SOCIETY OF ACTUARIES

Mortality Improvement Scale MP-2018

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Mortality Improvement Scale MP-2018

AUTHOR Society of Actuaries
Retirement Plans Experience Committee

Contents

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Section 1: Executive Summary

This report presents Scale MP-2018, the latest iteration of the pension mortality improvement scales developed annually by the Retirement Plans Experience Committee (RPEC or "the Committee") of the Society of Actuaries (SOA). Scale MP-2018 is based on the same underlying methodology and committee-selected assumption set used to develop Scale MP-2017 (SOA 2017) and reflects historical U.S. population mortality experience through 2016. The Scale MP-2018 mortality improvement rates can be found on the SOA website at the following link: https://www.soa.org/resources/experience-studies/2018/mortality-improvement-scale-mp-2018/

The Scale MP-2018 mortality improvement rates presented in this report are slightly lower than the corresponding Scale MP-2017 rates. Table 1[1] of deferred-to-62 annuity values shows that starting with RP-2006[2] base mortality rates, most 2018 pension obligations calculated using Scale MP-2018 (with a discount rate of 4.0%) are anticipated to be 0.2% to 0.4% lower for females, and 0.3% to 0.6% lower for males, relative to their Scale MP-2017 counterparts.

Table 1

	Monthly Deferred-To-62 Annuity-Due Values Base Rates: **RP-2006** Generational @ 2018; Discount Rate = 4.0%					
	Females			Males		
Projection Scale →	MP-2017	**MP-2018**	*% Change*	MP-2017	**MP-2018**	*% Change*
Age						
25	3.7415	**3.7264**	*-0.40%*	3.5078	**3.4823**	*-0.73%*
35	5.4392	**5.4182**	*-0.39%*	5.0908	**5.0574**	*-0.66%*
45	7.9196	**7.8917**	*-0.35%*	7.3994	**7.3585**	*-0.55%*
55	11.5967	**11.5585**	*-0.33%*	10.8479	**10.7961**	*-0.48%*
65	14.2829	**14.2505**	*-0.23%*	13.4119	**13.3632**	*-0.36%*
75	10.3945	**10.3682**	*-0.25%*	9.5786	**9.5501**	*-0.30%*
85	6.3208	**6.3076**	*-0.21%*	5.6657	**5.6504**	*-0.27%*

Section 4 includes analysis of an alternative mortality improvement model based on the same methodological underpinnings as RPEC_2014 (the model used to produce Scale MP-2018), but

[1] It should be noted that unlike the corresponding tables of annuity values included in prior Scale MP reports, this report presents the female values before the male values.
[2] The RP-2006 Mortality Tables are based on the same data used to construct the RP-2014 Mortality Tables but as of 2006, the base year of the RP-2014 study. These were computed by backing out mortality improvement from 2007–2014 from the RP-2014 rates. The SOA formally published these tables in July 2018.

with rates projected from a smoother graduation[3] of historical U.S. population mortality improvement. In particular, the model introduced in Section 4 is anticipated to reduce volatility resulting from incorporating updated mortality experience each year, though its fit to historical mortality improvement rates is diminished relative to that of the RPEC_2014 model.

RPEC believes that Scale MP-2018 produces a reasonable mortality improvement assumption for measuring obligations for most retirement programs in the United States within the context of the "assumption universe" as described in Actuarial Standard of Practice No. 35 (ASOP No. 35) (ASB 2014). However, RPEC also believes that other mortality improvement scales, including those created with an assumption set different than that selected by RPEC or those based on an underlying model other than RPEC_2014 (such as that introduced in Section 4) also could fall within the ASOP No. 35 assumption universe. It is the responsibility of the actuary to determine which mortality improvement assumption is appropriate to use for a given purpose.

[3] Specifically, the smoother historical graduation is based on order-2 Whittaker-Henderson graduation, as contrasted to the order-3 Whittaker-Henderson graduation used to develop Scale MP-2018. See Section 4 and Appendix B for additional details.

Section 2: Data Sources, Underlying Model and Recent U.S. Mortality Experience

2.1 Data Sources

The historical mortality information published by the Social Security Administration (SSA) in conjunction with the 2018 Trustees' Report included rates that are smoothed across ages for each individual year through calendar year 2015 (SSA 2018). The data for calendar years 1950 through 2014 were taken directly from these SSA-published mortality rates.

Based on prior discussions with the SSA, RPEC was aware that the SSA-published mortality rates for calendar year 2015 reflected preliminary Medicare data for ages 65 and older. Final Medicare data for 2015 became available subsequent to the development of the SSA-published rates but prior to the finalization of Scale MP-2018. RPEC elected to develop updated 2015 mortality rates reflecting the more current Medicare data and using the graduation methodology described in Actuarial Study No. 120 (SSA 2005).

Estimated SSA-style[4] rates for 2016 were calculated using data developed by the Centers for Disease Control and Prevention (CDC), the U.S. Census Bureau and the Centers for Medicare and Medicaid Services (CMS). The methodology used to develop the estimated rates for calendar year 2016 was the same as that described in Appendix C of the Scale MP-2017 report for the estimated SSA-style rates for calendar year 2015.

2.2 Mortality Improvement Model

The 2018 version of the RPEC mortality improvement model, denoted RPEC_2014_v2018, is based on the original RPEC_2014 model updated to reflect the historical mortality data through calendar year 2016 as described in Section 2.1. As in all prior MP scales, historical rates were calculated using a two-dimensional Whittaker-Henderson graduation of the natural logarithm of U.S. population mortality rates with smoothness components based on the sum of the squares of third finite differences. Scale MP-2018 rates were developed from this RPEC_2014_v2018 model, applying the same committee-selected assumption set used to develop Scale MP-2017; specifically:

- Long-term rate of mortality improvement: flat 1.0% rate to age 85, decreasing linearly to 0.85% at age 95, then decreasing linearly to 0.0% at age 115
- Horizontal convergence period (along fixed ages): 10 years
- Diagonal convergence period (along fixed year-of-birth cohorts): 20 years
- Horizontal/diagonal blending percentages: 50%/50%
- Initial slope constraint: 0.

