Modern Deterministic Scenarios for Interest Rates

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Section 1: Author's Notes and Acknowledgments

Appointed actuaries are required to opine that reserves are adequate under moderately adverse conditions, but have no generally accepted basis for determining what constitutes moderately adverse conditions with respect to interest rates or other economic variables. This report and its appendices aim to advance actuarial practice in this regard and to provide a framework that actuaries can use to evaluate moderately adverse conditions and to provide practical tools to assist actuaries in computing interest rate scenario sets that might reasonably and collectively be considered moderately adverse. In most states, interest rate scenarios for Asset Adequacy Testing are not prescribed by regulation, and it is not the intent of the Society of Actuaries (SOA) or the author to recommend or promote any change to insurance regulation as a result of this research.

The report is based on empirical analysis of historical interest rates and other economic variables. These historical data are, by their very nature, limited and noisy. In certain cases, we have applied practical expedients in our choice of analytic methods or assumptions—approaches that may not be theoretically "pure" but that we believe reasonably and appropriately address the data limitations. These practical expedients are described in the body of the report. In using the report and its appendices, the reader should evaluate the practical expedients in the context of the overall analysis. Our suggestions for future research include recommendations for future research in these areas.

Moderately adverse scenarios for interest rates and other economic variables are largely unstudied in the actuarial literature. The SOA's motto, well known to members, is this quote from Ruskin: "The work of science is to substitute facts for appearances and demonstrations for impressions." This report takes a first step in doing this work with respect to interest rates, but should be considered a first step. I encourage practitioners and researchers to further develop and strengthen the weak points in my analysis, and Section 9 includes suggestions for future research.

I would like to acknowledge several individuals and groups who were instrumental to completing this report. To the sponsoring SOA sections—the Financial Reporting Section and the Smaller Insurance Company Section: thanks for choosing to research this topic and for giving me the opportunity to work on one of the most enjoyable and rewarding projects in my career as an actuary. To the Project Oversight Group—Pam Hutchins, Steve Cheung, Duncan Cook, Kip Headley, Eric Janecek, Don Walker, Zhixin Wu and Corwin Zass—and to SOA research staff, especially Jan Schuh, Teri Slager and Ronora Stryker: thanks for your guidance throughout the project, your thoughtful review of the final report and scenario calculation tools, and for recognizing both when scope needed to expand and when it didn't. To Don Walker and Lori Helge, my co-panelists in presenting preliminary results at a session of the 2016 Valuation Actuary Symposium, along with the attendees: your contributions and the enthusiastic feedback, comments and questions provided important input into the final report. Finally, to my son and research assistant, Dylan Alberts: thanks for your support in tracking down, selecting and analyzing the data, and for what may be our only opportunity to work together in a professional capacity.

Section 2: Executive Summary

This report summarizes the methods and results of the SOA's *Modern Deterministic Scenarios for Interest Rates* research project, produced in response to a request for proposals issued in January 2016. As the name suggests, the objective of *Modern Deterministic Scenarios for Interest Rates* was to develop deterministic scenarios, specifically scenarios that can be considered moderately adverse and, therefore, appropriate for Asset Adequacy Analysis under the U.S. Standard Valuation Law and Valuation Manual. The term "Modern" is used because the scenarios were designed to reflect the current market environment and to be dynamic to future changes in the environment. The deterministic scenario set most commonly used for Asset Adequacy Analysis is the New York 7 scenario set (NY7). These were developed more than 25 years ago in a period when interest rates were much higher than those of the last several years, and therefore may not reflect moderately adverse conditions in the current environment.

The fundamental methodology was to perform an empirical data analysis on historical interest rate data using statistical measures. No existing interest rate data sets met our needs, so we compiled a historical interest rate series from multiple sources, referred to as the MDS Interest Rate Series and extending back to the 1700s, to serve as the basis of the empirical analysis. The empirical analysis used a conditional tail expectation (CTE) framework to define moderately adverse conditions. The results of the data analysis were used to determine a set of scenario modeling parameters. These parameters, along with initial interest rate values on a scenario start date, are used as inputs to a set of scenario calculation algorithms that generate the MDS Scenario Set. The development of the MDS Interest Rate Series is described in Section 4, the empirical analysis is described in Section 5, and the development of the parameters and algorithms to generate the MDS Scenario Set is described in Section 6.

The MDS Scenario Set includes 16 scenarios; eight scenarios are low-rate scenarios and eight are high-rate scenarios. These scenarios can be grouped by type, or analytic framework, summarized as follows:

1. CTE Reversion Target Scenarios (MDS1–MDS8)—These scenarios establish long-term reversion targets and grade interest rates toward the targets over a defined period. The reversion targets are designed to be moderately adverse targets, rather than mean or best estimate targets.

2. Rate Change CTE Scenarios (MDS9–MDS12)—These scenarios quantify, for each scenario year up to 30 years, a moderately adverse change from the initial rate, and apply that change to the initial rate to generate the projected scenario rate. The moderately adverse rate changes vary with the initial interest rate level.

3. Interest Rate Cycle Scenarios (MDS13–MDS14)—These scenarios project future interest rates cyclically, with the start of the cycle tied to the recent interest rate environment.

4. AIRG-Consistent Scenarios (MDS15–MDS16)—These scenarios are developed from a set of stochastic scenarios generated from the American Academy of Actuaries Interest Rate Generator (AIRG) and are designed to produce results consistent with stochastic results generated from the AIRG scenarios.

We developed, and included as appendices, Excel workbooks allowing the reader to compute the MDS scenarios for a given scenario start date. Appendix J computes scenarios MDS1–MDS14 based on selected user inputs. Appendix K computes the AIRG-based scenarios MDS15–MDS16 based on user input of 1,000 stochastic scenarios generated from the AIRG. To assist the reader in assessing the MDS scenarios, we used these workbooks to generate the MDS Scenario Set as of December 31, 2015, and compared them to the NY7. Use of the workbooks to generate the MDS Scenario Set and discussion of the December 31, 2015, scenarios are presented in Section 7. This MDS Scenario Set might be described as more moderate than the NY7, in the sense that the MDS interest rate changes are generally not as large or as rapid as the NY7 changes. In the December 31, 2015, low-rate MDS Scenarios, interest rates do not drop as low as in the NY7 decreasing rate scenarios and generally do not remain at their floors indefinitely. In the December 31, 2015, high-rate MDS Scenarios, interest rates increase to ultimate levels comparable to the NY7 increasing rate scenarios, but generally increase more slowly. Because the MDS Scenario Set was designed to be dynamic with respect to the initial interest rate environment, however, this relationship between the MDS scenarios and the NY7 Scenarios cannot be generalized to different interest rate environments. As discussed in Section 7.3, final publication of this report occurred after December 31, 2016, data were available, but the report was not updated to include analysis of December 31, 2016, scenarios compared with the NY7 because the yield curve changes between the two dates were relatively small and do not produce substantially different results. However, Appendix J, the scenario calculator for Scenarios MDS1–MDS14, has been updated to include data through December 31, 2016, which the reader can use to evaluate the 2016 scenarios.

Finally, in addition to the development of interest rate scenarios, *Modern Deterministic Scenarios for Interest Rates* includes analysis of historical data on inflation rates, corporate bond spreads and common stock returns. We did not explicitly include these variables in the MDS Scenario Set, but we did identify a number of considerations for the practitioner in modeling these items in a moderately adverse context consistent with the MDS Scenario Set. These factors are discussed in Section 8.

Section 3: Background and Objective

Under U.S. insurance regulation, a life insurance company is required to appoint an actuary ("Appointed Actuary") to issue an annual statement of opinion based on asset adequacy analysis ("Actuarial Opinion") with respect to its reserves that includes the statement:

> The reserves and related items, when considered in light of the assets held by the company with respect to such reserves and related actuarial items including, but not limited to, the investment earnings on the assets, and the considerations anticipated to be received and retained under the policies and contracts, make adequate provision, according to presently accepted actuarial standards of practice, for the anticipated cash flows required by the contractual obligations and related expenses of the company. (National Association of Insurance Commissioners, 2010, p. 9)

To support the Actuarial Opinion, the Appointed Actuary is required to complete an asset adequacy analysis. The most widely used method for performing asset adequacy analysis, applicable when results are sensitive to changes in interest rates, is cash flow testing, whereby asset and liability cash flows are tested over multiple interest rate scenarios. Actuarial Standard of Practice (ASOP) No. 22 establishes standards to be followed in performing asset adequacy analysis and stipulates that "when forming an opinion, the actuary should consider whether the reserves and other liabilities being tested are adequate under moderately adverse conditions" (Actuarial Standard of Practice No. 22: Statements of Opinion Based on Asset Adequacy Analysis by Actuaries for Life or Health Insurers, 2001, p. 8).

The first requirements for Appointed Actuaries of U.S. life insurers to perform asset adequacy analysis using cash flow testing were promulgated in 1986 under New York Regulation 126 (Curiale, 1994). Seven required deterministic interest rate scenarios were specified under Regulation 126 and have become known as the New York 7 scenarios (NY7). At one time, the NY7 were also required under the Standard Valuation Law (SVL) promulgated by the National Association of Insurance Commissioners (NAIC). Under 2001 amendments to the SVL, reference to a specific scenario set was eliminated, and the Appointed Actuary now has responsibility to determine the scenario set to be tested as part of his or her asset adequacy criteria, subject to the requirements of ASOP No. 22. The 2001 amendments to the SVL have been widely adopted by U.S. regulators, and currently only New York and a few other states continue to require testing of the NY7.

As computing power and modeling techniques have evolved, some actuaries have begun to use stochastic analysis in their cash flow testing, and some believe that stochastic analysis provides a more robust basis for determining moderately adverse conditions. However, there is currently a wide range of practice, with other actuaries believing that deterministic scenarios provide a more appropriate basis for evaluating asset adequacy. Many actuaries continue to use deterministic scenarios for modeling interest rates in performing asset adequacy analysis, either exclusively or in conjunction with stochastic scenario sets. As a practical matter, the NY7 remain the dominant scenario set for asset adequacy testing. The 2012 Asset Adequacy Testing Survey sponsored by the Financial Reporting and Smaller Insurance Company Sections of the SOA found that in performing cash flow testing, 87% of respondents run the NY7, 14% run a modified NY7 set, 73% run other deterministic sets and only 36% run any sort of stochastic scenarios. Moreover, while actuaries might consider a variety of scenarios in their analysis, 66% of respondents indicated that they not only run the NY7, but rely on the NY7 results to determine asset adequacy (Asset Adequacy Analysis Committee of the American Academy of Actuaries, 2014, pp. 28–29).

While the NY7 continue to be widely used for cash flow testing, those scenarios were developed to meet the needs of actuaries in an environment very different than the current low-rate environment that has existed from 2008 through the date of this report. Therefore, appointed actuaries may not consider NY7 to be moderately adverse. In the author's experience as a practicing actuary, some actuaries have considered the decreasing rate scenarios, and even the level scenario, to be more than "moderately" adverse at some recent valuation dates. To be sure, the NY7 Scenarios were developed under very different conditions than the recent environment, and other deterministic scenarios used by many actuaries may have been as well. For example, the 10-year Treasury rate in 1986 averaged more than 7.5%, after having been in double digits for almost the entire period 1980–1985. By contrast, the 10-year Treasury rate averaged only slightly above 2.0% in 2015 and has not exceeded 3% for a sustained period since the middle of 2010. In addition, since 1986, U.S. actuarial practice has progressed in formally defining "moderately adverse conditions." A CTE framework, and more specifically a CTE70 measure, has come to be accepted in the United States as a measure of moderately adverse conditions. The CTE70 measure has been formally adopted by the NAIC in reserving for variable annuities under Actuarial Guideline 43 (now incorporated in the Valuation Manual as VM-21: Requirements for Principle-Based Reserves for Variable Annuity Products) and, beginning in 2017, in reserving for life insurance policies under VM-20: Requirements for Principle-Based Reserves for Life Products. In these contexts, the CTE framework has been applied in evaluating results of stochastically generated scenarios, but is also a useful framework for evaluating deterministic scenarios.

To summarize this background: (1) the interest rate scenario is a critical assumption to cash flow testing results, (2) deterministic scenarios are used by most actuaries for evaluating asset adequacy and (3) the most widely used deterministic scenario set may not represent current moderately adverse conditions (with points 2 and 3 based on the 2012 Asset Adequacy Testing Survey). In spite of these facts, the current actuarial literature includes little formal exploration of alternative scenarios or even what constitutes a moderately adverse deterministic scenario set. In this light, the Financial Reporting Section, Smaller Insurance Company Section, Modeling Section and Committee on Life Insurance Research of the SOA initiated *Modern Deterministic Scenarios for Interest Rates* to develop a deterministic interest rate scenario set reflective of the current interest rate environment and current actuarial practice with respect to defining moderately adverse conditions, and to compare and contrast this set to the NY7. The resulting scenario sets are intended to help actuaries enhance current cash flow testing models to provide insurers, regulators and rating agencies with insightful information about risks and asset adequacy and are intended to be applicable across a wide range of interest rate environments. A secondary objective is to examine considerations for moderately adverse deterministic modeling of other economic variables such as investment spreads, inflation rates and equity returns.

3.1 Description of Methodology

Modern Deterministic Scenarios for Interest Rates develops interest rate scenarios for U.S. Treasury rates, utilizing the actual rates on a specified start date as inputs. It also develops key considerations in establishing a framework for deterministic modeling of equity returns, inflation rates and investment spreads. *Modern Deterministic Scenarios for Interest Rates* utilizes an empirical analysis of historical data on interest rates, inflation rates, common stock returns and fixed income investment spreads as the basis of its conclusions. The primary outputs of this analysis are (1) a set of interest rate scenarios, provided in the form of two Excel calculation workbooks, Appendices J and K, that may be considered to represent moderately adverse conditions and (2) a set of considerations or recommendations, provided in Section 8 of the body of the report, for the treatment of inflation rates, common stock returns and fixed income investment spreads in a moderately adverse modeling context.

The project plan for *Modern Deterministic Scenarios for Interest Rates* consisted of five distinct phases, which are covered in the report sections as follows:

> **Section 4: Construction of Historical Data Series—**Potential sources of historical data were identified and evaluated. The most useful sources of data were selected as the basis for analysis. The historical time period to be analyzed was chosen. For interest rates in particular, no single source of data proved to be the best source of data for the entire analysis period, so interest rate series were constructed from multiple sources. These constructed series are referenced throughout this report as the MDS Interest Rate Series, consisting of a long-term interest rate series—the MDS Long Interest Rate Series—and a short-term interest rate series—the MDS Short Interest Rate Series.
>
> **Section 5: Empirical Analysis of Historical Interest Rates—**The MDS Interest Rate Series were analyzed using a CTE framework. This analysis developed CTE metrics consistent with moderately adverse conditions using historical interest rate levels and changes in interest rates. Moderately adverse was defined using a CTE70 standard.
>
> **Section 6: Interest Rate Scenario Construction—**Using the results of the Empirical Analysis, the MDS Scenario Set was developed. Algorithms and parameters were constructed to generate those scenarios using a defined set of initial conditions. The Scenario Construction process also involved smoothing of the historical data and testing of internal consistency.
>
> **Section 7: Use of the MDS Interest Rate Scenarios—**Using the scenario parameters and calculation methods developed in Section 6, Excel workbooks were developed to calculate the MDS Scenario Set. These workbooks were used to calculate December 31, 2015, scenarios, which were analyzed and compared to the NY7 Scenarios calculated on the same date.
>
> **Section 8: Non-interest Rate Modeling Considerations—**Empirical analysis was conducted for inflation, fixed income spreads and common stock returns using similar methods to those used for interest rates. CTE analysis was also used, but in a more limited and specific way. Using the results of the empirical analysis, a set of considerations and/or suggested modeling approaches were suggested. These considerations do not result in specific scenarios for inflation rates, spreads or common stock returns, but rather context for treating these items consistent with the interest rate scenarios.

Section 4: Construction of Historical Data Series

4.1 Identification of Data Sources

The first step in constructing the historical data series was evaluating and selecting the specific source data. Particularly for interest rates and common stock returns, there are many historical sources of data. The number of potential sources for corporate bond data and inflation data may be more limited, but is still extensive. A number of different sources were considered for each of the data elements to be studied. For the reader's benefit, a complete listing of the sources evaluated is included as Appendix B. Ultimately, a limited number of these sources were actually used in our analysis; the data series used in our empirical analysis are listed in Appendix C. Appendices B and C are also available in Excel format to allow for additional analysis by the reader.

Criteria in selecting the source data included the following:

1. Public availability of the source—Only publicly available, not proprietary, data were considered.
2. Clear documentation of the original source—many sets of financial and economic data are reused or repackaged by an author from an original source. In such cases, the original source was required to be clearly identifiable.
3. Internal consistency of the series
4. Broad applicability of the underlying data
5. Appropriateness of the series to the objectives of *Modern Deterministic Scenarios for Interest Rates.*

Many private vendors offer financial data and indices on a subscription or other nonpublic basis. It was critical that sources be clear and transparent to the user, so such private sources were not considered. In some cases, we chose data for which electronic copies of the series require a nominal fee for purchase, but the source data are otherwise available to the practitioner at no cost.

Often a data series available from one source is reused or repackaged from a different original source. In the interest of transparency and adequate evaluation of the quality of data, we considered only data where the original source was clear and verifiable and could be clearly communicated to the user. A second consequence of the repackaging of data is some data series might be internally inconsistent in some way or inconsistent with the objectives of *Modern Deterministic Scenarios for Interest Rates.* Data sources were evaluated to identify any such inconsistencies. One key element of consistency was treatment of averaging. Most series were presented on or could be converted to an annual basis. However, some data were presented as yearly averages, while others were presented as values for a given day. As discussed later, yearly average data were determined to be the preferred basis for our analysis, so data series needed to be available on such a basis.

Many data series are highly specific, particularly as one looks at a longer historical period. For instance, *The Statistical History of the United States* includes a table, "Table Cj1223–1237: Money Market Rates: 1831–1997," that contains 15 separate data series that might be construed to represent money market rates over a portion of this period. The series include rates for commercial paper, financial paper, stock exchange time loans, stock exchange call loans, prime bankers' acceptances, three-month Treasury bills (new issue rates and market rates), Federal Reserve Bank of New York discount rate, Federal Funds Rate and selected bond and note issues. None of these individual series was available for the whole period 1831–1997, and the authors' analysis indicates that different rates might represent a best representation of market rates at different points in time. In such cases, we needed to evaluate which rate series offered the best representation of broad market rates at which points in time.

Finally some data may be of historical interest, but may not be useful as a basis of projecting interest rates. For instance, Officer (2003, p. 81) notes that when evaluating U.S. long-dated market interest rates, "it is logical to begin with the market yield on federal-government bonds," but "the date when the representativeness of the federal-securities market yield ceases and, more generally, the periods when there is a lack of such representativeness, are well-recognized in the literature." Including references to Homer and Sylla (1991), Mitchell (1911), Friedman and Schwartz (1982) and Macaulay (1938), Officer describes several reasons why U.S. government bond yields have not accurately represented market interest rates in various periods. At various times, "United States government bond yields were distorted by gold premiums" (Homer and Sylla, 1991, p. 290), "backing of national-bank notes by government bonds again enhanced the price, and reduced the yield, of such bonds" (Officer, 2003, p. 81), and "many United States government bonds were partially tax exempt" (Friedman and Schwartz, 1982, p. 120, n. 23), along with a number of other issues. Suffice to say, such issues had to be considered in determining what rates best represented market interest rates for our analysis.

We considered many data series from many different sources, as shown in Appendix B. Ultimately we identified a much smaller number of data series that best met the criteria for inclusion in our analysis, and these data series are detailed in Appendix C. The sources of these data series, along with some other particularly valuable literature sources, are worthy of some discussion and are described here as a direct reference for the reader:

> *www.measuringworth.com*—This website proved to be our single most useful source of data. It includes the reports "What Was the Interest Rate Then? A Data Study" (Officer, 2003) and "What Was the Consumer Price Index Then? A Data Study " (Officer, 2006) by Lawrence Officer, along with the accompanying interest rate and consumer price index data series that he developed. The interest rate report is an excellent distillation of the interest rate history of the United States and the United Kingdom since the 1700s, developing series of market interest rates from various sources. The report is referenced extensively in *Modern Deterministic Scenarios for Interest Rates,* and the interest rate series provided a primary input into the MDS Interest Rate Series. The CPI report and related series of U.S. CPI data since 1774 was our main source of inflation data. While all of the data originated from other sources, Officer's work to distill disparate data into a single market view of interest and inflation rates and the accompanying analysis were invaluable.

> *Historical Statistics of the United States* (HSUS, various editions)—The U.S. Census Bureau formerly published the *Historical Statistics* as a supplement to the *Statistical Abstract of the U.S.* It is now maintained by Cambridge University Press and is a wide-ranging reference including data tables and related essays covering all facets of life in the United States. All data originated from other original sources, and in turn the HSUS was a key data source for Officer. For *Modern Deterministic Scenarios for Interest Rates,* we utilized the HSUS both directly and indirectly, using its essays for valuable context and some of its data tables for our empirical analysis. The sheer magnitude and comprehensive nature of the data made it a critical resource for a project of this type.

> *Federal Reserve Board Selected Interest Rates–H.15*—We consider this the most comprehensive source of contemporary interest rate data in the United States. The Federal Reserve maintains a daily, monthly and annual interest rate history of interest rates and interest rate indices. We utilized Fed data for more recent yield rates on U.S. government instruments and for corporate bond yield indices. We note that since the completion of our work, the Fed has discontinued a number of data series that are available from other sources, but maintains the history utilized in our analysis.

