0.51
30% Treasury bonds, 20% corporate bonds, 25% oil, 12.5% gold, 12.5% commodities	0.33	1.14	0.80	0.44
30% Treasury bonds, 20% corporate bonds, 25% public equity, 12.5% real estate, 6.25% private equity, 6.25% infrastructure	0.29	1.14	0.83	0.50
30% Treasury bonds, 20% corporate bonds, 25% public equity, 12.5% oil, 6.25% gold, 6.25% commodities	0.26	1.15	0.86	0.56

Asset Allocation Plan	Average – 99th Percentile Funding Ratio	Modified Average Funding Ratio	Minimum Funding Ratio	Modified Sharpe Ratio
30% Treasury bonds, 20% corporate bonds, 12.5% real estate, 6.25% private equity, 6.25% infrastructure, 12.5% oil, 6.25% gold, 6.25% commodities	0.28	1.14	0.85	0.50
15% Treasury bonds, 10% corporate bonds, 75% public equity	0.37	1.21	0.79	0.55
15% Treasury bonds, 10% corporate bonds, 37.5% real estate, 18.75% private equity, 18.75% infrastructure	0.36	1.16	0.78	0.45
15% Treasury bonds, 10% corporate bonds, 37.5% oil, 18.75% gold, 18.75% commodities	0.44	1.18	0.72	0.41
15% Treasury bonds, 10% corporate bonds, 50% public equity, 12.5% real estate, 6.25% private equity, 6.25% infrastructure	0.37	1.19	0.78	0.51
15% Treasury bonds, 10% corporate bonds, 50% public equity, 12.5% oil, 6.25% gold, 6.25% commodities	0.35	1.20	0.81	0.56
15% Treasury bonds, 10% corporate bonds, 25% public equity, 25% real estate, 12.5% private equity, 12.5% infrastructure	0.37	1.17	0.77	0.47
15% Treasury bonds, 10% corporate bonds, 25% public equity, 25% oil, 12.5% gold, 12.5% commodities	0.39	1.19	0.77	0.48
15% Treasury bonds, 10% corporate bonds, 25% public equity, 12.5% real estate, 6.25% private equity, 6.25% infrastructure, 12.5% oil, 6.25% gold, 6.25% commodities	0.36	1.18	0.79	0.51
75% public equity, 12.5% real estate, 6.25% private equity, 6.25% infrastructure	0.46	1.24	0.74	0.53
75% public equity, 12.5% oil, 6.25% gold/6.25% commodities	0.47	1.25	0.73	0.52

Figure 7 shows the risk and return trade-off of each asset allocation plan (17 individual asset classes and one liability replicating portfolio, as in Table 6, and 40 balanced plans, as in Table 7). Twenty asset allocation plans that do not meet the minimum funding ratio requirement are shown as red dots. As expected, most of them are heavily invested in equities, real estate or commodities with high volatility. The remaining 38 asset allocation plans are shown as blue dots. The top six allocation plans are labeled in the figure. Except for the plans with 100% government bonds and 100% high-dividend public equities, the other four of the top six plans are balanced plans and are more likely to be used in reality. The allocation plan with 30% Treasury bonds, 20% corporate bonds and 50% public equity is the balanced plan with the highest modified Sharpe ratio. Based on the limited samples used here, it is the best choice of the sample DB plan.

Figure 7
Asset Allocation Risk-Return Analysis



5.3 Risk Capital

With the stochastic analysis ready, required risk capital can be calculated for different LDI strategies. Required risk capital is the amount of capital that a plan sponsor wants to hold as a protection against adverse changes in funding status. In this example, a simplified approach is taken without requiring much extra modeling other than the LDI benchmark model.

1. The risk capital considers three major risk categories: economic risk, insurance risk and operational risk. Market risk and credit risk are combined as economic risk because they are integrated in the economic scenario generator based on the same macroeconomic model.
2. A one-year mark-to-market approach is used for calculating required risk capital. The required capital at the end of the first year is discounted back to the valuation date, using the liability discount rate.
3. A confidence level of 99% is used in the example. The capital is expected to cover the loss occurred in a once-in-100-years event.
4. The liability discount curve is the AA-rated corporate bond yield curve, for simplicity. In practice, it could be changed to the risk-free curve plus the liquidity premium. The example tries to avoid the additional complexity of determining the liquidity premium, which is another big topic.
5. This plan has a cap of 5% for COLA adjustments. This embedded option is not separately evaluated but is reflected in stochastic analysis as part of the total liability.
6. Economic risk capital is calculated as the funding shortfall at the chosen confidence level.
7. Insurance risk capital is calculated based on a stress scenario including a 10% decrease of mortality rates and a 10% increase of the mortality improvement rate at the same time.
8. Operational risk capital is calculated as 10% of the required capital for economic and insurance risk.
9. The correlation coefficient is assumed to be 15% between economic risk capital and insurance risk capital and 10% between operational risk capital and required capital due to other risks.

Table 8 shows the required risk capital as a percentage of plan liability for each asset class, liability replicating portfolio and top-performing balanced plans.

Table 8
Sample Risk Capital Result

Allocation Plan	Risk Capital/Plan Liability			
	Economic Risk	Insurance Risk	Operational Risk	Total
100% Treasury bonds	14.5%	13.1%	2.1%	21.3%
100% corporate bonds	15.5%	13.1%	2.2%	22.1%
100% large-cap public equity	34.3%	13.1%	3.8%	39.1%
100% mid-cap public equity	55.9%	13.1%	5.9%	60.2%
100% small-cap public equity	45.5%	13.1%	4.9%	49.9%
100% high-dividend public equity	22.4%	13.1%	2.8%	28.0%
100% equity REITs	39.8%	13.1%	4.4%	44.4%
100% mortgage REITs	36.1%	13.1%	4.0%	40.8%
100% private equity	43.0%	13.1%	4.7%	47.5%
100% infrastructure fund	40.0%	13.1%	4.4%	44.5%
100% crude oil	86.1%	13.1%	8.9%	90.4%
100% gold	52.0%	13.1%	5.5%	56.3%
100% commodity index	58.3%	13.1%	6.2%	62.6%
Liability replicating portfolio	14.9%	13.1%	2.1%	21.6%
30% government bonds, 20% corporate bonds, 50% public equity	23.4%	13.1%	2.9%	28.9%
30% government bonds, 20% corporate bonds, 25% public equity, 12.5% oil, 6.25% gold, 6.25% commodities	31.7%	13.1%	3.6%	36.6%
15% government bonds, 10% corporate bonds, 75% public equity	30.7%	13.1%	3.5%	35.7%
15% government bonds, 10% corporate bonds, 50% public equity, 12.5% oil, 6.25% gold, 6.25% commodities	39.1%	13.1%	4.3%	43.7%

Assessment of capital adequacy requires that we quantify the impact on the available capital at the same confidence level. The plan sponsor's equity return can be used to roughly measure the change in available capital. The plan sponsor's equity return can be simultaneously generated along with asset returns in the LDI benchmark model. Table 9 lists some descriptive statistics of the plan sponsor's first-year stochastic equity returns generated by the LDI benchmark model.

Table 9
Simulated Plan Sponsor First-Year Equity Return

Minimum	**−48.3%**
5th percentile	−38.0%
10th percentile	−29.5%
25th Percentile	−14.3%
Average	3.6%
Median	4.1%
75th percentile	19.3%
90th percentile	35.4%
95th percentile	46.9%
Maximum	63.5%

However, the available capital that a plan sponsor holds will meet the requirements not only from its DB plan but also from its business operations. A pension plan sponsor would normally prefer to allocate a certain amount of capital as a provision for adverse deviations inside the pension fund and keep the balance outside of the pension fund in combination with other enterprise capital. Therefore, with the help of the LDI benchmark model, capital adequacy can be assessed at a higher level than the DB plan for the plan sponsor. Capital adequacy could be used as another constraint when choosing the asset allocation plan.

Section 6: Conclusion

LDI, as one of several approaches to mitigate pension liability risk, involves asset modeling at a more detailed level than common market indices. Alternative assets and unique asset selection strategies are used in LDI, and their impact can be reflected with a more comprehensive and detailed analytic framework.

By incorporating macroeconomic factors, the LDI benchmark model is intended to support more consistent and reasonable scenarios that reflect dependency among asset classes and between assets and liabilities. The example of an LDI benchmark model developed in this report uses fundamental economic factors and asset subclass level information to better capture the reality of LDI strategies and cyclical economic patterns. A model of this type can be used to construct the benchmark return of LDI strategies, inform the choice of asset allocation plans, and predict the possible financial outcomes and future contributions for a chosen strategy. It is helpful for investment decisions and risk analysis.

The model that accompanies this report supports different views of and assumptions about real-world markets and is not confined to a single model-based optimal asset allocation plan. The individual parameter of a pension plan such as current funding ratio, target funding ratio and risk appetite can be reflected in the model as well.

Endnotes

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[1] Milliman's corporate pension funding study tracks the 100 largest DB plans sponsored by U.S. public companies. For details, visit http://us.milliman.com/PFS.

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Appendix A: Economic Scenario Generation for LDI

Unlike capital management that usually has a one-year time horizon, the evaluation of LDI strategies may take a much longer period covering an entire economic cycle. Most real-world scenario generators rely on either historical data or current market data for calibration. They may be sophisticated enough to reflect period-to-period movements and linkages among the economic variables. However, the following factors may be missing from these models:

1. Existing scenario generators used in capital management rarely consider the economic fundamentals of the stochastic scenarios directly, but focus on the information derived from asset market prices. Given a scenario, it is difficult to say whether it is caused by economic growth, consumption, investment, labor market shocks, monetary policies or fiscal policies. For investment strategy analysis, understanding the causes of a scenario is important, because people will know or assess whether the scenario is reasonable.
2. The cyclical pattern of the economy is not often considered in existing scenario generators. The economic cycle is an important consideration when designing investment strategies. A market shock as in the 2008 financial crisis might be significant, but the losses are recoverable if there is a diversified equity portfolio and a long enough time horizon. It is better to reflect the cyclical patterns in real-world scenarios.
3. Interest rate, inflation rate, credit spread, credit default and equity return are all linked together in the economic system. Reflecting their dependency is important for maintaining the logic of economic development and macroeconomic policy making in the scenarios.

A.1 Economic Scenario Generation for Fundamental Economic Factors

In this report, a vector autoregressive (VAR) model is used to generate scenarios for fundamental economic factors, including real GDP growth rate, inflation rate, unemployment rate, short-term interest rate, long-term interest rate, credit spread, personal consumption growth rate and investment growth rate. Asset return scenarios are determined based on their relationship with these fundamental economic factors. These fundamental economic factors are chosen to capture the economic situation in a succinct and manageable way.

1. **Real GDP growth rate** is a direct reflection of real economic activities excluding price changes. It is an aggregate indicator of consumer spending, private- or public-sector investment, and imports/exports.
2. **Inflation rate** affects economic activities in various ways. In general, the inflation rate is determined by the money supply growing faster than the economy in the long term. It can reduce the burden of public and private debt. However, hyperinflation and unexpected inflation can be harmful to the economy, discourage investment and exports, and even cause social unrest. A very low or negative inflation rate usually accompanies economic recession.
3. **Unemployment rate** is another important indicator of the economy. It reflects the balance of the labor market. An economic recession usually comes with a high unemployment rate.
4. **Short-term interest rate** is largely determined by central-bank monetary policies, which are used to smooth the impact of economic volatilities. In an economic recession, the short-term interest rate is usually low to spur economic growth. In an economic expansion, the short-term interest rate is usually high to cool the economy down.
5. **Long-term interest rate** is affected by the long-term view of economic growth, inflation expectations, market liquidity and the short-term interest rate.
6. **Credit spread** usually contracts during economic expansions and widens during economic recessions.
7. **Consumption growth rate** is affected by personal income, which grows faster during an economic expansion. Higher consumption also means higher spending and a higher GDP growth rate.
8. **Investment growth rate** is an indicator of the investment portion of the economy. A higher investment growth rate means a higher economic growth rate, ceteris paribus.

Although these factors are used in this report for illustration, other factors can be included as well, depending on the features of the economy and purpose of a model. For example, to model an export-driven economy, exports/imports and foreign investment positions can be included in the list of fundamental economic factors. The economic scenario generator presented here is only one of the many possible economic scenario generators that could be used with an LDI benchmark model. Table A.1 describes the historical data of fundamental risk factors used in this example.

Table A.1
Fundamental Economic Factor Historical Data

Fundamental Economic Factor	Indicator	Notation	Data Source
Real GDP growth rate	U.S. quarterly notional GDP growth rate—inflation rate	gdpgr	Bureau of Economic Analysis (seasonally adjusted at annual rates)
Inflation rate	CPI—all urban consumers (current series)	cpi	Bureau of Labor Statistics
Unemployment rate	U-3 rate	unemploy	Bureau of Labor Statistics
Short-term interest rate	U.S. 3-month Treasury bill rate	m3tb	Bloomberg (USGG3M Index)
Long-term interest rate	U.S. 10-year Treasury bond yield	tb10y	Bloomberg (USGG10Y Index)
Credit spread	U.S. AA-rated finance corporate bond 10-year credit spread	aa10y	Bloomberg (c02310Y Index)
Consumption growth rate	U.S. personal consumption expenditures	pconsump	Bureau of Economic Analysis (seasonally adjusted at annual rates)
Investment growth rate	U.S. gross private domestic investment	gpdinv	Bureau of Economic Analysis (seasonally adjusted at annual rates)

Quarterly historical data from 1991Q1 to 2016Q4 are used. This analysis covers three full economic cycles. A vector autoregression (VAR) model is used to describe the relationship of the fundamental economic factors based on this historical data. By incorporating lagging variables into the analysis through VAR, relationships among leading, coincident and lagging economic factors can be better reflected. For example, the short-term interest rate is largely controlled by the Fed, after reviewing economic growth, unemployment and other economic conditions. Time is needed before making rate decisions. For simplicity, VAR(1) is used so that the evolution of fundamental economic factors is affected by their values in the previous quarter. A quarter is likely to be enough for the interaction among fundamental economic factors. Having a higher order of VAR model can only improve the results marginally in this example:

$$\mathbf{F}_t = \mathbf{c} + \mathbb{A}_1 \mathbf{F}_{t-1} + \mathbf{e}_t$$

where

$\mathbf{F}_t = (\mathrm{gdpgr}_t, \mathrm{cpi}_t, \mathrm{unemploy}_t, \mathrm{m3tb}_t, \mathrm{tb10y}_t, \mathrm{aa10y}_t, \mathrm{pconsump}_t, \mathrm{gpdinv}_t)^T$, a column vector with eight elements as the value of fundamental economic factors at time t or during period t;
$\mathbf{c} =$ a column vector with eight elements to represent the constant terms of the eight fundamental economic factors;
$\mathbb{A}_1 =$ an 8×8 matrix containing the model parameters describing the linear dependence of fundamental economic factors; and
$\mathbf{e}_t =$ a column vector with eight elements to store the error terms that cannot explained by linear models.

Table A.2 shows the fitted model parameters ($\mathbb{A}_1$ and $\mathbf{c}$) based on the historical data. It also shows σ, the standard deviation of error vector $\mathbf{e}_t$.

Table A.2

VAR(1) Model Parameters

Variable	$\mathbb{A}_1$								c	σ
	unemploy	gpdinv	pconsump	gdpgr	m3tb	aa10y	tb10y	cpi		
gdpgr	−0.04	0.09	0.49	−0.48	−0.07	−0.25	0.06	−0.45	0.83	0.59
cpi	0.09	−0.06	0.05	0.14	0.13	−0.22	−0.09	−0.03	0.27	0.59
unemploy	0.94	−0.01	−0.13	−0.02	−0.03	0.29	0.09	0.02	−0.12	0.20
m3tb	−0.01	0.01	0.04	0.08	0.92	−0.15	0.03	−0.09	0.22	0.43
tb10y	0.03	−0.02	0.00	0.02	0.07	−0.14	0.86	−0.06	0.41	0.52
aa10y	−0.07	0.04	0.09	−0.09	−0.03	1.02	0.02	−0.03	0.30	0.31
pconsump	0.01	0.00	−0.03	0.08	0.06	−0.54	−0.02	0.10	1.58	0.50
gpdinv	0.71	0.15	3.67	−1.24	0.06	−1.62	−0.26	−1.79	−3.08	1.95

Based on the fitted VAR(1), the stable values of fundamental risk factors $\bar{\mathbf{F}}$ can be derived.

$$\bar{\mathbf{F}} = \mathbf{c} + \mathbb{A}_1 \bar{\mathbf{F}}$$

Table A.3 lists the stable values based on VAR(1), along with the historical mean and standard deviation. The VAR(1) suggests a lower future economic growth rate, inflation rate, interest rates, consumption and investment growth rate than in the past 26 years. Credit spread is expected to be a little bit higher, and the unemployment rate is expected to stay at the same level. These model-implied expectations are good checkpoints to assess the model's reasonableness against a model user's view on future economic development.

Table A.3

VAR(1) Stable Values

Variable (Quarterly)	VAR(1)	Historical Data	
	Stable Value (%)	Mean (%)	Standard Deviation (%)
gdpgr (quarterly)	0.48	0.54	0.68
cpi (quarterly)	0.44	0.57	0.59
unemploy	6.06	6.06	1.58
m3tb	0.94	2.62	2.18
tb10y	3.17	4.51	1.75
aa10y	1.29	1.09	0.67
pconsump (quarterly)	0.99	1.18	0.63
gpdinv (quarterly)	0.69	1.23	3.05

The error terms of fundamental economic factors are expected to be zero. However, the error terms are not independent of each other. It is important for the economic scenario generator to capture the correlation when developing future scenarios. Table A.4 shows the correlation matrix of the error vector $\mathbf{e}_t$.

Table A.4
VAR(1) Error Term Correlation Matrix

	gdpgr	cpi	unemploy	m3tb	tb10y	aa10y	pconsump	gpdinv
gdpgr	1.00	−0.68	−0.07	0.15	0.03	−0.01	−0.14	0.45
cpi	−0.68	1.00	−0.17	0.18	0.33	−0.24	0.62	0.04
unemploy	−0.07	−0.17	1.00	−0.33	-0.12	0.20	−0.22	−0.26
m3tb	0.15	0.18	−0.33	1.00	0.48	−0.31	0.33	0.23
tb10y	0.03	0.33	−0.12	0.48	1.00	−0.39	0.37	0.24
aa10y	−0.01	−0.24	0.20	−0.31	−0.39	1.00	−0.34	−0.14
pconsump	−0.14	0.62	−0.22	0.33	0.37	−0.34	1.00	−0.01
gpdinv	0.45	0.04	−0.26	0.23	0.24	−0.14	−0.01	1.00

Based on VAR(1), stochastic scenarios of fundamental economic factors can be constructed as

$$\mathbf{F}_t = \mathbf{c} + \mathbb{A}_1 \mathbf{F}_{t-1} + \boldsymbol{\sigma} \cdot \mathbb{L}\boldsymbol{\varepsilon}_t$$

where

$\boldsymbol{\sigma}$ is a column vector containing the standard deviation of error terms of 8 fundamental economic factors;
$\boldsymbol{\varepsilon}_t$ is a column vector containing eight independent random variables following standard normal distribution; and
$\mathbb{L}$ is an 8×8 lower triangular matrix so that the error term correlation matrix can be decomposed as $\mathbb{L} \times \mathbb{L}^T$.

By using Cholesky decomposition, a correlation matrix $\mathbf{CM}$ such as that in Table A.4, can be decomposed as the product of a lower triangular matrix $\mathbb{L}$ and its transpose $\mathbb{L}^T$, given that the correlation matrix is positive definite. $\mathbb{L}\boldsymbol{\varepsilon}_t$ has the same correlation matrix from which $\mathbb{L}$ is derived, as shown below:

$$\mathrm{Cov}(\mathbb{L}\boldsymbol{\varepsilon}_t, \mathbb{L}\boldsymbol{\varepsilon}_t) = \mathbb{E}(\mathbb{L}\boldsymbol{\varepsilon}_t(\mathbb{L}\boldsymbol{\varepsilon}_t)^T) = \mathbb{E}(\mathbb{L}\boldsymbol{\varepsilon}_t(\boldsymbol{\varepsilon}_t)^T\mathbb{L}^T) = \mathbb{L}\mathbb{E}(\boldsymbol{\varepsilon}_t(\boldsymbol{\varepsilon}_t)^T)\mathbb{L}^T = \mathbb{L} \times \mathbb{I} \times \mathbb{L}^T = \mathbb{L} \times \mathbb{L}^T = \mathbf{CM}$$

where $\mathbb{I}$ is an 8×8 identity matrix, because $\boldsymbol{\varepsilon}_t$ contains independent random variables, all of which have an expected value of zero.