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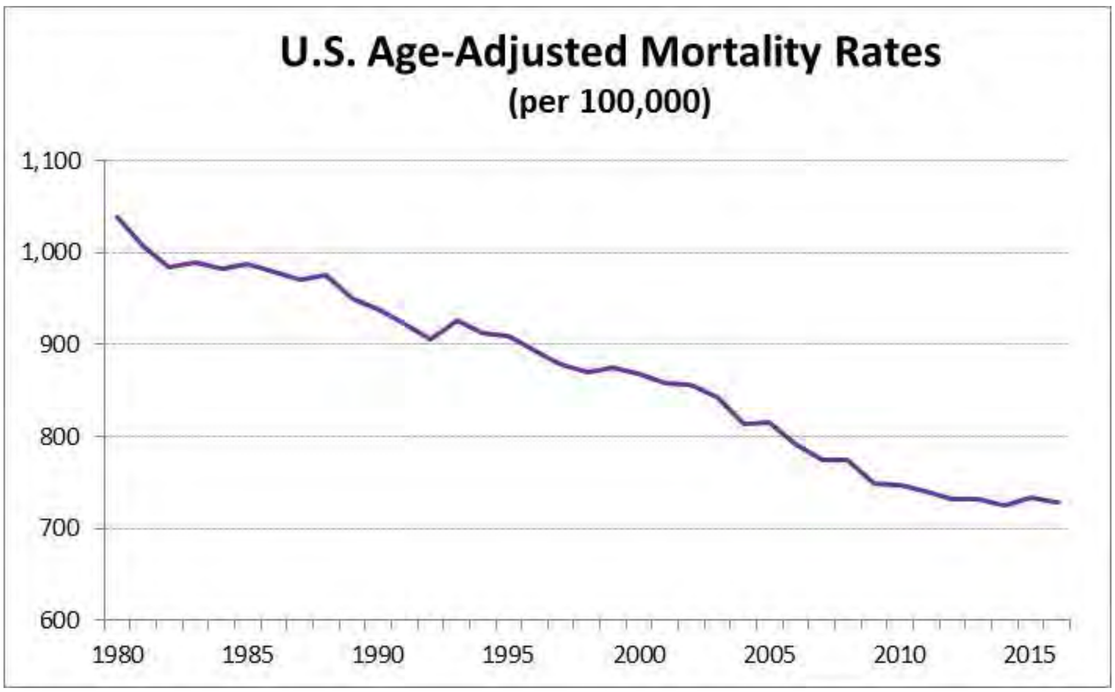
[4] "SSA-style" rates refer to mortality rates developed by RPEC using the same data sources and methodology used by the Social Security Administration to develop the mortality rates that are published along with the annual Trustees' Report.

Continuing to apply a two-year step-back from 2016 (the most recent year of mortality data) along with a 20-year diagonal convergence period results in Scale MP-2018 long-term rates that are fully attained in calendar year 2034.

2.3 Recent U.S. Population Mortality Experience

The age-adjusted mortality rate for 2016 was 728.8 (per 100,000), a *decrease* of 0.6% from the 2015 rate of 733.1 but still slightly higher than the 2014 rate of 724.6. Figure 1 shows the total (males and females combined) age-adjusted mortality rates in the United States for calendar years 1980 through 2016 (NVSS 2016, 2018a).

Figure 1



Mortality rates in calendar year 2016 were lower than those in 2015 for seven of the 10 leading causes of death in the United States: −1.8% for heart disease, −1.7% for cancer, −2.4% for chronic lower respiratory diseases, −0.8% for stroke, −1.4% for diabetes, −11.2% for influenza and pneumonia and −2.2% for kidney disease. Mortality rates were higher for the other three causes: +9.7% for unintentional injuries, +3.1% for Alzheimer's disease and +1.5% for suicide (NCHS 2017).

The age-adjusted mortality improvement rate averaged approximately 0.5% per year over the period 2009 to 2016, compared to an average of approximately 1.6% per year between 2000 and 2009. It is not yet clear whether this recent slower pace of U.S. mortality improvement represents a temporary phenomenon (similar to what occurred in the mid-1980s) or if it suggests a more fundamental shift in longer-term U.S. mortality patterns.

Preliminary analysis by the National Vital Statistics System (NVSS 2018b) indicates that the average age-adjusted death rate in the United States (per 100,000 of population) was 732.1 during 2017, which was 0.5% higher than the corresponding value of 728.8 in 2016. It should be noted that this preliminary information for calendar year 2017 was not reflected in any of the mortality improvement scales presented in this report.

The above mortality improvement statistics illustrate age-adjusted mortality improvement rates for the U.S. population as a whole. The trends of mortality improvement vary significantly by gender and age group.

Section 3: Impact of Scale MP-2018

3.1 Comparison of 2018 Annuity Values

Table 2 presents a comparison of monthly deferred-to-age-62 annuity-due values, all calculated generationally as of 2018 ("Generational @ 2018") with the following:

- RP-2006 rates; Employee rates for ages below 62 and Healthy Annuitant rates for ages 62 and older
- Mortality projection using Scale MP-2017 for the first columns of annuity values and using Scale MP-2018 for the second columns of annuity values
- A discount rate of 4.0%.

Table 2

	Monthly Deferred-To-62 Annuity-Due Values Base Rates: **RP-2006** Generational @ 2018; Discount Rate = 4.0%					
	Females			Males		
Projection Scale →	MP-2017	**MP-2018**	*% Change*	MP-2017	**MP-2018**	*% Change*
Age						
25	3.7415	**3.7264**	*-0.40%*	3.5078	**3.4823**	*-0.73%*
35	5.4392	**5.4182**	*-0.39%*	5.0908	**5.0574**	*-0.66%*
45	7.9196	**7.8917**	*-0.35%*	7.3994	**7.3585**	*-0.55%*
55	11.5967	**11.5585**	*-0.33%*	10.8479	**10.7961**	*-0.48%*
65	14.2829	**14.2505**	*-0.23%*	13.4119	**13.3632**	*-0.36%*
75	10.3945	**10.3682**	*-0.25%*	9.5786	**9.5501**	*-0.30%*
85	6.3208	**6.3076**	*-0.21%*	5.6657	**5.6504**	*-0.27%*

Therefore, 2018 measurements of retirement plan obligations calculated using Scale MP-2018 with RP-2006 base rates (and a 4.0% discount rate) are anticipated to be 0.2% to 0.4% lower for females, and 0.3% to 0.6% lower for males, relative to their Scale MP-2017 counterparts. Measurements of obligations for annuitants are expected to be at the lower end of those ranges.