> Sidney Homer and Richard Sylla (2005), *A History of Interest Rates,* 4th ed.—This text is a comprehensive and much-cited treatment of historical interest rates. It provides numerous tables

of historical interest rates and valuable historical analysis and context. While not directly contributing data to the MDS Interest Rate Series, it was a key source of data and analysis both for Officer and the HSUS, and was also valuable in our assessment of various data series.

Robert Shiller, *Irrational Exuberance* Data Tables—Created to support Shiller's book *Irrational Exuberance,* these tables include useful long-term data for interest rates, inflation and common stock returns. Ultimately, we used the Shiller data, which were not original to Shiller, only for common stock prices and dividends.

4.2 Selection of Source Data and Construction of Data Series

The empirical analysis conducted for *Modern Deterministic Scenarios for Interest Rates* required a data series that covered a long period of time. We did not attempt to predefine the period, other than to say that it should be as long a period as relevant data were available. Relevance of more remote historical data is arguable. Indeed, it is commonly argued that current times are fundamentally different from historical periods with respect to drivers of interest rates, limiting the applicability of older historical data for setting future expectations. Such arguments might cite differences in economic growth rates, inflation rates, financial market efficiency, government or central bank policies and risk of default, among other factors. While analysis of interest rates in relation to other factors was outside the scope of *Modern Deterministic Scenarios for Interest Rates,* we did consider several factors that might limit the applicability of more remote historical data.

The Industrial Revolution, beginning around 1760, "marks a major turning point in history; almost every aspect of daily life was influenced in some way. In particular, average income and population began to exhibit unprecedented sustained growth" ("Industrial Revolution," n.d., in Wikipedia). Therefore, data much before the start of the Industrial Revolution might be less applicable with respect to economic growth rates.

Default risk on government bonds has changed over time:

> The trend over the past seven centuries has been for bond yields to decline. This can't be attributed to lower economic growth or lower inflation, but must clearly be attributed to lower risk of default. Between 1285 and the mid-1600s, yields on government bonds fluctuated between 6% and 10% and in some cases were around 20% … Since the mid-1600s, the average yield on government bonds has been around 4%. Before the 1600s, high interest rates were driven by risk; since the 1600s, high interest rates have been driven by inflation. (Taylor, 2013, p. 2)

Recent economic events notwithstanding, U.S. Treasury securities are perceived to be free of default risk and can reasonably be expected to remain so under moderately adverse conditions. Therefore, data prior to the 1600s might be less applicable with respect to default risk.

The United States has been considered the world center of financial power since approximately the end of World War I and was preceded in that role by Great Britain (Taylor, 2013, p. 1): "The country at the center of economic power can issue more bonds at a lower cost." In addition, it should be recalled that, far from being the world center of financial power, the United States was a new nation in the late 1700s and remained a developing nation throughout much of the 1800s. Homer and Sylla (2005, pp. 270–322) provide an excellent summary of the history of U.S. interest rates in the context of the broader development of the nation and its government, financial and monetary institutions. This history includes rapidly expanding settlement of North America, periods of significant upheaval including civil war and ebb and flow in state-centered and federal-centered institutions. For the purposes of this study, it is sufficient to say that U.S. data prior to approximately 1920 may be less relevant to future expectations.

Considering these factors, our working assumption was that interest rate data back to the 1700s would be relevant for our purposes, but that, prior to about 1920, U.K. data were more relevant than U.S. data.

At the outset of *Modern Deterministic Scenarios for Interest Rates,* we hoped that a single existing data set covering the entire time period would be available to use as the basis of analysis. Ideally, this data set would include data on various tenors and time steps, allowing robust analysis of the full yield curve and analysis of daily, monthly and annual changes in interest rates. Unfortunately, no such data set exists. As Appendix B shows, the Federal Reserve maintains a robust repository of modern interest rate data, including Constant Maturity Treasury (CMT) rates for 90-day, 180-day, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year maturity periods, presented in daily, monthly and annual time steps, but these data go back, at most, to 1953. Before the mid-20th century, data are generally available only on an annual basis and do not provide information for specific maturity periods.

In addition, the instruments that best represent market interest rates have changed over time, for various reasons. The 10-year CMT is currently considered a key benchmark long-term interest rate. However, the yield curve exhibits considerable upward slope beyond 10-year maturities, meaning that 20- and 30-year maturities provide a better measure of long-term interest rates. However, even in the last 30 years, 20- and 30-year Treasury bonds have been discontinued and reintroduced, so that neither of them alone can serve as a measure of long-term interest rates even back to 1980. Even the 10-year Treasury suffers similar limitations in availability or applicability over a longer historical period.

These limitations in the data led us to make several key decisions:

1. We would construct data series from various existing data series or data sets to use as the basis for our analysis. These constructed series are referenced herein as the MDS Interest Rate Series.

2. We would construct two series—a short-term interest rate series and a long-term interest rate series. These series are referenced as the MDS Short Rate Series and the MDS Long Rate Series. For scenario projections, the entire yield curve would be constructed from these two rates.

3. The MDS Interest Rate Series would be based on yearly average interest rate data. Some historical data series present point-in-time data, while others present yearly average data. For deterministic scenarios, where projected rates do not vary on a daily time step, we judged that yearly average presents a more useful measure than point-in-time values.

4. Prior to 1920, U.K., not U.S., data would be the primary basis of the MDS Interest Rate Series.

In light of these decisions, we constructed the MDS Long and Short Interest Rate Series as we describe next.

4.3 MDS Long Interest Rate Series

Table 1 summarizes the construction of the MDS Long Interest Rate Series, showing the component interest rates and their sources. Following the table is discussion of the series construction.

TABLE 1 - MDS LONG INTEREST RATE SERIES CONSTRUCTION					
	Time Period				
Segment	Start	End	Interest Rate Description	Source	Web Link
1	1729	1921	Yield on Bank of England Consolidated Annuities (consuls), compiled as Measuring Worth UK Long, contemporary series	What Was the Interest Rate Then? A Data Study	https://www.measuringworth.com/interestrates/
1Trans	1922	1922	Average of 1922 rates for segments 1 and 2		
2	1923	1941	Moody's AAA Corporate Bond Yield, less 0.31% credit spread	Federal Reserve Board Selected Interest Rates - H.15	https://www.federalreserve.gov/releases/h15/data.htm
3	1942	1952	Yield on fully taxable US Government bonds due or callable after 15 years	Historical Statistics of the United States Millenium Edition, Table Cj1192	http://hsus.cambridge.org/HSUSWeb/index.do
4	1953	2015	Simple average of 20 year CMT and 30 year CMT from Federal Reserve Board	Federal Reserve Board Selected Interest Rates - H.15	https://www.federalreserve.gov/releases/h15/data.htm

Over the period 1953–2015, CMT data are available from the Federal Reserve for either or both 20- and 30-year maturities. Table 2 summarizes the availability and relationship of the two rates over this period. Both rates have been available for approximately half of this 63-year period. In the periods when both rates were available, no significant term structure was observed, so we concluded that the simple average of the two rates provided a good measure of the long-term rate.

TABLE 2 - 20 YEAR AND 30 YEAR CONSTANT MATURITY TREASURY RATES - 1953-2015			
Time	Avg Rate During Time Period (%)		
Period	20-year	30-year	30yr - 20yr
Apr 1953-Jan 1977	4.99	NA	NA
Feb 1977-Dec 1986	10.64	10.54	(0.10)
Jan 1987-Sep 1993	NA	8.22	NA
Oct 1993-Feb 2002	6.44	6.28	(0.16)
Mar 2002-Jan 2006	4.99	NA	NA
Feb 2006-Dec 2015	3.72	3.87	0.15

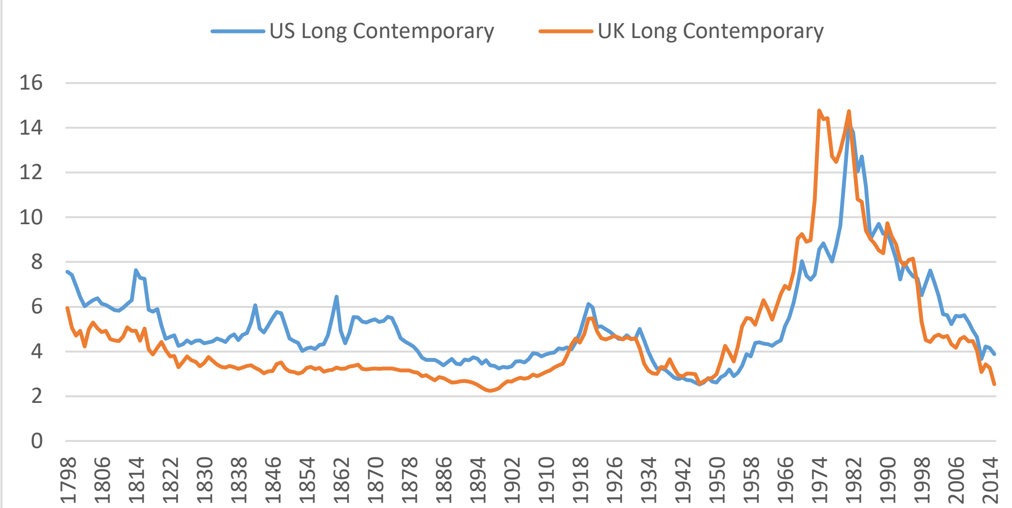
Before 1953, Fed data on Treasury rates were unavailable. Over the period 1942–1952, Table Cj1192 of the HSUS captures the yield on fully taxable U.S. government securities due or callable after 15 years and represents the best available measure of long-term interest rates for this period. The MDS Long Interest Rate Series uses this rate for this period.

Before 1942, U.S. government bonds were partially or fully tax exempt and therefore do not represent an accurate measure of market interest rates. In addition, before 1942, the government bond data captured in HSUS table Cj1192 captured shorter instruments and are therefore not as representative of long-term rates. As a result, we needed a different source for earlier rates. Several authors (Homer and Sylla, 2005; Officer, 2003) have concluded that high-grade corporate bond yields, measured by Moody's AAA seasoned corporate bond yields for the period 1919 forward, represent the best measure of long-term U.S. interest rates. We did not fully concur, because Moody's AAA rates exhibit a positive spread to government bond yields in all periods that comparable U.S. government bond yields are available. Therefore, while we concurred that Moody's AAA rates provided the best basis for long-term rates, we found it necessary to adjust those rates for an implicit credit spread. Table 3 shows the spread between the Moody's AAA rates and long-term U.S. government bond yields from HSUS table Cj1192 rates over the period 1942–1952. The spread fell within a tight range from 0.24% to 0.38%, averaging 0.31%. For the period 1923–1941, therefore, the MDS Long series uses the Moody's AAA rates, less a constant spread of 0.31%.

TABLE 3 - MOODY'S AAA SEASONED CORPORATE BOND SPREADS 1942-1952			
Year	Moody's AAA	LT Govt Bond Yield HSUS Table Cj1192	Moody's AAA Spread
1942	2.83	2.46	0.37
1943	2.73	2.47	0.26
1944	2.72	2.48	0.24
1945	2.62	2.37	0.25
1946	2.53	2.19	0.34
1947	2.61	2.25	0.36
1948	2.82	2.44	0.38
1949	2.66	2.31	0.35
1950	2.62	2.32	0.3
1951	2.86	2.57	0.29
1952	2.96	2.68	0.28
Average Spread			**0.31**

As noted above, the United States supplanted the United Kingdom as the global center of finance after World War I. The effect on interest rates is illustrated in Figure 1. This figure compares U.S. and U.K. long-term interest rates as compiled by Officer for the period 1798–2015, the entire period for which U.S. rates are available. This chart clearly demonstrates that before 1920, U.S. rates consistently exceeded U.K. rates, often by a significant amount. Since then, U.S. rates have generally been lower than U.K. rates, consistent with the U.S. leadership position, but varying with economic conditions in each country.

Fig 1 - US vs. UK Long-term Interest Rates since 1798



Average	1798-1920	1920+	Sources:
US Long	4.78	5.81	Measuringworth.com
UK Long	3.49	6.18	Measuringworth.com

On the basis of this data, U.K. long-term interest rates as compiled by Officer were used as the basis of MDS Long Interest Rate Series before 1922. Officer's U.K. rate series go back to 1729, and we chose to extend the MDS series back that far, consistent with our objective of capturing the maximum amount of relevant data. For 1753–1918, which covers most of this period, Officer's representative rate is the yield on

Bank of England consolidated annuities (consols), and for 1729–1752, he chose the rate on preconsolidated annuities. Many of the difficulties discussed above regarding U.S. government bond yields as a measure of market interest rates are not applicable to U.K. rates. As Officer notes (2003, p. 58), "From the time consols came into existence, in the mid-18th century, until World War I, the yield on consols was the representative British long-term interest rate. Acceptance of the representativeness of the consols yield was universal"; he further noted that "prior to the availability of the consols, the rate on (non-consolidated) annuities legitimately serves as the reference rate." However, Officer (p. 59) goes on to note that, "after World War I, consols lost their exclusive representativeness of the long-term, interest rate," due to broader U.K. government debt offerings, and therefore he selected broader compilations of yields on U.K. government (gilt-edged) securities as the basis of his U.K. long-term rate series for 1919 and later. For purposes of *Modern Deterministic Scenarios for Interest Rates,* we concurred that the rates selected by Officer represented the best U.K. reference rates for our analysis.

At the transition between segment 1 and segment 2 of the MDS Long Interest Rate Series, the difference between the two component rates was significant enough that we chose to smooth the transition by averaging the two rates in the transition year. No such smoothing was needed between the later segments.

We should note, for the benefit of the reader who might be interested in studying our sources, that for both long- and short-term interest rates, Officer constructed a "Contemporary" series and a "Consistent" series. His Contemporary series represents historical rates without adjustment. His Consistent series attempts to adjust for differences in each pair of adjacent component series at the transition points. For purposes of *Modern Deterministic Scenarios for Interest Rates,* the true historical rates in the Contemporary series provided the more appropriate basis, primarily for consistency with the nature of our analysis, but also because we disagreed with some assumptions in the development of his Consistent series. With respect to the first point, we considered it crucial that our analysis use true historical data, and if the market itself exhibits discontinuities over time, these discontinuities are reality and should not be smoothed out in considering future interest rates. Our quibbles with Officer's Consistent series assumptions only reinforced this fundamental concern and are not discussed in detail here.

4.4 MDS Short Interest Rate Series

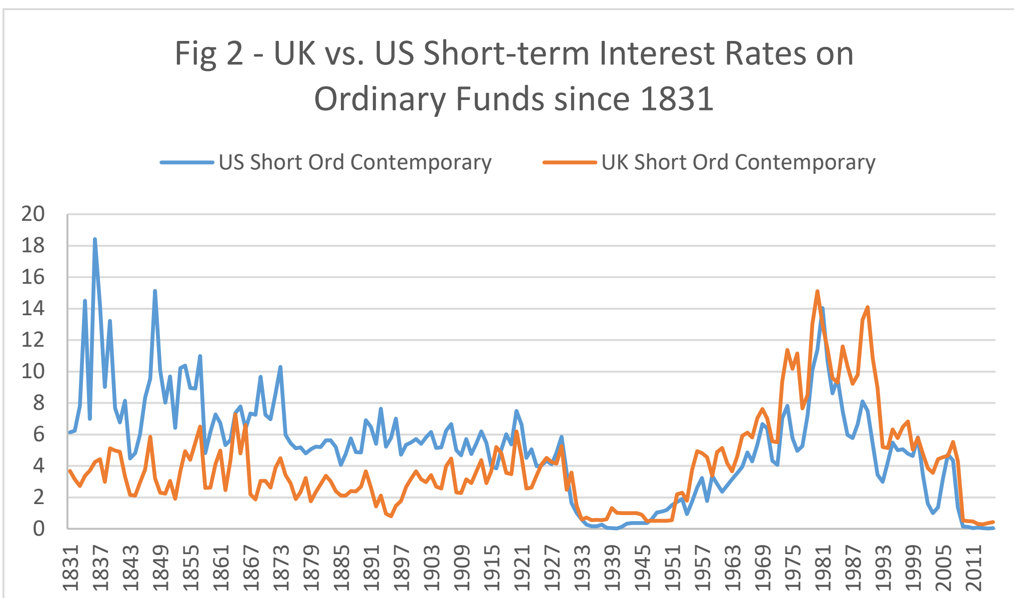
Table 4 summarizes the construction of the MDS Short Interest Rate Series, showing the component interest rates and their sources. Following the table is discussion of the series construction.

TABLE 4 - MDS SHORT INTEREST RATE SERIES CONSTRUCTION					
	Time Period				
Segment	Start	End	Interest Rate Description	Source	Web Link
1	1825	1912	London market discount rate on bills of exchange	What Was the Interest Rate Then? A Data Study	https://www.measuringworth.com/interestrates/
1T	1913	1917	London market discount rate on bills of exchange, increased by 10bps in 1913, 20bps in 1914... 50bps in 1917	What Was the Interest Rate Then? A Data Study	https://www.measuringworth.com/interestrates/
2	1918	1930	Prime bankers' acceptances, 90 days	Historical Statistics of the United States, series Cj1230 - Prime Bankers Acceptances,	http://hsus.cambridge.org/HSUSWeb/index.do
3	1931	1933	3-Month Treasury Bill: New Issues Rate	What Was the Interest Rate Then? A Data Study	https://www.measuringworth.com/interestrates/
4	1934	2015	3-Month Treasury Bill: Secondary Market Rate	What Was the Interest Rate Then? A Data Study	https://www.measuringworth.com/interestrates/

Short-term U.S. Treasury rates are available farther back than long-term rates, with market rates on 90-day Treasury Bills available back to 1934 and rates on new issues back to 1931. The 90-day Treasury rates were available from several sources, including the Federal Reserve, HSUS table Cj1231–1232 as well as Officer's series U.S. Short-Term Interest Rates on Ordinary Funds, Contemporary Series (see discussion of the "Contemporary Series" in Section 4.3. In addition, for short-term interest rates, Officer distinguished between Ordinary Funds and Surplus Funds, with the former more representative of rates on secured borrowing, which is more representative of a risk-free rate. We chose Officer's data because the various series have slight differences, and his series included an investigation to determine the most reliable sources.

For the period 1918–1930, we departed from the series selected by Officer. He considered rates on prime four-to-six-month commercial paper as well as rates on 90-day prime bankers' acceptances, and selected the commercial paper rate for his series, primarily because they had modestly higher transaction volume. The commercial paper rates are available in HSUS table Cj1224, and the bankers' acceptance rates are available as HSUS table Cj1230, along with a number of other short-term interest rates. We observed that the bankers' acceptance rates aligned more closely with 90-day Treasury rates over the period 1931–1934 and were consistently lower than commercial paper rates over the period 1918–1930, implying lower risk. Therefore, we selected the bankers' acceptance rates for the MDS Short Interest Rate Series over this period.

Using the same considerations as for the MDS Long Interest Rate Series, we judged that for periods before World War I, U.K. interest rates were more useful than U.S. interest rates. Figure 2 demonstrates the relationship between U.S. and U.K. interest rates for the period 1831–2015 as compiled by Officer in his U.K. Short-Term Interest Rates on Ordinary Funds: Contemporary Series and U.S. Short-Term Interest Rates on Ordinary Funds: Contemporary Series.



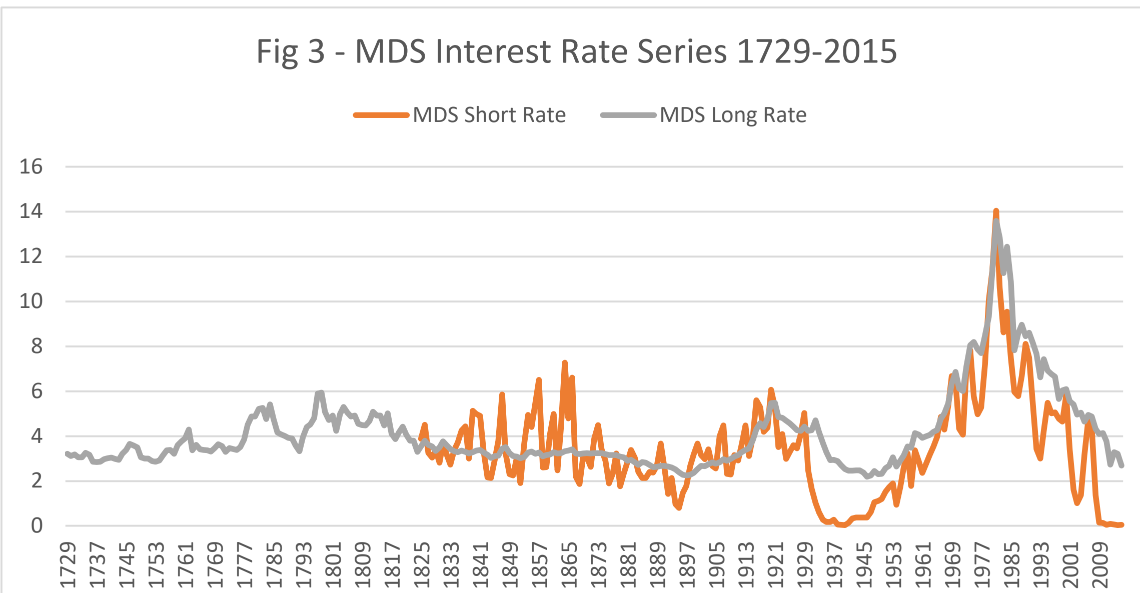
Average	1831-1920	1921-2015	Sources:
US Short	6.94	3.61	Measuringworth.com
UK Short	3.33	4.85	Measuringworth.com

Again, we relied on the work of Officer to identify the most representative U.K. interest rates, and again the task is more straightforward than for U.S. interest rates. His series U.K. Short Term Interest Rates on Ordinary Funds: Contemporary Series is represented by the London market discount rate on bills of exchange for the period 1790–1918. He compiled the rates from various sources, and his compilation is the basis of the MDS Short Interest Rate Series prior to 1918. Unlike the Long Term Rate Series, which was available back to 1729, Officer was able to compile short-term rates only back to 1790. In reviewing the data, we observed very little variability in the rates before 1825, with rates remaining constant for years at a time. This sort of stability is not a characteristic of current short-term interest rates, so we chose to exclude the period 1790–1824 and begin the MDS Short Interest Rate Series in 1825.