Figures A.1 to A.8 show the statistics of 100 generated stochastic scenarios of fundamental economic factors, including the expected value, minimum, maximum, and 10th, 25th, 75th, 90th and 99th percentiles. Linear dependences among fundamental economic factors lead to a stable range of prediction after five years. This is helpful for designing extreme and also plausible and reasonable scenarios governed by patterns found from historical data. It is quite different from market-data-driven economic scenario generators, where the range of prediction usually explodes with time.

Figure A.1
Stochastic Scenarios of Real GDP Growth Rates (gdpgr)

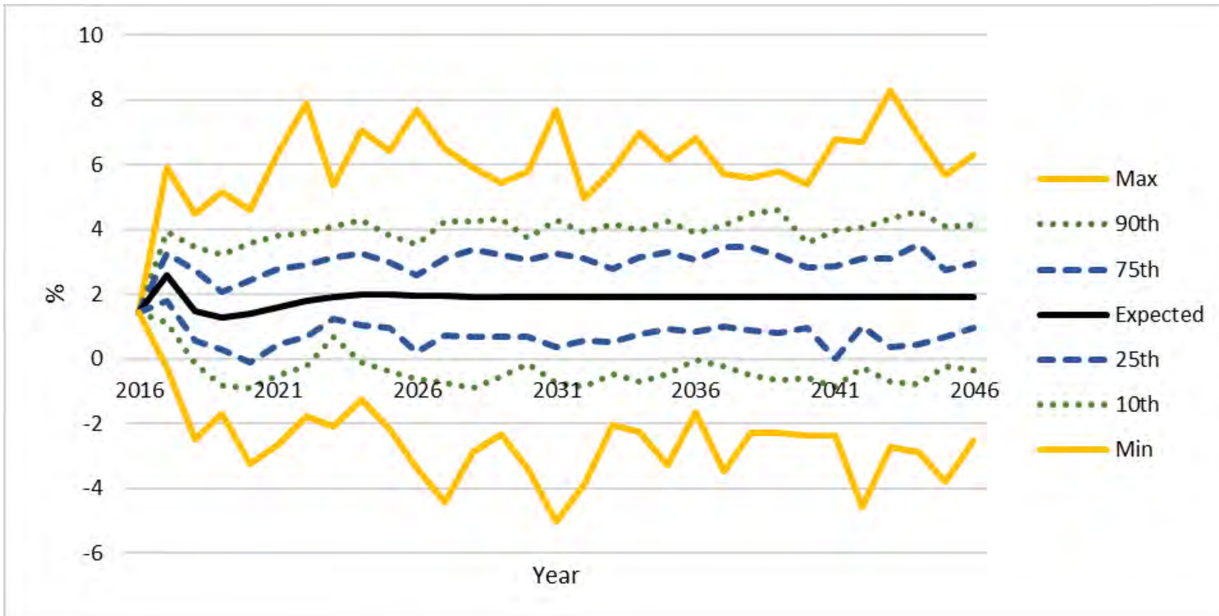


Figure A.2
Stochastic Scenarios of Inflation Rates (cpi)

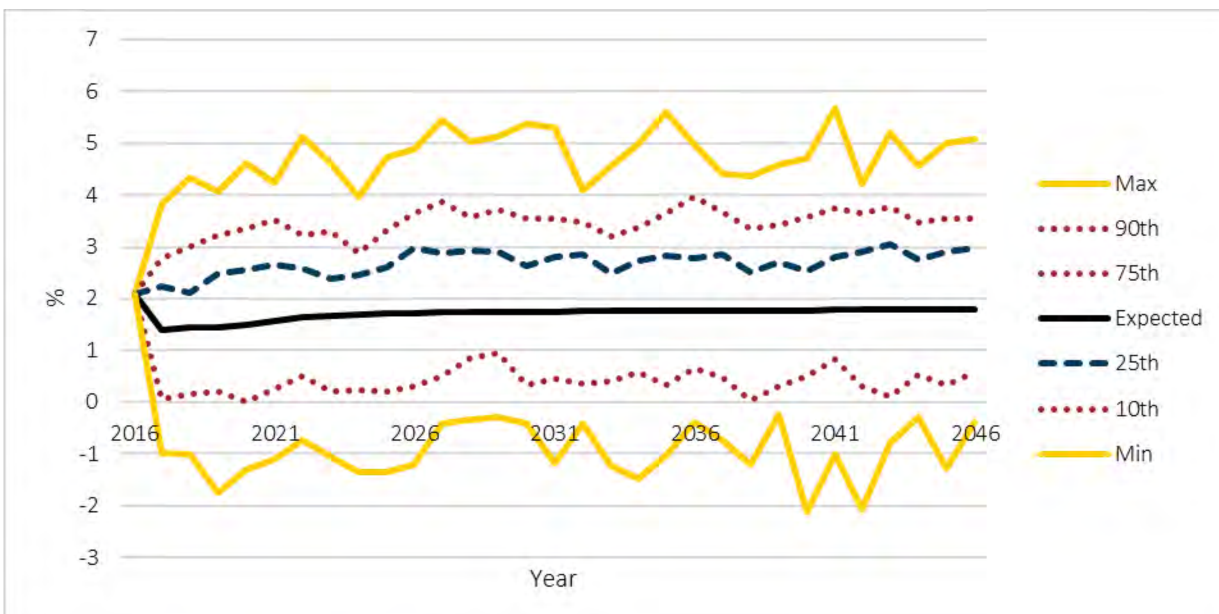


Figure A.3

Stochastic Scenarios of Unemployment Rates (unemploy)

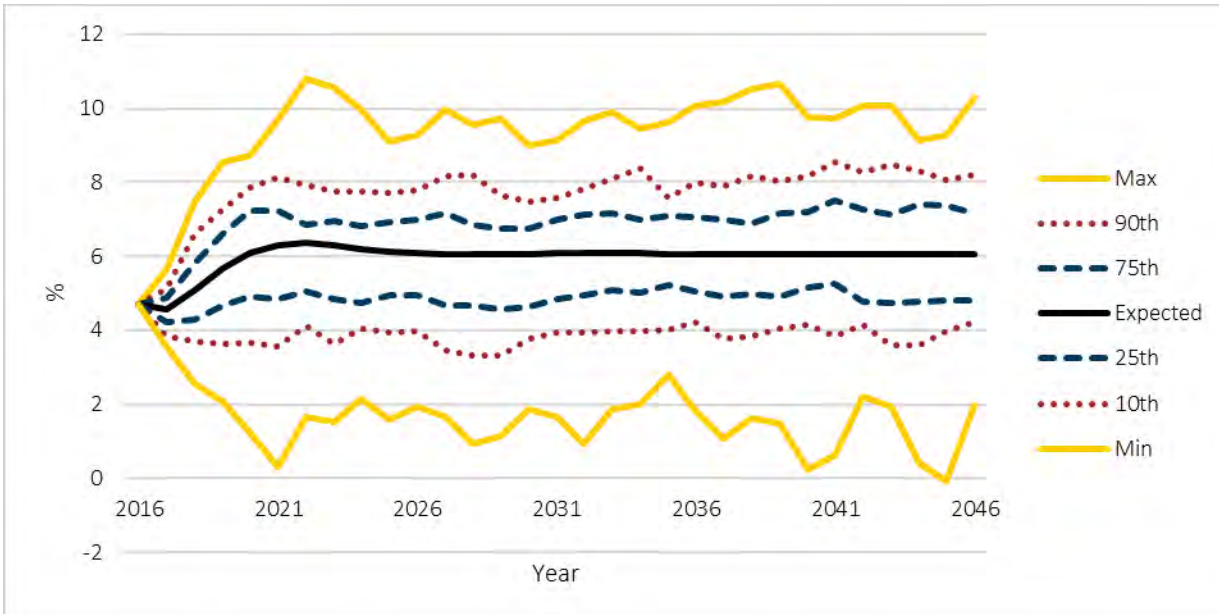


Figure A.4

Stochastic Scenarios of 3-Month Treasury Bill Yields (m3tb)

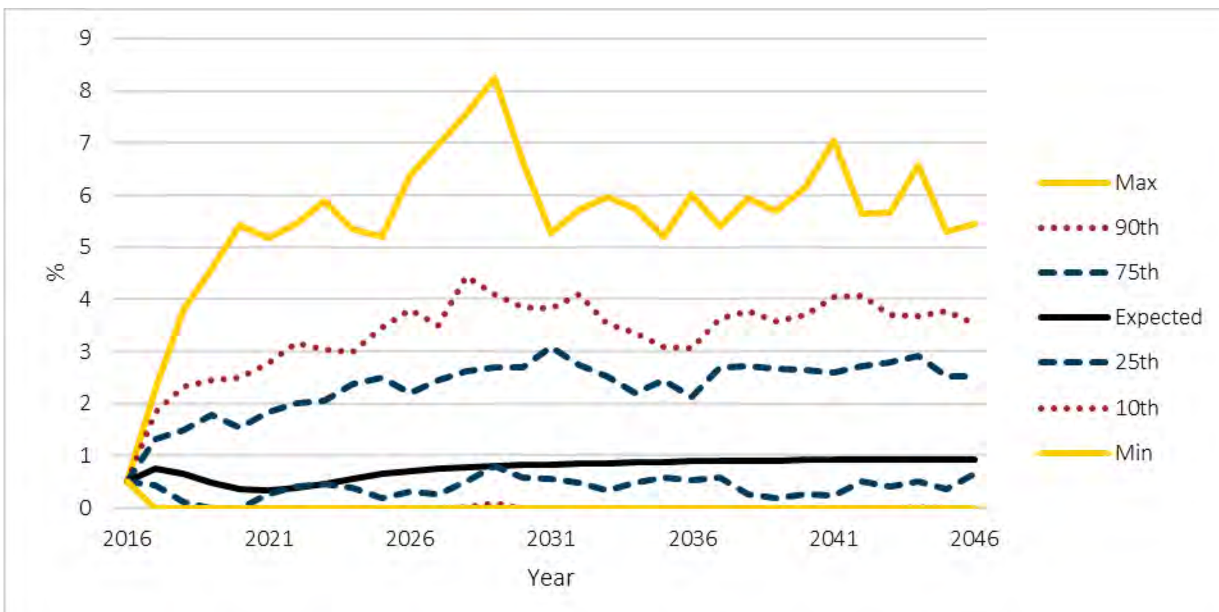


Figure A.5
Stochastic Scenarios of 10-Year Treasury Bond Yields (tb10y)

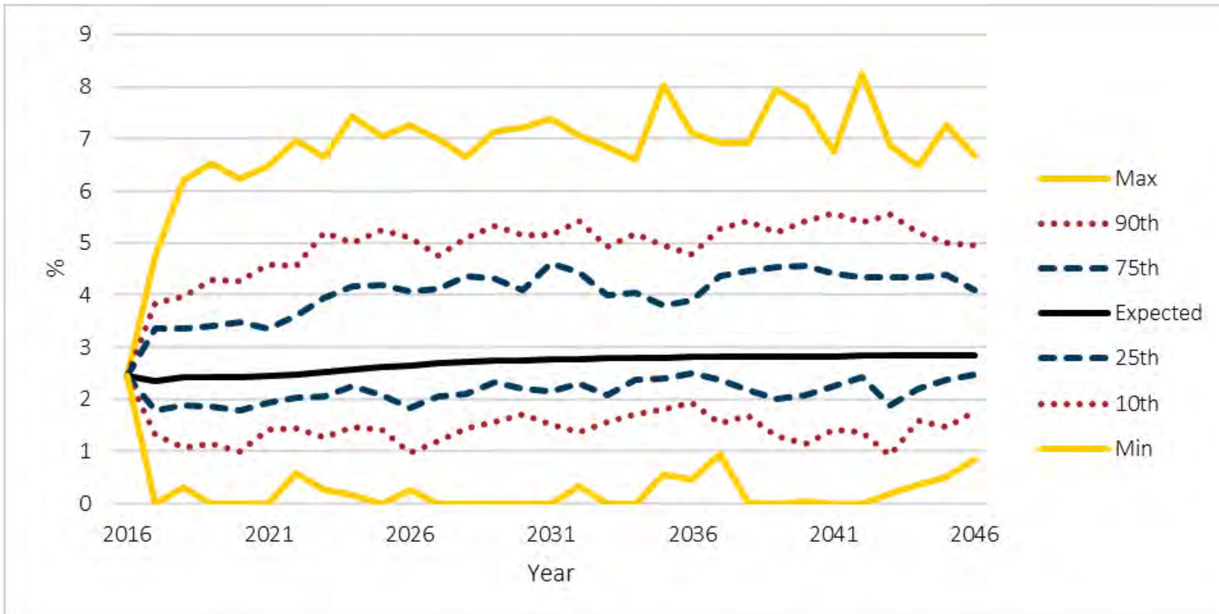


Figure A.6
Stochastic Scenarios of 10-Year AA-Rated Corporate Bond Credit Spreads (aa10y)

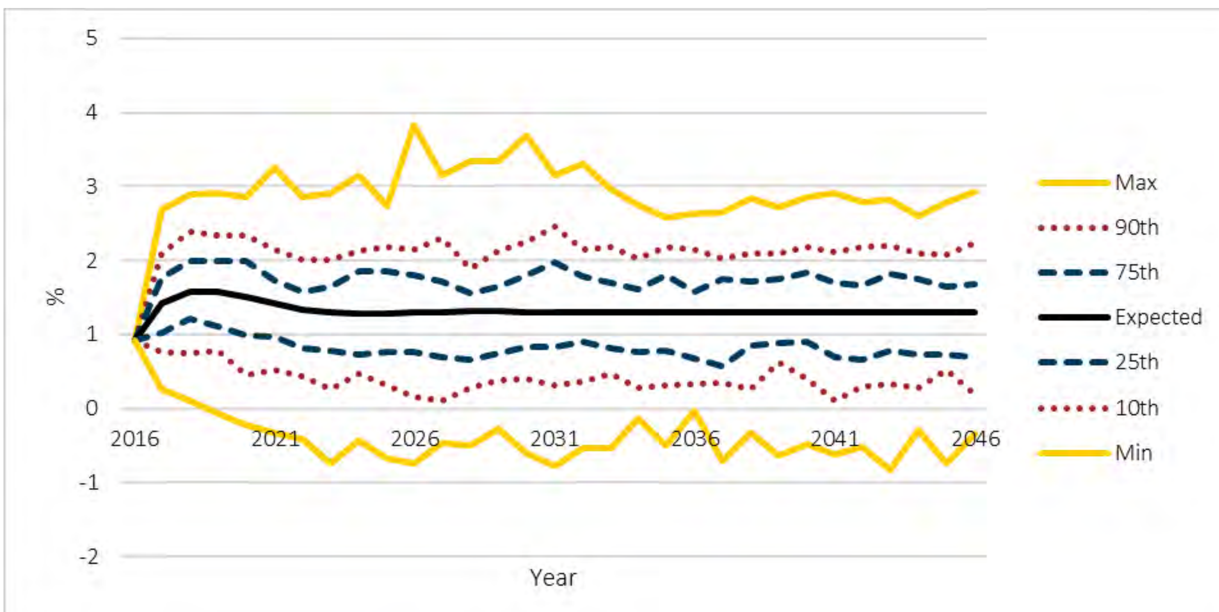


Figure A.7
Stochastic Scenarios of Personal Consumption Growth Rates (pconsump)

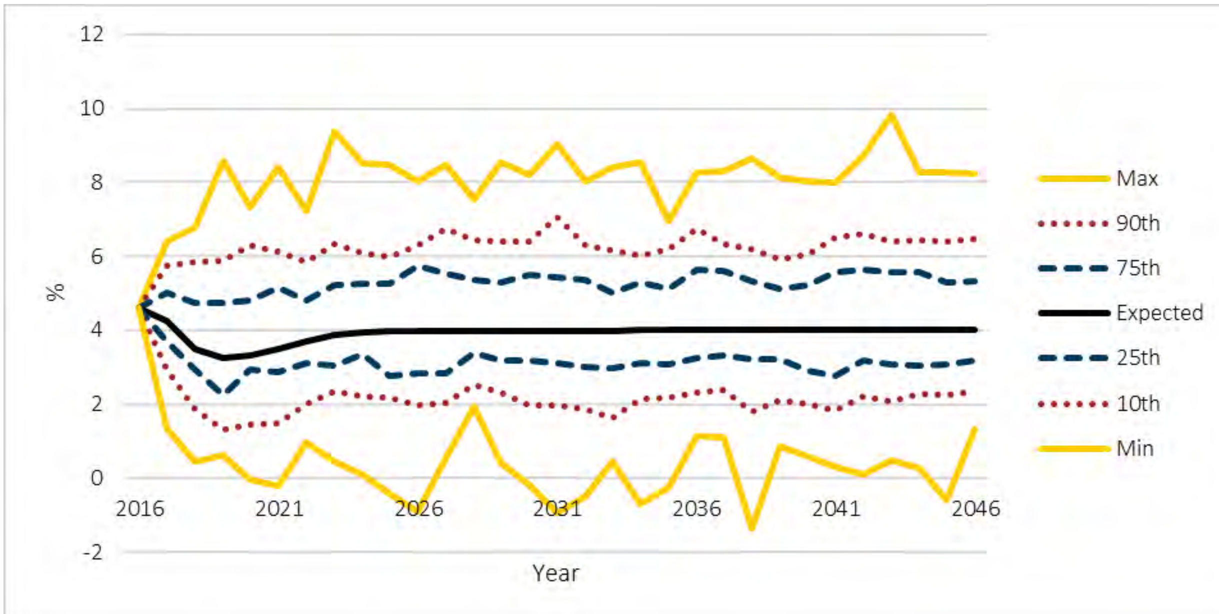
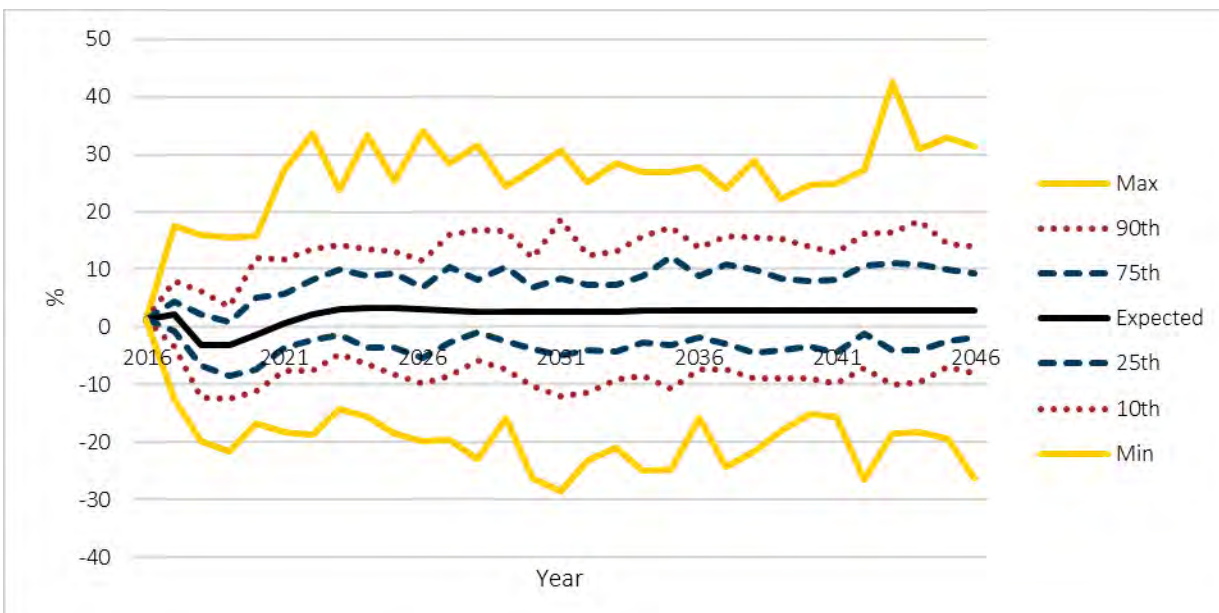