3.2 Comparison of 2018 Cohort Life Expectancies

Table 3 presents a comparison of 2018 complete cohort life expectancy values[5] at the indicated ages, all calculated assuming:

- Base mortality rates equal to RP-2006 Employee rates for ages below 62 and RP-2006 Healthy Annuitant rates for ages 62 and older and
- Mortality projection starting in 2006 using Scale MP-2017 for the first columns of annuity values and using Scale MP-2018 for the second columns.

Table 3

	2018 Cohort Life Expectancies (Complete)					
	Females			Males		
Projection Scale →	MP-2017	**MP-2018**	*% Change*	MP-2017	**MP-2018**	*% Change*
Age						
25	64.39	**64.23**	*-0.26%*	61.78	**61.49**	*-0.47%*
35	53.63	**53.48**	*-0.28%*	51.10	**50.86**	*-0.46%*
45	42.95	**42.83**	*-0.29%*	40.48	**40.30**	*-0.44%*
55	32.47	**32.35**	*-0.36%*	30.12	**29.97**	*-0.50%*
65	22.68	**22.61**	*-0.33%*	20.71	**20.60**	*-0.51%*
75	14.34	**14.29**	*-0.34%*	12.89	**12.84**	*-0.40%*
85	7.68	**7.66**	*-0.25%*	6.75	**6.73**	*-0.34%*

Based on these assumptions, age-65 cohort life expectancy decreased slightly less than one month for females and slightly more than one month for males.

[5] The life expectancy values presented in this report were calculated as complete cohort life expectancies, which are smaller than the corresponding 0.0% monthly annuity values by a constant 1/24th of a year.

Section 4: Alternative Model Based on Smoother Historical Graduation

4.1 Introductory Comments

For the past few years, the Committee has been conducting an ongoing project to assess the overall effectiveness of alternative approaches for the projection of future U.S. mortality rates relative to the following three characteristics that the Committee believes are important objectives for mortality improvement models in general:

- *Stability:* Periodic updates to the historical data should not have undue influence on liabilities generated by the new scale.
- *Forecast accuracy*: The ultimate goal of a mortality improvement scale is to estimate future trends in mortality. There is tremendous year-over-year variability in mortality improvement rates, so no projection methodology can be expected to track very closely with evolving future experience. However, to the extent achievable, minimizing errors between actual and projected mortality improvement rates over short- and long-term forecasts is desirable.
- *Fit*: Models typically smooth historical rates of mortality (or mortality improvement) in an attempt to distinguish trends from random noise. Ideally the smoothed data should not stray too far from the raw data. Fit measures how close the smoothed rates are to the underlying raw data.

This section assesses the effectiveness and impact of a variation of the current RPEC model that is based on a smoother graduation of historical U.S. population mortality improvement data.

4.2 Model Based on Order-2 Historical Graduation

All of the RPEC models to date (including Scale MP-2018) have been based on historical U.S. population mortality rates that have been graduated with a two-dimensional "order-3" Whittaker-Henderson model. In this context, order-3 refers to the degree of the finite difference operators used in the smoothness components of the two-dimensional Whittaker-Henderson objective function; see Appendix B for additional details.

Except for the fact that the finite differences used in the smoothness components of the Whittaker-Henderson objective function are of order 2 rather than order 3,[6] the alternative model being introduced, denoted RPEC_O2_v2018,[7] is the same as that which underpins Scale MP-2018. This change in finite difference operators produces a generally smoother two-dimensional surface of mortality improvement rates. Although the two sets of heat maps[8] bear a

[6] The RPEC_2014_v2018 smoothness parameters of 100 in the calendar year direction and 400 in the age direction were determined to work well wihin the RPEC_O2_v2018 model and were therefore left unchanged.
[7] Changing the graduation methodology affects both the historical rates as well as the projected improvement factors and thus cannot be accomplished by modifying parameters within the existing RPEC_2014_v2018 model.
[8] See Appendix A for comparisons of the order-3 (i.e., Scale MP-2018) and order-2 heat maps. The historical graduation component of the heat maps are the smoothed rates for calendar years prior to 2015.

large-scale resemblance, the order-2 graduation produces a considerably smaller range of variation between the lows and highs and more gradual transitions in both the horizontal and vertical directions. As an example of one of the more extreme differences, between 2004 and 2014 the smoothed rates for age-70 males decreased from 2.41% to 0.95% (a drop of 1.46%) under the order-2 graduation, compared to a decrease from 2.83% to 0.51% (a drop of 2.32%) under the order-3 graduation.

4.3 Effect on 2018 Annuity Values

Table 4 presents a comparison of 2018 deferred-to-62 annuity values developed using the Scale MP-2018 rates to those developed by applying the same committee-selected assumption set described in Section 2.2 to the RPEC_O2_v2018 model. For purposes of this Section 4 and the heat maps in Appendix A, the order-2 mortality improvement rates so calculated have been denoted O2-2018.

Table 4

	Monthly Deferred-To-62 Annuity-Due Values Base Rates: **RP-2006** Generational @ 2018; Discount Rate = 4.0%					
	Females			Males		
Projection Scale →	MP-2018	**O2-2018**	*% Change*	MP-2018	**O2-2018**	*% Change*
Age						
25	3.7264	**3.7472**	*0.56%*	3.4823	**3.5181**	*1.03%*
35	5.4182	**5.4494**	*0.58%*	5.0574	**5.1100**	*1.04%*
45	7.8917	**7.9385**	*0.59%*	7.3585	**7.4346**	*1.03%*
55	11.5585	**11.6306**	*0.62%*	10.7961	**10.9132**	*1.08%*
65	14.2505	**14.3244**	*0.52%*	13.3632	**13.4927**	*0.97%*
75	10.3682	**10.4631**	*0.92%*	9.5501	**9.6896**	*1.46%*
85	6.3076	**6.3262**	*0.29%*	5.6504	**5.7382**	*1.55%*

In this comparison as of 2018, the O2-2018 annuity values are consistently larger than their Scale MP-2018 counterparts. That would not have necessarily been the case for comparable deferred annuity values determined as of prior calendar years, as discussed in Section 4.4 and illustrated in Appendix C.