The transition from the U.K. Bill of Exchange Rates to the U.S. Bankers' Acceptance rate in 1918 required a transition period. In 1918, the date we transitioned to the U.S. Bankers' Acceptance rate, the latter rate was 62 basis point higher. To avoid this discontinuity, we increased the U.K. Bill of Exchange Rate by 10 basis points in 1913 and added another 10 basis points each year through 1917. Thus, we graded in the 62 basis point change over six years. No such transition adjustments were required for the remainder of the series.

4.5 Final MDS Interest Rate Series

Figure 3 shows the final MDS Long and Short Interest Rate Series. Appendix D provides details of the rates and their components, and is also available in Excel format to allow for additional analysis by the reader. Appendix D.1 details the construction of the MDS Long Interest Rate Series, and Appendix D.2 details the construction of the MDS Short Interest Rate Series.



Following are some observations based on visual review of the series, which inform the empirical analysis to follow:

- Long Rates have exhibited an effective floor between 2% and 3% and have approached this level several times, and for extended periods of time.
- Short Rates have approached zero in only two periods—the Great Depression of the 1930s and the Great Recession beginning in 2008.
- Prior to the 1970s, there was no precedent for Long Rates above 6% or Short Rates above 8%. The double-digit interest rates of the early 1980s appear extreme in historical terms.
- A cyclical pattern of interest rates can be observed to some degree, but there is little consistency in the cycles to make them useful for prediction.
- Short Rates exhibit much greater volatility than Long Rates in all observed periods.

- Prior to 1920, no consistent term structure is observed. There were frequent and sustained inversions, and little correlation was observed between Short and Long Rates. Beginning in the 1920s, we see a consistent upward slope of the yield curve, with Long Rates consistently higher than Short Rates. In this same period, we also observe greater correlation between Long and Short Rates, although Short Rates remain much more volatile. Although detailed analysis of this shift was outside of our scope, central bank policies provide one possible explanation. As noted in a Federal Reserve educational document (History of the Federal Reserve, https://www.federalreserveeducation.org/about-the-fed/history), "during the 1920s, the Fed began using open market operations as a monetary policy tool." It appears likely that central bank open market operations have played a role in this notable change in the shape of the curve. Interestingly, while there have been significant changes in Fed policies and priorities over this period, the positive yield curve shape has remained consistent. Based on our analysis, we believe the change in shape has primarily impacted Short Rates—pushing down Short Rates relative to Long Rates, but not significantly impacting the level of Long Rates.

Section 5: Empirical Analysis of Historical Interest Rates

5.1 Analytic Frameworks

We considered a number of different approaches to analyzing the historical data and settled on three as being the most useful bases for our scenarios. These analytic frameworks are described briefly below and developed more fully in Section 5.5. We developed scenarios based on each of these analytic frameworks, and the further step of converting the analysis into scenarios is covered in Section 6:

> *CTE Reversion Target Analysis Framework*—This framework develops scenarios that revert to a moderately adverse long-term target independent of the initial rate.
>
> *Rate Change CTE Analysis Framework*—This framework develops scenarios based on moderately adverse changes in interest rates, where the moderately adverse changes are tied to the initial rate level.
>
> *Interest Rate Cycle Analysis*—This framework develops scenarios explicitly considering the cyclical nature of interest rates. While the CTE Reversion Target and Rate Change CTE frameworks both grade toward an ultimate level of interest rates, the Interest Rate Cycle framework utilizes cyclical changes over the entire scenario horizon.

5.2 Sample CTE Analysis Methodology

The general methodology used for the empirical analysis can be termed Sample CTE analysis or Empirical CTE analysis. As the name Conditional Tail Expectation connotes, CTE analysis is generally performed using an expected distribution of results. In stochastic simulation testing, although the population distribution of results is not known, a large number of simulations are performed to generate a sample distribution that reliably represents the population distribution. For *Modern Deterministic Scenarios for Interest Rates,* CTE analysis was applied to empirical data and therefore generates CTE metrics for a sample distribution rather than a population distribution.

Our analysis applied the CTE methodology to the observed distribution of historical interest rates. This approach carries some inherent limitations. Most notably, our sample is limited in size. The MDS Long and Short Interest Rate Series contains 287 and 191 yearly observations, respectively. While these are long periods relative to the lifetime of a person or the projection horizon of a typical actuarial model, they represent a small sample set for performing CTE analysis, particularly because the tail to be measured represents only a subset of the total observations. This is particularly true for the Rate Change CTE analysis, which as discussed in Section 5.5.2.2, was stratified by interest rate level.

We tried in various ways to fit the sample distribution of interest rate statistics to theoretical probability distributions. Success in these efforts would have been very helpful in understanding the shape of the tail and quantifying the expected CTE values. We do not present these fitting efforts in detail, but they were unsuccessful. In general, we were unable to identify theoretical distributions that reasonably fit the sample distribution of interest rates, so we directly used the Sample CTE values to estimate the parameters used in the scenario development. This section describes the Sample CTE analysis, and Section 6 describes how the results of the CTE analysis were used to set the scenario parameters.

5.3 Moderately Adverse Conditions

Our objective was to generate a scenario set that might reliably capture moderately adverse conditions. Section 7 includes discussion of stress-testing scenarios, but our fundamental objective was not to develop stress scenarios. Similarly, our objective was not to develop expected or best-estimate scenarios. This objective required a definition of moderately adverse conditions as well as an assumption about the relationship between interest rates as model inputs and projected financial results as model outputs.

Section 3.4.2 of ASOP 22 requires that "when forming an opinion, the actuary should consider whether the reserves and other liabilities being tested are adequate under moderately adverse conditions," defined in Section 2.15 as "Conditions that include one or more unfavorable, but not extreme, events that have a reasonable probability of occurring during the testing period." In U.S. life insurance regulation, the CTE framework (also known as tail value at risk) has been codified as a measure of moderately adverse conditions. Actuarial Guideline 43 (National Association of Insurance Commissioners, 2012) and VM-20 (National Association of Insurance Commissioners, 2016) utilize a CTE70 standard as the definition of moderately adverse. CTE70 is a statistical measure and, as such, is typically applied to stochastic projections. As typically used in actuarial modeling applications, CTE70 is a single-tail measure representing the mean loss amount in the 30% of scenarios with the greatest losses. In keeping with this definition, we performed an empirical CTE analysis, applied to the MDS Interest Rate Series, to develop our scenarios. As discussed further below, our analysis required some modification to consider both tails of the interest rate distribution, since interest rates are not losses and since either tail of the interest rate distribution may produces losses in actuarial models.

However, in using the CTE framework to develop deterministic interest rate scenarios, there are some assumptions and definitional adjustments required.

First, this approach relies on the assumption that CTE70 interest rate inputs will result in CTE70 model results. That is, we assume that if our scenarios represent the CTE70 tail of the distribution of interest rates, the resulting model results will be consistent with the CTE70 tail of actuarial projection results. Intuitively, this is a reasonable assumption. Practitioners know that the greatest losses tend to occur in scenarios where interest are relatively high or relatively low, and moderate interest rate scenarios tend to produce the most favorable results. It therefore follows that scenarios that represent the tails of the interest rate distribution will tend to produce adverse results. However, whether an interest rate scenario that represents an average of the most adverse X% of possible scenarios will generate a result that approximates the average of the results for those scenarios is a difficult question that would require testing outside the scope of *Modern Deterministic Scenarios for Interest Rates.* Stated formulaically, does mean[F(x)] = F[mean(x)]?

Second, adverse model results may be generated across the entire distribution of interest rate scenarios—for example, high-rate scenarios, low-rate scenarios or scenarios with rapid changes in rates. When measuring results, the practitioner does not need to know which scenarios generated which results to rank them and compute the CTE results. However, in developing deterministic scenarios, we do not know a priori which scenarios will generate the greatest losses. The answer will vary considerably due to product and company-specific factors. In *Modern Deterministic Scenarios for Interest Rates,* we wished to produce a scenario set that would be useful for a wide range of product types and company factors. Therefore, we had to make an assumption about how the tails of the interest rate distributions would translate into the 30% tail of model results. Generally speaking, some adverse results may come from the highest interest rate scenarios, and some may come from the lowest interest rate scenarios, so we needed to generate both high-rate scenarios and low-rate scenarios, whatever the initial rate environment. We had no practical way to determine, for any given product in any given initial environment, whether high-rate scenarios would tend to be more adverse than low-rate scenarios, or vice versa. Therefore, to be consistent with the CTE70 standard, we determined that our analysis should draw equally from the left tail and the right tail of the interest rate distribution, capturing the 15% left tail and the 15% right tail.

Finally, the standard CTE terminology is based on consideration of only one tail and requires adjustment when considering two tails. To avoid new terms entirely, we have introduced subscripts H and L to denote the high and low tails (or left and right tails) of the interest rate distribution. So CTE_H85 represents the right tail of the interest rate distribution—the average of the highest 15% of values for a given interest rate measure. Conversely, CTE_L85 represents the left tail, or the average of the lowest 15% of values for a given interest rate measure.

5.4 Empirical Analysis Period

The MDS Interest Rate Series extend back to 1729 for the Long Rate and 1825 for the Short Rate. Section 4.2 discusses the decision of how much history to capture in the MDS Interest Rate Series. In addition, in performing the empirical analysis, one necessary decision was whether to include the entire available history in our analysis. We considered three arguments in evaluating whether to exclude or otherwise adjust the historical period considered in the analysis:

Argument 1—Older data are not relevant because the world has changed.

This argument posits that modern conditions are distinct enough from historical conditions that data before some date are not relevant for use in building prospective interest rate scenarios. We discussed this argument to an extent in Section 4, but it is an important enough question to revisit in developing our empirical analysis. One version of this argument is used to explain why recent interest rate levels, although common in earlier periods, should be considered too low to continue for a sustained period. Reasons underlying this argument might include structural changes in the economy (e.g., the change from an agrarian economy to an industrial economy) making older history irrelevant; globalization in financial markets altering the "natural" level of interest rates; evolution of financial markets resulting in greater market efficiency but less stability; and central bank monetary policy fundamentally alters the level of interest rates.

To be sure, the MDS Interest Rate Series indicate that average interest rates since the beginning of the 20th century have been higher than in the 18th and 19th centuries. Table 5 breaks the MDS Long Interest Rate Series into three equal, nearly century-long segments and shows average rates, along with selected percentile statistics, for the three segments.

Table 5 - MDS Long Interest Rate Statistics by Era			
	1729-1824	1825-1919	1920-2015
Mean	3.91	3.15	5.27
10th Percentile	3.00	2.64	2.51
20th Percentile	3.18	2.80	2.93
30th Percentile	3.37	2.99	3.73
40th Percentile	3.47	3.12	4.19
50th Percentile	3.71	3.20	4.61
60th Percentile	4.01	3.24	5.01
70th Percentile	4.45	3.28	6.06
80th Percentile	4.76	3.36	7.43
90th Percentile	5.01	3.54	8.53
90th Percentile - 10th Percentile Gap	2.01	0.90	6.03

Table 5 shows that average interest rates have been 5.27% in the modern segment, much higher than the earlier two segments. It also demonstrates that the entire distribution has not shifted upward, only the right half, which is in turn driven by the period of the 1970s–1990s. From 1969 through 2000, which constitutes 32 years or 33% of the 1920–2015 period, every year except 1998 experienced a Long Rate higher than the maximum rate before or since. A visual review of the rates indicates that this high interest rate period started in 1966 and continued through 2007, constituting 42 years or 44% of the 1920–2015 period. This indicates to us that the higher average rate levels since 1920 are driven primarily by the 1970s–1990s. In fact, the 10th percentile of interest rates is lower in the modern segment than in the earlier segments. Arguments that interest rates in preindustrial periods should be ignored are often premised on an assumption that interest rates correlate to economic growth rates and that economic growth has been greater since the Industrial Revolution. This would suggest that 1825–1919 interest rates should have been higher than 1729–1824 interest rates, which was not the case. It is also notable that the high interest rate period beginning in the 1970s was not driven by high economic growth. Finally, even granting that higher economic growth rates should lead to higher levels of interest rates, it is hardly clear that economic growth over the next century will continue the trend of the last century. Such factors as population aging, environmental resource sustainability and the continued shift from an industrial economy to an information economy are some factors that could contribute to significantly different patterns of growth. These considerations together led us to conclude that supposed shifts in growth patterns were not a reason to exclude past data.

Similarly, such factors as globalization and evolution of financial markets are generally thought to improve market efficiency. One aspect of improved market efficiency should be reduced friction, which we believe would tend to reduce interest rates, not increase them. Another aspect of improved market efficiency would be that a market price reflects greater information and more accurately represents the “true” price of an instrument. More efficient markets might thus result in more price volatility as new information is reflected more quickly in market prices, but it does not follow that this should cause risk-free rates to be higher. Therefore, we concluded that these factors did not provide a basis to exclude past, lower interest rate, data.

We found central bank policies to be a more compelling reason why the modern era might differ from prior eras. As previously noted, the emergence of a consistently positively sloped yield curve coincides with the beginning of the Federal Reserve’s (and other central banks’) use of interest rates to manage monetary policy. The Fed’s primary monetary tool is the short-term Federal Funds rate, and short-term Treasury rates correlate closely with the Federal funds rate. Fabozzi and Fabozzi (1995, ch. 37) discuss various hypotheses of the term structure of interest rates. The liquidity premium or interest-rate-risk hypothesis is commonly used to explain the positively sloped term structure. Referring back to Figure 3, the shift to the yield curve shape now considered normal occurred around the end of World War I. Under the liquidity premium hypothesis, one might argue that before World War I, investors did not consider interest rate risk or the need for liquidity premia. Under this hypothesis, one might expect short-term rates to be similar before and after this shift, but long-term rates to be higher to reflect these factors. Alternatively, one might argue that Fed policies are generally accommodative, suggesting that short-term rates have generally been below market levels since the beginning of Open Market Operations, but that long-term rates more fairly represent long-term market rates in all periods. We did not attempt to resolve the reason for the sudden appearance of the term structure, but we did consider it a characteristic of modern interest rates that we should reflect in our analysis. With respect to long-term interest rates, we did not find it compelling to think that investors would have suddenly “discovered” the liquidity premium. Combined with the fact that long-term rates are generally less subject to Fed management than short-term rates, we took the view that the steepening of the yield curve since 1920 is more a consequence of lower short-term rates than of higher long-term rates. Thus, in considering earlier periods, we adjusted older short-term rates downward to consider the term structure rather than increasing older long-term rates. Finally, on this topic we must note that some authors, including Taleb—“the U.S. Government (or rather the Federal Reserve) has been trying for years to iron out the business cycle, leaving us exposed to a severe disintegration” (Taleb, 2010, p. 329)—have argued that Fed policies to maintain economic stability result in unintended consequences that increase the likelihood of major shocks to the system, including either the period of the late 1970s and early 1980s or the financial crisis of 2008. This may be the case, but we note that these two periods represent the two extreme ends of the interest rate spectrum and do not suggest an inherent upward bias in interest rates due to Fed policies. For purposes of developing moderately adverse interest scenarios, we considered these periods to be appropriately weighted when considered against the longer historical period.

Given all of this, we concluded that it was appropriate to include in our analysis data back to the beginning of the MDS Interest Rate Series. We also concluded that the modern term structure of interest rates should be reflected in our scenarios.

Argument 2—Extreme periods should be excluded from the analysis.

We quickly rejected this argument, concluding that it would be inappropriate to exclude from our analysis periods that might be considered extreme. The CTE framework does not ignore results that might be considered extreme, but considers extreme results in the context of more moderate but more likely results. The consideration of this argument was an additional factor for including in our analysis data back to the beginning of the MDS Interest Rate Series. While an extreme period should not be excluded, it is also important that an extreme period not be overweighted simply because it is recent. The high interest rate period of the 1970s could be considered to have begun in 1970 and lasted until 2000, a full 30-year period. Given the extremely high rates of this period, it is important to analyze a period long enough that this period does not dominate the analysis.

Argument 3—The different time periods covered by the MDS Long and MDS Short Interest Rate Series make them noncomparable.

We were very cognizant of this argument and considered it throughout our analysis. The MDS Long Interest Rate Series covers 287 years, while the MDS Short Interest Rate Series covers only 191 years. In considering the relationship of long-term and short-term interest rates, we consistently performed our analysis so as to ensure that no bias was introduced by the differences in these periods.

5.5 Details of the Empirical Analysis

The methods and results of our analyses under all three analytic frameworks—the CTE Reversion Target framework, the Rate Change CTE framework and the Interest Rate Cycle framework—are presented in Sections 5.5.1, 5.5.2 and 5.5.3, respectively. In addition, both the CTE Reversion Target and the Rate Change CTE frameworks require a transition step; Section 5.5.4 describes the analysis supporting this transition step. The further step of converting the analysis into scenarios is covered in Section 6.

5.5.1 CTE Reversion Target Analysis

The reversion target analysis is based on the mean reversion concept used in many stochastic scenario generators. However, rather than using a long-term mean as a reversion target, this analysis generates long-term measures of the sample CTE_L85 and CTE_H85 statistics as reversion targets. This analysis posits that a scenario in which interest rates revert to the desired sample CTE level may be considered a moderately adverse scenario.

Table 6 shows the values of CTE_L85 and the CTE_H85 for the MDS Long and Short Interest Rate Series, measured over various periods. This analysis shows that the left tail of interest rates, CTE_L85, does not vary greatly by measurement period. Perhaps contrary to expectation, for both Long and Short Rates, the left tail values are somewhat higher over the longer historical periods. CTE_H85 does show significant variation by measurement period, and the differences are consistent with intuitive expectations. This right tail value decreases as one adds more historical data, diluting the impact of the 1970s–1980s period.

Table 6 - MDS Interest Rate Reversion Statistics

Rate Series	CTE Level	1729-2015	1825-2015	1920-2015
MDS Long	CTE_L85	2.59	2.48	2.46
	CTE_H85	7.56	8.52	9.94
MDS Short	CTE_L85		0.40	0.12
	CTE_H85		7.31	8.61
Spread - Long vs. Short	CTE_L85		2.08	2.34
	CTE_H85		1.21	1.33

The data for various time periods illustrate the need to adjust for the difference in period between the MDS Long and MDS Short Interest Rate Series. The MDS Short CTE_H85 value over its entire available period 1825–2015 is 7.31%, nearly equal to the MDS Long CTE_H85 value of 7.56% over its entire historical period 1729–2015. When measured over consistent periods, however, the spreads between the MDS Long and MDS Short Rates are much greater. In constructing the Reversion Target Scenarios, as discussed in Section 6.2, the Short Rate targets were adjusted to account for the difference in observation periods.

Another element required in developing reversion scenarios is the time of reversion. To determine time of reversion, we considered the correlation between an initial interest rate i_X and the interest rate *t* years later, i_{X+t}. Table 7 shows the R^2 value for various values of t for the MDS Short and Long Interest Rate Series. This demonstrates that after 15–20 years, correlation with the beginning rate becomes very small. In other words, the starting rate has little bearing on the rate 15–20 years later. We used this observation to set our reversion period assumptions.

Table 7 - R-sq of i_x vs i_{x+t}		
t	MDS Short	MDS Long
1	0.74	0.94
5	0.33	0.69
10	0.21	0.42
15	0.03	0.19
20	0.01	0.05

See Appendix E for additional details of the Reversion Target analysis, which is also available in Excel format to allow for additional analysis by the reader.

5.5.2 Rate Change CTE Analysis

Unlike the reversion analysis, which posits a reversion to a specified moderately adverse long-term target independent of the initial level of interest rates, the Rate Change CTE analytic framework applies CTE analysis to changes in interest rates from their initial levels over a t-year period. This framework posits that applying the Sample CTE analysis to t-year interest rate changes, we can quantify moderately adverse interest rate changes from the initial rate. Scenarios can then be constructed based on these rate change statistics. We encountered several challenges in this process, and following is a discussion of these challenges and our methods for addressing them.

5.5.2.1 Adjustment for Differences in Long and Short Observation Periods

For the MDS Long Interest Rate Series, analysis was based on CTE_L85 and CTE_H85 statistics, consistent with the discussion above.

For the MDS Short Interest Rate Series, it was necessary to consider the impact of the shorter observation period—191 years of data compared with 287 years for the MDS Long Interest Rate Series. Given the 287 MDS Long data points, the 15% left and right tails each included 43 observations. We observed that most of these Long Rate tail observations points are concentrated in the period since 1825, when both series were available. By assuming the same for Short Rates, we concluded that extending the MDS Short Interest Rate series from 1825 back to 1729 would add 96 years of exposure without adding any new tail observations. Spread over the shorter 191 years of the MDS Short Interest Rate series, the 43 tail observations represent the 23rd percentile. Rounding up, we therefore assumed that CTE_L75 and CTE_H75 results for the MDS Short Interest Rate analysis would be equivalent to CTE_L85 and CTE_H85 results for the MDS Long Interest Rate analysis, given the difference in observation period.