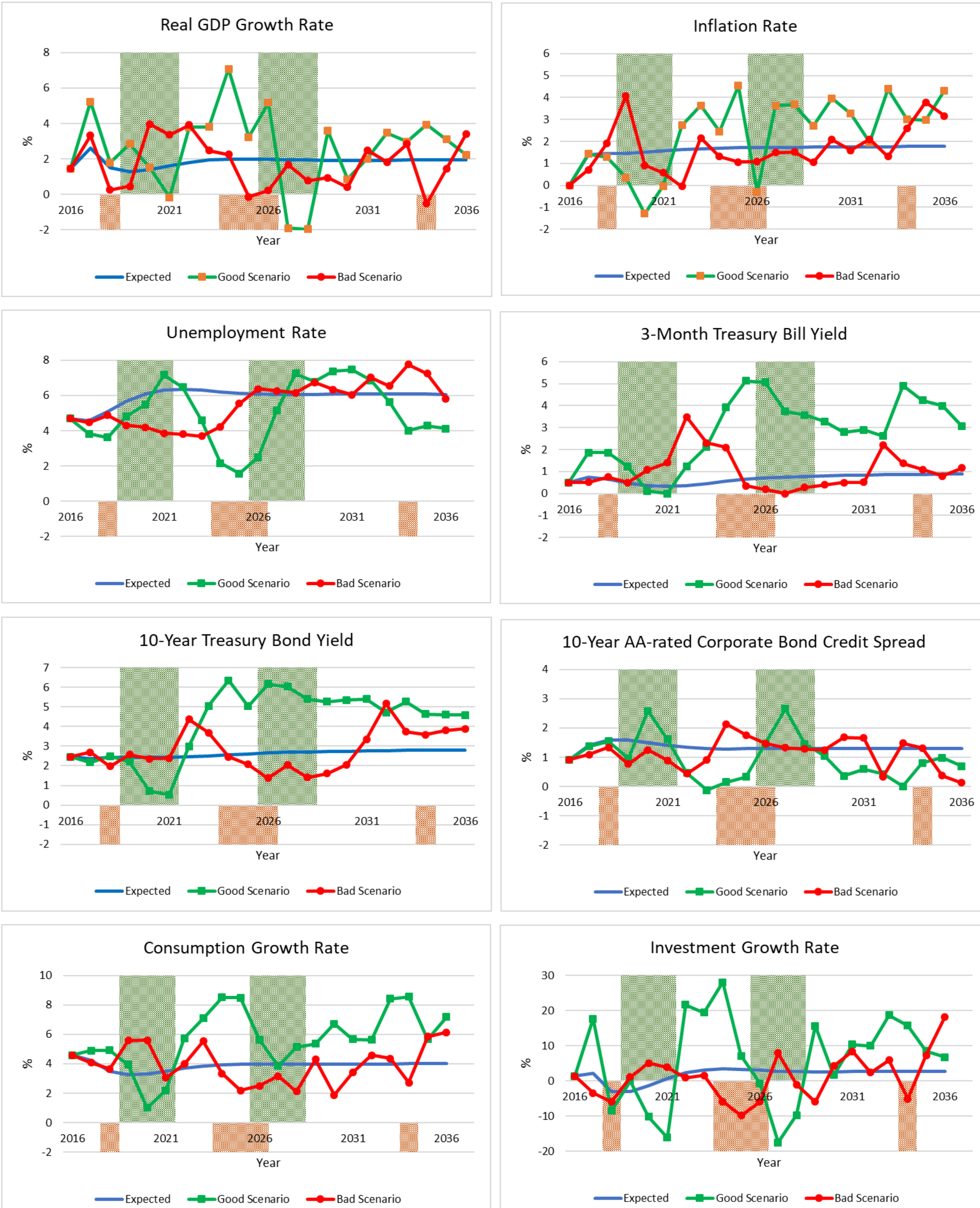
Figure A.8
Stochastic Scenarios of Investment Growth Rates (gpdinv)



It is also important to check the reasonableness of individual scenarios regarding whether basic economic patterns are followed. For example, in Figure A.9, two scenarios of fundamental economic factors are shown: a good scenario and a bad scenario. The projected funding ratio at the end of 2036 is 146% for the good scenario and 80% for the bad scenario, using a balanced asset allocation plan of 25% Treasury bonds, 45% corporate bonds, 18% public equity, 6% real estate and 6% other alternative investments.

Figure A.9

Sample Scenarios of Fundamental Economic Factors



Note: In each graph, the green shaded areas represent periods when the economy is in recession under the good scenario, and the orange shaded areas represent periods when the economy is in recession under the bad scenario.

Both scenarios have economic cycles as governed by the VAR(1) model. During economic recession, unemployment rate and credit spread go up. GDP growth rate, interest rates, consumption growth and investment growth go down. The bad scenario is not worse than the good scenario at all times. The major difference between the two scenarios is that under the bad scenario, the economy is running at a much slower pace than under the good scenario. The average annual real GDP growth rate is 1.75% under the bad scenario and 2.57% under the good scenario. Unemployment rates are higher under the bad scenario on average. Consequently, interest rates are lower to stimulate the economy, which is detrimental to the funding sufficiency of DB plans. In each case, the stochastic scenarios are consistent with general economic patterns and can be explained. Investment decisions are then made upon plausible scenarios.

As shown in Figure A.9, the economic status is projected for each quarter under each scenario. This not only is helpful for understanding the scenarios but also is critical for asset return generation, because it can differentiate between economic recession and expansion. During an economic recession, higher volatility and correlation are usually observed and need to be reflected in stochastic asset returns. Real GDP growth rate, unemployment rate, consumption growth rate and investment growth rate are used in the following logistic model to predict whether the economy is in recession or not:

$$R_t = \frac{1}{1 + e^{-\boldsymbol{\beta}\mathbf{F}_t}}$$

where

R_t = the probability that the economy is in recession during period t;

$\mathbf{F}_t = \begin{pmatrix} 1, \mathrm{gdpgr}_t, \mathrm{gdpgr}_{t-1}, \mathrm{gdpgr}_{t-2}, \mathrm{unemploy}_t, \mathrm{unemploy}_{t-1}, \mathrm{unemploy}_{t-2}, \\ \mathrm{pconsump}_t, \mathrm{pconsump}_{t-1}, \mathrm{pconsump}_{t-2}, \mathrm{gpdinv}_t, \mathrm{gpdinv}_{t-1}, \mathrm{gpdinv}_{t-2} \end{pmatrix}^T$, a column vector with 13 elements containing the constant term and fundamental economic factors during periods t, $t-1$ and $t-2$.

$\boldsymbol{\beta}$ is a row vector with 13 elements containing the model parameters for variables in $\mathbf{F}_t$.

Historical data of U.S. economic cycles (1999Q1 to 2016Q4) from the National Bureau of Economic Research (NBER) and fundamental economic factors are used to calibrate the logistic model, with parameters shown in Table A.5. Current and previous two quarters' values of fundamental economic factors are used to determine the current status of the economy. Changes from quarter to quarter are more important than the absolute value of economic factors for predicting the economic status. The calibrated logistic model has 100% accuracy in matching the history of the past 26 years.

Table A.5

Economic Recession Prediction: Logistic Model Parameter

Variable	Period	Parameter (β)
Intercept		78.0
Unemployment rate	Current quarter *t*	66.3
	Previous quarter *t* − 1	−51.3
	Previous quarter *t* − 2	−32.2
Investment growth rate	Current quarter *t*	0.3
	Previous quarter *t* − 1	−3.4
	Previous quarter *t* − 2	4.3
Consumption growth rate	Current quarter *t*	−1.2
	Previous quarter *t* − 1	−9.9
	Previous quarter *t* − 2	0.1
Real GDP growth rate	Current quarter *t*	−25.3
	Previous quarter *t* − 1	−8.4
	Previous quarter *t* − 2	−17.0

A.2 Economic Scenario Generation for Asset Return

With the fundamental stochastic scenarios generated, return scenarios of each asset classes can be constructed based on their relationships with fundamental economic factors. Linear models are used to describe the relationships based on historical data:

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\phi}_1 \mathbf{y}_{t-1} + \boldsymbol{\phi}_2 \mathbf{y}_{t-2} + \mathbb{B}_0 \mathbf{F}_t + \mathbb{B}_1 \mathbf{F}_{t-1} + \mathbb{B}_2 \mathbf{F}_{t-2} + e_t$$

where

$\mathbf{y}_t$ is a column vector containing the returns of all asset classes during period t;
$\boldsymbol{\alpha}$ is a column vector containing the constant terms of all asset classes;
$\boldsymbol{\phi}_i$ is a column vector containing parameters to govern the relationship between current return and return for all asset classes during period $t - i$;
$\mathbb{B}_i$ is a matrix with eight columns that contains all asset classes' model parameters for the eight fundamental economic factors during period $t - i$; and
$\mathbf{F}_t = (\mathrm{gdpgr}_t, \mathrm{cpi}_t, \mathrm{unemploy}_t, \mathrm{m3tb}_t, \mathrm{tb10y}_t, \mathrm{aa10y}_t, \mathrm{pconsump}_t, \mathrm{gpdinv}_t)^T$, a column vector with eight elements as the value of fundamental economic factors at time t or during period t.

Other models, such as generalized linear models (GLM), classification and regression trees (CART), *k*-nearest neighbors (KNN) and artificial neural networks (ANN), were tested but did not show material improvement above linear models. Given that only 26 years of historical data (1991Q1 to 2016Q4) are used, linear models are sophisticated enough to describe the relationships embedded in the historical data. Table A.6 lists the asset classes and historical data used to calibrate the linear models.

Table A.6
Asset Return Historical Data

Asset Class	Return Type	Time Period	Data Source
Treasury bond yield curve (terms: 1, 2, 3, 5, 7, 10, 20 and 30 years)	Yield	1991Q1–2016Q4 except for 20-year bond yield starting from 1993Q4	U.S. Department of the Treasury
AAA-, AA-, A- and BBB-rated corporate bonds[1]	Credit spread	1996Q4–2016Q4	BofA Merrill Lynch US Corporate AAA, AA, A, BBB Effective Yield (BAMLC0A3CAEY, BAMLC0A2CAAEY, BAMLC0A1CAAAEY, BAMLC0A4CBBBEY) *Federal Reserve Economic Data*
	Default rate	1991Q1–2016Q4	*2016 S&P Annual Global Corporate Default Study and Rating Transitions Report*
Public equity, large cap	Dividend yield	1991Q1–2016Q4	S&P 500 Index (^GSPC) *Yahoo Finance*
	Capital return		
Public equity, mid cap	Dividend yield	2000Q3–2016Q4	iShares Core S&P Mid-Cap (IJH) *Yahoo Finance*
	Capital return		
Public equity, small cap	Dividend yield	2000Q3–2016Q4	iShares Core S&P Small Cap (IJR) *Yahoo Finance*
	Capital return		
Public equity, high dividend yield	Dividend yield	2004Q1–2016Q4	iShares Select Dividend ETF (DVY) *Yahoo Finance*
	Capital return		
Equity REITs	Cap rate	1991Q1–2016Q4	FTSE NAREIT US Real Estate Index—All Equity REITs
	Capital return		
Mortgage REITs	Cap rate	1991Q1–2016Q4	FTSE NAREIT US Real Estate Index—Mortgage REITs
	Capital return		
Infrastructure project index	Dividend yield	2007Q1–2016Q4	SPDR S&P Global Infrastructure ETF (GII) *Yahoo Finance*
	Capital return		

Private equity index	Dividend yield	2006Q4–2016Q4	PowerShares Global Listed Private Eq ETF (PSP) *Yahoo Finance*
	Capital return		
Crude oil	Total return	1991Q1–2016Q4	WTI Crude Oil Price (DCOILWTICO) *Federal Reserve Economic Data*
Gold	Total return	1991Q1–2016Q4	London Bullion Market Association (LBMA) Gold Price (GOLDPMGBD228NLBM) *Federal Reserve Economic Data*
Commodity index	Total return	1992Q1–2016Q4	Global Price Index of All Commodities (PALLFNFINDEXQ) *Federal Reserve Economic Data*
Wage index[2]	Total return	1991Q1–2016Q4	U.S. Compensation/Employed Compensation of Employees, Received: Wage and Salary Disbursements (A576RC1) and All Employees: Total Nonfarm Payrolls (PAYEMS) *Federal Reserve Economic Data*
Plan sponsor equity	Total return	1991Q1–2016Q4	GE stock price (used as an example) *Yahoo Finance*

[1]Although high-yield, high-risk bonds may be excluded from pension asset allocation, they can be easily incorporated into the economic scenario generator.
[2]The wage index has a high correlation with the inflation index, so it may be replaced with the inflation index. However, the wage index can behave quite differently by industry, company, and occupation.

Table A.7 lists the parameters of linear models for asset return economic scenario generation used in this report. To avoid overfitting, we removed from the final models explanatory variables with a parameter not statistically different from zero (*p*-value of *t*-test > 0.3). The last column of Table A.7 shows the adjusted R^2 of the linear models. It indicates the portion of asset return volatility that can be explained by the linear relationships. Certain asset classes, including infrastructure project index, private equity, mortgage REITs, oil and gold, have a low adjusted R^2. They are more driven by idiosyncratic factors than by fundamental economic factors. The second-to-last column of Table A.7 contains the standard deviation of the idiosyncratic factors that cannot be explained by the linear models. Stochastic scenarios of asset returns can be generated as

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\phi}_1 \mathbf{y}_{t-1} + \boldsymbol{\phi}_2 y_{t-2} + \mathbb{B}_0 \mathbf{F}_t + \mathbb{B}_1 \mathbf{F}_{t-1} + \mathbb{B}_2 \mathbf{F}_{t-2} + \boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon}_t$$

where

$\boldsymbol{\sigma}$ is a column vector containing the standard deviation of error terms of all asset return models; and

$\boldsymbol{\varepsilon}_t$ is a column vector containing independent random variables following a standard normal distribution for all asset return models.

Table A.7

Asset Return Model Parameters (Returns in % Format)

			Autocorrelation		Fundamental Risk Factors																									Adjusted R^2
Lag (Quarter)			2	1	2	1	0	2	1	0	2	1	0	2	1	0	2	1	0	2	1	0	2	1	0	2	1	0	0	
Asset Class		Intercept	ϕ_2	ϕ_1	gdpgr			cpi			unemploy			m3tb			tb10y			aa10y			pconsump			gpdinv			σ	
Treasury bond zero rate (term)	1	(0.09)	0.00	0.75	(0.07)	0.00	0.00	(0.01)	0.00	0.00	0.00	0.00	0.00	0.00	(0.68)	0.88	0.00	(0.27)	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.02	(0.01)	0.00	0.14	100%
	2	(0.20)	0.00	0.80	0.00	0.00	0.00	0.00	0.00	(0.08)	0.00	0.00	0.00	0.15	(0.75)	0.72	0.00	(0.53)	0.63	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.16	99%
	3	(0.23)	0.00	0.75	0.00	0.00	0.00	0.00	0.00	(0.08)	0.00	0.00	0.00	0.17	(0.66)	0.61	0.00	(0.62)	0.77	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.15	99%
	5	(0.24)	0.00	0.74	0.00	0.00	0.00	0.00	0.00	(0.06)	0.00	0.00	0.00	0.08	(0.34)	0.31	0.00	(0.73)	0.97	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00	(0.02)	0.10	100%
	7	(0.05)	0.00	0.79	0.00	0.00	(0.04)	0.00	0.00	(0.07)	0.00	0.00	0.00	0.00	(0.11)	0.13	0.00	(0.82)	1.02	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.06	100%
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100%
	20	0.03	0.00	0.85	0.00	0.00	0.00	0.00	(0.06)	0.00	0.00	0.00	0.00	(0.05)	0.23	(0.18)	0.00	(0.76)	0.92	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	100%
	30	0.22	0.00	0.78	0.00	0.00	(0.07)	0.00	(0.05)	0.00	0.00	0.00	0.00	(0.08)	0.28	(0.22)	0.00	(0.65)	0.88	0.00	0.09	(0.08)	0.05	(0.10)	0.00	0.00	0.00	0.03	0.09	100%
AAA-rated corporate bond	Credit Spread	0.18	0.00	0.87	(0.16)	(0.11)	0.00	(0.16)	(0.06)	0.01	0.00	0.00	0.00	0.00	(0.22)	0.25	0.00	0.14	(0.17)	0.02	(0.77)	0.76	(0.03)	0.14	0.00	0.02	0.00	0.00	0.15	92%
	Default Rate	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100%
AA-rated corporate bond	Credit Spread	(0.06)	0.00	0.76	(0.17)	0.00	0.00	(0.16)	0.00	0.00	0.00	0.00	0.00	0.00	(0.20)	0.27	0.00	0.16	(0.23)	0.00	(0.65)	0.91	0.00	0.11	0.00	0.02	0.00	0.00	0.17	95%
	Default Rate	0.01	(0.21)	0.92	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.07	(0.07)	0.00	0.05	(0.05)	0.00	0.00	0.01	(0.03)	0.05	0.00	(0.01)	0.00	0.00	0.00	0.00	0.00	0.04	73%
A-rated corporate bond	Credit Spread	(0.10)	0.00	0.59	(0.14)	(0.13)	(0.02)	(0.15)	(0.18)	0.00	0.00	0.00	0.00	0.00	(0.19)	0.28	0.00	0.18	(0.26)	0.00	(0.69)	1.25	0.00	0.21	0.00	0.01	0.00	0.00	0.16	96%
	Default Rate	(0.09)	0.00	0.62	0.01	0.00	(0.02)	0.02	0.00	(0.03)	0.06	0.00	(0.08)	0.00	0.05	(0.06)	0.00	0.00	0.03	0.00	0.05	0.05	(0.03)	0.00	0.05	0.00	0.00	0.00	0.05	79%
BBB-rated corporate bond	Credit Spread	0.10	0.00	0.52	0.00	(0.13)	(0.11)	(0.03)	(0.26)	(0.28)	0.00	0.00	0.00	0.00	(0.21)	0.25	0.00	0.39	(0.43)	(0.26)	0.00	0.98	0.00	0.31	0.00	0.00	0.00	0.00	0.20	96%
	Default Rate	0.20	(0.10)	0.65	(0.04)	0.00	(0.11)	(0.05)	0.00	(0.10)	0.00	0.00	(0.07)	0.00	0.00	(0.05)	0.08	0.00	0.00	0.15	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.03	0.11	76%
Public equity, large cap	Dividend Yield	0.26	0.00	0.91	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	(0.02)	0.06	0.00	(0.08)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	88%
	Capital Return	(0.22)	0.00	(0.16)	6.73	2.08	1.24	5.91	1.06	0.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00	(4.45)	4.05	0.00	0.00	(5.15)	(3.49)	0.00	0.00	(1.39)	(0.21)	0.00	5.87	32%
Public equity, mid cap	Dividend Yield	2.35	0.00	0.00	0.37	0.00	0.00	0.40	0.00	(0.23)	0.00	0.00	0.00	0.00	0.00	0.09	0.00	(0.33)	0.00	0.00	0.00	(0.01)	(0.24)	0.00	0.00	(0.06)	0.00	0.00	0.33	43%
	Capital Return	(15.06)	(0.14)	(0.34)	5.44	0.00	0.00	4.01	0.00	0.00	0.00	0.00	2.46	0.93	0.00	0.00	(5.78)	(5.04)	8.55	(0.29)	0.00	0.00	0.00	0.00	6.63	(0.88)	0.00	0.00	5.86	52%
Public equity, small cap	Dividend Yield	2.13	(0.13)	0.00	0.53	0.00	0.00	0.37	0.00	0.00	0.00	(0.46)	0.56	0.15	(0.28)	0.25	0.00	(0.41)	(0.01)	0.00	(0.57)	0.36	(0.33)	0.00	0.00	(0.05)	0.00	0.00	0.29	51%
	Capital Return	(19.71)	(0.18)	(0.50)	8.08	3.27	3.61	7.53	0.31	(2.07)	1.07	(2.05)	3.50	1.77	3.66	(4.45)	(6.57)	(6.58)	10.41	7.64	(4.42)	(2.42)	(2.00)	3.11	6.03	(0.91)	(0.49)	0.00	4.85	59%
Public equity, high dividend	Dividend Yield	1.42	0.00	0.41	0.00	0.00	(0.16)	0.00	0.00	(0.14)	0.00	0.00	0.00	0.00	0.20	(0.19)	0.00	0.00	0.00	0.00	0.42	0.00	0.00	0.00	0.25	0.00	0.00	0.03	0.19	80%
	Capital Return	3.42	0.00	(0.14)	0.00	0.00	0.00	0.00	0.00	0.74	0.00	0.00	0.00	(6.45)	5.81	0.00	0.00	0.00	0.00	9.05	(10.17)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.97	55%
Equity REITs	Cap Rate	0.05	0.42	0.48	0.00	0.00	0.00	0.00	0.00	(0.09)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(0.24)	0.27	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.19	79%
	Return	(6.10)	0.00	(0.16)	3.81	2.23	0.00	6.28	4.77	0.00	0.00	0.00	0.00	(3.28)	6.55	(4.11)	0.00	0.00	0.00	16.04	0.00	(15.38)	0.00	0.00	0.00	0.00	0.00	0.00	6.45	49%
Mortgage REITs	Cap Rate	2.56	0.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00	(0.03)	0.12	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	(0.91)	0.00	0.00	0.00	0.04	1.15	11%
	Return	1.85	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.55	0.00	(8.12)	0.00	0.00	0.00	0.00	0.00	(0.74)	9.82	14%
Infrastructure project index	Cap Rate	4.07	0.00	0.00	(0.36)	(0.23)	(0.00)	(0.33)	(0.59)	(0.06)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(0.14)	0.20	0.00	0.00	0.00	0.00	0.79	4%
	Return	0.58	0.00	0.00	1.07	0.00	0.00	(2.97)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.15	10%
Private equity index	Cap Rate	9.43	0.00	0.00	0.00	(1.58)	4.28	0.00	0.00	4.69	0.00	0.00	0.00	(1.50)	0.00	0.00	(2.50)	0.00	4.03	0.00	(6.02)	2.80	0.00	(1.26)	(4.10)	0.00	0.27	(0.82)	4.37	2%
	Return	3.81	0.00	0.01	25.51	0.00	0.00	14.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(4.18)	0.00	0.00	4.26	0.00	0.00	(15.68)	0.00	0.00	(3.91)	2.83	0.00	9.51	52%
Crude oil	Return	(7.65)	(0.17)	(0.34)	0.00	(3.93)	0.00	0.00	0.00	18.39	0.00	0.00	0.00	0.00	0.00	0.00	(9.62)	9.15	0.00	0.00	4.16	0.00	0.00	0.00	0.00	0.00	0.75	0.00	12.36	38%
Gold	Return	(0.14)	0.13	0.00	0.00	0.00	0.25	0.00	0.00	3.36	0.19	0.00	0.00	0.00	0.00	0.00	(0.38)	3.49	(3.59)	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	6.35	5%
Wage index	Inflation	1.05	0.00	(0.41)	0.00	0.09	0.00	0.00	0.00	0.00	0.27	(0.36)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(0.29)	0.39	0.35	0.00	0.00	0.00	0.67	29%
Commodity index	Index Return	(9.79)	0.00	0.14	1.74	(2.04)	0.00	0.00	0.00	9.46	(4.48)	4.86	0.00	0.00	0.00	0.00	(3.90)	3.39	0.00	0.00	1.67	0.00	0.00	0.00	3.06	(0.28)	0.64	0.00	3.98	75%
Plan sponsor equity	Equity Return	(1.29)	0.00	(0.17)	4.16	0.00	0.00	3.69	0.00	0.00	0.00	13.19	(12.92)	0.00	0.00	0.00	0.00	(2.84)	3.43	7.47	0.00	(9.68)	0.00	0.00	0.00	(1.13)	0.00	0.00	9.95	30%