4.4 Effectiveness Relative to General Model Objectives

RPEC investigated the anticipated effectiveness of a variety of order-2 models relative to order-3 models using various committee-developed backtesting spreadsheets. Overall, that research has shown that relative to the order-3 models, the order-2 models tend to improve stability metrics and worsen fit metrics. Depending on the specific model parameters and future timeframe selected, the order-2 impact on the forecast accuracy metrics tended to be neutral. Additional comments regarding each of these model objectives are discussed below.

Stability

RPEC's backtesting runs have indicated that year-over-year mortality improvement rates—and resulting annuity values—tend to be considerably less volatile under order-2 models compared to order-3 models. Of course, this improvement in year-over-year stability must be balanced against the reduced sensitivity of the order-2 model to future changes in U.S. mortality patterns.

While it does not provide conclusive evidence for the enhanced stability of all order-2 models, it is interesting to compare some order-3 and order-2 annuity values over a number of consecutive years. For example, RPEC calculated order-2 deferred-to-62 annuity values for years 2014 through 2018 reflecting the same underlying methodology, historical data and committee-selected assumption set as used in each of those year's corresponding MP scale. For each age that the Committee tested, the order-2 annuity values started off lower than the corresponding (order-3) MP annuity values in 2014 and ended up higher than those MP annuity values by 2018. Overall, these order-2 annuity values exhibited considerably less year-over-year volatility than their order-3 counterparts. See Appendix C for additional details.

Forecast Accuracy

The order-2 forecast accuracy metrics were very similar to those produced by the order-3 model. Certain combinations of gender and projection timeframes produced slightly improved results, and other combinations produced slightly worse results. In all cases, the differences in the forecast accuracy metrics were modest.

Fit

Not surprisingly, the order-2 graduated fit to the individual ungraduated historical mortality improvement rates was not as tight as it is using order-3. As an example of this phenomenon, Figures 2 and 3 compare five-year compound average[9] mortality improvement rates at age 65 for years 1994 through 2014 under three bases:

- SSA: The age-65 mortality rates published by the Social Security Administration in connection with annual Trustees' Reports
- The (order-3) Scale MP-2018 age-65 mortality mortality improvement rates and

[9] The averages being shown are based on a geometric average of improvement rates over the period of five years ending with each applicable year.

- The order-2 age-65 mortality mortality improvement rates.

Figure 2

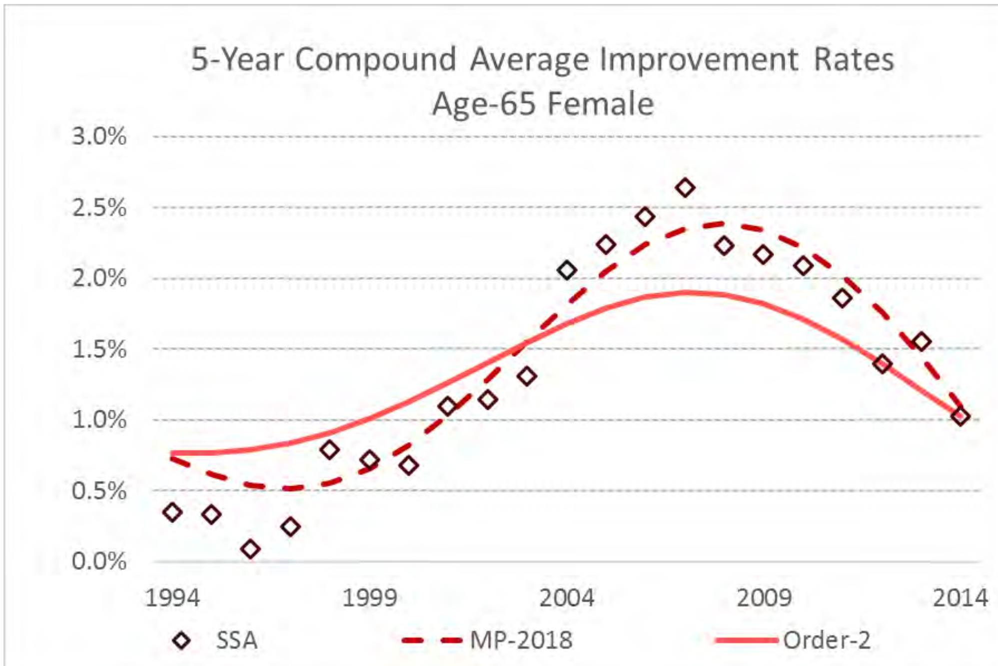
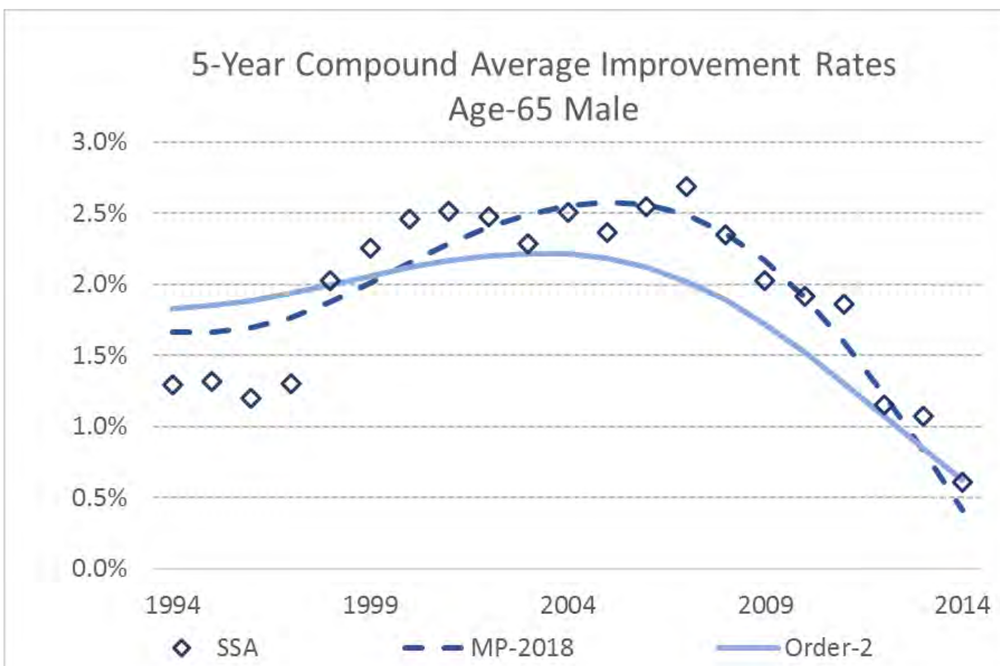