5.5.2.2 Interest Rate Grouping

As discussed above, a key initial hypothesis was that the rate change CTE statistics would vary according to the initial level of interest rates, in both magnitude and direction. We hypothesized that the mean value of the change in rates over a t-year period, denoted $E(\Delta i_t)$, would exhibit an inverse relationship with the initial interest rate, denoted i_x, which would become stronger as t increased (i.e., a reversion effect that would increase with the change period t, whereby low rates would tend to increase and high rates would tend to decrease). We also hypothesized that the variance of the change in rates over a t-year period, $var(\Delta i_t)$, would exhibit a direct relationship with i_x (i.e. higher initial interest rates would tend to exhibit larger changes and a wider range between the CTE_L and CTE_H values). We were optimistic that these relationships would be strong enough and consistent enough that we could regress the Rate Change CTE statistics to some function of the initial rate, allowing us to build scenarios from regression formulas. In fact, the data did support our hypotheses regarding these relationships, as shown in Table 8 and Table 9.

These tables show, separately for the MDS Long and MDS Short Series, the mean and variance of changes in interest rates—E(Δi_t) and var(Δi_t)—for various t-year change periods, both in total and by Interest Rate Group. In these tables, the mean changes in interest rates clearly demonstrate the expected reversion patterns over time, with lower Interest Rate Groups showing positive average changes and higher Interest Rate Groups showing negative average changes. Over shorter change periods, these tables also demonstrate that the range of variance in rate changes is smaller when initial interest rates are lower, as expected. Unfortunately, these tables also demonstrate a significant amount of sampling noise in the data. Partly as a result of this noise, we were not able to develop regression formulas that we considered to be sufficiently predictive.

Table 8 - MDS Long Interest Rate Series
Mean and Standard Variation of t-year Interest Rate Absolute Changes by Initial Rate Group

Initial	Init Rate Range		E(Δi_t)				var(Δi_t)			
Rt Group	Low	High	t=1	t=10	t=20	t=30	t=1	t=10	t=20	t=30
Total	2.19	13.58	0.00	0.05	0.15	0.35	0.19	2.23	5.09	6.79
Group 1	2.00	2.75	0.04	0.49	1.96	4.06	0.02	0.39	1.67	7.51
Group 2	2.75	3.75	0.01	0.26	0.38	0.55	0.03	0.53	1.19	2.16
Group 3	3.75	6.00	(0.02)	(0.21)	0.07	(0.53)	0.17	2.06	7.38	2.59
Group 4	6.00	10.00	0.09	0.14	(2.11)	(2.37)	0.52	9.74	5.59	2.45
Group 5	10.00	15.00	(0.59)	(4.49)	(6.77)	(8.75)	3.19	0.84	0.87	1.08

Table 9 - MDS Short Interest Rate Series
Mean and Standard Variation of t-year Interest Rate Absolute Changes by Initial Rate Group

Initial	Init Rate Range		E(Δi_t)				var(Δi_t)			
Rt Group	Low	High	t=1	t=10	t=20	t=30	t=1	t=10	t=20	t=30
Total	2.19	13.58	(0.00)	(0.08)	(0.08)	0.07	1.38	5.22	11.13	14.74
Group 1	2.00	2.75	0.01	0.96	2.63	5.08	0.01	0.38	0.87	1.64
Group 2	2.75	3.75	0.30	0.85	2.46	2.84	0.54	2.02	5.88	10.88
Group 3	3.75	6.00	0.18	0.07	0.25	(0.04)	0.77	3.15	6.92	3.93
Group 4	6.00	10.00	(0.23)	(0.67)	(1.96)	(2.07)	2.10	7.57	4.67	2.82
Group 5	10.00	15.00	(0.93)	(2.47)	(5.18)	(7.03)	4.27	11.70	8.94	11.40

As an alternative to regressing the rate changes on the initial level of interest rates, we stratified the interest rate data into the Interest Rate Groups. The objective of this stratification was to place our initial interest rate observations into relatively homogeneous groups within which the Rate Change CTE_L and CTE_H values would be independent of the initial rate level. We applied three primary criteria in choosing the Interest Rate Groups: (1) meaningful number of observations in the group, (2) a range of interest rates in the group that was not too large and (3) low R^2 value for a linear regression between the initial interest rate i_x and the interest rate change Δi_t (the Interest Rate Groups were selected so that within a Group the rate changes would be independent of the initial rate; a low R^2 value was indicative of this independence). These criteria were applied judgmentally, and in some cases, all three criteria could not be reasonably satisfied. In particular, the number of observations of high interest rates is so few that there was no way to create a group with a meaningful number of observations or a small range of interest rates. However, we are satisfied that we were ultimately able to develop Interest Rate Groups for the MDS Long and the MDS Short Interest Rate Series that reasonably satisfied these criteria in most cases.

Table 10 summarizes the Interest Rate Groups used for the Long Rate Change CTE analysis, including the number of observations for each Group and the R^2 values for various change periods. For Groups 1, 2 and 3, we were generally happy with the results. Group 1 includes fewer observations than we would have liked, but the homogeneity would have suffered by increasing the upper end of the range. This table clearly demonstrates the shortcomings of the groupings for high interest rate environments, due to sparse data. There are only 31 years out of 286 (note that the number of observations is one less than our total, because 2015 has not yet experienced any changes) with Long Rates above 6%, and these 31 observations are distributed over a wide range—from 6.00% to 13.58%. We attempted to address these shortcomings in the scenario construction process, but this paucity of data reiterates the extreme nature of the 1970s–1980s environment. If and when interest rates reenter a Group 4 or Group 5 environment, the user should consider these shortcomings carefully.

Table 10 - MDS Long Interest Rate Series Interest Rate Groupings for Rate Change CTE Analysis							
	Initial Rate Range		# of	R^2 of Δi_t vs. i			
Group #	Low	High	Observations	t=1	t=10	t=20	t=30
Group 1	2.00	2.75	34	0.01	0.26	0.15	0.02
Group 2	2.75	3.75	131	0.01	0.01	0.02	0.02
Group 3	3.75	6.00	91	0.02	0.14	0.12	0.21
Group 4	6.00	10.00	25	0.02	0.14	0.44	0.96
Group 5	10.00	15.00	6	0.03	0.55	0.77	0.75
Total	2.19	13.58	286	0.01	0.15	0.36	0.49

Table 11 similarly summarizes the Interest Rate Groups used for the MDS Short Rate Change CTE analysis. The distribution of observations is much more symmetric than for Long Rates, and the R^2 results are generally better. Group 1, which is where the Short Rate falls at the time of this report, has very sparse data. In addition, six of the 19 data points cover the period 2009–2014; given that those points are so recent, they contribute to the change statistics for only change periods of six years or less. Like the Long Rate data for Group 1, we felt that expanding Group 1 to capture additional data points would adversely affect the homogeneity of the group and increase the bias in our estimates.

Table 11 - MDS Short Interest Rate Series Interest Rate Groupings for Rate Change CTE Analysis							
	Initial Rate Range		# of	R^2 of Δi_t vs. i			
Group #	Low	High	Observations	t=1	t=10	t=20	t=30
Group 1	-	0.50	19	0.00	0.05	0.11	0.00
Group 2	0.50	2.50	44	0.00	0.02	0.08	0.25
Group 3	2.50	4.00	63	0.00	0.01	0.02	0.04
Group 4	4.00	6.00	45	0.07	0.04	0.04	0.11
Group 5	6.00	15.00	19	0.00	0.35	0.41	0.84
Total	0.03	14.04	190	0.04	0.19	0.50	0.69

5.5.2.3 Modeled vs. Sample Rate Change CTE Statistics

As discussed above, we were generally unsuccessful in regressing or otherwise modeling our CTE statistics formulaically. However, there was one instance where such an effort yielded a useful result. Due to small sample sizes, the sample CTE_L and CTE_H statistics are subject to significant sampling error, more so than the mean or standard deviation. As such, we sought to relate the CTE_L and CTE_H statistics to the mean and standard deviation to reduce the impact of sampling error. Under an assumption of normality

$$P(X > x) = P(Z > z)$$

where

$$z = (x - \mu) / \sigma$$

and

$$P(X < x) = P(Z < z)$$

where

$$z = (x - \mu) / \sigma.$$

By defining x to be equal to the sample CTE_L and CTE_H values, we can compute the corresponding z value for each *t*-year Interest Rate Change period within each Interest Rate Group. The z value can be interpreted as the number of standard deviations, or number of sigmas, by which the CTE value differs from the mean. As shown in Table 12, we observed that the z statistics associated with the CTE values were generally consistent across Interest Rate Groups and Rate Change periods, that is, that the number of sigmas represented by CTE value was invariant across Interest Rate Group and Rate Change Period.

Table 12 - MDS Long and Short Rates Z-factors for Modeling Rate Change CTE				
	MDS Long Rates		MDS Short Rates	
Interest Rate Group	Z-factor$_L$(G)	Z-factor$_H$(G)	Z-factor$_L$(G)	Z-factor$_H$(G)
Group 1	(1.24)	1.72	(1.09)	1.23
Group 2	(0.98)	1.79	(1.15)	1.33
Group 3	(1.08)	1.91	(1.16)	1.29
Group 4	(1.11)	1.68	(1.13)	1.31
Group 5	(1.37)	1.42	(1.15)	1.27
Weighted Average	(1.07)	1.81	(1.14)	1.29

We applied the weighted average values $z(CTE_L)$ and $z(CTE_H)$ to the Interest Rate Group–specific and Rate Change Period–specific sample standard deviations to produce "modeled" CTE_L and CTE_H values with a reduced impact of sampling error:

$${}_{modeled}CTE_L(IG,t) = \mu(IG,t) + z(CTE_L) \times \sigma(IG,t)$$

and

$${}_{modeled}CTE_H(IG,t) = \mu(IG,t) - z(CTE_H) \times \sigma(IG,t)$$

where

$\mu(IG,t)$ = sample mean interest rate change;

$\sigma(IG,t)$ = sample standard deviation of interest rate change;

IG = Interest Rate Group;

t = rate change period; and

$z(CTE_L)$ and $z(CTE_H)$ are the weighted average z values.

This modeling approach recognizes that sampling error has a greater impact on sample CTE statistics than on sample mean and variance statistics, because CTE statistics measure only a portion of the sample distribution while mean and variance apply to the entire distribution. This approach has limitations, discussed below, but we considered it a useful tool in spite of these limitations:

1. Assumption of normality—Use of the z-statistics reflects an implicit assumption that rate changes within an Interest Rate Group are normally distributed, which we do not believe to be the case. In particular, we computed separate z statistics for CTE_L and CTE_H explicitly because we observed skew in the distribution. While this assumption may have introduced bias into our estimates, we believe this approach provided a practical benefit in reducing sampling error in the tail that outweighs any such bias. That is, we believe it is reasonable to express CTE level in relation to the standard deviation and do not believe the "number of standard deviations" represented by a given CTE level is misstated in any significant way due to this assumption. We did not explicitly test this belief.

2. This approach does not explicitly address sampling error in the sample mean and sample standard deviation. While such sampling error clearly exists, given the sample sizes of the Interest Rate Groups, we were unsuccessful in identifying a modeling approach to address this modeling error; the data clearly indicated that the mean and variance varied with both the Interest Rate Group and Rate Change Period, yet we were unable to establish a reasonable regression formula, so we had no mechanism to isolate the sampling error.

The Modeled Rate Change CTE values differed from the directly calculated Sample Rate Change CTE values primarily for high Interest Rate Groups—Interest Rate Groups 4 and 5—where data are most limited. For the MDS Long Interest Rate Series, Interest Rate Groups 4 and 5, we also observed that the sample standard deviation indicated significant sampling error, which we did not observe in other groups. As a result, for these two groups we used combined σ(IG,t) values in computing the Modeled Rate Change CTE values. As further discussed in Section 4, the Modeled and Sample CTE values were reviewed together in setting the Rate Change CTE scenario parameters.

Appendix F includes the calculated z values for each Interest Rate Group and Rate Change Period, the weighted averages z values and the development of the Modeled Rate Change CTE values for both the MDS Long and Short Interest Rate Series, on both an absolute change basis and a relative change basis.

5.5.2.4 Absolute vs. Relative Changes in Rates

Over any t-year period, we can quantify the change from the initial rate i_X in either absolute (i.e., straight difference) or relative (i.e., percent difference) terms, where

$$\Delta i_t^{abs} = i_{x+t} - i_x$$

and

$$\Delta i_t^{rel} = (i_{x+t} - i_x) / i_x.$$

The CTE statistics can similarly be computed on either absolute or relative bases. While we had an initial hypothesis that both the direction and magnitude of interest rate changes were dependent on the initial interest rate, we did not know whether absolute or relative change statistics would have greater predictive value, so we computed the CTE statistics on both bases. Ultimately, we concluded, on the basis of correlation statistics, that the absolute change statistics provided the more useful basis for scenario construction in most cases. The primary exception was the left tail of rate changes over long change periods, primarily for Short Rates, that is, $CTE_L(\Delta i_t)$ when $t \geq 15$. Often these left tail changes were large enough decreases that when the absolute change is applied to interest rates at the low end of an Interest Rate Group, it could push projected rates negative. While we think negative interest rates are possible, we also believe that sustained nominal interest rates below zero would be beyond moderately adverse. Therefore, we used the relative change statistics in cases where this issue might arise.

The Rate Change CTE Analysis was performed for change periods of one to 30 years. As indicated in the discussion of the Reversion Analysis, correlation of i_{X+t} to i_X declines substantially beyond year 15 or 20, but the Rate Change Analytics clearly indicated that when considered at the level of Interest Rate Groups, interest rates could not be considered to approach a reasonable ultimate level until closer to 30 years. Said differently, we did not believe 15 years of rate changes captured a wide enough dispersion between the low and high scenarios, leading us to extend the analysis period under this analytic framework. Tables 13 and 14 show summary rate change CTE results on an absolute change basis for the MDS Long and Short Interest Rate Series, respectively. In Table 13, $CTE_L85(\Delta i_t^{abs},g)$ and $CTE_H85(\Delta i_t^{abs},g)$ denote the low- and high-tail CTE values for the absolute change in MDS Long Interest Rates over time *t* for Interest Rate Group g. In Table 14, $CTE_L75(\Delta i_t^{abs},g)$ and $CTE_H75(\Delta i_t^{abs},g)$ denote the same for MDS Short Interest Rates, but with the adjustment in CTE level from CTE85 to CTE75 as discussed in Section 5.5.2.1. These tables show the directly computed Sample CTE values, along with the Modeled CTE values as described above. In most cases, the Sample CTE values and the Modeled CTE values are close to one another. The primary exceptions are the Group 4 and Group 5 values for Long Rates, due to the impact of limited sample sizes as previously discussed. Note that relative change CTE values are not shown in the tables, but are included in Appendix F. Refer to Tables 10 and 11 for the interest rate ranges represented by the Interest Rate Groups.

Table 13 - MDS Long Interest Rate Series
Rate Change CTE Values by Interest Rate Group - Absolute Changes

	$CTE_L85(\Delta i_t^{abs},g)$				$CTE_H85(\Delta i_t^{abs},g)$			
Interest Rate Group	t=1	t=10	t=20	t=30	t=1	t=10	t=20	t=30
<u>Sample Values</u>								
Group 1	(0.14)	(0.34)	0.28	1.54	0.33	1.55	4.17	9.40
Group 2	(0.25)	(0.50)	(0.66)	(0.74)	0.29	1.65	2.26	3.12
Group 3	(0.67)	(1.87)	(2.30)	(2.28)	0.60	2.60	5.55	2.61
Group 4	(0.83)	(2.71)	(4.61)	(4.69)	1.23	5.79	1.89	(0.43)
Group 5	(3.06)	(5.44)	(8.02)	(10.11)	2.24	(2.73)	(5.25)	(7.20)
<u>Modeled Values</u>								
Group 1	(0.13)	(0.18)	0.59	1.13	0.32	1.62	4.30	9.01
Group 2	(0.18)	(0.52)	(0.79)	(1.02)	0.33	1.58	2.35	3.21
Group 3	(0.46)	(1.74)	(2.83)	(2.24)	0.71	2.38	4.98	2.38
Group 4	(0.99)	(2.88)	(4.38)	(3.86)	1.94	5.26	1.75	0.17
Group 5	(1.67)	(7.51)	(9.04)	(10.25)	1.26	0.63	(2.91)	(6.22)

Table 14 - MDS Short Interest Rate Series Rate Change CTE Values by Interest Rate Group - Absolute Changes								
	$CTE_L75(\Delta i_t^{abs},g)$				$CTE_H75(\Delta i_t^{abs},g)$			
Interest Rate Group	t=1	t=10	t=20	t=30	t=1	t=10	t=20	t=30
Sample Values								
Group 1	(0.09)	0.21	1.54	3.70	0.12	1.63	3.63	6.77
Group 2	(0.63)	(1.16)	(0.03)	(0.41)	1.22	2.42	5.75	7.73
Group 3	(0.92)	(2.12)	(2.57)	(2.51)	1.26	2.35	3.65	2.56
Group 4	(2.05)	(3.68)	(4.68)	(4.01)	1.59	2.55	0.79	0.13
Group 5	(3.27)	(6.29)	(8.67)	(11.42)	1.84	2.24	(1.49)	(2.89)
Modeled Values								
Group 1	(0.10)	0.26	1.56	3.61	0.14	1.76	3.83	6.73
Group 2	(0.54)	(0.78)	(0.31)	(0.93)	1.24	2.68	5.60	7.11
Group 3	(0.83)	(1.96)	(2.75)	(2.31)	1.31	2.36	3.66	2.53
Group 4	(1.89)	(3.82)	(4.43)	(3.99)	1.64	2.89	0.83	0.10
Group 5	(3.29)	(6.38)	(8.59)	(10.89)	1.75	1.96	(1.31)	(2.66)

Appendix F provides the details of the Rate Change CTE analysis and is also provided in Excel format to allow additional analysis by the reader. It includes statistics for MDS Long and MDS Short Interest Rate Series; all change periods from one to 30 years; statistics in total and by Interest Rate Group; and statistics including mean, variance, Sample CTE_L and CTE_H and Modeled CTE_L and CTE_H. Appendix F is divided into four sections, as follows:

Appendix F.1—Long Interest Rates, absolute changes
Appendix F.2—Short Interest Rates, absolute changes
Appendix F.3—Long Interest Rates, relative changes
Appendix F.4—Short Interest Rates, relative changes

5.5.3 Interest Rate Cycle Analysis

The data to support the Interest Rate Cycle analytic framework were much less robust than either the Reversion Target or Rate Change CTE framework. While we have limited data points with respect to annual interest rates, the number of interest rate cycles observed in the period covered by the MDS Interest Rate Series is in the single digits. The observed interest rate cycles are wildly diverse in duration and in range of interest rate levels. As a result, developing cyclical scenarios based on rigorous data analysis is impossible. Instead, we used some fairly general observations.

Tables 15 and 16 capture the historical interest rate cycles for the MDS Long and Short Rate Series. The definition of “cycle” is loose, but generally we attempted to capture periods which exhibited an initial peak, a decline in rates for some period until a bottom, or trough value, was reached, and then increased toward another peak. We tried to ensure that the cycles were of significant duration and reflected a meaningful change from the peak to the trough.

Table 15 - MDS Long Interest Rate Series Historical Interest Rate Cycles								
	Beginning Peak		Trough		Ending Peak		Total	
Cycle #	Year	Rate	Year	Rate	Year	Rate	Duration	Range
1	1746	3.65	1753	2.86	1762	4.29	16	1.43
2	1762	4.29	1772	3.30	1784	5.41	22	2.11
3	1784	5.41	1792	3.33	1798	5.94	14	2.61
4	1798	5.94	1897	2.25	1921	5.48	123	3.69
5	1921	5.48	1946	2.19	1981	13.58	60	11.39
6	1981	13.58	2015	2.69				10.89

Table 16 - MDS Short Interest Rate Series Historical Interest Rate Cycles								
	Beginning Peak		Trough		Ending Peak		Total	
Cycle #	Year	Rate	Year	Rate	Year	Rate	Duration	Range
1	1826	4.50	1833	2.73	1839	5.13	13	2.40
2	1839	5.13	1844	2.13	1847	5.85	8	3.72
3	1847	5.85	1852	1.91	1857	6.50	10	4.59
4	1857	6.50	1862	2.47	1864	7.27	7	4.80
5	1864	7.27	1868	1.87	1873	4.49	9	5.40
6	1873	4.49	1895	0.80	1920	6.06	47	5.26
7	1920	6.06	1940	0.04	1981	14.04	61	14.00
8	1981	14.04	2014	0.03				14.01

Here are some observations regarding these cycles:

- Before the late 1800s, Short Rate cycles were short and reflected significant variation in rates. These cycles were generally not accompanied by cycles in Long Rates.
- The 1700s exhibited fairly regular cycles of Long Rates, but the period 1798–1920 effectively reflects one long cycle.
- Particularly for Long Rates, the severity of the cycle, measured by the full range of interest rates, appears to correlate with the duration of the cycle. The cycle that peaked in 1981 with a Long Rate of 13.58% and a short rate of 14.04% was 60 years in length. We are now 35 years into the next cycle and have just reached its bottom.
- As can be seen by looking back at Figure 3, the cycles resemble ocean waves more than sine waves. Rather than a smooth cycle between low rates and high rates, it is more common for rates to increase rapidly to a peak and then fall rapidly from the peak, remaining near the trough for a much longer period.