Based on the fitted linear models, the stable values of asset return $\bar{y}$ can be derived as follows:

$$\bar{\mathbf{y}} = \boldsymbol{\alpha} + \boldsymbol{\phi}_1 \bar{\mathbf{y}} + \boldsymbol{\phi}_2 \bar{\mathbf{y}} + \mathbb{B}_0 \bar{\mathbf{F}} + \mathbb{B}_1 \bar{\mathbf{F}} + \mathbb{B}_2 \bar{\mathbf{F}}$$

Table A.8 compares the stable values with average historical returns. As expected, differences are noticeable for asset classes with a low adjusted R^2. For asset classes with a high adjusted R^2, historical trends could also lead to difference. For example, the stable values of interest rates are much lower than historical averages because of the downward trend in the historical data. These need to be checked to make sure the models are consistent with the model user's view of the future economy.

Table A.8
Asset Return Linear Model Stable Value (%)

Asset Class		Linear Model	Historical Data	
		Stable Return (%)	Mean Return (%)	Standard Deviation (%)
Treasury bonds, 0 rate (for given term)	1 year	1.24	2.93	2.23
	2 years	1.58	3.25	2.22
	3 years	1.84	3.49	2.15
	5 years	2.42	3.93	1.98
	7 years	2.84	4.26	1.85
	10 years	3.17	4.51	1.75
	20 years	3.72	4.82	1.54
	30 years	3.82	5.05	1.50
AAA-rated corporate bonds	Credit spread	0.42	0.47	0.59
	Default rate	0.00	0.00	0.00
AA-rated corporate bonds	Credit spread	0.44	0.58	0.79
	Default rate	0.02	0.02	0.08
A-rated corporate bonds	Credit spread	0.99	1.04	0.94
	Default rate	0.06	0.06	0.11
BBB-rated corporate bonds	Credit spread	1.90	1.83	1.08
	Default rate	0.18	0.18	0.25
Public equity, large cap	Annualized dividend yield	2.05	2.00	0.52
	Capital return (quarterly)	1.76	2.04	7.66
Public equity, mid cap	Annualized dividend yield	1.36	1.29	0.46
	Capital return (quarterly)	2.38	2.32	9.39
Public equity, small cap	Annualized dividend yield	1.13	1.12	0.48
	Capital return (quarterly)	2.78	2.57	9.92
Public equity, high dividend	Annualized dividend yield	3.57	3.60	0.45
	Capital return (quarterly)	1.49	1.27	7.87
Equity REITs	Cap rate (quarterly)	1.29	1.56	0.43
	Capital return (quarterly)	1.52	1.42	9.53

Asset Class		Linear Model	Historical Data	
		Stable Return (%)	Mean Return (%)	Standard Deviation (%)
Mortgage REITs	Cap rate (quarterly)	2.89	2.98	1.26
	Capital return (quarterly)	−0.81	−0.85	10.79
Infrastructure project index	Annualized dividend yield	3.41	3.51	0.91
	Capital return (quarterly)	−0.21	−0.18	7.74
Private equity	Annualized dividend yield	6.40	5.72	5.32
	Capital return (quarterly)	−1.70	−0.80	15.57
Crude oil	Total return (quarterly)	2.00	2.37	16.33
Gold	Total return (quarterly)	1.40	1.37	6.80
Commodity index	Total return (quarterly)	0.72	0.79	0.82
Wage index	Total return (quarterly)	0.49	1.12	8.50
Plan sponsor equity	Total return (quarterly)	1.92	3.16	12.51

The relationship between asset returns and fundamental economic factors is not always linear. During an economic recession, higher volatility and correlation are often observed. Asset return economic scenarios need to be further adjusted to reflect the nonlinear relationship. Table A.9 shows the volatility of idiosyncratic factors (error terms) and the correlation between systemic factors (prediction by linear models) and idiosyncratic factors, using either all the data or the data in recession. Volatility and correlation behaved quite differently in recession.

Table A.9
Asset Return Linear Model Idiosyncratic Factors: Volatility and Correlation

Asset Class		Idiosyncratic Factor Volatility (%)		Correlation With Systemic Factors	
		All Periods	Recession	All Periods	Recession
Treasury bond, 0 rate (for given term)	1 year	0.1	0.1	0.0%	27.2%
	2 years	0.2	0.1	0.0%	−21.9%
	3 years	0.2	0.2	0.0%	−24.9%
	5 years	0.1	0.1	0.0%	3.7%
	7 years	0.1	0.1	0.0%	18.0%
	10 years	0.0	0.0	0.0%	0.0%
	20 years	0.1	0.1	0.0%	−39.9%
	30 years	0.1	0.1	0.0%	−28.8%
AAA-rated corporate bonds	Credit spread	0.1	0.2	0.0%	−1.5%
	Default rate	0.1	0.2	0.0%	−1.5%
AA-rated corporate bonds	Credit spread	0.0	0.0	0.0%	0.0%
	Default rate	0.0	0.1	0.0%	32.2%
A-rated corporate bonds	Credit spread	0.2	0.2	0.0%	−4.8%
	Default rate	0.0	0.1	0.0%	−9.2%

Asset Class		Idiosyncratic Factor Volatility (%)		Correlation With Systemic Factors	
		All Periods	Recession	All Periods	Recession
BBB-rated corporate bonds	Credit spread	0.2	0.1	0.0%	8.7%
	Default rate	0.1	0.1	0.0%	−33.1%
Public equity, large cap	Dividend yield	0.2	0.4	0.0%	0.1%
	Capital return	5.9	5.5	0.0%	38.0%
Public equity, mid cap	Dividend yield	0.3	0.4	0.0%	18.6%
	Capital return	5.9	5.6	0.0%	6.9%
Public equity, small cap	Dividend yield	0.3	0.3	0.0%	−16.7%
	Capital return	4.9	4.1	0.0%	4.5%
Public equity, high dividend	Dividend yield	0.2	0.2	0.0%	11.8%
	Capital return	5.0	6.2	0.0%	21.3%
Equity REITs	Cap rate	0.2	0.2	0.0%	−53.0%
	Capital return	6.5	7.3	0.0%	50.1%
Mortgage REITs	Cap rate	1.2	2.4	0.0%	8.9%
	Capital return	9.8	10.7	0.0%	−10.1%
Infrastructure projects	Cap rate	0.8	0.5	0.0%	72.1%
	Capital return	7.2	9.8	0.0%	17.4%
Private equity	Cap rate	4.4	4.0	0.0%	−75.3%
	Capital return	9.5	15.1	0.0%	6.1%
Crude oil	Total return	12.4	12.6	0.0%	7.5%
Gold	Total return	6.3	6.1	0.0%	−11.9%
Commodity index	Total return	0.7	0.8	0.0%	−3.0%
Wage index	Total return	4.0	5.1	0.0%	−8.3%
Plan sponsor equity	Total return	10.0	12.5	0.0%	−2.5%

Therefore, the idiosyncratic part of asset return economic scenarios $\boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon}_t$ needs to be adjusted to reflect non-constant volatility and nonlinear relationships, with the following steps:

Step 1. Predict whether or not the economy is in recession for each period under each scenario, based on fundamental economic factors, such as the logistic model described in A.1 Economic Scenario Generation for Fundamental Economic Factors.

Step 2. Construct two correlation matrices from the error terms of asset return models. The first correlation matrix, $\mathbf{CM}_{\text{All}}$, describes the general relationships among error terms of all asset classes, using all historical data. Theoretically, it is better to use only data during economic expansions. In this specific case, using data during economic expansions generates a correlation matrix that is not positive semidefinite, a condition that ensures consistency among correlations. The issue can be addressed using all historical data. The resulting correlation matrix is only slightly more conservative. The second correlation matrix, CM_{Res}, describes the relationships among error terms only in economic recessions. Cholesky decomposition can be then performed to get the lower triangular matrices $\mathbb{L}_{\text{All}}$ and $\mathbb{L}_{\text{Res}}$ to generate correlated idiosyncratic factors.

Step 3. Generate correlated idiosyncratic factors $\mathbf{I}_t^i$ for all asset returns during period t under scenario i:

$$\mathbf{I}_t^i = \begin{cases} \boldsymbol{\sigma}_{\mathbf{All}} \cdot \mathbb{L}_{\mathbf{All}} \boldsymbol{\varepsilon}_t^i & \text{if the economy is not in recession during period } t \text{ under scenario } i \\ \boldsymbol{\sigma}_{\mathbf{Res}} \cdot \mathbb{L}_{\mathbf{Res}} \boldsymbol{\varepsilon}_t^i & \text{otherwise} \end{cases}$$

where

$\boldsymbol{\sigma}_{\mathbf{All}}$ is a column vector containing the standard deviation of error terms of all asset return models in normal periods, as shown in column 3 of Table A.9;

$\boldsymbol{\sigma}_{\mathbf{Res}}$ is a column vector containing the standard deviation of error terms of all asset return models in normal periods, as shown in column 4 of Table A.9;

$\boldsymbol{\varepsilon}_t^i$ is a column vector containing independent random variables following a standard normal distribution for all asset return models;

$\mathbb{L}_{\mathbf{All}}$ is a lower triangular matrix so that the error term correlation matrix $\mathbf{CM}_{\mathbf{All}}$ can be decomposed as $\mathbb{L}_{\mathbf{All}} \times \mathbb{L}_{\mathbf{All}}{}^T$; and

$\mathbb{L}_{\mathbf{Res}}$ is a lower triangular matrix so that the error term correlation matrix $\mathbf{CM}_{\mathbf{Res}}$ can be decomposed as $\mathbb{L}_{\mathbf{Res}} \times \mathbb{L}_{\mathbf{Res}}{}^T$.

Till this step, non-constant volatility and nonlinear relationships among idiosyncratic factors of asset return models have been taken care of. The economic scenario generation formula becomes

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\phi}_1\mathbf{y}_{t-1} + \boldsymbol{\phi}_2\mathbf{y}_{t-2} + \mathbb{B}_0\mathbf{F}_t + \mathbb{B}_1\mathbf{F}_{t-1} + \mathbb{B}_2\mathbf{F}_{t-2} + \mathbf{I}_t^i$$

$$\text{Let } \mathbf{y}_t^p = \boldsymbol{\alpha} + \boldsymbol{\phi}_1\mathbf{y}_{t-1} + \boldsymbol{\phi}_2\mathbf{y}_{t-2} + \mathbb{B}_0\mathbf{F}_t + \mathbb{B}_1\mathbf{F}_{t-1} + \mathbb{B}_2\mathbf{F}_{t-2}$$

$$\mathbf{y}_t = \mathbf{y}_t^p + \mathbf{I}_t^i$$

Step 4. Adjust $\mathbf{I}_t^i$ to reflect nonzero correlation between idiosyncratic factors and systemic factors during recessions:

$$\mathbf{J}_t^i = \begin{cases} \left(\boldsymbol{\rho}_r \cdot \mathbf{y}_t^p + \sqrt{1 - \boldsymbol{\rho}_r{}^2} \cdot \mathbf{I}_t^i\right) \dfrac{\boldsymbol{\sigma}_{\mathbf{Res}}}{\sqrt{\boldsymbol{\rho}_r{}^2\left(\boldsymbol{\sigma}_{\mathbf{Res}}^p\right)^2 + (1 - \boldsymbol{\rho}_r{}^2)(\boldsymbol{\sigma}_{\mathbf{Res}})^2}} & \text{if in recession during period } t \text{ under scenario } i \\ \mathbf{I}_t^i & \text{otherwise} \end{cases}$$

where $\boldsymbol{\rho}_r$ is a column vector containing the nonzero correlation between idiosyncratic factors and systemic factors, as shown in column 6 of Table A.9.

The economic scenario generation formula becomes

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\phi}_1\mathbf{y}_{t-1} + \boldsymbol{\phi}_2\mathbf{y}_{t-2} + \mathbb{B}_0\mathbf{F}_t + \mathbb{B}_1\mathbf{F}_{t-1} + \mathbb{B}_2\mathbf{F}_{t-2} + \mathbf{J}_t^i$$

Figures A.10 to A.33 show the statistics of 100 generated stochastic scenarios of returns of some asset classes and wage inflation, including the expected value, minimum, maximum, and 10th, 25th, 75th, 90th and 99th percentiles. As with the stochastic scenarios for fundamental economic factors, the range of prediction does not increase rapidly with the projection year, which indicates that economic cyclical patterns have been reflected in the scenarios.