Figure 3



These figures show how much closer the age-65 Scale MP-2018 mortality improvement rates track with the SSA rates compared to the order-2 rates. This reduction in fit might or might not be an issue, depending on the user's opinions with respect to (1) how much "noise" is inherent in historical U.S. population mortality datasets and (2) the importance of tight historical fit within the context of the application at hand.

4.5 Modification of Order-2 Model Parameters

It is the SOA's current intention to update both of the underlying order-3 (RPEC_2014) and order-2 (RPEC_O2) models annually. In the past, the SOA has made various spreadsheet tools available to users who wish to use the RPEC_2014 models with assumption sets other than those selected by the Committee. Starting with this 2018 release, that same spreadsheet functionality will be available to those who wish to modify the committee-selected assumption set for either of the RPEC_2014 or RPEC_O2 models; see Section 5 for the applicable SOA links.

4.6 Interaction of Historical Graduation and Projection Methodolgies

Mortality improvement scales used in retirement-related applications, such as Scale MP-2018, have typically been applied to both the period from the year of the base table to the valuation date and the period after the valuation date. In bridging the gap between base table date and valuation date, fidelity to the twists and turns of historical experience might motivate a relatively granular graduation approach such as one based on order 3. On the other hand, projections into the future can exhibit more volatility than desired, and there may be reason to consider a more stable method; e.g., launching forecasts off an order-2 graduation.

There is no requirement that the model used to project estimates of future rates be based on the same graduation used for smoothing historical data. In 2018, for example, one could consider using the order-3 graduation for mortality improvement rates through 2014 (the last year available after two-year step-back) and the order-2 model for projected rates for 2015 and beyond. Of course, such an approach would create discontinuities between the 2014 and 2015 rates. If desired, these discontinuities could be smoothed by interpolating the rates slightly before and slightly after the transition years.

Other options for historical mortality improvement rates that could be used in conjunction with projections based on either the order-2 or order-3 model include ones based on the mortality rates published by the Social Security Administration[10] (e.g., SSA 2018) or on the crude U.S. historical population mortality rates published by the SOA (SOA 2018).

[10] The SSA mortality rates published annually in conjunction with the Trustees' Reports are smoothed within calendar years but not between calendar years.

Section 5: Online Tools

The SOA has made available the following Excel workbooks that users may find helpful:

- Scale MP-2018 rates can be downloaded in Excel format at: https://www.soa.org/resources/experience-studies/2018/mortality-improvement-scale-mp-2018-rates.xlsx

- The RPEC_2014_ v2018 tool can be used to reconstruct Scale MP-2018 or construct alternative scales based on the same underlying order-3 graduated historical mortality data; see the workbook for instructions at: https://www.soa.org/resources/experience-studies/2018/rpec-2014-v2018-model-implementation-tool.xlsx

- The RPEC_O2_v2018 tool can be used to construct alternative scales based on the order-2 graduated historical mortality data; see the workbook for instructions at: https://www.soa.org/resources/experience-studies/2018/rpec-o2-v2018-model-implementation-tool.xlsx

Appendix A: Heat Maps

The next two pages compare the MP-2018 and O2-2018 gender-specific heat maps for calendar years 1951 through 2035. Because of the continued use of a two-year step-back in both sets of rates, 2014 is the final year of graduated historical data included explicitly[11] and 2015 is the first year of the projected rates. The vertical dashed white lines on the heat maps distinguish between the historical and projected rates, and the thin vertical gray lines indicate the 2018 rates.

[11] The historical graduation processes (both order-3 and order-2) reflected mortality rates for 2015 and 2016, and hence those years implicitly influenced the final RPEC_2014_v2018 and RPEC_O2_v2018 models.