5.5.4 Interest Rate Transition Analysis

One last item had to be addressed under the CTE Reversion Target and Rate Change CTE frameworks. The MDS Interest Rate Series represent calendar year average interest rates, which is the appropriate basis from which to develop scenarios with annual time steps. Given that a projected rate is designed to apply for a full projection year, basing the analysis on single-day values could introduce inappropriate volatility in rates from year to year.

That said, the input value for a projection scenario is a daily rate, not a yearly average rate, and that initial rate represents a best estimate of interest rates over the near term. Our scenarios therefore require an approach for transitioning from daily interest rates on the scenario start date to projected yearly average values.

Using the period for which daily Treasury rate data are available from the Federal Reserve, we performed a Sample CTE analysis of the change from a daily interest rate to the average interest rate over the following year, using the Rate Change CTE analysis methodology. Table 17 shows the result of this analysis for the MDS Long and Short Interest Rate Series. This analysis was performed by Interest Rate Group, using the same interest rate groupings discussed above (with the top end of Group 5 adjusted to reflect higher maximum daily values than maximum yearly average value). Refer to Tables 10 and 11 for the interest rate ranges represented by the Interest Rate Groups.

Table 17 - MDS Long and Short Interest Rate Series
Transitional Change Statistics - Change from Initial Rate to Next Yearly Average

Interest	MDS Long Series				MDS Short Series			
Rate Group	# of Obs	Mean	$CTE85_L$	$CTE85_H$	# of Obs	Mean	$CTE85_L$	$CTE85_H$
Group 1	177	0.35	0.05	0.74	1,654	(0.01)	(0.09)	0.08
Group 2	650	(0.04)	(0.52)	0.71	2,131	0.21	(0.74)	1.08
Group 3	4,980	0.01	(0.60)	0.60	3,052	0.14	(1.27)	1.13
Group 4	5,787	0.02	(0.78)	0.91	4,561	(0.00)	(1.32)	1.17
Group 5	1,512	(0.23)	(2.07)	1.50	3,945	(0.24)	(2.45)	2.15
Total	13,106	(0.01)	(1.00)	0.90	15,343	(0.01)	(1.62)	1.38

As discussed in Section 6, these transition statistics were used in constructing some of the MDS Scenarios. Additional details of the transition analysis are provided in Appendix H, which is also available as an Excel workbook to allow for additional analysis by the reader.

Section 6: Interest Rate Scenario Construction

Scenarios were constructed based on each of the three analytic frameworks discussed in Section 5.1. In addition, scenarios were constructed by applying the Rate Change CTE analysis framework to a set of stochastic scenarios generated using the American Academy of Actuaries Interest Rate Generator (AIRG). The scenario construction process was similar, but not identical, for each type of scenario. In total, we developed 16 scenarios: eight CTE Reversion Target scenarios, four Rate Change CTE scenarios, two Interest Rate Cycle scenarios and two AIRG scenarios.

We also developed a methodology for constructing the entire yield curve from the Long and Short Rates.

Finally, for the CTE Reversion Target Scenarios and the Rate Change CTE Scenarios, we developed four approaches to modeling the relationship between Short Rates and Long Rates.

This section discusses the specific scenarios developed under each analytic framework and describes the construction steps. The discussion of the final scenarios is descriptive. For the specific formulas used to generate the scenarios, we refer the reader to the Appendix J, an Excel Scenario Calculation Workbook.

6.1 Scenario Shape Considerations

Coming into *Modern Deterministic Scenarios for Interest Rates,* we had an expectation of the general characteristics or shapes we should capture in the scenarios. In many cases, those expectations were confirmed by the data analysis, while in other cases we modified those expectations based on the empirical data. In many cases, those expectations were informed by what we saw as strengths or weaknesses in the NY7 Scenarios. The following discussion covers many of these characteristics, how they were ultimately reflected in the MDS Scenario set and how they may relate to the NY7.

Term structure of interest rates—The NY7 uses a parallel shift convention, and we expected that the data would not support parallel shifts. In fact, we found that, by any measure, Short Rates are more volatile than Long Rates, so that parallel shifts tend to overstress Long Rates, understress Short Rates, or both. We concluded early on that none of the MDS Scenarios should reflect parallel shifts. In the author's experience, many actuaries incorporate nonparallel shifts into their Asset Adequacy Testing scenarios, often a single inversion scenario. In keeping with this practice, our initial expectation was to incorporate a steepening scenario, which could result in an inversion, based on CTE analysis of the relationship between Short and Long Rates. We planned to incorporate a flattening scenario as well using the same approach.

In analyzing yield curve shape, we relied on post–World War I data, given the abrupt change in term structure discussed in Section 4.5. As we analyzed yield curve shape, we observed a couple of interesting things. First, historically, inversions have occurred only as a result of Short Rates changing more quickly than Long Rates, and thus have occurred only when rates are increasing. Second, inversions have occurred in both low-rate and high-rate environments. These observations suggest that some inversion scenarios are not reasonable, for instance, scenarios where Long Rates drop rapidly without a corresponding drop in Short Rates or where Long Rates drop when Short Rates are rising.

Our empirical interest rate analysis of interest rates treated Long and Short Rates independently and so addressed the issues noted above. However, we realized that the risks associated with changes in the steepness of the yield curve might depend on the product and other scenario-specific factors. Therefore, we decided to modify our planned treatment of yield curve shape, developing four Short Rate projection options from which the practitioner can choose. Long Rates are the same under all approaches: these approaches vary only in how Short Rates are projected in relation to Long Rates. The first option, Independent Short Rates, projects Short Rates using the Short Rate parameters developed through the empirical analysis of the MDS Short Interest Rate Series. For the other three options, we analyzed historical yield curve shapes to quantify mean, moderately steep and moderately flat yield curve shapes, measured as the absolute spread between Short and Long Rates. Table 18 shows the development of the spread targets.

Table 18 - Development of Yield Curve Target Steepness Parameters - Spread between Long and Short Rates

Description	Formulation	Historical (1920-2015)	Scenario Parameter
Moderately Flat	$CTE_L85(i_L-i_S)$	(0.03)	-
Mean Steepness	$E(i_L-i_S)$	1.75	1.75
Moderately Steep	$CTE_H85(i_L-i_S)$	3.64	3.50

Therefore, the four Short Rate options are (1) independent projection of short rates, or reversion to a target spread of (2) 0.00%, (3) 1.75% or (4) 3.50% to Long Rates. The spread grades linearly from the initial actual spread to the target. In the Reversion Scenarios, the grading period equals the scenario reversion period, largely to avoid unusual patterns that might arise through the use of a different grading period. In the Rate Change CTE Scenarios, the grading period was set to 10 years. The choice of a 10-year grading period was not highly scientific, but was generally consistent with historical patterns of change.

Level scenario as a basis for projection—The NY7 keys all scenarios off of a level scenario, with an implicit assumption that the level scenario is a best estimate and the risk of increases and decreases is symmetrical. The historical data show that a level scenario does not reflect the best estimate, but that the best-estimate scenario varies with the initial rate. In addition, generally speaking, a level scenario is not itself moderately adverse, so we did not generate a level scenario. Level scenarios are useful as a baseline for practitioners, but are not included in the MDS Scenarios.

Symmetry of Increasing and Decreasing Scenarios—The NY7 Scenarios all use symmetric shifts, whereby increases equal decreases regardless of initial environment (subject to floors). The historical data show that the distribution of future interest rates is dependent on the starting level of rates and is generally not symmetric. Therefore, we developed the MDS Scenarios to reflect the likely magnitude of upward and downward changes separately.

Dependence on Starting Interest Rate Level—As discussed in Section 5.7.2, we observed clear correlation between the initial level of interest rates and subsequent rate changes. As such, the MDS Scenarios take into account initial interest rates in setting the scenario rates.

Pop-up and Pop-down Scenarios—We began *Modern Deterministic Scenarios for Interest Rates* with an expectation that pop-up and pop-down scenarios would be a significant element of the scenarios, like they are for the NY7. However, our analysis indicated that rate pops on the order of the 3% used in the NY7 are extreme events that rarely occur. Table 19 shows the CTE level corresponding to pop-ups and pop-downs of 3%. This table is based on our Transition analysis data, which captures Long Rate data back to 1962 and Short Rate data back to 1954, and defines a "pop" as the change from a daily value to the following one-year rolling average. For Long Rates, a 3% pop in either direction is a greater than CTE99 occurrence in any rate environment. In this time period, a 3% pop-up never occurred, with the maximum being 2.64%. We identified only four days, all in June of 1982, which experienced a Long Rate pop-down of 3%. For Short Rates, 3% pops have occurred, but rarely. When Short Rates are below 6%, a 3% pop in either direction has been a greater than CTE99 occurrence. With Short Rates above 6%, a Group that admittedly covers a wide range of rates, a 3% pop-up remains a beyond-moderate event, at CTE93, while a 3% pop-down has been common, driven by rates dropping from their highs in the early 1980s.

This analysis led us to rethink how we would handle pops in the MDS Scenarios. We concluded that pop-up and pop-down scenarios as used in the NY7 did not fit well into our framework for moderately adverse conditions. Therefore, rather than creating separate pop-up and pop-down scenarios, we incorporated moderately adverse pops into our Reversion and Rate Change CTE Scenarios.

Table 19 - CTE Analysis of NY7 Pop-up and Pop-down Scenarios Sample CTE Level for 3% Pops				
Rate Groupings	NY7 - Pop-down 3%	NY4 - Pop-up 3%	Min Pop	Max Pop
Short Rates:				
Total	CTE$_L$98	CTE$_H$98	(4.70)	6.46
0.00-0.50	> CTE$_L$99	> CTE$_H$99	(0.79)	1.88
0.50-2.50	> CTE$_L$99	> CTE$_H$99	(1.77)	1.66
2.50-4.00	> CTE$_L$99	> CTE$_H$99	(2.33)	2.06
4.00-6.00	> CTE$_L$99	> CTE$_H$99	(3.35)	6.46
6.00+	CTE$_L$60	CTE$_H$93	(4.70)	4.35
Long Rates:				
Total	> CTE$_L$99	> CTE$_H$99	(3.04)	2.64
2.00-2.75	> CTE$_L$99	> CTE$_H$99	(0.04)	1.36
2.75-3.75	> CTE$_L$99	> CTE$_H$99	(0.84)	1.31
3.75-6.00	> CTE$_L$99	> CTE$_H$99	(1.23)	1.24
6.00-10.00	> CTE$_L$99	> CTE$_H$99	(2.03)	2.64
10.00+	> CTE$_L$99	> CTE$_H$99	(3.04)	2.30
Note: Based on transition analysis of long and short treasury rates, initial daily values compared to subsequent 1 year average.				

Speed and duration of interest rate changes—The NY7 Scenarios reflect all changes over a 10-year period, with scenarios 2 and 5 increasing/decreasing rates 5% over 10 years, and scenarios 3 and 6 increasing/decreasing rates 5% over five years and then returning to their original level over the next five years. We modified the Rate Change CTE analysis to quantify the CTE level associated with a 5% change over 10 years, for both the MDS Long and MDS Short Interest Rate Series. This analysis is summarized in Table 20 and indicates that, generally speaking, a 5% change in interest rates over a 10-year period is an extreme event. For Long Rates, this sort of change has been a CTE98 or greater event overall. By Interest Rate Group, for only two Groups was such a change less than a CTE99 event, both driven by the 1970s–1980s period. For Group 5, with initial interest rates greater than 10%, the mean reduction in Long Rates over 10 years is greater than 5%, indicating a strong downward reversion when rates are that high. And for Group 3, a 5% increase has been a CTE92 event, far from unprecedented, but still beyond moderate. For Short Rates, not surprisingly, a 5% change over 10 years is not as extreme as for Long Rates, but still beyond moderate. Overall, a 5% reduction over 10 years has been a CTE93 event, and a 5% increase has been a CTE96 event. When the initial Short Rate is below 4%, such changes are unprecedented, with a maximum observed increase of 4.29% and a maximum observed decrease of 3.42%. This magnitude of change has been more likely when rates are higher, with the 1970s–1980s period producing instances where Short Rates increased nearly 10% over 10 years and decreased nearly 9% over 10 years.

Although not explicitly shown in Table 20, another key aspect of these findings was that we must consider interest rate change periods longer than 10 years. For the Reversion Scenarios, the change period is 15 years; for the Rate Change CTE Scenarios and the AIRG Scenarios, the change period is 30 years; and for the Interest Rate Cycle Scenarios, rates continue to change for the duration of the scenario. The findings shown in Table 20 did not modify our scenario construction in any significant way, but served to confirm our expectations.

Table 20 - CTE Analysis of NY7 Scenarios 2 and 5 Sample CTE Level for 5% change over 10 years				
Rate Groupings	NY5 - Down 5% over 10 yrs	NY2 - Up 5% over 10 yrs	Min 10 yr Chg	Max 10 yr Chg
Short Rates:				
Total	CTE$_{L}$93	CTE$_{H}$96	(8.66)	9.71
0.00-0.50	> CTE$_{L}$99	> CTE$_{H}$99	0.10	2.24
0.50-2.50	> CTE$_{L}$99	> CTE$_{H}$99	(2.44)	3.56
2.50-4.00	> CTE$_{L}$99	> CTE$_{H}$99	(3.42)	4.29
4.00-6.00	CTE$_{L}$93	CTE$_{H}$91	(5.68)	9.71
6.00-15.00	CTE$_{L}$50	CTE$_{H}$99	(8.66)	5.00
Long Rates:				
Total	CTE$_{L}$99	CTE$_{H}$98	(5.44)	7.46
2.00-2.75	> CTE$_{L}$99	> CTE$_{H}$99	(0.74)	1.82
2.75-3.75	> CTE$_{L}$99	> CTE$_{H}$99	(1.81)	3.92
3.75-6.00	> CTE$_{L}$99	CTE$_{H}$92	(2.68)	7.46
6.00-10.00	> CTE$_{L}$99	> CTE$_{H}$99	(3.31)	4.38
10.00+	CTE$_{L}$30	> CTE$_{H}$99	(5.44)	(2.73)
Note: Based on analysis of MDS Long and MDS Short interest rate series, yearly average for year X compared with yearly average for year X+10				

6.2 MDS CTE Reversion Target Scenario Development

Table 21 shows the CTE Reversion Target Scenarios (Reversion Scenarios). These scenarios are numbered MDS1–MDS8.

Table 21—CTE Reversion Target Scenarios		
Scenario Number	Scenario Name	Scenario Description
MDS1	Reversion: High	Grade linearly to an 85$_{H}$CTE (right tail) reversion target over a 15-year period
MDS2	Reversion: Low	Grade linearly to a 85$_{L}$CTE (left tail) reversion target over a 15-year period
MDS3	Delayed Reversion: High	Long and Short Rates level for five years, then grade linearly to 85$_{H}$CTE reversion target over a 10-year period
MDS4	Delayed Reversion: Low	Long and Short Rates level for five years, then grade linearly to 85$_{L}$CTE reversion target over a 10-year period
MDS5	Pop-up with Reversion: High	Initial pop-up, then grade linearly to 85$_{H}$CTE reversion target by year 15
MDS6	Pop-down with Reversion: Low	Initial pop-down, then grade linearly to 85$_{L}$CTE reversion target by year 15
MDS7	Delayed Pop-up with Reversion: High	Long and short rates level for five years followed by pop-up, then grade linearly to 85$_{H}$CTE reversion target by year 15
MDS8	Delayed Pop-down with Reversion: Low	Long and short rates level for five years followed by pop-down, then grade linearly to 85$_{L}$CTE reversion target by year 15

The necessary parameters for the Reversion Scenarios were the reversion targets, the reversion period and the reversion pattern.

Table 22 presents the development of the reversion targets. The Long Rate Reversion Targets were set directly from the empirical analysis, with rounding applied. The Short Rate Reversion Targets considered the difference in observation periods for the MDS Long and MDS Short Rate Series, along with the observed shift in term structure after World War I. To adjust, we quantified the average spread between Long and Short Rates over the period 1920–2015, and applied those spreads to the MDS Long Reversion Targets. The columns labeled Scenario Parameter show the parameters used in the scenario development.

Table 22 - Reversion Target Scenarios
Development of Reversion Targets

	Observed Values		Scenario Parameter	
Description	CTE$_{L}$85	CTE$_{H}$85	CTE$_{L}$85	CTE$_{H}$85
(1) Long Rate long-term CTE Values, 1729-2015	2.59	7.56	**2.60**	**7.50**
(2) Long Rate long-term CTE Values, 1920-2015	2.46	9.94		
(3) Short Rate long-term CTE Values, 1920-2015	0.12	8.61		
(4)=(2)-(3) Spread, 1920-2015	2.34	1.33	2.10	1.25
(5)=(1)-(4) Short Rate long-term CTE Values, extrapolated 1729-2015			**0.50**	**6.25**

We chose a consistent 15-year total reversion period for all the Reversion Scenarios, on the basis of correlation analysis indicating that the correlation with the initial rate has worn off significantly by year 15. The analysis could have supported a 20-year reversion period, but we opted for 15 years as being slightly more conservative in most cases. For the Reversion Scenarios, the year 15 rate should be held level for longer projection periods.

Scenarios MDS1 and MDS2 are the basic reversion scenarios, with simple linear reversion to the high-rate and low-rate targets, respectively, over the 15-year reversion period. Scenarios MDS3 through MDS8 use different reversion patterns to address different risks.

Scenarios 3 and 4 delay the start of the reversion by five years and shorten the reversion period to 10 years, reaching the reversion targets after 15 years. This scenario is designed to capture the likelihood that one remains in the current environment for a sustained period before reverting. The five-year delay was developed judgmentally, not through any particular data analysis, since the Rate Change CTE Scenarios capture these timing elements more robustly.

Scenarios 5 and 6 employ a pop-up and a pop-down, respectively, in year 1, then grading linearly to the reversion target over the next 14 years. The pop-up and pop-down are based on the Rate Change CTE Scenarios, and include two pieces. The first piece is a CTE85 transitional pop, reflecting the risk of a pop from the initial rate to the next year's average rate. The second piece is the one-year Rate Change used in the Rate Change CTE Scenarios. These two pieces were summed to equal a single pop-up amount. We considered including only the transitional pop, but decided it would be prudent to add the other piece, because the magnitudes of the pops were generally very modest.

Scenarios 7 and 8 combine the five-year delay and the initial pop-up or pop-down.

6.3 MDS Rate Change CTE Scenario Development

Table 23 presents the Rate Change CTE Scenarios, scenarios MDS9–MDS12.

Table 23—Rate Change CTE Scenarios		
Scenario Number	Scenario Name	Scenario Description
MDS9	85_HCTE Rate Changes	Change from initial rate based on 85_HCTE (right tail) historical change statistics for the applicable Interest Rate Group
MDS10	85_LCTE Rate Changes	Change from initial rate based on 85_LCTE (left tail) historical change statistics for the applicable Interest Rate Group
MDS11	85_HCTE Change w Transitional Pop-up	Change from initial rate based on 85_HCTE (right tail) historical change statistics for the applicable Interest Rate Group, with initial pop-up based on 85_HCTE transitional change
MDS12	85_LCTE Change w Transitional Pop-down	Change from initial rate based on 85_LCTE (left tail) historical change statistics for the applicable Interest Rate Group, with initial pop-down based on 85_LCTE transitional change

The Rate Change CTE Scenarios are based on the empirical rate change CTE analysis discussed in Section 5.5.2. That analysis developed statistics for the t-year change in interest rates from an initial rate level, with t ranging from 1 to 30 years. As a result the Rate Change CTE Scenarios project interest rates for scenario years 1 to 30 by developing rate change parameters for change periods of 1 to 30 years and applying those parameters to the initial interest rates on the scenario start date. The rate change parameters vary by Interest Rate Group, so that the projected changes in interest rates are dependent on the initial rate level. The Rate Change CTE Scenarios converge to an ultimate rate in year 31. For projection periods longer than 31 years, the year 31 rate should be held level.

MDS9 and MDS10 are the basic Rate Change CTE Scenarios and assume that the first year's average interest rate equals the initial interest rate, which represents a best estimate assumption for the transition effect.

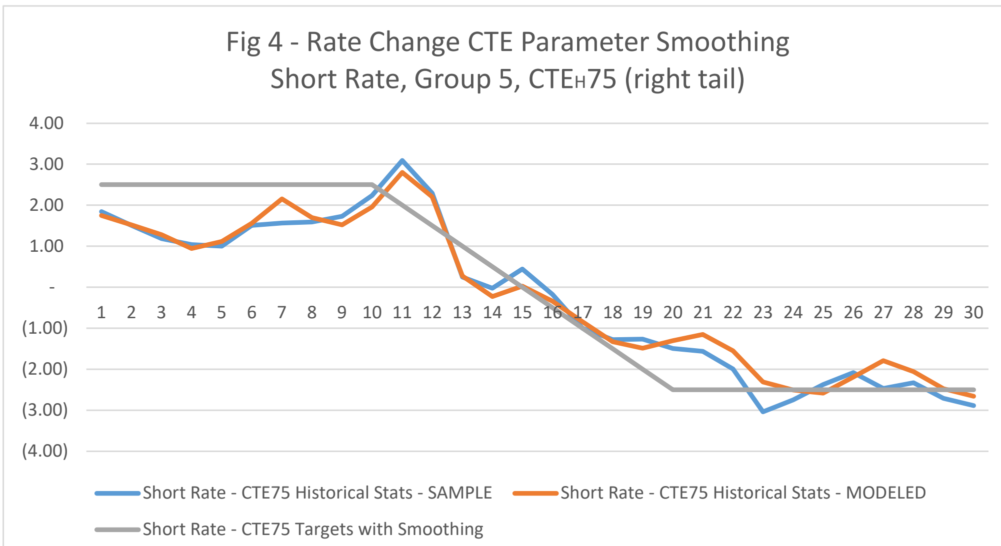
MDS11 and MDS12 add a transitional pop-up or pop-down in the first year, based on the CTE85 results of the transitional rate change analysis. The Interest Rate Group is reset after the transitional pop to determine the subsequent rate change parameters.