Figure A.10
Stochastic Scenarios of 1-Year Treasury Bond Yields

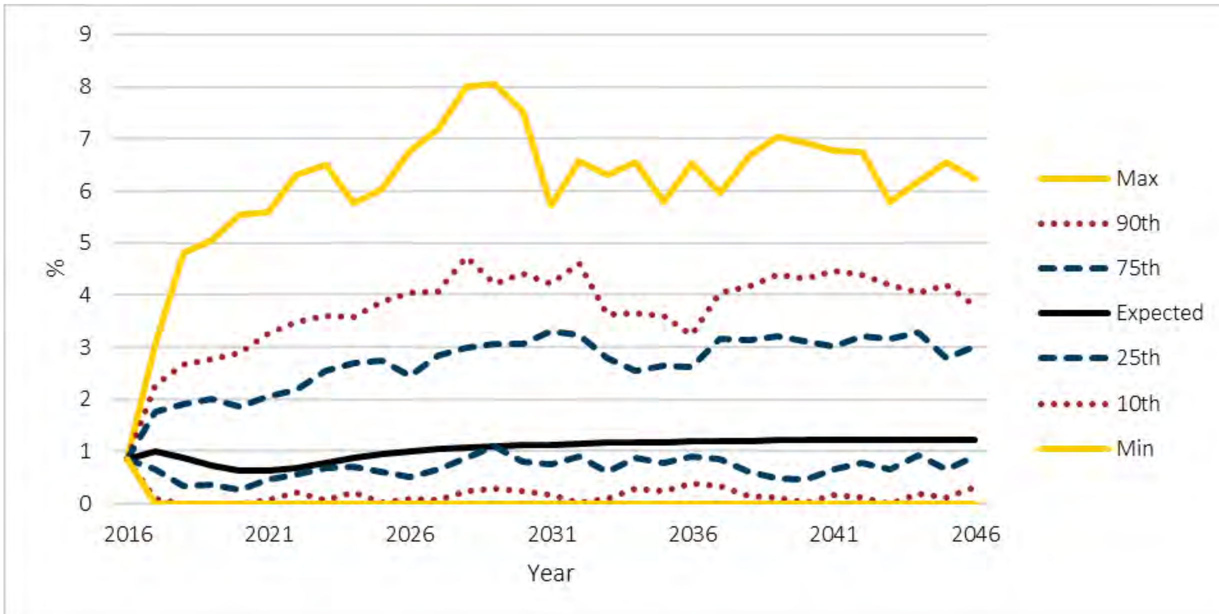


Figure A.11
Stochastic Scenarios of 20-Year Treasury Bond Yields

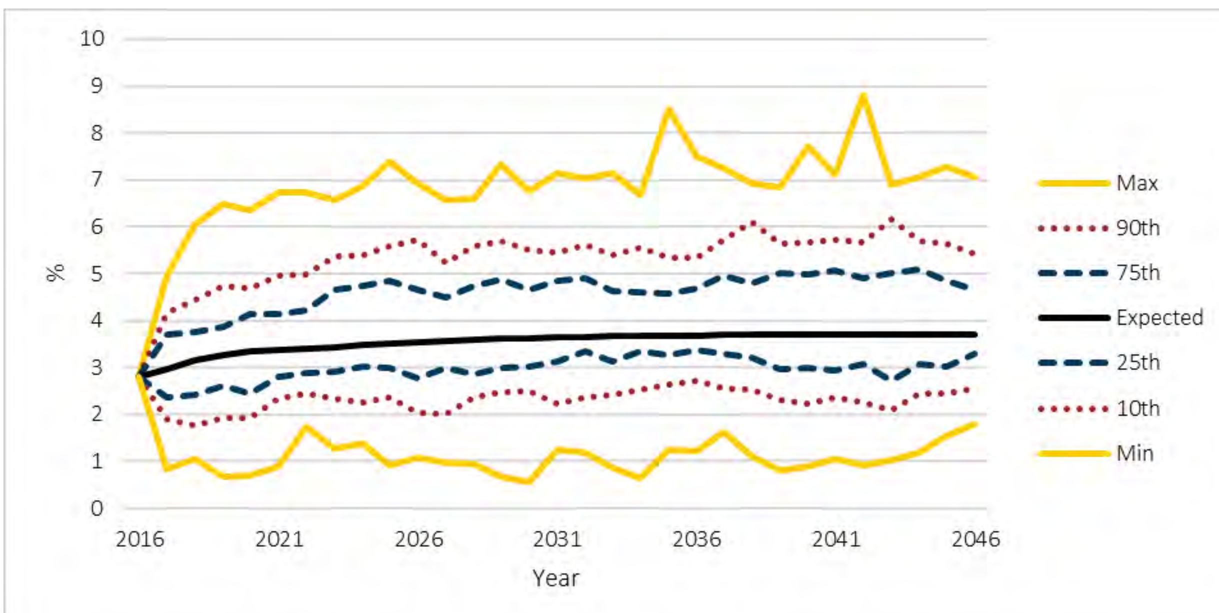


Figure A.12

Stochastic Scenarios of AA-Rated Corporate Bond Credit Spreads

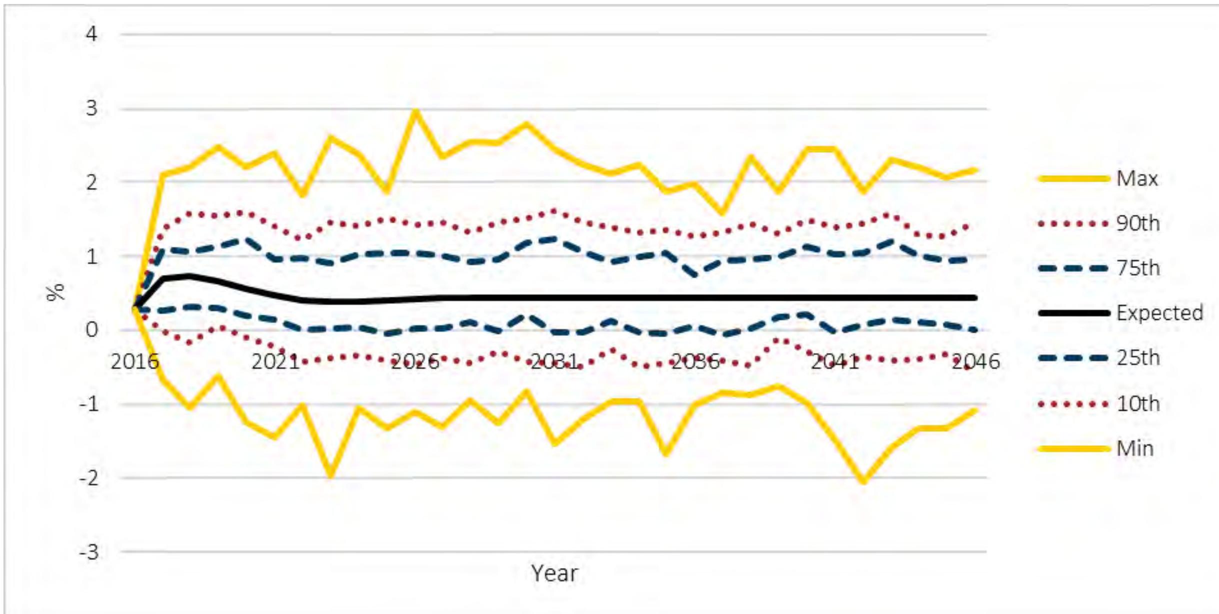


Figure A.13

Stochastic Scenarios of AA-Rated Corporate Bond Default Rates

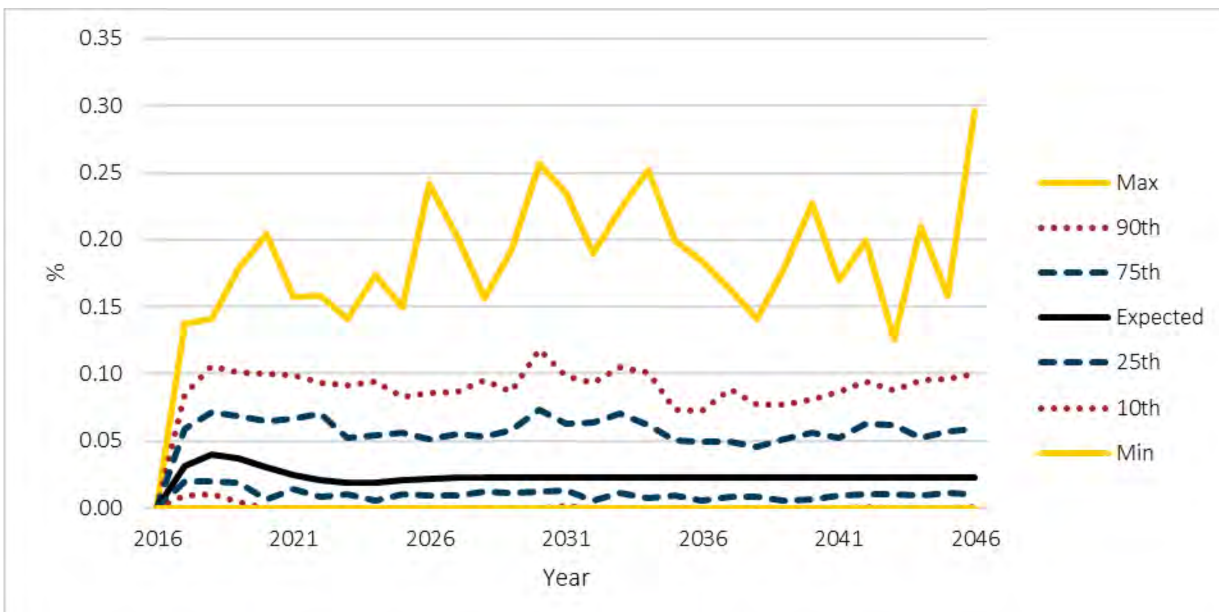


Figure A.14

Stochastic Scenarios of Large-Cap Public-Equity Dividend Yields (S&P 500)

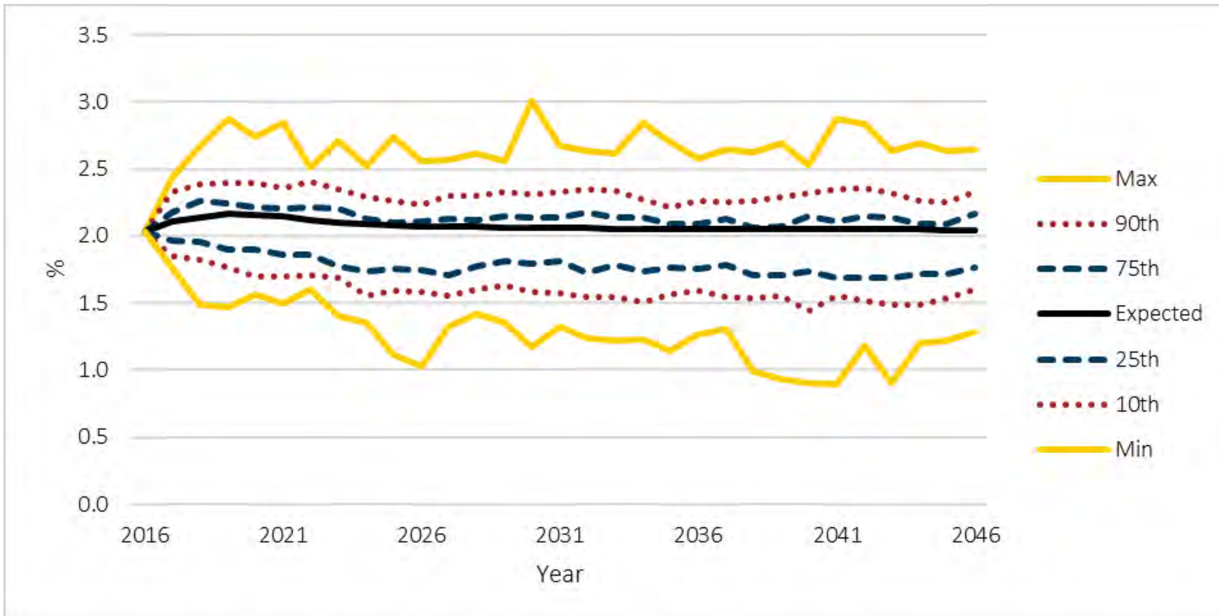


Figure A.15

Stochastic Scenarios of Large-Cap Public-Equity Capital Returns (S&P 500)