Figure 4

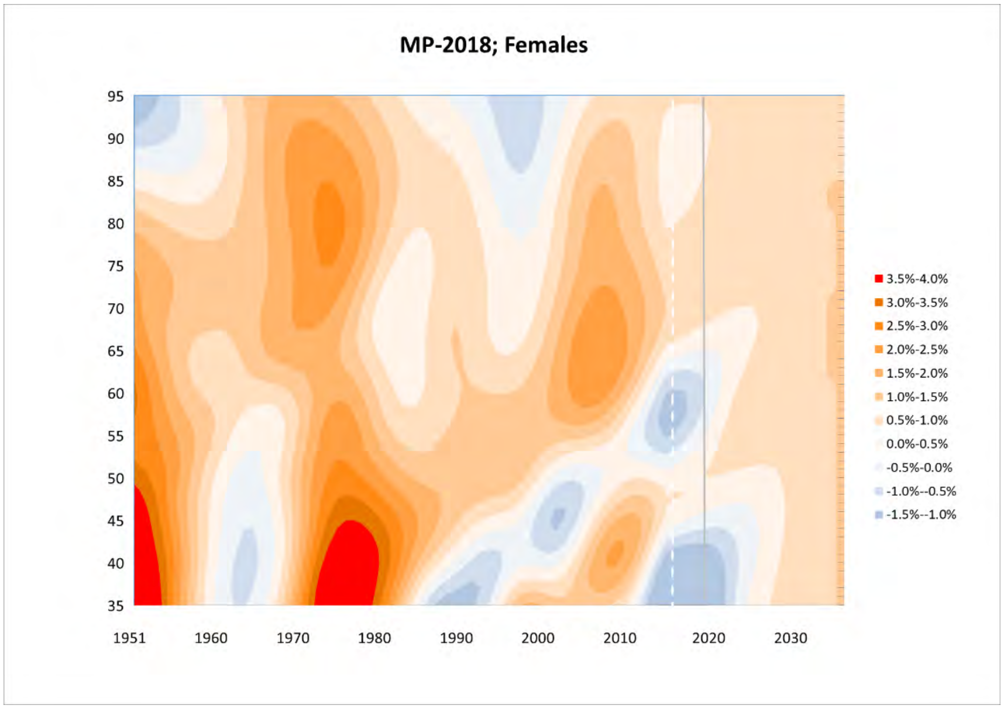


Figure 5

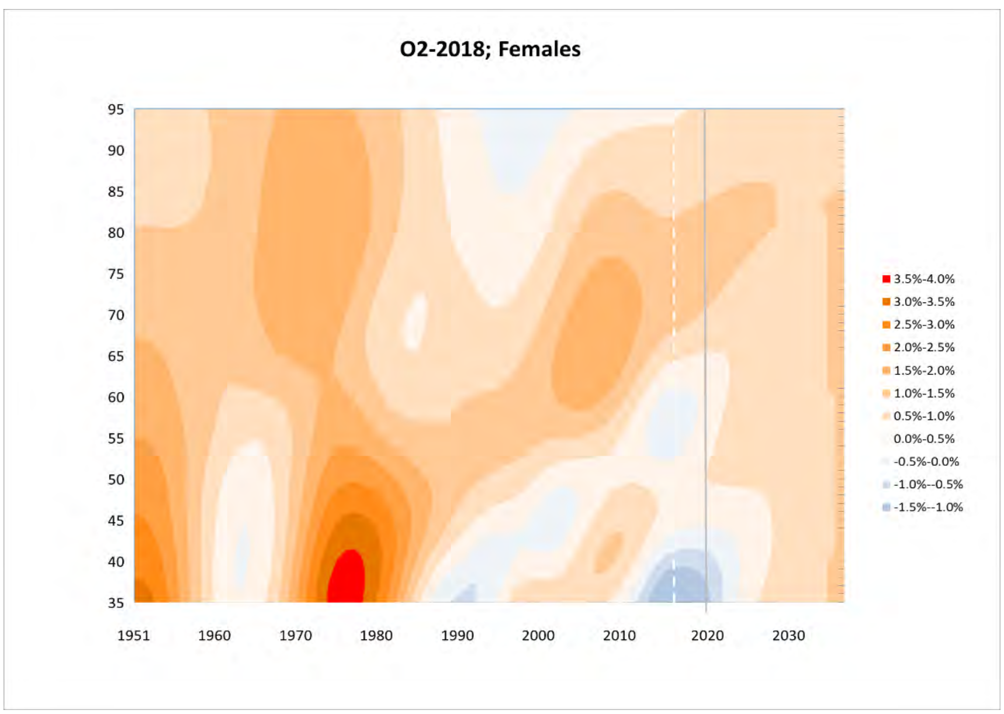


Figure 6

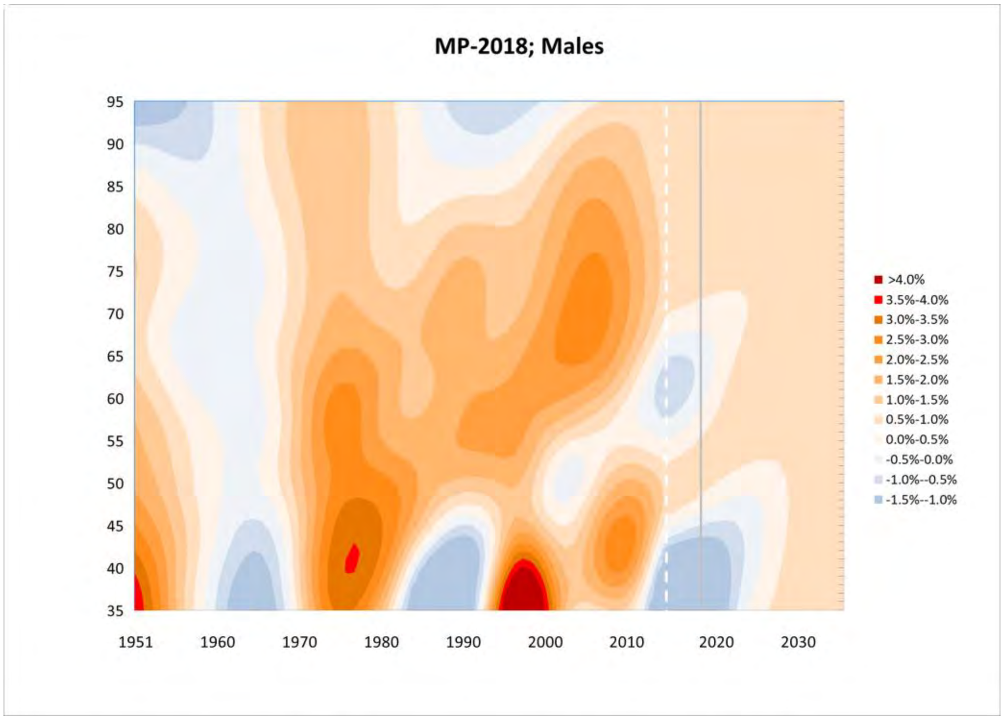
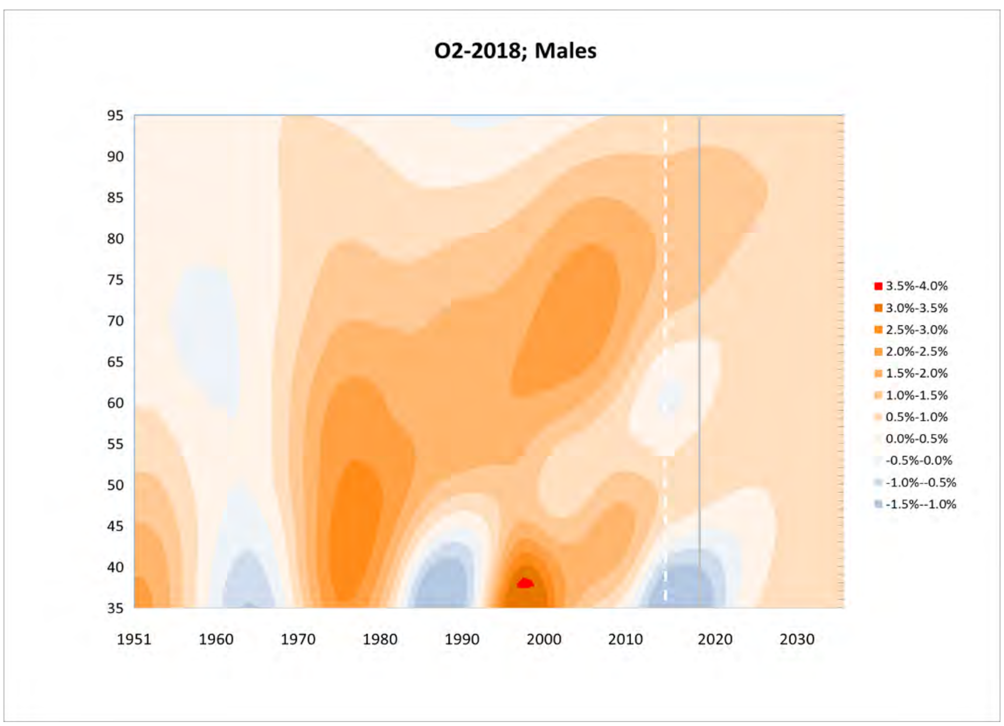