Two very important additional steps were necessary in constructing the Rate Change CTE Scenarios:

1. ***Smooth the Empirical Rate Change CTE values to generate scenario parameters*—**As discussed in Section 5.7.2, sample size was quite limited for many of the Interest Rate Groups, limiting the credibility of both the sample CTE values and the modeled CTE values. Therefore, the amount of volatility in the resulting values required smoothing. The smoothing process is described further below. Without such smoothing, there would be greater risk of instability in the Rate Change CTE Scenarios.

   The smoothing process considered several criteria: (1) consistency with the empirical values, (2) simplicity in the pattern of values with changes in time of change t, (3) consistency of results for adjacent Interest Rate Groups and (4) consistency of smoothed results for absolute and relative change statistics. The smoothing process involved art as well as science. It was performed manually and relied on our judgment of what was reasonable. The smoothing process was applied for absolute and relative rate change data.

Figure 4 shows graphically an example of the development of the smoothed rate change parameters, compared with the sample values and modeled values. This graph shows the parameter development for Short Rates, Interest Rate Group 5 (6%+), CTE_H75 (right tail). Recall that for short rates, we used a CTE75 standard rather than a CTE85 standard, to adjust for the difference in observation period between the MDS Long and MDS Short Interest Rate Series (see discussion in Section 5.7.1). The empirical data, represented by the blue and orange lines, show that for rate change periods of one to 12 years, CTE75 value of the right tail of the distribution reflected increases of 1% to 3%, with no strong directional pattern. For the next 10 years, the data reflect the mean reversion effect, with the net change falling to zero (i.e., rates back to their initial level) by year 15 or 16. Rates continue to fall from there, settling in at a net change from the initial rate of −2% to −3% beginning in year 22. For scenario projection purposes, we smoothed this to a level rate for the first 10 years 2.5% above the initial rate. Over the next 10 years, we graded linearly from a 2.5% net increase to a 2.5% net decrease, then holding the change flat for the last 10 years.



Considering both the Long and Short Rates, the five Interest Rate Groups, the two tails and absolute and relative changes, we performed a similar smoothing exercise approximately 30 times. To avoid overfitting, we tried to set a limit of no more than three linear segments for one fitting. Appendix G contains the details of the smoothing for all segments, presenting the data in both tabular and graphical form. Appendix G is presented in five sections, as shown below, and is also available as an Excel workbook to allow for additional analysis by the reader. The graphical Appendices G.2–G.5 also provide specific parameter fitting notes:

Appendix G.1—Tables of all Rate Change CTE parameter development
Appendix G.2—Graphs of Rate Change CTE_L parameter development, Long Rates
Appendix G.3—Graphs of Rate Change CTE_H parameter development, Long Rates
Appendix G.4—Graphs of Rate Change CTE_L parameter development, Short Rates
Appendix G.5—Graphs of Rate Change CTE_H parameter development, Short Rates

2. *Eliminate discontinuities at the boundary between Interest Rate Groups*—Because Rate Change parameters vary by Interest Rate Group, it is possible to generate very different scenarios due to small changes in the initial rate. For instance, the boundary between Long Rate Groups 1 and 2 is 2.75%, so that scenarios projected from a 2.74% initial rate could be very different from scenarios projected from a 2.76% initial rate, due to differences in Group 1 and Group 2 parameters. In considering this problem, we also recognized that the boundaries between Interest Rate Groups are somewhat arbitrary. Although we chose the boundaries so that groupings would be as nearly homogeneous as possible, the boundaries could easily have been shifted slightly. The grouping process reflects the assumption that there are factors specific to an Interest Rate Group that drive the rate change patterns for that Group. As rates move from one Group to another, the drivers of rate change behavior would be expected to gradually shift from one Group to the other rather than jump suddenly.

   Based on this thought process, we created a weighting process to eliminate discontinuities. This process is described and illustrated as follows:

   Example:
   Initial Long Rate 3.00%

   Long Rate Interest Rate Group Values:
   Group 1: Low Rate 2.00% High Rate 2.75% Midpoint 2.375%
   Group 2: Low Rate 2.75% High Rate 3.75% Midpoint 3.25%

   a. Determine the Interest Rate Group based on the initial rate. This is denoted the Primary Group. In the example, Group 2 is the Primary Group.
   b. Determine whether the initial rate is above or below the midpoint of the Primary Group. If it is below the midpoint, the next lower Group is denoted the Secondary Group; if above the midpoint, the next higher Group is the Secondary Group. In the example 3.00% is below the midpoint of Group 2, so Group 1 is the Secondary Group.
   c. Compute the weighting between the Primary and Secondary Groups based on where the initial rate falls between the midpoint and the boundary of the two Groups. If the initial rate falls right on the midpoint of the Primary Group, the Primary Group gets 100% weight. If the initial rate falls right on the boundary between the two Groups, the Groups each get 50% weight. Between the boundary and the midpoint, grade linearly. For the example, the initial rate of 3.00% is midway between the midpoint and the lower boundary of Group 2, so the Group 2 weight is 0.5 x 100% + 0.5 x 50% = 75%. Group 1 weight is then 25%.
   d. When computing the Rate Change Scenario values, compute the Rate Change parameters as the weighted average of the appropriate scenario parameters, using these weights. Group weighting is computed independently for Long and Short Rates, since the Long and Short Interest Rate Groups are independent of one another.

Developing the tables of annual Rate Change parameters for all the Interest Rate Groups constitutes most of the work of creating the Rate Change CTE Scenarios. The process to generate the scenarios is largely a lookup process, and the steps are as follows for the Long Rate in scenarios MDS9 and MDS10:

1. Look up the Primary Interest Rate Group and Secondary Interest Rate Group for the initial interest rate. The initial rate is used for scenario year 1.
2. Compute the primary and secondary weightings, as described above.
3. For t = 1-30, look up four values from the rate change parameter tables: (a) the absolute change parameter for the Primary Group, (b) the absolute change parameter for the

Secondary Group, (c) the relative change parameter for the Primary Group and (d) the relative change parameter for the Secondary Group.

4. For both the Primary and Secondary Interest Rate Groups, compute the floored Rate Change parameter. If the absolute change parameter is positive, the floored Rate Change parameter equals the absolute change parameter. If the absolute change parameter is negative, the Floored Rate Change parameter is the greater of the Absolute Change parameter and the Relative Change Parameter times the initial rate.
5. Apply the Primary and Secondary weightings to the Primary and Secondary Floored Rate Change parameters to generate the Weighted Floored Rate Change parameter.
6. The interest rate for scenario year t + 1 is set equal to the scenario year 1 rate plus the Weighted Floored Rate Change parameter for year *t* (note that changes are not applied until scenario year 2, so year t values from the Rate Change parameter tables are used to generate the rates for scenario year t + 1).

For the Short Rate for scenarios MDS9 and MDS10, if the "Independent" Short Rate option has been selected, the same steps are used to compute the scenario Short Rates. If the "Flat," "Steep" or "Mean" Short Rate options have been selected, the Short Rate is computed as a spread to the Long Rate as described in Section 6.1.

Scenarios MDS11 and MDS12 incorporate a transitional pop-up or pop-down, respectively. For these scenarios, the interest rate for scenario year 1 is computed by applying a transitional increase or decrease to the initial rate. The transitional increase/decrease is computed using steps 1–6 described above, but with parameter values looked up from the Transitional Rate Change parameter table. The scenario interest rates for years 2 through 31 are then computed as per steps 1–6, but the scenario year 1 interest rate (after the transitional change) is used as the initial rate for the purpose of establishing Primary and Secondary Interest Rate Groups and weightings.

6.4 MDS Interest Rate Cycle Scenarios

Table 24 presents the two Interest Rate Cycle Scenarios, MDS13 and MDS14.

Table 24—Interest Rate Cycle Scenarios

Scenario Number	Scenario Name	Scenario Description
MDS13	Cyclical, 20 year cycle	20-year cycles of interest rates: five years declining, 10 years flat, five years increasing
MDS14	Cyclical, 40 year cycle	40-year cycles of interest rates: 10 years declining, 20 years flat, 10 years increasing

As discussed in Section 5.8, the historical data are inadequate for setting Interest Rate Cycle parameters. The general shape of the scenarios—a flat bottom between peaks—is based on the observation that rates tend to remain near their bottom for a while, but tend to increase to a peak and retreat from the peak more quickly. The two cycle lengths are based on observations of some cycles of 20 years or less, and others with a much longer cycle period. Given that the projection period for many actuarial models is 50 years or less, we did not consider that a cycle period longer than 40 years would be useful.

Table 25 shows the complete scenario parameters for the cyclical scenarios. We set the peak and the valley rates based on the observation that shorter cycles have tended to be milder than longer cycles. However, we set the rates for both scenarios to be relatively consistent with moderately adverse levels based on our other analyses. For all cycles before the latter 20th century, rates peaked at or below 6%, which formed the basis of our peak for the shorter cycle. The trough rate for the 20-year cycle was set to be slightly milder than the CTE_L85 level. The peak and trough of the 40-year cycle were set to be modestly more extreme than the moderately adverse reversion targets.

Table 25 - Interest Rate Cycle Scenario Parameters						
		Cycle Segments - Years			Trough	Peak
Scenario	Rate	Decline	Flat	Increase	Rate	Rate
MDS13	Long	5	10	5	3.00	6.00
	Short	5	10	5	1.00	6.00
MDS14	Long	15	10	15	2.50	8.00
	Short	15	10	15	0.50	8.00

The last parameter needed for the cyclical scenarios is the entry point, which is based on user input. The user identifies and inputs the cycle segment that exists on the scenario start date—decreasing, flat or increasing—based on recent interest rate history. The user then estimates the period of time that rates have been in that segment. We have not developed any specific tools for this purpose, but the instructions tab in Appendix J, the Excel-based MDS Scenario Calculator, provides additional guidance. The scenario formulas complete the current cycle based on these inputs and then projects future segments.

The cyclical scenarios do not converge to an ultimate rate. The scenario calculations in tabs “MDS13” and “MDS14” of Appendix J project the cycles for 100 years. The “Scenario Output” tab includes only 31 years of output, so for longer projection periods, the user can pull the additional years of output directly from the “MDS13” and “MDS14” tabs.

6.5 MDS AIRG Scenarios

Table 26 presents the two AIRG Scenarios, MDS15 and MDS16.

Table 26 - AIRG-based Scenarios		
Scenario Number	Scenario Name	Scenario Description
MDS15	AIRG CTE_H85	Rates based on 1000 scenarios from Academy interest rate generator, CTE_H85 of cumulative average rates, annualized.
MDS16	AIRG CTE_L85	Rates based on 1000 scenarios from Academy interest rate generator, CTE_L85 of cumulative average rates, annualized.

The AIRG Scenarios utilize a calculation similar to the Rate Change CTE Scenarios, but applied to a set of stochastic scenarios generated by the Academy Interest Rate Generator. There are a couple of subtle differences between the two calculations.

First, the initial rate is known, so the CTE calculations can be applied to the projected rates rather than the change in rates. This simplifies the calculations, and it is easily shown that the result is the same.

Second, rather than computing the CTE value for the year t interest rate directly, we compute the CTE value for accumulated interest from the model start date through year t, and then divide by the prior year’s accumulated CTE value. We took this approach to address the volatility inherent in the stochastic scenarios. From one projection year to the next, the scenarios included in the interest rate tails may change, and if a scenario enters the tail computation on the basis of a single year of interest, we believe the distance of the CTE value from the mean may be overstated. We considered that results would be more stable and reasonable if a scenario is counted in the tail computation on the basis of its compounded interest rate entering the tail, not just a single year's value.

The full AIRG process is as follows:

1. Run the AIRG to generate the stochastic scenarios. Choose the annual time step, a 30-year projection period and nominal yields. We set up our computations based on the Academy's 1000 scenario subset. The user could choose a different number of scenarios if desired.
2. Compute the accumulated value ${}_iAV_t$ for each scenario i and projection year t.
3. For each projection year t, compute the CTE values for the accumulated values: $CTE_L85(AV_t)$ and $CTE_H85(AV_t)$.
4. For each projection year t, compute the CTE value for that year's interest rate as $CTE85(i_t) = CTE85(AV_t) / CTE85(AV_{t-1}) – 1$. This computation should be done for both the high and low CTEs. The resulting interest rate is the scenario rate for year t.

Appendix K is an Excel workbook that can be used to compute the MDS AIRG Scenarios. These scenarios converge to their ultimate rate by year 30, and Appendix K provides 30 years of output.

6.6 Completing the Yield Curve

The scenario computations described above generate two interest rates: a Short Rate equal to the constant maturity 90-day Treasury bill rate and a Long Rate equal to the average of the 20- and 30-year CMT rates. From these two rates, it is necessary to complete the full yield curve for use in cash flow testing models. For this purpose, we performed a two-factor linear regression against the Long and Short Rates for each of the following Treasury constant maturity rates: 90-day, 180-day, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year. The regression formula is of the form

$y = m_1x_1 + m_2x_2 + b$

where x_1 and x_2 are the Long and Short Rates and y is the calculated rate for the specific maturity

We performed the regressions using daily values for the given Treasury series, for as long as those series were available. Table 27 shows, for each Treasury maturity, the regression parameters and the R^2 values. The fit of the regression formulas were very strong, as indicated by the R^2 values, the lowest of which is 0.9897.

Table 27 - Full Yield Curve Construction Regression Parameters by Maturity				
	Regression Parameters			
US Treasury Maturity	Long Coefficient (m1)	Short Coefficient (m1)	Intercept (b)	Regression R-square
0.25	1.000	-	-	1.0000
0.5	1.019	0.067	(0.195)	0.9986
1	0.943	0.163	(0.294)	0.9960
2	0.713	0.444	(0.872)	0.9946
3	0.693	0.376	-	0.9965
5	0.432	0.673	(0.517)	0.9897
7	0.273	0.827	(0.606)	0.9918
10	0.186	0.869	(0.294)	0.9947
20	0.021	0.987	-	0.9999
30	(0.027)	1.017	-	0.9999

In spite of the strong fit of the regression formulas, on any scenario start date the actual rates for the various constant maturity Treasuries will not precisely equal the computed rates using the regression formulas. Therefore, we compute the difference between the actual and regressed initial CMT value for each maturity and grade that difference off linearly over 15 years.

The regression parameters are presented as well in Appendix I, which is also available as an Excel workbook to allow for additional analysis by the reader.

Section 7: Use of the MDS Interest Rate Scenarios

We have discussed extensively the development of the historical MDS Interest Rate Series, the empirical analysis and the construction of the MDS Scenario set. We now present the resulting scenarios. We will discuss calculation of the scenarios from the user's perspective, considerations in the use of the scenarios, and the December 31, 2015, MDS Interest Rate Scenarios.

7.1 User Calculation of Scenarios

The MDS Scenarios, while deterministic in nature, are more complicated to compute than the NY7 and involve multiple computation steps. If we presented them formulaically, they would be challenging to program accurately. Instead, we have developed two separate Excel workbooks to calculate all 16 MDS Interest Rate Scenarios. The first calculation workbook, provided as Appendix J, computes scenarios MDS1–MDS14, all of the scenarios except for the AIRG-based scenarios. The second workbook, provided as Appendix K, computes the AIRG-based scenarios MDS15–MDS16. Both workbooks include a full set of instructions, and parameters and computations are fully accessible.

Users who wish to program the scenario calculations themselves should be able to do so using the Excel workbooks and Section 6. This may not be necessary, however, as the Excel workbooks are straightforward to use. Below we provide an overview of the two workbooks.

7.1.1 Scenario MDS1-MDS14 Calculation Workbook-Appendix J

Appendix J is self-contained and requires limited user input. Calculating the MDS Scenario set with this workbook does not require prior knowledge of the *Modern Deterministic Scenarios for Interest Rates* report or of the scenario construction process. However, the user should first read the Purpose and Use on the "Read First" tab and the workbook instructions on the "Instructions" tab. These tabs are intended to provide all guidance required for a user to calculate scenarios MDS1–MDS14. The "Instructions" tab includes (1) descriptions of all the workbook tabs, (2) step-by-step instructions for computing a scenario set and (3) guidance in choosing user input parameters and using the scenario sets.

User inputs are very simple. For the initial yield curve input, one may input the CMT rates manually or may enter the valuation date and choose to look up the rates from a preloaded table. At the time of this initial report, interest rates had been loaded for all dates from January 1, 1982, to January 6, 2017. The user also selects the short rate modeling method: independent, flat, mean or steep. Finally, the user inputs two initialization parameters for the Interest Rate Cycle Scenarios MDS13 and MDS14: the initial/current cycle segment (increasing, decreasing or flat) and the period that rates have been in that segment.

No other inputs or calculation steps are required of the user. After the inputs are complete, the user can view the projected Short Rates and Long Rates, both in tabular form and graphically, in the "Scenario Summaries" tab. The "Scenario Output" tab provides the projection of the full yield curve in a form that can be readily converted to import into the user's actuarial modeling software. Thirty years of output are provided in the "Scenario Output" tab. To extend scenarios MDS1–MDS12 to longer projection periods, the year 31 rate should be held level. To extend scenarios MDS13–MDS14 for longer projection periods, the user may find rates projected to 100 years on the "MDS13" and "MDS14" tabs, We have not provided a utility to convert the output into any other file formats, so that step is left to the user.

<u>NOTE</u>: We have left all formulas and scenario parameters unprotected in Appendix J, allowing the user flexibility to modify the scenario parameter and even calculations as he or she sees fit. We believe this is critical to furthering the research objectives of *Modern Deterministic Scenarios for Interest Rates.* We do not recommend such changes, however, without a thorough understanding of the entire workbook and this report.

7.1.2 AIRG Scenario MDS15-MDS16 Calculation Workbook—Appendix K

The AIRG Scenario calculation workbook, Appendix K, uses as input 1,000 interest rate scenarios generated using the Academy Interest Rate Generator. It includes an Instructions tab that is intended to provide all guidance required by a user, including (1) descriptions of all the workbook tabs, (2) step-by-step instructions for computing a scenario set and (3) a description of the calculation algorithm.

The workbook is set up to use 1,000 AIRG scenarios projected monthly for 30 years. This is a large volume of input data, so the file is quite large. The calculations could be easily replicated in Access or another program. While the calculations are not complicated, they are unwieldy due to the volume of data. It is important to follow the steps precisely to avoid errors. For projection periods longer than 30 years, the year 30 rate should be held level.

Output is provided in the same format as the Appendix J workbook.

7.2 Considerations in the Use of the MDS Scenarios

Annual time step—The scenarios have been constructed to use an annual time step. The rate change from the initial rate to the year 1 rate should occur immediately after the model start date and should be held constant for 12 full months. The user may wish to use a shorter time step (e.g., monthly or quarterly). In that case, we recommend interpolating in such a way that the average rates for each projection year are preserved.

Method for Computing Short Rates—If the user's results are sensitive to yield curve shape, the selection of the Method for Computing Short Rates may be important to results. The user should understand the four available methods (independent, flat, mean, steep) and should select a method appropriate to his or her liabilities. When in doubt, sensitivity testing with alternate methods may be appropriate.

Implications of two-tailed CTE analysis—As discussed in Section 5.3, the two-tailed CTE analysis underlying the MDS Scenarios presumes that adverse scenarios are relatively evenly distributed between high-rate scenarios and low-rate scenarios. In reality, this will often not be the case, due to liability characteristics and/or the initial interest rate environment. In an extreme case, where adverse projection results are all attributable to one tail of the interest rate distribution, the MDS Scenarios may be considered more than moderately adverse, representing a CTE85 result rather than a CTE70 result. This result is not unique to the MDS Scenarios, but is a characteristic of virtually any deterministic scenario set developed to be applied broadly across product types. In most cases, we do not believe this would introduce undue conservatism, but the user should consider that possibility.

Model Input/Output Equivalence Assumption—As discussed in Section 5.3, the methodology of Modern Deterministic Scenarios for Interest Rates implicitly assumes that by applying a CTE measure to a model assumption such as the interest rate scenario, the projection model will produce results consistent with that CTE level. This is an untested assumption, and the user should validate it before relying on the MDS Scenarios.

Reasonableness Checking—The MDS Scenarios are relatively complicated as deterministic scenarios go. As such, there is a risk that certain input conditions will produce scenarios that are unreasonable in some way. We have tested to ensure that the calculations do not produce any unintended consequences, but the user should always review the resulting scenarios for reasonableness.

7.3 December 31, 2015, MDS Scenarios

We have computed interest scenarios as of December 31, 2015. The scenarios are presented graphically in Figures 5–8. Figures 5 and 6 show the high-rate and cyclical scenarios, along with NY7 Scenarios 2–4. Figure 5 contains Long Rates and Figure 6 contains short rates. Figures 7 and 8 show the low-rate scenarios alongside NY7 Scenarios 5–7: Figure 7 with the Long Rates and Figure 8 with the Short Rates. See Sections 6.2–6.5 for the MDS Scenario descriptions.