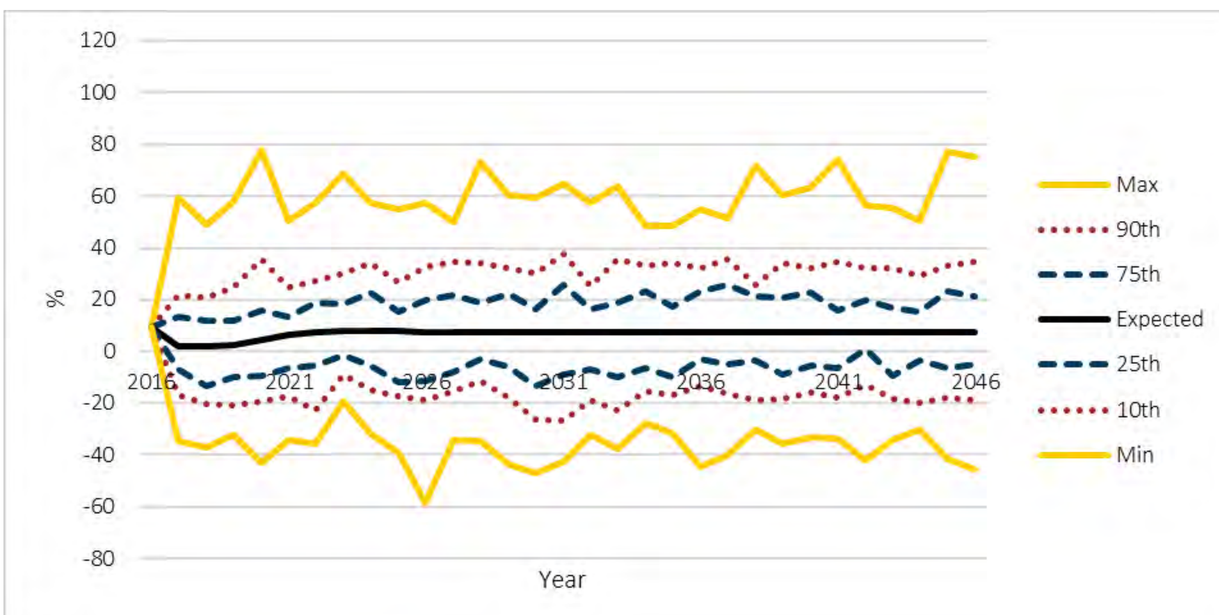


Figure A.16
Stochastic Scenarios of Mid-Cap Public-Equity Dividend Yields

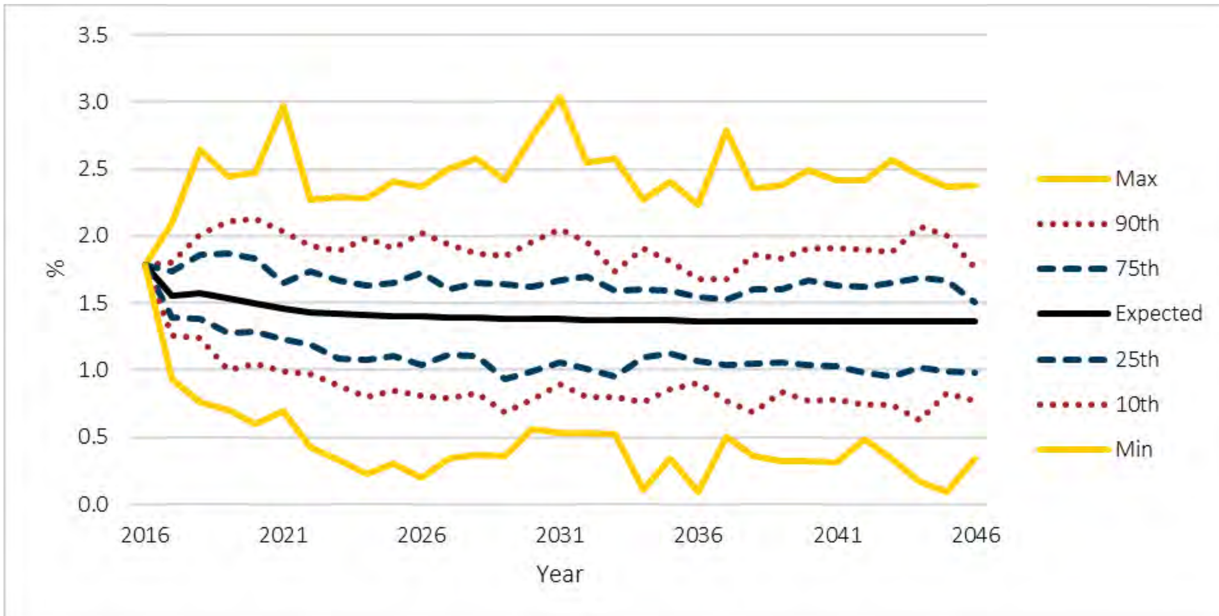


Figure A.17
Stochastic Scenarios of Mid-Cap Public-Equity Capital Returns

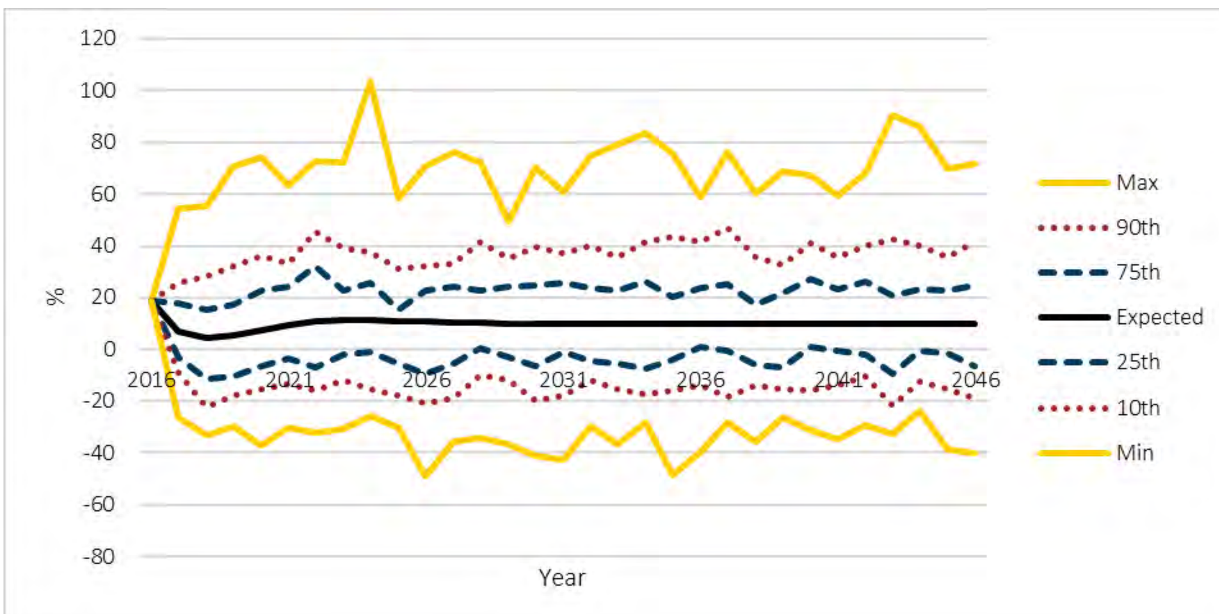


Figure A.18

Stochastic Scenarios of Small-Cap Public-Equity Dividend Yields

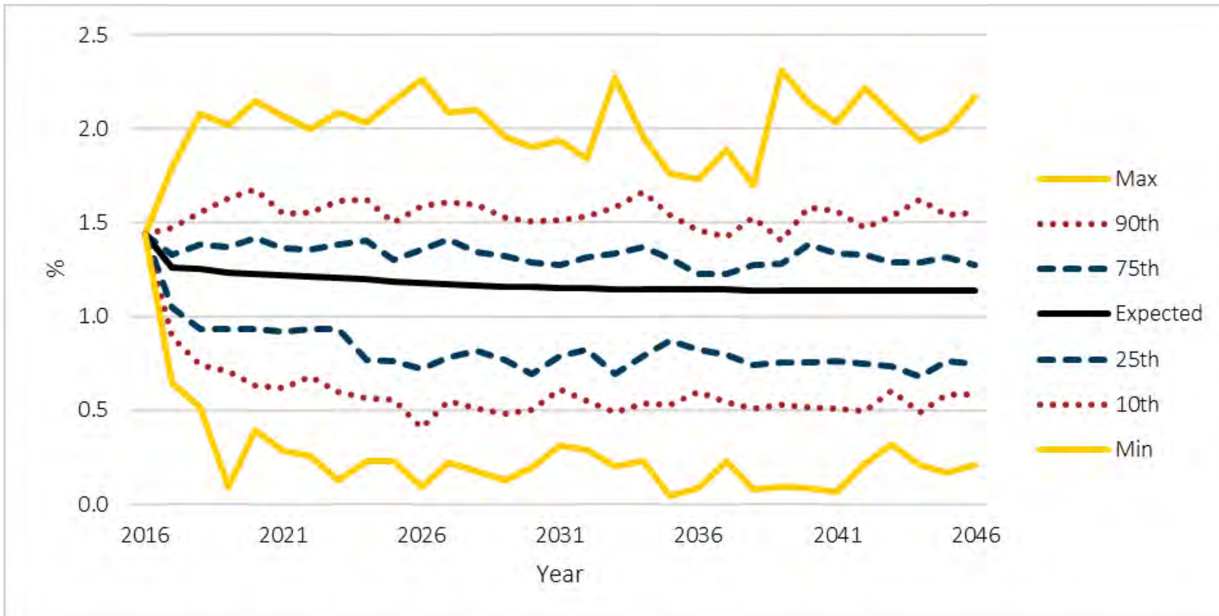


Figure A.19

Stochastic Scenarios of Small-Cap Public-Equity Capital Returns

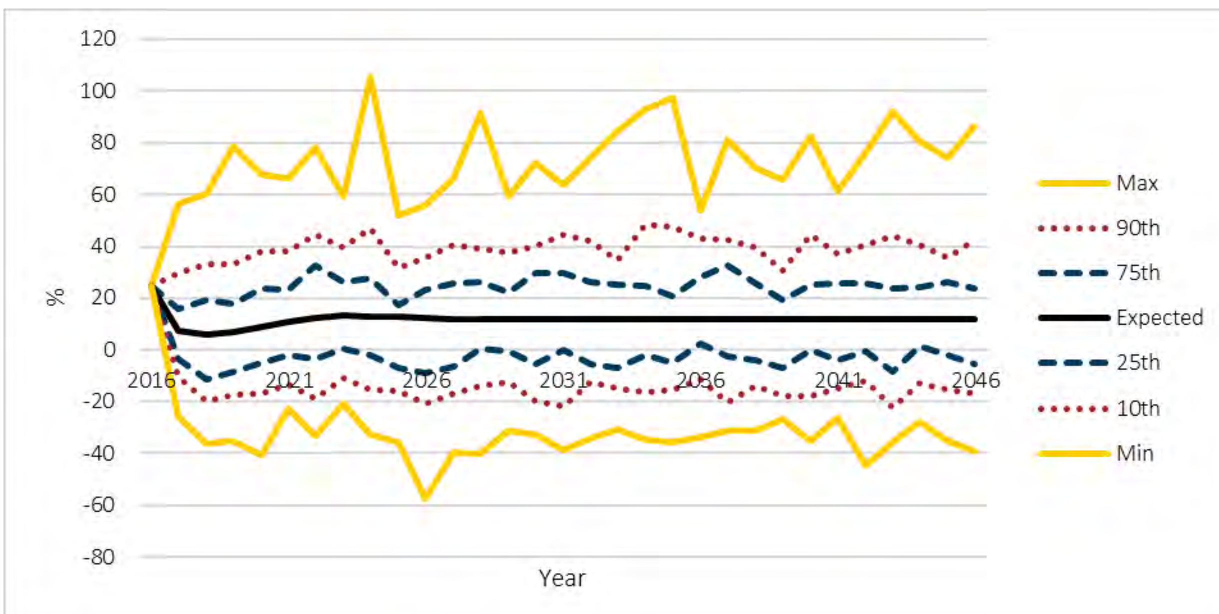


Figure A.20

Stochastic Scenarios of High-Dividend Public-Equity Dividend Yields

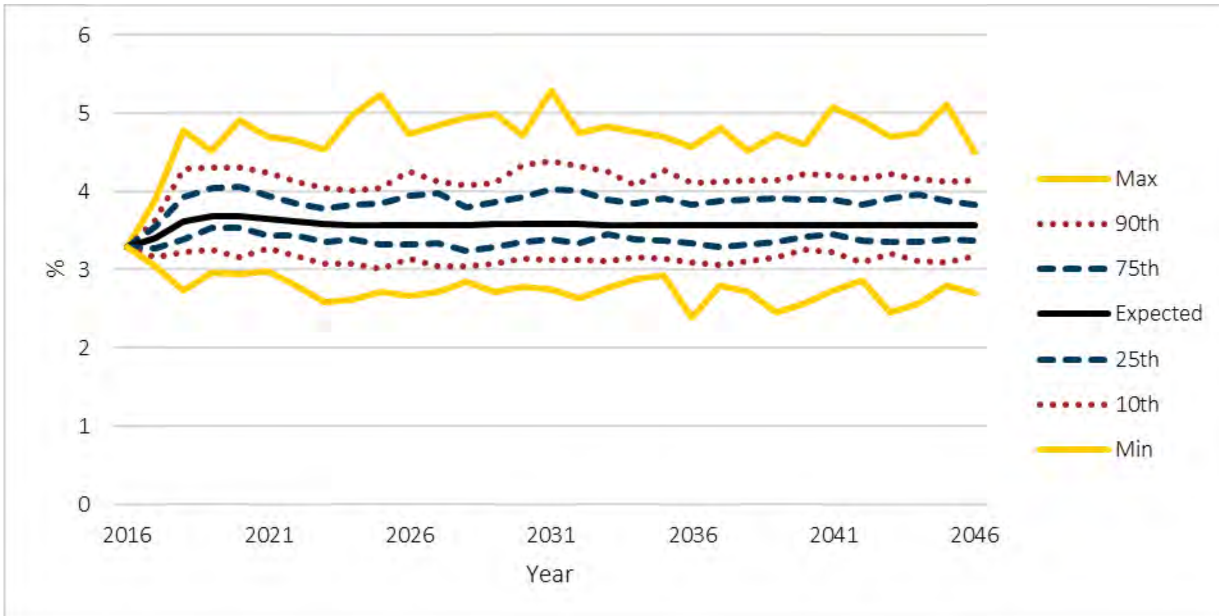


Figure A.21

Stochastic Scenarios of High-Dividend Public-Equity Capital Returns

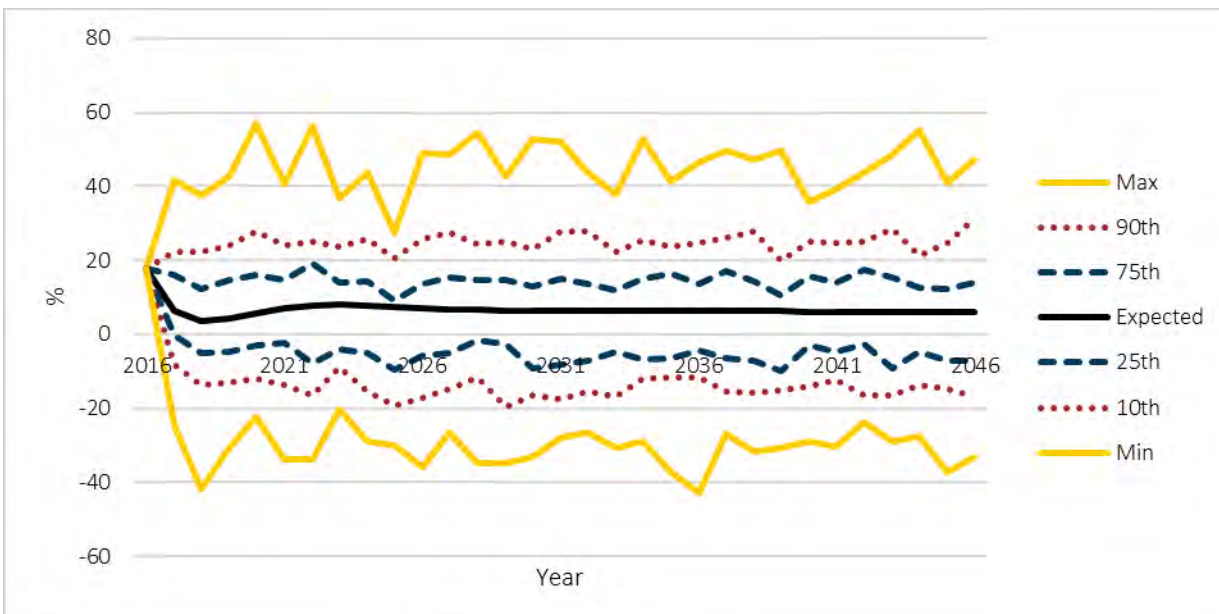


Figure A.22

Stochastic Scenarios of Equity REIT Cap Rates

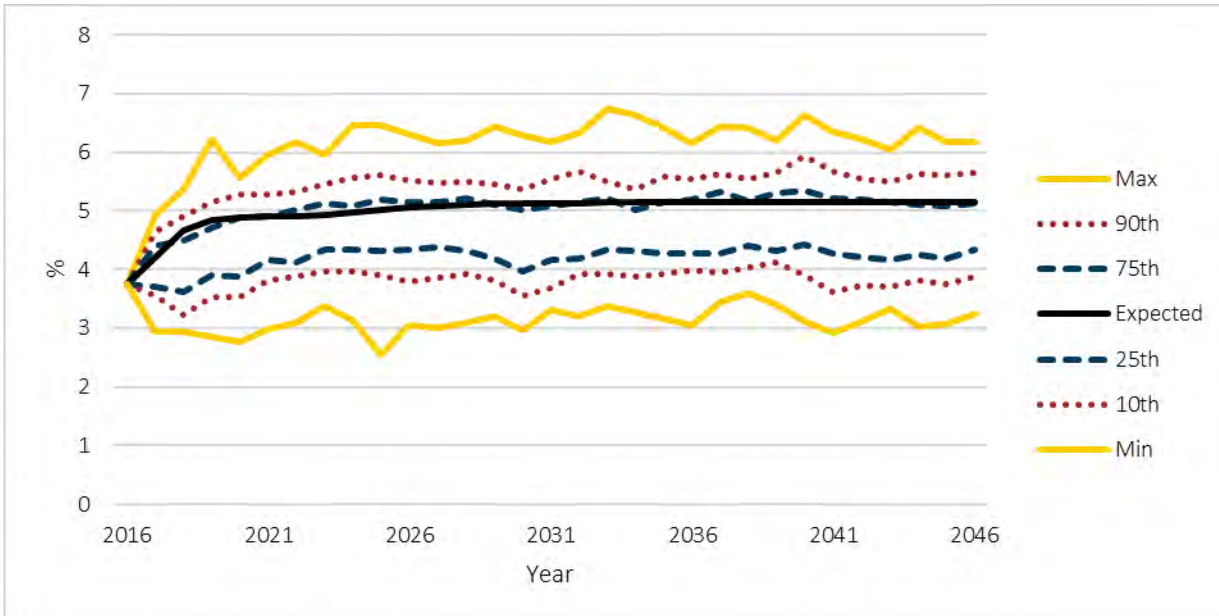


Figure A.23

Stochastic Scenarios of Equity REIT Capital Returns

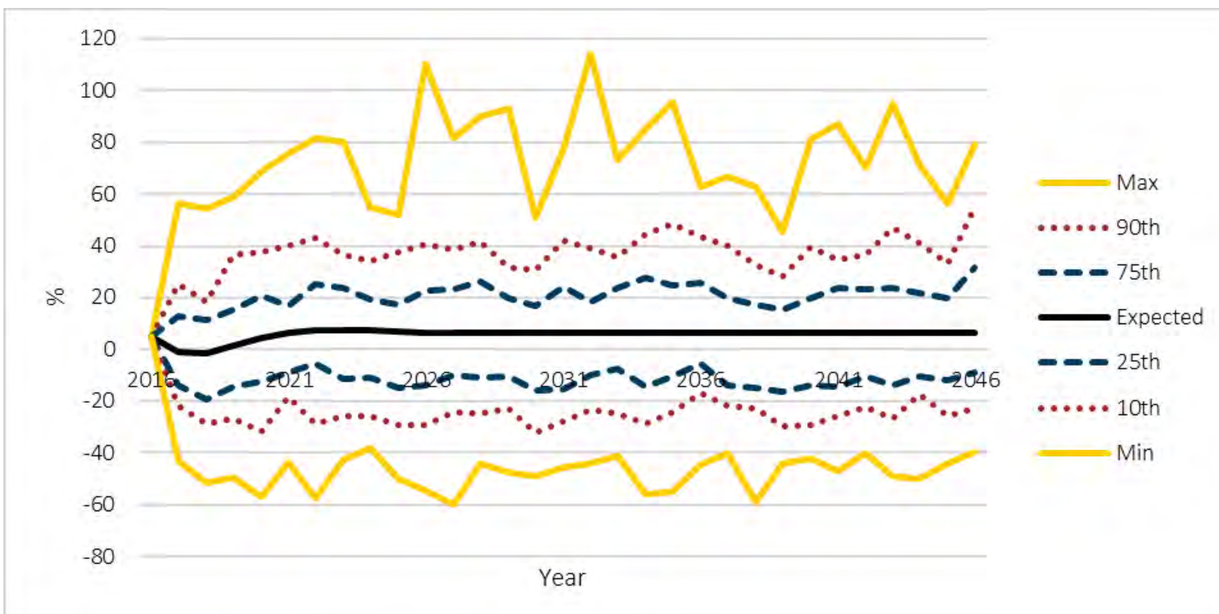


Figure A.24
Stochastic Scenarios of Mortgage REIT Cap Rates

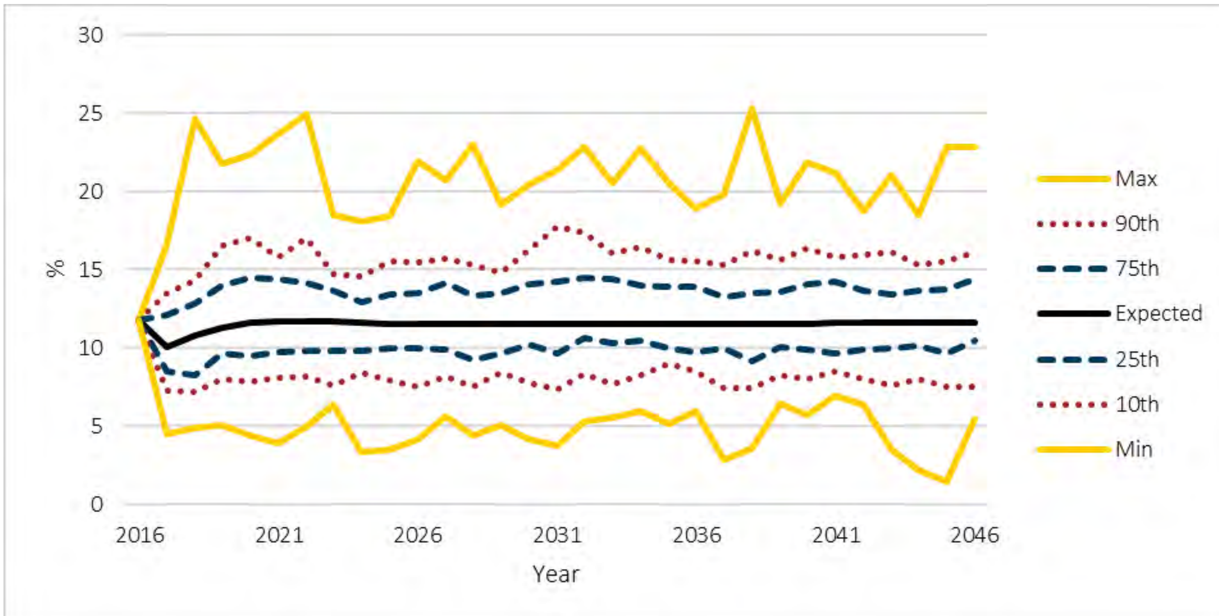


Figure A.25
Stochastic Scenarios of Mortgage REIT Capital Returns

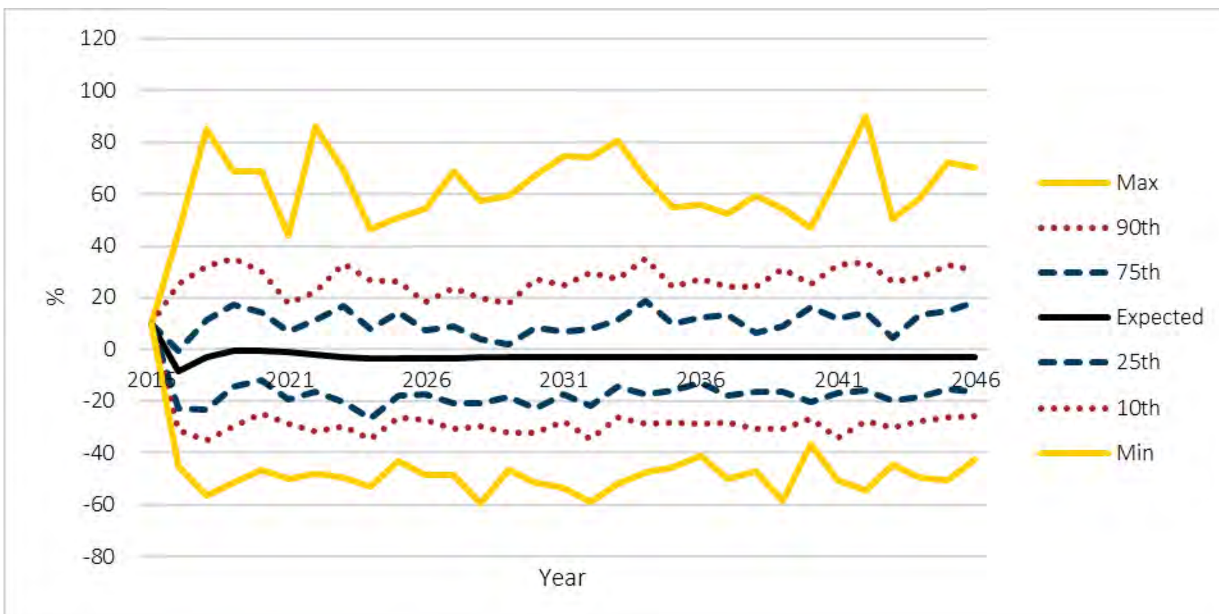


Figure A.26
Stochastic Scenarios of Infrastructure Project Index Dividend Yields

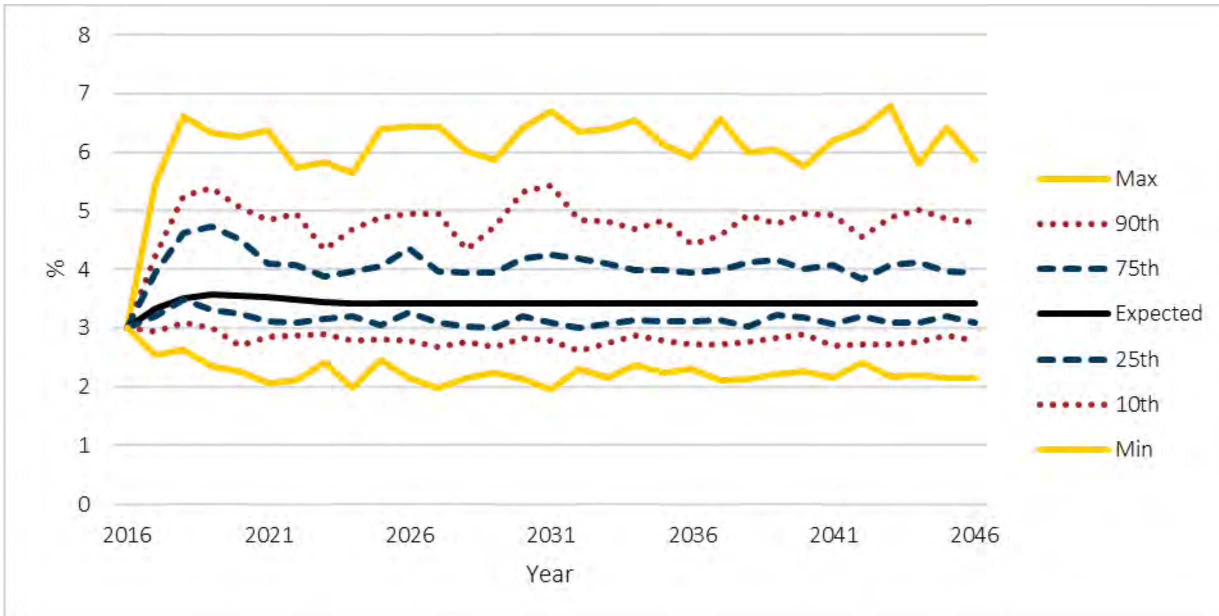


Figure A.27
Stochastic Scenarios of Infrastructure Project Index Capital Returns

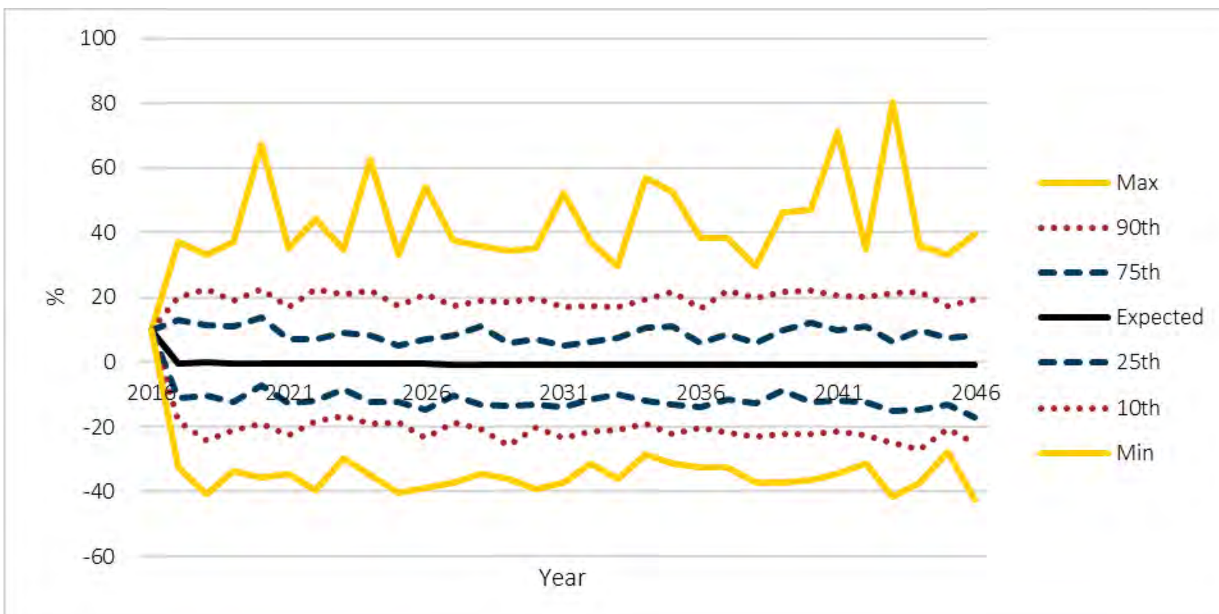


Figure A.28
Stochastic Scenarios of Private-Equity Index Dividend Yields

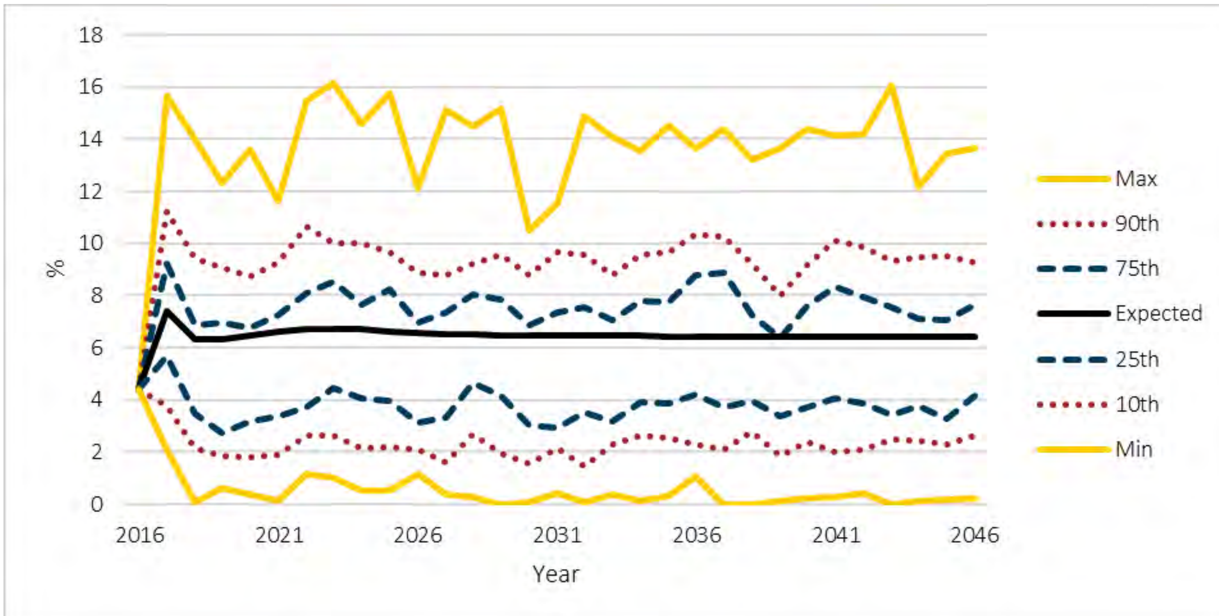


Figure A.29
Stochastic Scenarios of Private-Equity Index Capital Returns

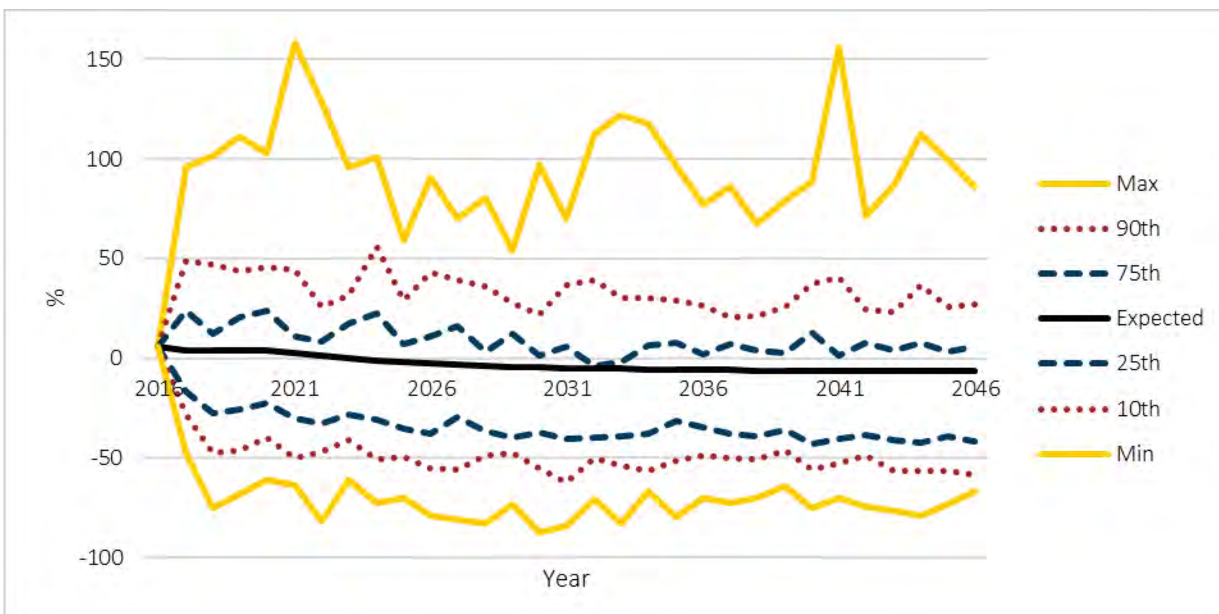


Figure A.30
Stochastic Scenarios of Crude-Oil Returns

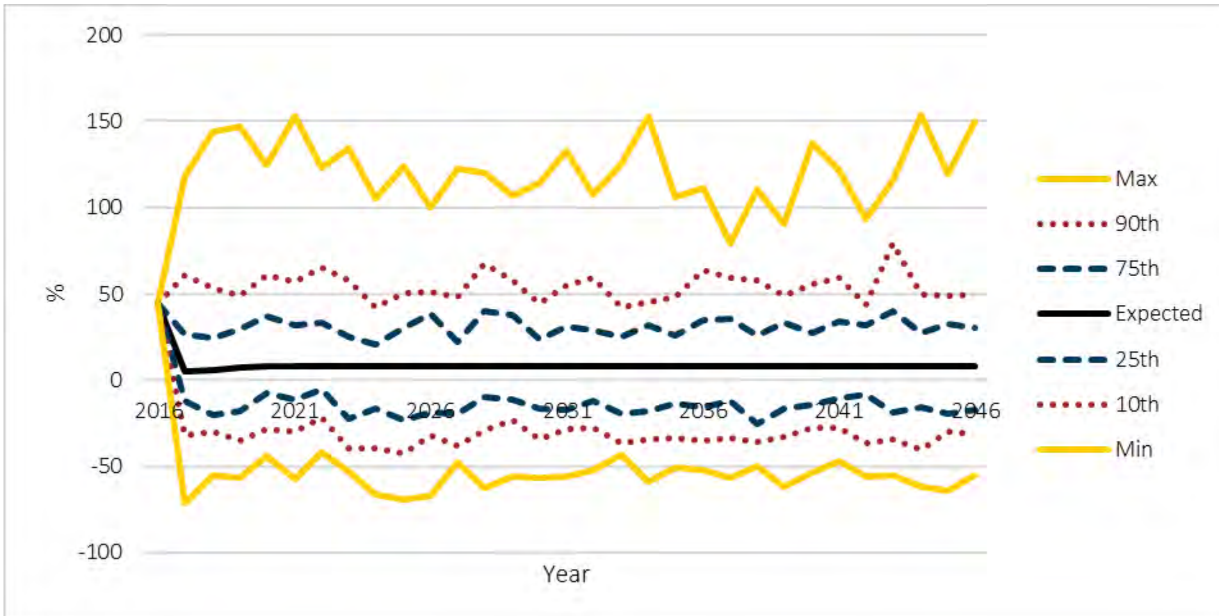


Figure A.31
Stochastic Scenarios of Gold Returns

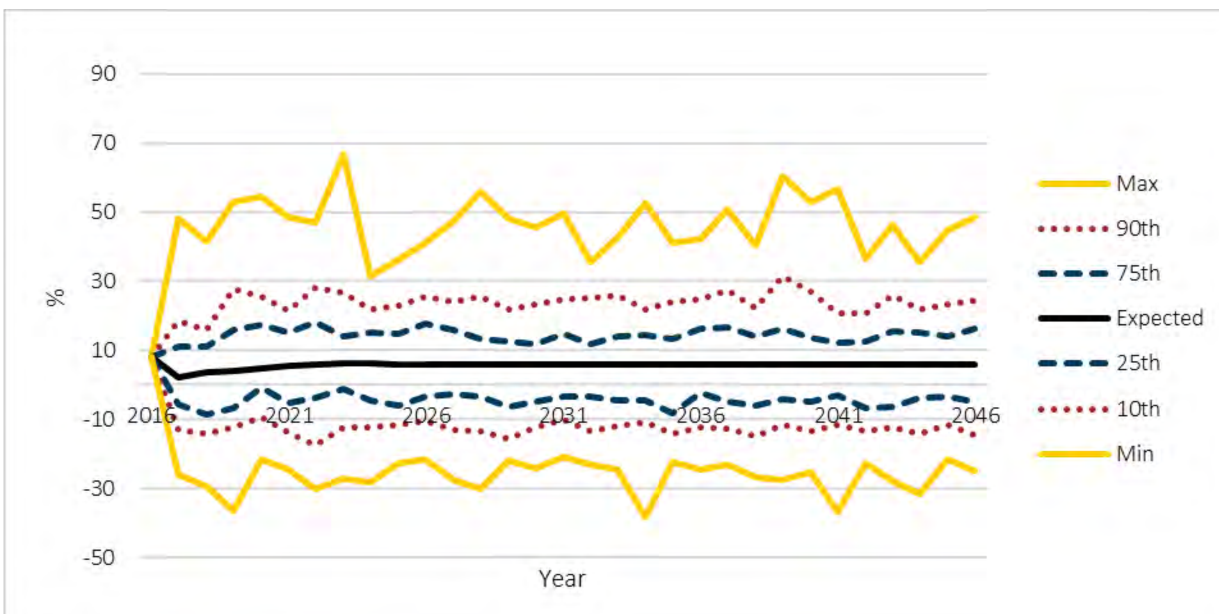


Figure A.32
Stochastic Scenarios of Commodity Index Returns