Figure 7



Appendix B: Two-Dimensional Whittaker-Henderson Graduation

Graduation of raw data is essentially a mathematical exercise in identifying a new set of data points that appropriately balances (1) fit to the underlying dataset and (2) some measure of overall smoothness. The process by which this is typically accomplished is through the minimization of a so-call objective function, which is composed of a fit component plus a penalty for bumpiness.

RPEC used a two-dimensional version of the Whittaker-Henderson graduation technique to develop both the RPEC_2014_v2018 and RPEC_O2_v2018 models. The following terms are needed to describe the applicable Whittaker-Henderson objective function:

- $w_{x,y}$: The weight applied to the fit component at age x in year y
- $r(x,y)$: The raw (ungraduated) value at age x in year y
- $s(x,y)$: The graduated value at age x in year y
- h: The smoothness parameter in the horizontal direction
- Δ_h^m: The finite difference operator[12] of order m in the horizontal direction
- v: The smoothness parameter in the vertical direction
- Δ_v^n: The finite difference operator of order n in the vertical direction.

The two-dimensional Whittaker-Henderson objective function can now be defined as

$$\sum_x \sum_y w_{x,y} \left(r(x,y) - s(x,y)\right)^2 + h \sum_x \sum_y (\Delta_h^m s(x,y))^2 + v \sum_x \sum_y (\Delta_v^n s(x,y))^2$$

The fit component is represented by the first double summation, and the bumpiness penalties in the horizontal and vertical directions are represented by the second and third double summations, respectively. For the RPEC_2014_v2018 model, both m and n were set equal to 3, whereas for the RPEC_O2_v2018 model, both values were set equal to 2.

The reader interested in additional information about Whittaker-Henderson graduation techniques is directed to the website maintained by Robert C. W. Howard, from which much of the material included in this appendix has been excerpted (Howard 2018).

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[12] The first order finite difference operator, $\Delta = \Delta^1$ is defined by $\Delta[f(x)] = f(x) - f(x-1)$. The higher order finite difference operators are defined recursively as $\Delta^n = \Delta[\Delta^{n-1}]$.

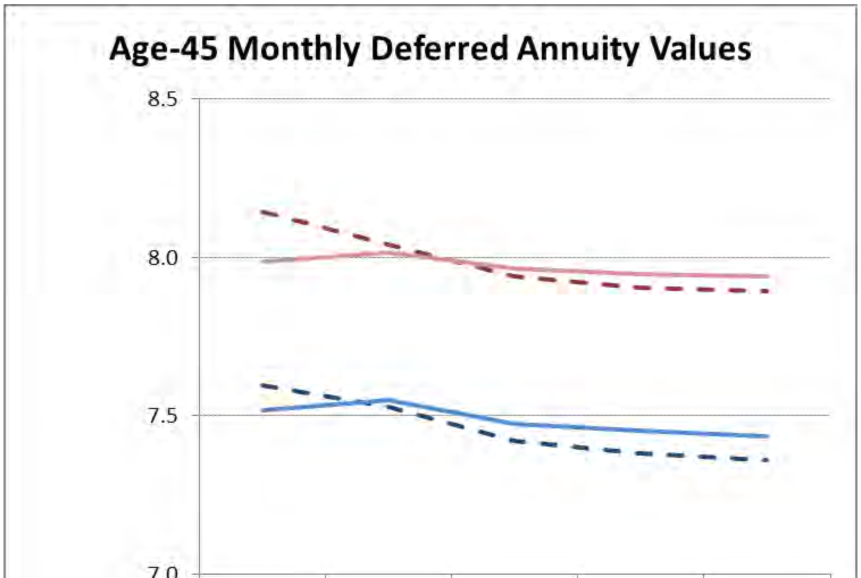
Appendix C: Additional Annuity Comparisons

While it does not provide conclusive evidence for the enhanced stability of all order-2 models, it is interesting to compare some order-3 and order-2 annuity values over a number of consecutive years. To that end, Figures 8–10 present comparisons of deferred-to-62 annuity values calculated for each calendar year going back to 2014. Specifically, the deferred annuity values for ages 45, 65 and 85 were constructed assuming the following:

- A discount rate of 4.0%
- RP-2006 mortality rates; Employee rates prior to age 62 and Healthy Annuitant rates for age 62 and older
- For each calendar year yyyy (yyyy = 2014, 2015,…, 2018), generational projection of base mortality rates using
    - For the annuity values in the "MP" rows of the tables: Scale MP-yyyy
    - For the annuity values in the "O2" rows of the tables: a hypothetical mortality projection scale developed using the same underlying data and the same committee-selected assumption set[13] as those used to construct MP-yyyy, but reflecting an order-2 Whittaker-Henderson graduation in lieu of order-3.[14]

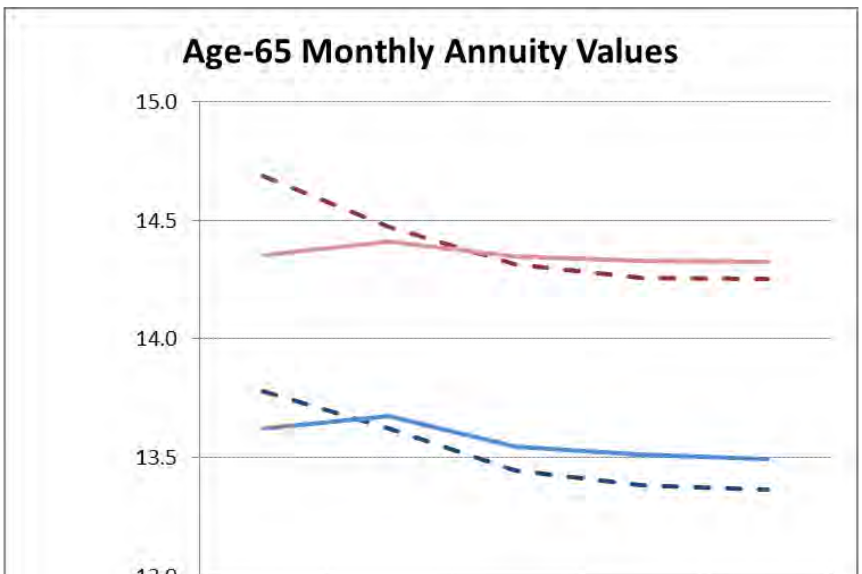
[13] Beginning in 2016, changes were made to the committee-selected assumption set to improve stability; see Section 4.2 of the MP-2016 Report (SOA 2016) for details.
[14] Note that for *yyyy* = 2018, the annuity values so constructed are precisely the O2-2018 values described in Section 4.3.

Figure 8



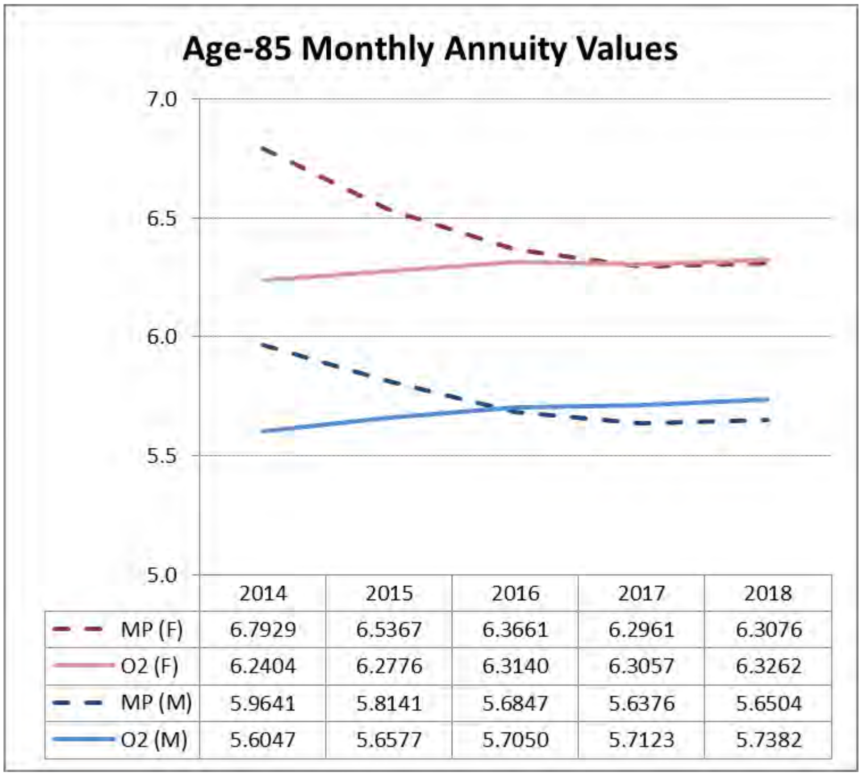
	2014	2015	2016	2017	2018
MP (F)	8.1425	8.0392	7.9398	7.9026	7.8917
O2 (F)	7.9865	8.0135	7.9647	7.9471	7.9385
MP (M)	7.5946	7.5290	7.4202	7.3803	7.3585
O2 (M)	7.5177	7.5484	7.4743	7.4521	7.4346

Figure 9



	2014	2015	2016	2017	2018
MP (F)	14.6860	14.4734	14.3142	14.2554	14.2505
O2 (F)	14.3513	14.4105	14.3502	14.3272	14.3244
MP (M)	13.7797	13.6198	13.4451	13.3842	13.3632
O2 (M)	13.6207	13.6736	13.5436	13.5111	13.4927

Figure 10



	2014	2015	2016	2017	2018
MP (F)	6.7929	6.5367	6.3661	6.2961	6.3076
O2 (F)	6.2404	6.2776	6.3140	6.3057	6.3262
MP (M)	5.9641	5.8141	5.6847	5.6376	5.6504
O2 (M)	5.6047	5.6577	5.7050	5.7123	5.7382

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The Society of Actuaries (SOA), formed in 1949, is one of the largest actuarial professional organizations in the world dedicated to serving 32,000 actuarial members and the public in the United States, Canada and worldwide. In line with the SOA Vision Statement, actuaries act as business leaders who develop and use mathematical models to measure and manage risk in support of financial security for individuals, organizations and the public.

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