We note that although this report was published in 2017, it was substantially completed in 2016 before December 31, 2016, data were available. We considered updating this section to reflect a December 31, 2016, date, but chose not to, because the interest rate changes between December 31, 2015, and December 31, 2016, were generally small and would produce similar results. Appendix J has been updated to include data through December 31, 2016, so the reader may make this comparison using this Excel workbook.

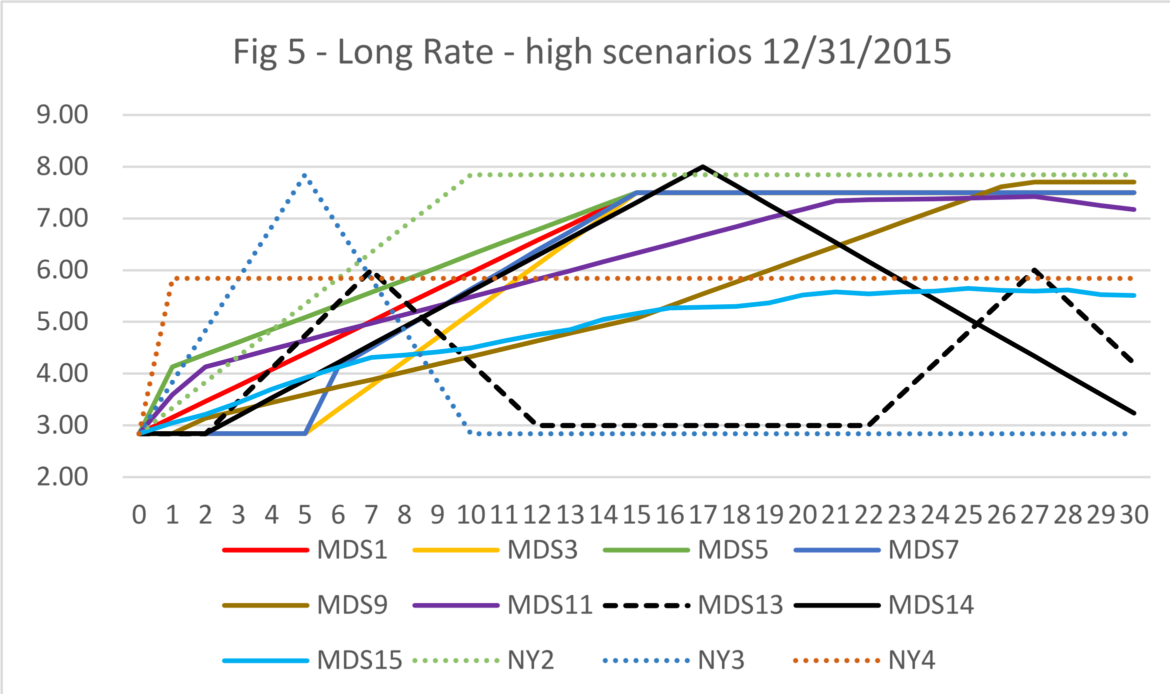


Fig 5 - Long Rate - high scenarios 12/31/2015
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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
MDS1
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MDS9
MDS11
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MDS14
MDS15
NY2
NY3
NY4

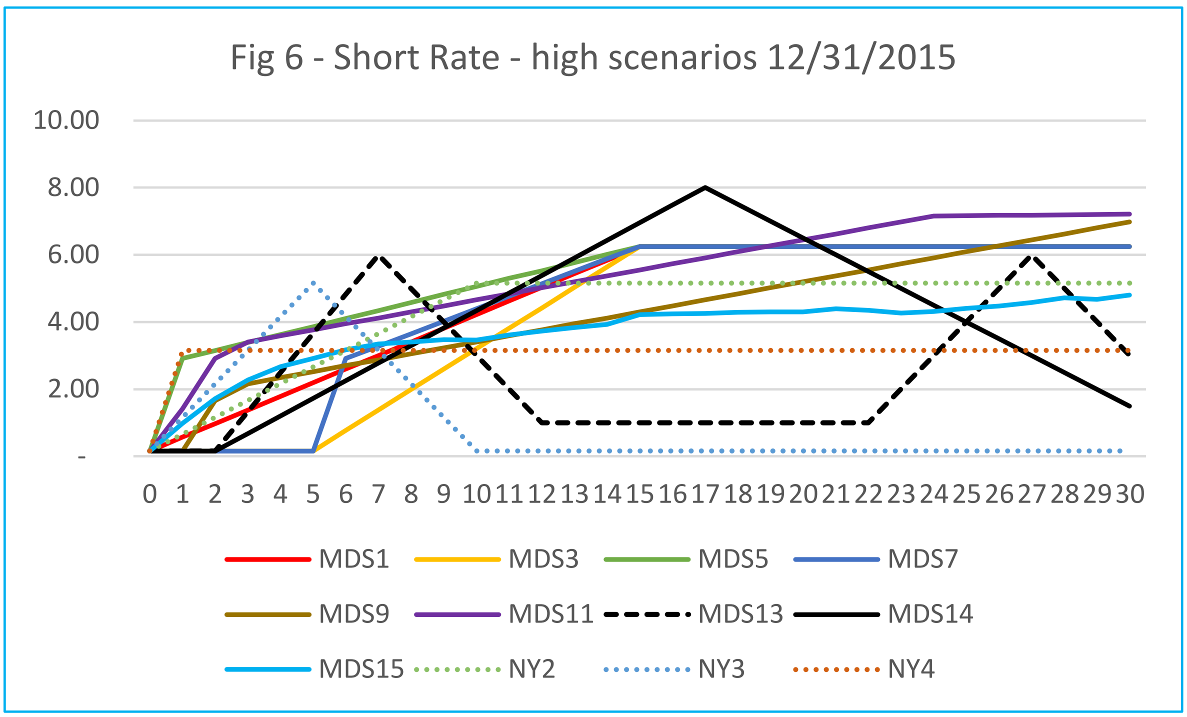
Fig 6 - Short Rate - high scenarios 12/31/2015
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-
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
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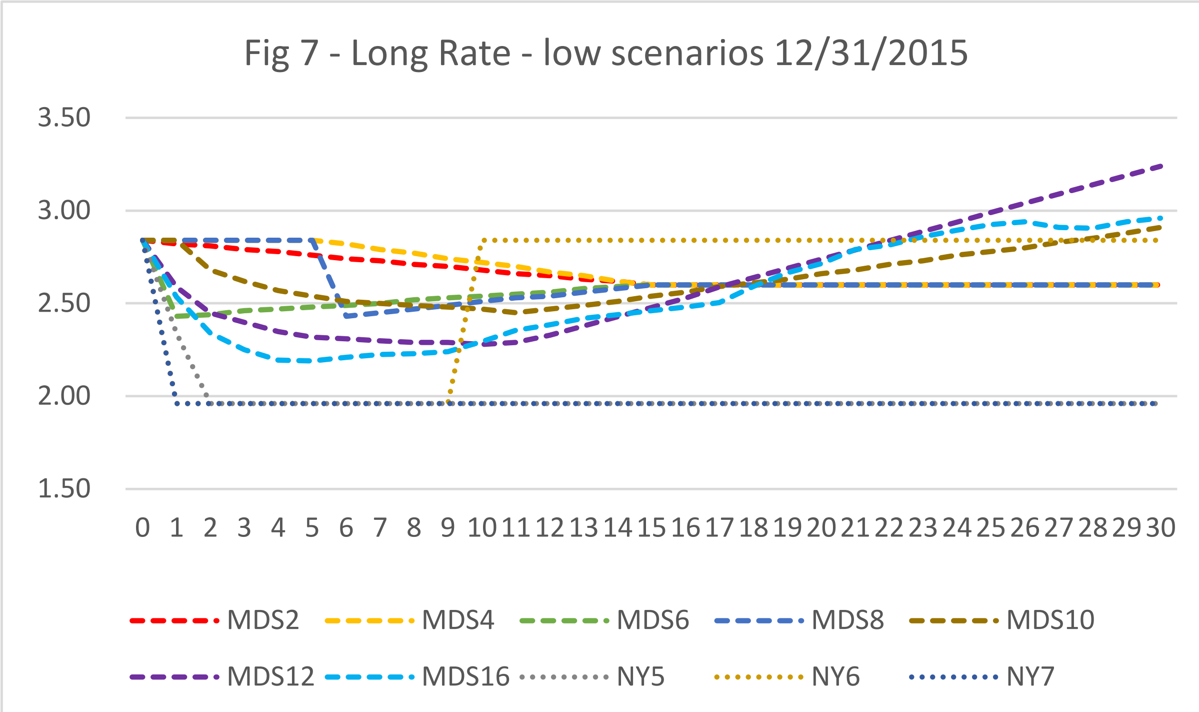
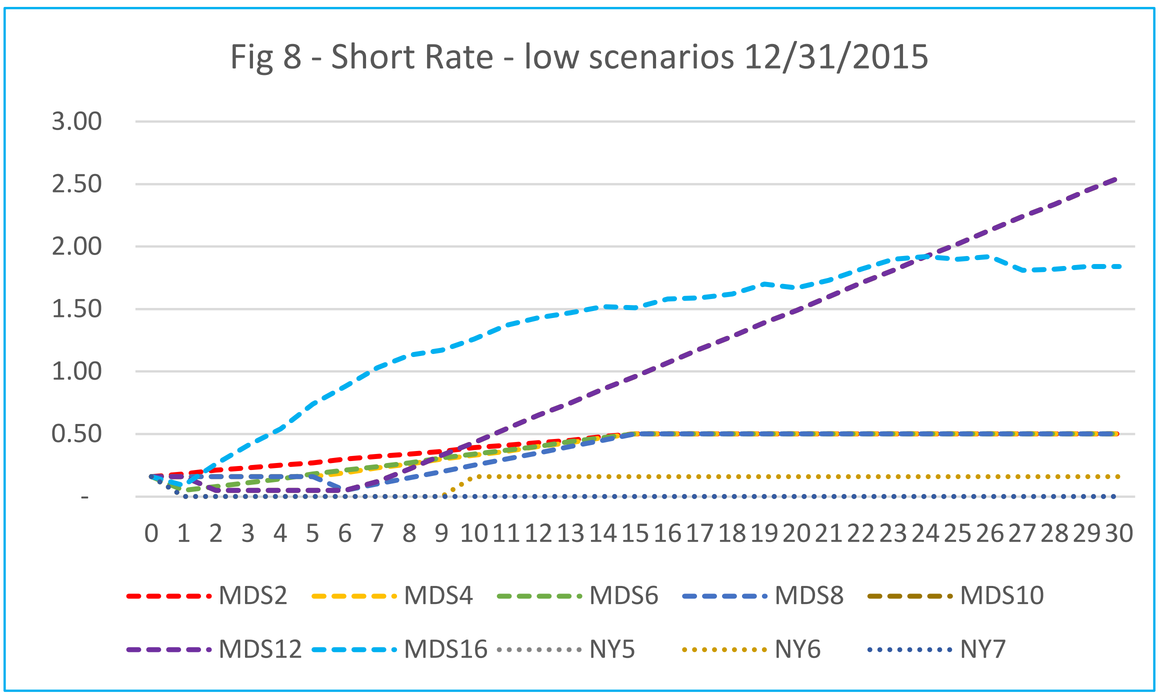
MDS15
NY2
NY3
NY4





These graphs are useful to understand the range of MDS Scenarios, but are perhaps not very useful to understand the scenario dynamics in a deeper way. Appendix O includes graphs that are more useful for this purpose. The graphs in Appendix O separate the MDS Scenarios by analytic framework, that is, separate graphs for the Reversion Target Scenarios, the Rate Change CTE Scenarios and the Interest Rate Cycle Scenarios, with each compared with only the most comparable scenarios from the NY7. From studying the graphs above and those in Appendix O, we can make a number of observations regarding the scenarios. Note that these observations are specific to a December 31, 2015, scenario start date and that different initial conditions may yield different relationships. In addition, note that the Short Rates use the independent calculation method in all of these graphs. Selection of a different Short Rate modeling method would result in different Short Rates.

- For Long Rates, the MDS scenarios exhibit more moderate changes in rates than the NY7 at December 31, 2015. The ultimate rates for the high-rate scenarios are similar between NY7 and MDS, but the NY7 rates increase more rapidly. In addition, the MDS low-rate Scenarios do not decrease as much as the NY7 decreasing rate scenarios, and the MDS Scenarios with the greatest decreases show a recovery in rates over time.
- Comparing the various MDS Scenarios for Long Rates, the Rate Change CTE Scenarios increase more slowly than the Reversion Target Scenarios in the high-rate subsets. In the low-rate subsets, the Rate Change CTE Scenarios show greater initial rate decreases than the Reversion Target Scenarios, but ultimately rebound to a higher ultimate rate level. For the low-rate subset, the AIRG scenario is very comparable to the Rate Change CTE Scenarios, but for the high-rate subset, the AIRG Scenario shows significantly smaller increases than any of the others.
- For Short Rates, the NY7 Scenarios and MDS Scenarios appear more comparable than for Long Rates. The MDS high-rate Scenarios increase at least as rapidly as the NY7 and to a higher ultimate level. The MDS low-rate Scenarios are less extreme than the NY7, showing reversion to a level higher than the starting rate. This reflects the fact that the MDS Scenarios do not employ parallel shifts.
- Comparing the various MDS Scenarios for Short Rates, the relationships are similar to those described above for Long Rates. In the high-rate subset, the Rate Change CTE Scenarios increase more slowly than the Reversion Target Scenarios, and the AIRG Scenario exhibits considerably smaller increases than the other scenarios. In the low-rate subset, the Rate Change CTE Scenarios decline more than the Reversion Target Scenarios but later rebound to a higher ultimate rate level, and the AIRG Scenario is very comparable to the Rate Change CTE Scenarios.
- The MDS Interest Rate Cycle Scenarios are not directly comparable to any of the NY7 Scenarios, but the most apt comparisons are to scenarios 3 and 6 of the NY7. The change patterns are very different, but the MDS Scenarios generally cycle to a higher level of interest rates, since the rate levels in future cycles are independent of the starting rate level.
- The magnitude of pop-ups and pop-downs in the MDS Scenarios differs considerably from the NY7, as discussed in Section 6.1. This is particularly true for the Long Rates, where pop-ups are half as large as those in NY7 Scenarios 4 and 7. For Short Rates, the MDS pop-ups are much more comparable to NY7 Scenarios 4 and 7.
- The MDS Scenarios do not support a view that the level scenario is more than moderately adverse at December 31, 2015. While producing more moderate rate decreases than the NY7 decreasing Scenarios, the MDS Scenarios show considerable decreases from the starting level.

These observations are consistent with our expectations based on earlier discussion.

Section 8: Non-Interest Rate Modeling Considerations

8.1 Common Stocks

Common stocks and other equity securities may comprise a portion of a life insurance company's general account investment portfolio, and the Appointed Actuary may deem common stock or other equity securities appropriate to include in the asset portfolio modeled for cash flow testing. Modeling equity returns can be a challenge in a deterministic context. The objective of this section is to discuss some factors that should be considered when including common stocks and other equity investments in deterministic projections performed under moderately adverse conditions, such as Asset Adequacy Analysis.

The considerations discussed in this section should not be used as a basis for modeling separate account accounts, equity derivatives or other funds where price risk is a dominant risk. In such cases, deterministic scenarios cannot substitute for stochastic equity return scenarios to quantify price risk. The considerations in this section were developed for the benefit of the actuary whose company holds equity assets in its general account and who considers it appropriate to include those assets as part of his or her cash flow testing portfolio. Equities often offer attractive expected returns relative to other asset classes, and any cash flow testing model that reflects equity returns must also reflect equity risks.

We did not consider it meaningful to attempt to create a scenario of equity returns tied to a scenario of interest rates, because correlation is not strong enough to provide reliable results. Therefore, we considered moderately adverse common stock modeling in terms of the following:

1. Average holding period returns
2. Price shocks
3. Available cash flows and
4. Portfolio diversification.

8.1.1 Moderately Adverse Holding Period Returns

Historical returns for a broad common stock index can be analyzed using empirical CTE analysis, similar to the analysis that was performed for interest rates. For analysis of historical common stock returns, we relied on historical Standard & Poor's composite stock index data compiled by Robert Shiller for *Irrational Exuberance* and maintained since then on his website. His data capture monthly average values beginning in 1871. Because we were not trying to develop common stock scenarios in the same way we were trying to develop interest rate scenarios, we did not consider it necessary to go back further in time.

Using the Shiller data, we computed monthly total returns including price returns and dividends. We then computed accumulated returns, implicitly assuming reinvestment of dividends. For each monthly initial index value, we then computed holding period total returns for holding periods of 10, 20, 30 and 40 years. Using these series of holding period returns, we then performed CTE analysis at CTE60, CTE70 and CTE80 levels. We considered only long positions, and since the impact of equity returns is single-tailed in this case, we did not need to bifurcate the tails as we did with interest rates.

Over the entire period 1871–2015, the average annualized return was 8.96%. The CTE results for the 10-, 20-, 30- and 40-year holding period returns over this period were as shown in Table 28.

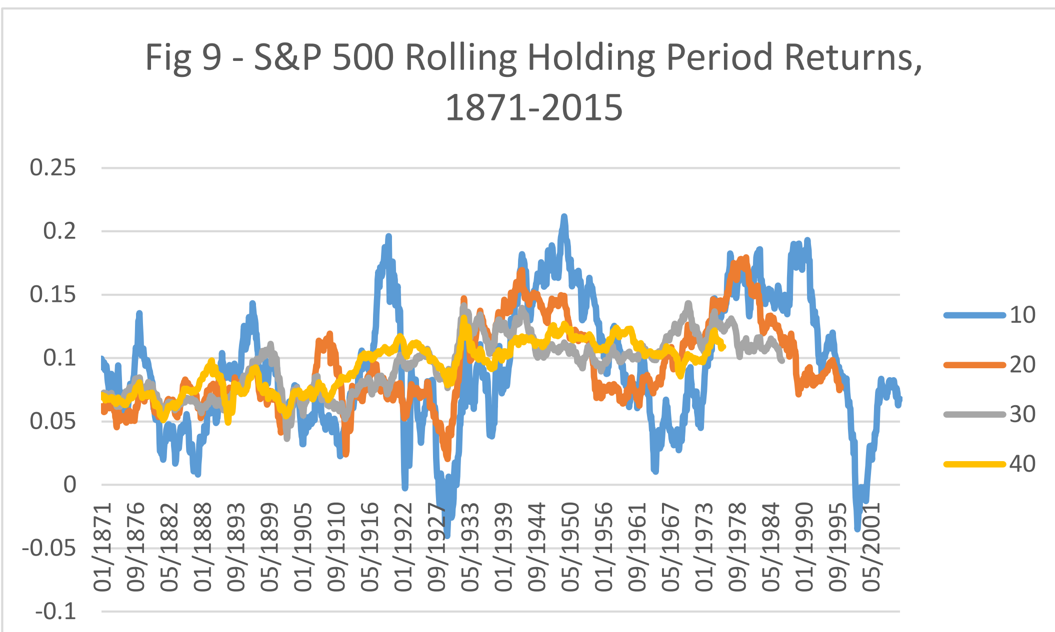
Table 28 - Standard & Poor's 500
Holding Period Total Return CTE Statistics 1871-2015

	Annualized Holding Period Return			
CTE level	10	20	30	40
CTE60	4.4%	6.4%	7.0%	7.2%
CTE70	3.5%	6.1%	6.6%	6.8%
CTE80	2.5%	5.6%	6.3%	6.5%

Some observations are as follows:

- The moderately adverse holding period return varies significantly with the holding period. This is expected, because price volatility has a relatively greater impact over shorter holding periods. This suggests that it may be appropriate to consider the projection period in setting equity return assumptions.
- The absolute difference between the returns at the different CTE levels also declines as holding period increases. At a 10-year holding period, the difference between CTE60 and CTE80 is 1.9%. For holding periods of 20 years and beyond, this difference does not vary significantly.

We considered whether older data are applicable for assumption-setting purposes. Figure 9 presents the rolling average holding period returns. This chart shows a general increase in the average holding period returns over time, but also greater volatility. In particular, it should be noted that the 10- and 20-year holding period returns have declined significantly in recent years, falling to the same range as the late 1800s. Thirty- and 40-year holding period returns have not fallen similarly, but we believe it is prudent, given the patterns in the shorter holding periods, to consider the longer historical period.



Overall, considering the CTE70 level, it appears that moderately adverse average holding period returns for a well-diversified common stock portfolio should not exceed 6.5–7.0%. If the projection horizon or the average life of the liabilities is less than 30 years, the holding period returns should be lower. A less diversified portfolio should use lower average holding period returns. Other types of equity securities would use similar considerations.

8.1.2 Moderately Adverse Price Shocks

To model common stocks without considering price shocks does not adequately consider the risks associated with these investments. We also computed historical CTE values for price shocks, to provide the reader with some input on price shocks that might be considered moderately adverse. Similar to our treatment of average returns, we considered price shocks over a holding period.

Using the Shiller data, we computed the one-year monthly average price return for initial month t as the average price for months t + 1 to t + 12, divided by the average price for month t. The price shock was defined to be the negative of the one-year monthly average price return. The price shock was computed separately for each monthly data point. Then the maximum holding period price shock was identified for each initial month t, for 10-, 20-, 30- and 40-year holding periods. Using each initial month, this process produced a series of holding period price shocks. We then performed empirical CTE analysis on this series.