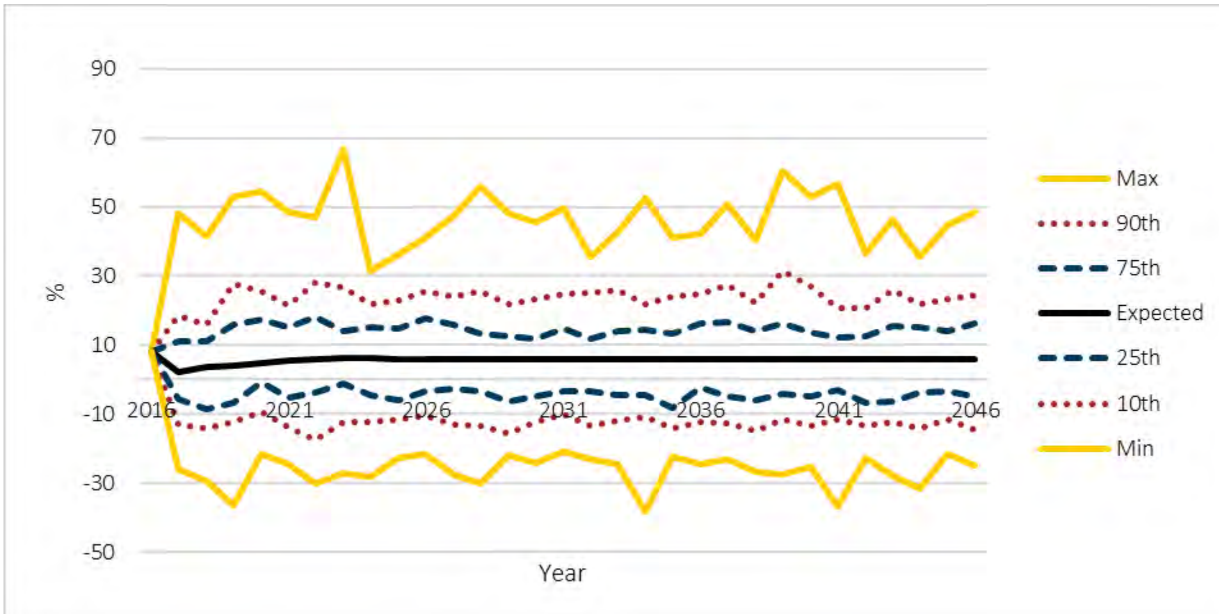
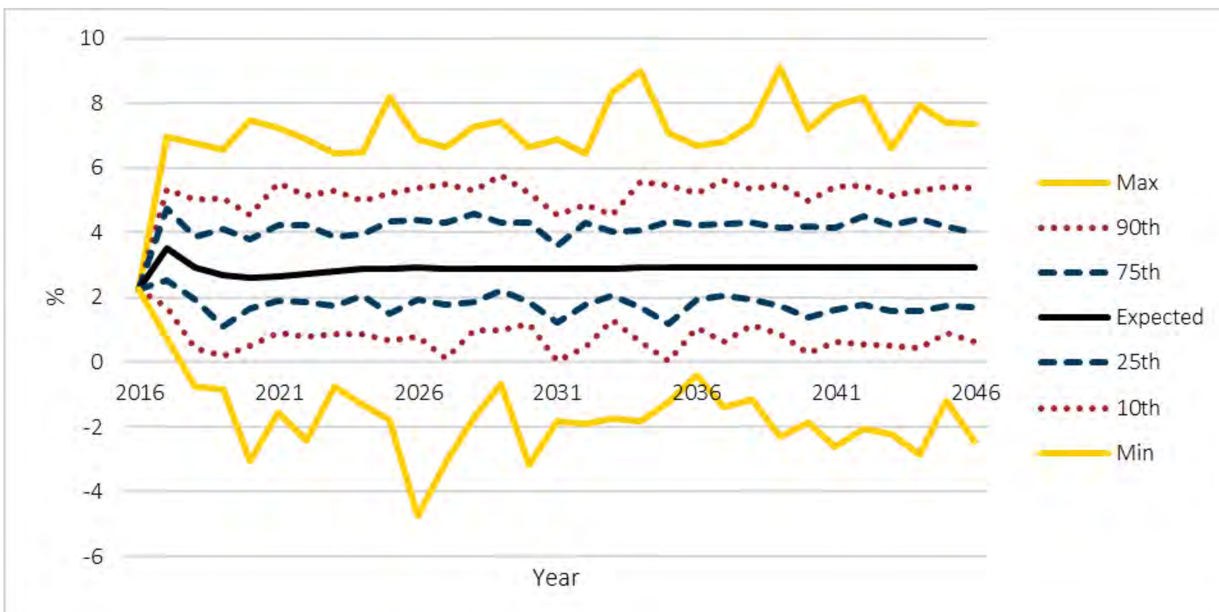


Figure A.33
Stochastic Scenarios of Wage Inflation



It is helpful to check the reasonableness of individual scenarios regarding whether basic economic patterns are followed, as Section A.1 did for fundamental economic factors. Using the same "good" and "bad" scenarios as in Figures A.9, Figures A.34 to Figure A.39 show the generated asset returns. Both the good scenario and the bad scenario have several economic cycles (recession and expansion). The good scenario is not always superior to the bad scenario.

During economic recessions, Treasury bond yields go down. Credit spread and default rate go down. Public equities (large cap, mid cap, small cap and high dividend class) behave similarly but with different volatility during recessions. Both dividend yields and capital returns go down. Equity REITs have a much stronger correlation with the macro economy than mortgage REITs. Cap rate and capital returns drop drastically for equity REITs during recessions. However, mortgage REITs are less correlated with the general economic cycles and could move in the opposite direction of equity REITs. These patterns are consistent with the correlation numbers shown in Table A.9. Infrastructure dividend yield may increase during economic recessions, likely as a result of greater infrastructure investment during recessions to spur the economy. As with public equity, private-equity dividend yields and capital returns decrease during recessions. Commodity investment returns have a weak relationship with the general economic cycle, except that gold's price usually increases during a recession. Wage inflation has a strong correlation with the economic cycle, with low wage inflation during recessions. Wage inflation is a factor that may affect pension liabilities if the benefit is linked to the future wage.

Figure A.34

Sample Scenarios of Bond Yields, Credit Spread and Default Rates

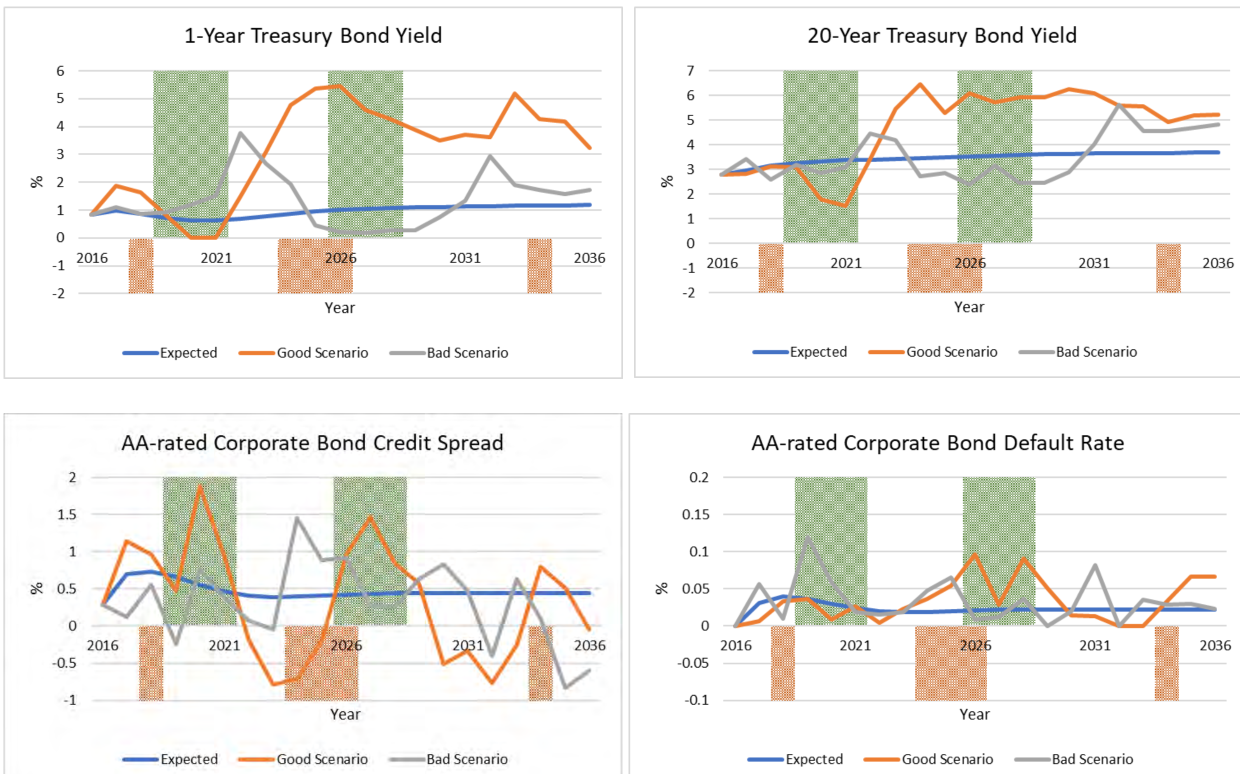


Figure A.35

Sample Scenarios of Public-Equity Dividend Yields and Capital Returns

Figure A.36

Sample Scenarios of Real Estate Investments

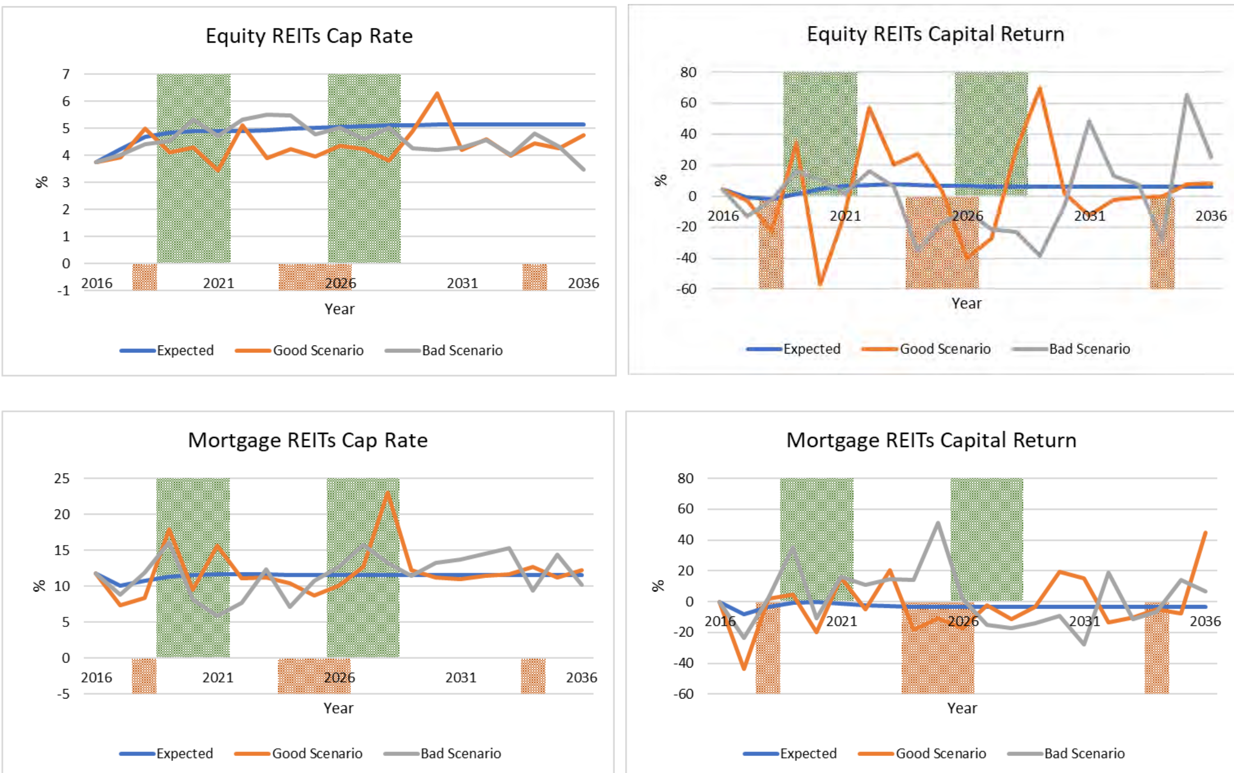


Figure A.37

Sample Scenarios of Infrastructure Investments

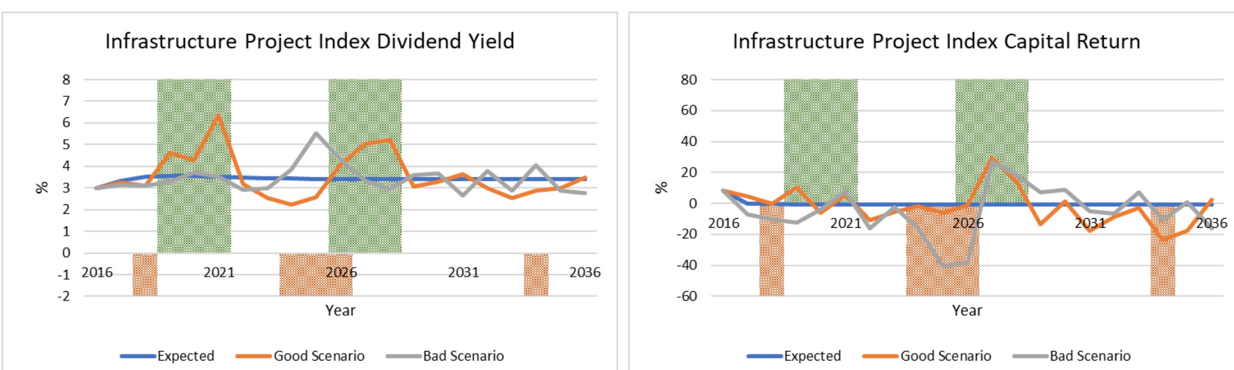


Figure A.38

Sample Scenarios of Private-Equity Investments

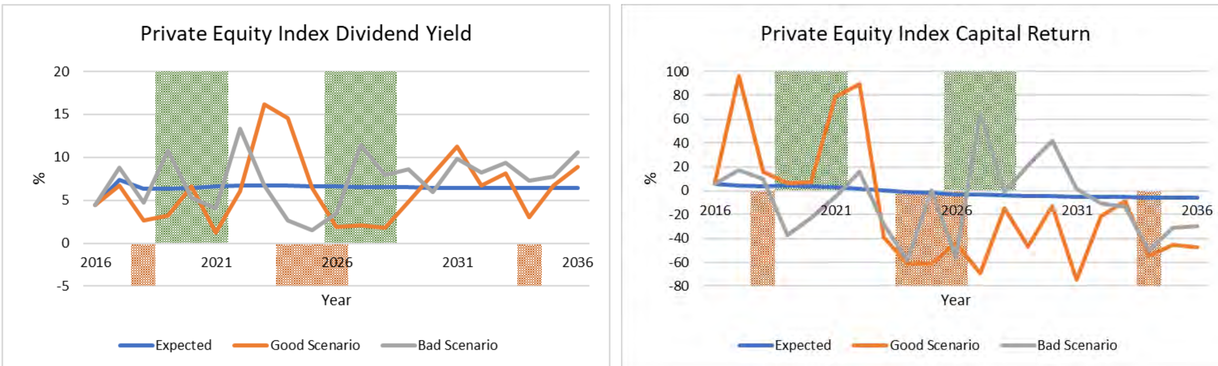
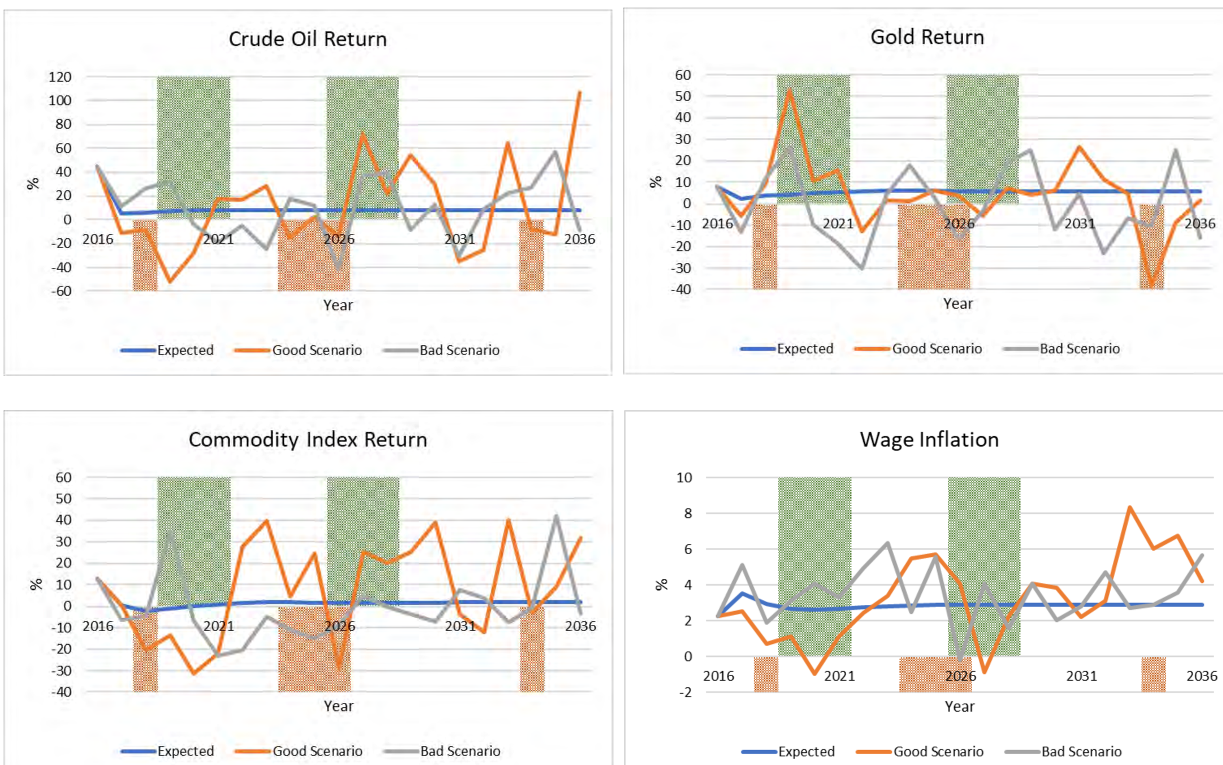


Figure A.39

Sample Scenarios of Commodity Investments



A.3 Economic Scenario Generation for Bond Fund Return

With generated scenarios of the Treasury bond yield curve, credit spread and default rate, the model permits calculation of bond fund income rates and capital returns, based on bond fund investment strategy. Bond fund income rates can be used to project the positive cash flows (coupon payments) from bond investments. These cash flows can be used to meet benefit payout requirements or be reinvested. Bond fund capital returns are useful to project the bond fund values. This report uses five bond funds constructed to invest in Treasury bonds and AAA-, AA-, A- and BBB-rated corporate bonds, respectively. Their returns are generated using the following steps.

Step 1. Set the term mix of each bond fund. To match the long-term nature of the liability, all bond funds contain the mix of maturities shown in Table A.10. The bonds have an average maturity of 19 years and will be rebalanced each year to maintain the term mix.

Table A.10
Bond Fund Term Mix Assumption

Bond Maturities (Years)	Mix (% of Fund)
1	2.5%
3	2.5%
5	2.5%
7	2.5%
10	15%
20	50%
30	25%

Step 2. Using the assumption that the bonds have an annual coupon rate same as the bond yield at the same term, calculate the coupon payments and income rates for each projection period. Coupon payments and redemptions are then reinvested in the same term mix.

Step 3. Evaluate the existing bonds each year with the new yield curve. In addition to the new yield curve and shorter maturity that could change the bond value from the beginning of the period, default and credit rating migration also are reflected in the revaluation. As part of the economic scenario generator, default rates are scenario dependent. Rating migration probabilities and recovery rates are kept fixed, as shown in Table A.11. If a bond is in default, its remaining value will be determined by the product of the recovery rate and the bond value. If a bond changes its rating, the value will be changed based on the yield curve of bonds with the new credit rating. The capital return is then calculated as the percentage change of total bond value.

Table A.11
Rating Migration and Recovery Rate Assumptions

	Rating Migration (%)					Default Rate (Scenario Dependent)	Recovery Rate (%)
	AAA	AA	A	BBB	Junk[1]		
AAA	87.05	9.03	0.53	0.05	3.34—Default $Rate_{AAA}$	Default $Rate_{AAA}$	49
AA	0.52	86.82	8.00	0.51	4.15—Default $Rate_{AA}$	Default $Rate_{AA}$	49
A	0.03	1.77	87.79	5.33	5.08—Default $Rate_{A}$	Default $Rate_{A}$	37
BBB	0.01	0.10	3.51	85.56	10.82—Default $Rate_{BBB}$	Default $Rate_{BBB}$	25

[1]Bonds downgraded below rating BBB (junk bonds) are assumed to be reinvested into investment-grade bonds. The recovered value from defaulted bonds also is assumed to be reinvested into investment-grade bonds.

Sources: The rating migration assumption is set according to the *2016 S&P Annual Global Corporate Default Study and Rating Transitions Report.* The recovery rate assumption is set according to *Moody's Corporate Default and Recovery Rates, 1920–2010* (Moody's Investors Service, February 28, 2011).

Step 4. After the valuation, rebalance the bond fund to the target term mix and credit rating.

Table A.12 lists the average simulated bond fund return for each bond fund. The bond fund return generation approach described here is only one possible approach. Many other bond fund investment strategies exist and can be applied. In practice, if the bond fund has enough historical data, linear models can be used to simulate future income rates and capital returns, as was done for other asset classes in A.2 Economic Scenario Generation for Asset Return.

Table A.12
Average Simulated Bond Fund Return

Fund Type	Average Return (%)
Government bond fund	4.22
AAA-rated corporate bonds	3.60
AA-rated corporate bonds	4.59
A-rated corporate bonds	4.31
BBB-rated corporate bond	5.13

Appendix B: Construction of the Liability Replicating Portfolio

As one of the candidates for LDI, the liability replicating portfolio is the optimal investment portfolio to minimize the difference between pension assets and pension liabilities, as described in Section 2.1.3. In this section, the process of deriving the liability replicating portfolio is explained. It is also one of the asset allocation plans analyzed for LDI in Section 5.1.

In this example, the liability replicating portfolio is the optimal asset portfolio that has the same value as the pension liability at time 0, matches the liability sensitivity to fundamental economic factors and asset returns, and matches the liability values in the future. It can be considered an encompassing approach of asset and liability management, as it incorporates duration matching, convexity matching, cash flow matching and so on in a holistic way, not only at the current time point but also in the future. The optimization problem can be formalized as below:

$$\min_x \|Ax - b\|$$

$$\text{s.t.} \qquad \textstyle\sum_{i=1}^{ni} x_i = L(0)$$

$$x_i \geq 0 \text{ for } i = 1, 2, \ldots, NI$$

$$\|Ax-b\| = \begin{bmatrix} a_1^{S_1}(0) & a_2^{S_1}(0) & \cdots & a_{NI-1}^{S_1}(0) & a_{NI}^{S_1}(0) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1^{S_{NS}}(0) & a_2^{S_{NS}}(0) & \cdots & a_{NI-1}^{S_{NS}}(0) & a_{NS}^{S_{NS}}(0) \\ a_1^1(1) & a_2^1(1) & \cdots & a_{NI-1}^1(1) & a_{NI}^1(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1^n(1) & a_2^n(1) & \cdots & a_{NI-1}^n(1) & a_{NI}^n(1) \\ a_1^1(t) & a_2^1(t) & \cdots & a_{NI-1}^1(t) & a_{NI}^1(t) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1^n(t) & a_2^n(t) & \cdots & a_{NI-1}^n(t) & a_{NI}^n(t) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{NI-1} \\ x_{NI} \end{bmatrix} - \begin{bmatrix} L^{S_1}(0) \\ \vdots \\ L^{S_{NS}}(0) \\ L^1(1) \\ \vdots \\ L^n(1) \\ L^1(t) \\ \vdots \\ L^n(t) \\ \vdots \end{bmatrix}$$

where

$x = (x_1, x_2, x_3, \cdots, x_{NI})^T$, a column vector that contains the initial amount of investment on each asset class;
NI = total number of asset classes available for investment;
$L(0)$ = initial pension liability value;
b is a column vector that contains the liability values or sensitivities that need to be matched under certain scenarios;

$L^{S_i}(0)$ = initial pension liability value under shock scenario S_i;
NS total number of shock scenarios;
$L^i(t)$ = projected liability value at time t under scenario i;
A is a matrix with NI columns that includes the unit values and sensitivities of each asset class under the same set of scenarios used for calculating vector b (the number of rows depends on how many matching conditions are required in the optimization process);
$a_j^{S_i}(0)$ = initial unit value of asset class j under shock scenario S_i;
$a_j^i(t)$ = projected unit value of asset class j at time t under scenario i; and
$x_i \geq 0$ signifies there is no short selling for asset class i.

The optimization problem can be easily converted to a quadratic programming problem of minimizing a quadratic function subject to linear constraints:

$$\min_x \|Ax - b\|$$

$$=> \min_x x^T A^T Ax - 2b^T Ax + b^T b$$

$$=> \min_x x^T A^T Ax - 2b^T Ax$$

Table A.13 lists the shock scenarios that have been used to match the exposures to immediate changes of fundamental economic factors in this report.

Table A.13
Initial Shock Scenarios (in basis points)

Shock Scenario (S_i)	Real GDP Growth Rate	Inflation Rate	3-Month Treasury Bill Yield	10-Year Treasury Bond Yield	Credit Spread (AA-Rated Corporate Bond – Government Bond)
Interest rate (IR) level up	0	0	25	25	0
IR level down	0	0	−25	−25	0
Credit spread up	0	0	0	0	25
Credit spread down	0	0	0	0	−25
Inflation rate up	0	25	0	0	0
Inflation rate down	0	−25	0	0	0
GDP growth rate up	25	0	0	0	0
GDP growth rate down	−25	0	0	0	0
Short-term IR up	0	0	25	0	0
Short-term IR down	0	0	−25	0	0
Long-term IR up	0	0	0	25	0
Long-term IR down	0	0	0	−25	0

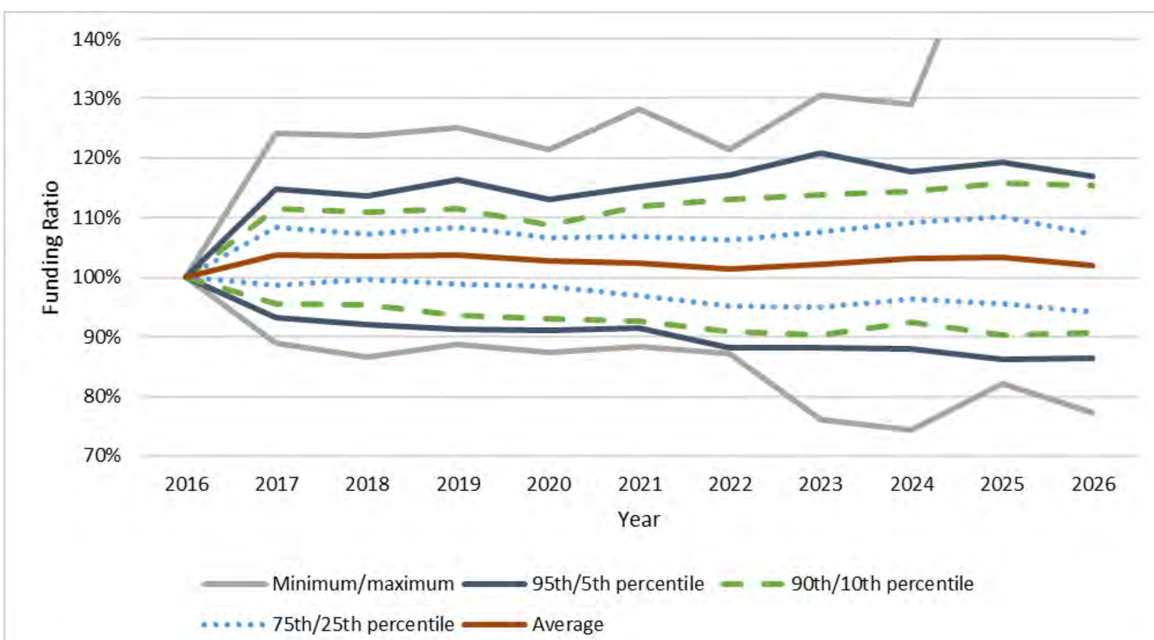
In addition to shock scenarios, the example also attempts to match the liability values under each stochastic scenario at the end of the first, third, fifth and 10th years. Table A.14 shows the solved liability replicating portfolio for the DB plan in Section 4, which has an initial liability value of $10 million. The majority investment of the liability replicating portfolio is in AA-rated corporate bonds, as expected. With the discount curve set as the AA-rated corporate bond yield curve, the liability is likely to be better matched with the AA-rated corporate bond investment.

Table A.14
Liability Replicating Portfolio

Type	Market Value ($)	Percentage
Government bonds	0	0.0%
Corporate bonds, AAA	0	0.0%
Corporate bonds, AA	9,234,961	92.3%
Corporate bonds, A	0	0.0%
Corporate bonds, BBB	0	0.0%
Large-cap equity	0	0.0%
Mid-cap equity	0	0.0%
Small-cap equity	0	0.0%
High-dividend equity	193,300	1.9%
Equity REIT	129,661	1.3%
Mortgage REIT	129,990	1.3%
Infrastructure fund	0	0.0%
Private-equity fund	124,291	1.2%
Oil	9,940	0.1%
Gold	177,857	1.8%
Commodities	0	0.0%

A basic way to assess how well the replicating portfolio matches the liability is to compare the projected funding ratios with the target 100% funding ratios for all stochastic scenarios. Figure A.40 shows the projected funding ratio of the DB plan in Section 4, assuming pension assets are all invested in the replicating portfolio. Of the scenarios, 90% (those in the fifth percentile to 95th percentile) have a funding ratio no less than 85% for the next 10 years.

Figure A.40
Funding Ratio Projection Using Liability Replicating Portfolio



In the example, cash flow matching is omitted when deriving the liability replicating portfolio, because the DB plan in this example is an ongoing plan with future plan contributions. This makes liquidity issues less of a concern. However, for a closed DB plan with large payouts, cash flow matching is more important and can be added to the objective function.

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