We opted to use the monthly average price return instead of end-of-period price return to measure price shocks, so as to maintain consistency with the annual time step of the MDS Scenarios. Often a price shock is followed by a rebound. Measuring the shock using the end-of-period price return would tend to miss the effect of the rebound, overstating the net impact of the price shock on a full year's results. This is illustrated in the data for 1931. The Shiller data show an S&P500 price of 8.26 for March 1931, dropping to 4.77 at June 1931 and rebounding back to 8.26 at September 1931. By using monthly average price returns, we avoid overstating the effect of the price drop between March and June.

Table 29 shows the price shock CTE results. It does not show as much dispersion of results as one might expect, because the same price shock is applicable to holding periods beginning on a number of different dates. Generally, these data support an assumption that a price shock of 30% is consistent with moderately adverse conditions for a holding period of 10–20 years, compared with 40% or more for projection periods of 30–40 years.

Table 29 - S&P 500 Historical Price Shock Statistics, CTE Values by Holding Period

	Holding Period Price Shock			
CTE level	10	20	30	40
CTE60	-26.2%	-31.3%	-31.4%	-41.7%
CTE70	-28.7%	-31.3%	-41.3%	-43.2%
CTE80	-31.1%	-40.6%	-43.2%	-43.2%

This quantification may understate moderately adverse price shocks for two reasons: (1) this analysis uses monthly average values as initial values, which results in less volatility than if daily values were used, and (2) this analysis considers only a single-year price shock, when price drops in multiple years are not unheard of. While the reader may want to consider whether these factors call for larger price shock assumptions, we believe these data provide a reasonable basis for our conclusions.

One difficulty with modeling a price shock is that investment/disinvestment assumptions may significantly change the asset mix over the course of a projection, so the impact of a price shock may vary substantially depending on when in the projection the shock is assumed to occur. Following is a procedure that could be used to model the impact of a price shock without dependence on a specific shock date. This procedure would likely need to be applied outside the modeling system and can also be applied to other types of equity assets besides common stocks:

1. Determine the average holding period return assumption and the holding period price shock assumption. Using the data above, and assuming a 40-year projection, one might assume an average holding period common stock return of 6.8%, inclusive of the impact of a one-time price shock of 45%.

2. Calculate the holding period common stock return assumption excluding price shocks. For our example, the holding period return excluding price shocks is $[1.068^{40}/(1-0.45)]^{(1/40)} = 8.4\%$.
3. Run the asset/liability projection for a given interest rate scenario using the common stock return assumption excluding price shocks, that is, 8.4%.
4. Generate a report showing the common stock balances for each year of the projection, reflective of beginning common stock assets along with any projected purchases.
5. For each projection year, quantify the surplus impact if a price shock occurred in that year, by applying the price shock percentage to the projected common stock balance.
6. Compute present values of the surplus impacts, so that the impacts for all years are comparable.
7. Perform a CTE70 calculation on the price shock PV impacts. For a 40-year projection, the CTE70 would use the 12 years with the worst price shock impacts.
8. Applying an appropriate tax effect, adjust the PV for surplus for the scenario by the CTE70 shock impact as quantified in step 7.

This approach provides a reasonable approach to considering the price shock risk in a deterministic context.

8.1.3 Other Equity Projection Considerations

1. Available cash flows—Generally, dividends provide the only available cash flows from common stock and other equity investments, short of selling shares. Models of equity assets should be careful to consider what cash flows are available, and not to consider the total return as available cash flow or to treat the asset as having a maturity date. Dividend assumptions should reflect the track record as well as stated dividend policies.
2. Sale of equities to fund cash flows—The framework described above for handling price shocks does not contemplate sale of equity assets to fund product cash flow needs. It assumes implicitly that the equity assets need never be liquidated. Equity investments give rise to greater price risk if they need to be liquidated at a particular time than if the investor can ride out the price fluctuations. Therefore, if the scenario cash flows require the sale of equity assets to meet cash flow needs, this framework will require modification to consider the timing of those cash flow needs. For instance, if the cash flow need arises in a year when the impact of the price shock is particularly high, one might assign greater weight to that year's price shock in computing the price shock CTE result described above. Alternatively, the underlying model could be set up to reflect a price discount on the sale of any common stock asset to capture this risk.
3. Portfolio rebalancing—Even if equity asset sales are not required to fund cash flows, sales may be necessary to maintain an asset mix that complies with regulatory requirements or the company's investment guidelines. In scenarios requiring disinvestment, sale of asset classes other than equities may result in an overweighting of common stock assets in the projected asset portfolio. Even without asset sales, models with reinvestment in equities may project an overweighting because all other asset classes roll over at defined maturity dates, while the equity assets just accumulate. One should always review portfolio distributions over the entire projection to avoid unreasonable results.
4. Portfolio diversification—The analytics above relate to the S&P 500, a diversified stock index. For a smaller, less diversified portfolio, price volatility and total return volatility would be greater. A robust analysis of diversification benefits was beyond the MDS scope, but these risks should be considered when modeling a less diversified equity portfolio.
5. Choice of holding period in setting assumptions—As described above, moderately adverse return and price shock assumptions should vary with the holding period. The appropriate holding period for setting these assumptions should consider the model projection period and the characteristics of the modeled liabilities. Generally, the holding period assumed in setting these assumptions

should not exceed the model projection period. In cases where a model includes liabilities with significantly shorter average lives than the projection period, it may be appropriate to assume a shorter holding period.

Summaries of the common stock price shock and total return analyses are provided in Appendix L, which is also available as an Excel workbook to allow for additional analysis by the reader. Details of these analyses are provided in the Excel workbook version of Appendix L.

8.2 Corporate Bond Spreads

Our investigation into corporate bond spreads focused on the question of how bond spreads might be projected in a given interest rate scenario. It is our experience that there is little consistency of actuarial practice in this regard. Some actuaries project initial spreads to be constant, some project a reversion to long-term average spreads, some project spreads to increase with the level of interest rates, and some project spreads to decrease with the level of interest rates.

In studying spreads, we focused on two questions:

1. What level of corporate spreads might be considered moderately adverse in a projection scenario?
2. Can corporate bond spreads be correlated to the level of interest rates in a scenario?

We did not attempt to create bond spread scenarios or a model for corporate bond spreads, much less spreads on other fixed income instruments. Portfolio mix and portfolio management practices may vary significantly from company to company, making such an effort nearly impossible. However, we were able to identify some of the factors that one should consider. While our empirical analysis focused on historical data for publicly traded corporate bonds, we believe the considerations are also applicable to other fixed income asset classes.

We used Moody's historical corporate bond yield data for our analysis of spreads. Monthly average data for Moody's AAA and Baa Seasoned Corporate Bond Yields were taken from the Federal Reserve (see Appendix C). The data are available monthly back to 1919. We computed yearly averages and compared the yields to the MDS Long Interest Rate series to determine spreads.

Table 30 summarizes this information, both in total and by Interest Rate Group. This information indicates that there may be some tendency toward wider spreads when interest rates are higher. However, any such correlation appears to be weak and inconsistent. Therefore, we conclude that there is not a strong basis for modeling spreads that vary with interest rates, either positively or negatively.

Table 30 - Historical Analysis of Corporate Bond Spreads,
Moody's Seasoned Corporate Bond Spreads to MDS Long Rate

	Moody's AAA				Moody's Baa			
Interest Groups	Mean	Variance	CTE_L85	CTE_H85	Mean	Variance	CTE_L85	CTE_H85
Total	0.55	0.33	0.23	1.17	1.74	0.74	0.81	2.97
Interest Group 1	0.40	0.26	0.25	0.84	1.39	0.58	0.85	2.26
Interest Group 2	0.39	0.27	0.15	0.95	1.65	0.78	0.65	2.77
Interest Group 3	0.54	0.33	0.24	1.15	1.87	0.90	0.81	3.44
Interest Group 4	0.73	0.33	0.30	1.30	1.69	0.46	1.13	2.43
Interest Group 5	0.62	0.23	0.28	0.95	2.33	0.54	1.76	3.27

Based on this data analysis, following are some factors we believe the reader should consider in modeling spreads on fixed income assets:

- Mean spreads are similar across all Interest Rate Groups, so it is appropriate to project a reversion of spreads from their starting levels. Using these data, a mean reversion target for AAA credits might be 0.50–0.60% and for Baa credits might be 1.60–1.80%.
- 2015 spreads averaged 1.20% for AAA credits and 2.31% for Baa credits, wider than the historical averages. Similarly, at the time of this report, 2016 spreads have been wider than the historical averages. Therefore, at this date it might be appropriate to project spread tightening in one's projections.
- Whether wider or tighter spreads is more conservative depends on many factors. Therefore, we do not propose a moderately adverse CTE target for spreads. The actuary should consider the risks in his or her block and whether any modifications to mean spread targets are needed to represent moderately adverse conditions.
- It is important to note that spread data were not available for the entire history of the MDS Long Interest Rate Series or for Short Interest Rates. Therefore, the spread information above should be considered indicative. Corporate bond spreads before 1919 may or may not have followed similar patterns. For instance, one might hypothesize that lower interest rates in earlier periods were accompanied by larger corporate bond spreads. This is an interesting question that was outside the scope of *Modern Deterministic Scenarios for Interest Rates,* but that may be a subject for future research.

The Spread Analysis and the underlying data are shown in Appendix M, which is also available as an Excel workbook to allow for additional analysis by the reader.

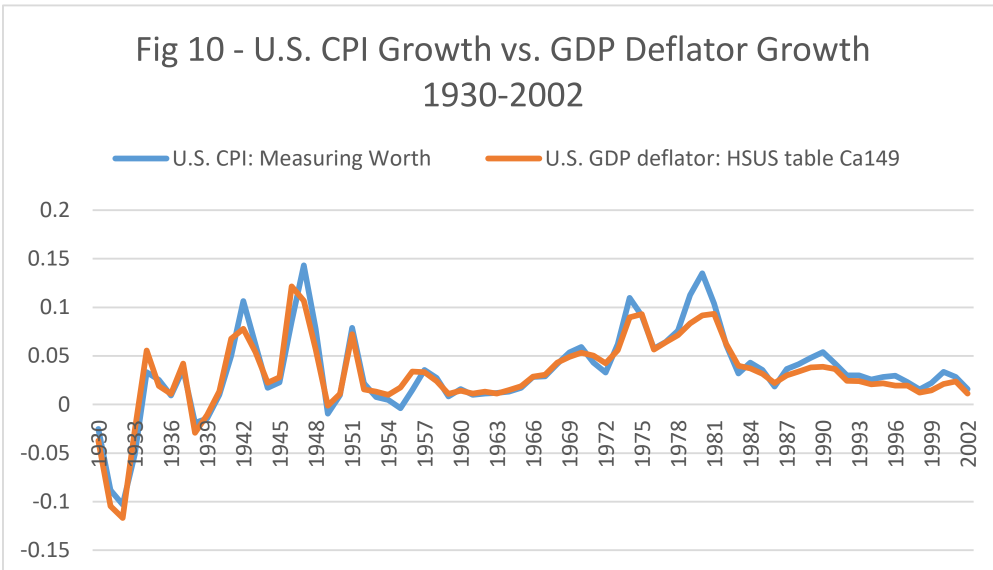
8.3 Inflation Rates

Our investigation into inflation rates focused on the question of how one might relate inflation rates to interest rates in a projection scenario. Economists are accustomed to thinking of the difference between a nominal interest rate and the inflation rate as the real rate of interest. For actuarial cash flow testing models, the primary use for the inflation rate is the projection of general expenses, and projection scenarios use nominal interest rates for projecting investment returns and for discounting present values. This difference in perspective is useful in considering the terminology below.

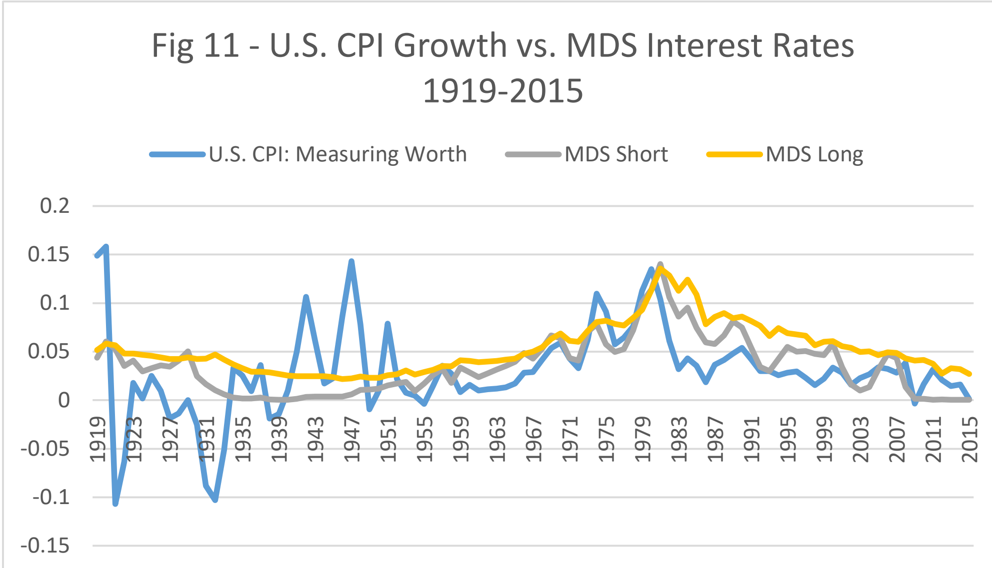
We considered two types of data for measuring historical inflation rates: consumer price indices (CPI) and gross domestic product (GDP) deflators. Perhaps neither source provides a perfectly satisfying basis for measuring inflation rates on insurance company expenses. CPI measures capture price inflation at the consumer level. GDP deflators measure the overall impact of inflation on economic output measured in nominal dollars. Generally, an insurance company's largest expense component is employee salary and benefit expenses, whose growth is measured by wage inflation and would not necessarily equal either CPI growth or GDP deflators. However, as discussed further below, we ultimately judged these theoretical concerns to be of little practical import in choosing an inflation basis.

Price deflators for gross domestic product and its major components are maintained by the U.S. Bureau of Economic Analysis (BEA). The *Historical Statistics of the United States Millennial Online Edition* includes GDP deflators for the period 1929–2002 as table Ca149, and in a more user-friendly form than we found on the BEA website, so we utilized the HSUS as our source for price deflator data. Various consumer price indices are maintained by the U.S. Department of Labor. The *Historical Statistics of the United States Millennial Online Edition* also includes a CPI series for the period 1774–2002 as table Cc1. Measuringworth.com (Williamson, 2016) also maintains a historical U.S. CPI series back to 1774 and including data through 2015. The two CPI sources differed only due to rounding, so we used the MeasuringWorth series to include data through 2015. Both series capture CPI for urban consumers. Because the MDS Interest Rate series do not use U.S. data until 1919–1920, we have limited our analysis of historical inflation data to the period beginning in 1919, to ensure comparability.

Figure 6 shows a graphical comparison of annual inflation rates based on GDP deflators and annual inflation rates based on CPI, for the period 1929–2002. It is visually apparent that the two measures track very closely to one another. The average effective rate of CPI growth over this period is 3.3%, while the average effective rate of GDP deflator growth is 3.0%. We chose the CPI as our inflation basis for further analysis, for primarily practical reasons: (1) CPI would be a more conservative basis for modeling expense inflation and (2) the CPI series included data through 2015, whereas a different source would have been needed to capture deflator data for 2003–2015.



We attempted to build a regression model to relate the CPI formulaically to the MDS Short Rate, the MDS Long Rate or both the MDS Long and Short Rates together. While the CPI is clearly correlated with both rates, as seen in Figure 7, the R^2 values are less than 0.50, which we considered to be too low to produce a predictive regression.



Instead, we opted for a simpler treatment. Intuitively, we expect CPI inflation rates to correlate more closely to short-term interest rates than to long-term interest rates, because they reflect a measure of current short-term conditions. We computed spreads of the MDS Short Interest Rate Series to CPI inflation rates for the period 1919–2015, and then computed the mean, standard deviation and CTE70 statistics for the spread. For reference, we did the same computations for the MDS Long Interest Rate. This analysis is presented in Table 31 and indicates that, on average, CPI inflation rates are 0.57% lower than the MDS Short Interest Rate. In a projection where moderately adverse expense levels are tested via a separate sensitivity test, this would provide a reasonable basis for projecting best estimate inflation levels over the course of a projection. However, there are a number of instances where real short-term interest rates are negative, that is, inflation rates exceed the MDS Short Interest Rate, including most of the period since 2008. The empirical CTE70 result would indicate that a moderately adverse inflation rate would exceed the Short Rate by about 4%. This result is driven by the World War II period, which had a number of years of very high rates of CPI inflation while Short Interest Rates were maintained near zero. Given the limited historical period considered in Table 31, the empirical CTE70 result appears to overweight this period and would appear to reflect more than moderately adverse inflation rates over an entire projection period. One would expect that in the face of sustained high rates of inflation, interest rates would rise such that this relationship would not exist for long.

Table 31 - Historical Spread Statistics, Interest Rates vs. CPI Inflation Rates, 1919-2015

Spread Statistic	MDS Short	MDS Long
Mean Spread to CPI	0.57%	2.29%
Standard Deviation	4.30%	4.39%
30th percentile	-0.82%	1.26%
CTE70	-3.99%	-2.14%

Given the limited data, we are unable to establish very precise guidance with respect to a moderately adverse rate of inflation in relation to interest rates. It does appear that inflation rates exceeding short-term interest rates by 0–2% might fall in the range of moderately adverse conditions.

Summary and detail of the inflation analysis are provided in Appendix N, which is also available as an Excel workbook to allow for additional analysis by the reader.

Section 9: Suggestions for Future Research

We consider the MDS Scenarios to represent an important step forward for practicing actuaries in modeling moderately adverse interest rate conditions. As well, we believe the considerations for common stock, investment spread and inflation modeling capture important fundamental risk factors for these items. However, the existing actuarial literature includes limited research into these factors, so in many ways we consider *Modern Deterministic Scenarios for Interest Rates* to be a first word rather than a final word. We recommend additional future research to validate and extend our analysis, as well as to apply alternate analytic methods and approaches.

A number of areas where practicing actuaries would benefit from additional research include the following:

- A critical untested assumption of *Modern Deterministic Scenarios for Interest Rates* is the assumption that the CTE level of the interest rate scenario equates to the CTE level of the model output. A field test would be very valuable to understand the impact of the MDS Scenarios on cash flow testing results.
- The analytic approach used for *Modern Deterministic Scenarios for Interest Rates* was intentionally agnostic to the drivers of interest rates, and whether the drivers of interest rates at a given time may lead to different expectations than the empirical analysis of historical interest rates. Additional research could attempt to link future interest rates to economic drivers, potentially yielding driver-based deterministic scenarios.
- We attempted unsuccessfully to build regression models to project future interest rates or interest rate changes as a function of the initial interest rate. Failing this, we developed relatively complex scenario parameters for *Modern Deterministic Scenarios for Interest Rates.* Through additional study of the distribution interest rates and the distribution of interest rate changes, more robust scenarios might be developed.
- Throughout the report we discuss instances where we addressed data limitations or uncertainty in a judgmental way. Examples include the data series transition rules used in the construction of the MDS Interest Rate Series (Section 4.2); methods used to address differences in time period covered by the MDS Long and Short Interest Rate Series (Sections 5.5.1 and 5.5.2.1); judgment applied in Rate Change CTE analysis and scenario development (Sections 5.5.2 and 6.3): for example, the choice of interest rate groupings, the parameter smoothing process and the method for applying both absolute and relative rate change parameters in the scenario calculation, and the method for eliminating discontinuities at the boundaries of interest rate groups. All these cases provide opportunities for future research increasing the analytical rigor or applying the judgment of other actuaries.
- The Rate Change CTE analysis considers rate change periods of one to 30 years and includes overlapping time periods. A body of literature addresses overlapping data in the context of hypothesis testing and regression, generally concluding that overlapping data causes parameter estimates to be inefficient and hypothesis tests to be biased—see Harri and Brorsen (1998, p. 1) and Britten-Jones and Neuberger (2011)—and proposing methods to address overlapping data. Our Rate Change CTE analysis did not involve either hypothesis testing or regression, and we did not make any adjustments to address overlapping data. We recommend further research to explore the impact of overlapping data on CTE analysis and methods to address it.
- We observed, but did not study, the fact that the upward sloping term structure of interest rates arose about the same time the Federal Reserve began using open market operations as a monetary policy tool. Further study of this relationship, and the term structure under various Fed policymakers, might yield important insights into the term structure.
- *Modern Deterministic Scenarios for Interest Rates* addresses fixed income spreads in only a very rudimentary way. The interaction of interest rates and spreads is critical and understudied. For

instance, in the financial crisis of 2008, declines in interest rates were accompanied by increases in spreads, with many unanticipated consequences. A set of interest rate scenarios is useful only insofar as it has a consistent set of spread assumptions. Some potential future studies could include (1) a study of corporate bond spreads in the period prior to 1919, (2) additional study into the correlation of spreads and interest rates, perhaps incorporating other economic measures as well and (3) spread studies for multiple fixed income rating and asset classes.

References

Actuarial Standard of Practice No. 22: Statements of Opinion Based on Asset Adequacy Analysis by Actuaries for Life or Health Insurers (2001, rev. 2012), Washington, DC: Actuarial Standards Board. http://www.actuarialstandardsboard.org/wp-content/uploads/2014/02/asop022_167.pdf.

Asset Adequacy Analysis Committee of the American Academy of Actuaries, Jeffrey R. Lortie chair (2014), *Asset Adequacy Analysis Practice Note August 2014— EXPOSURE DRAFT*. Washington, DC: American Academy of Actuaries.

Britten-Jones, Mark and Anthony Neuberger (2011), "Improved Inference and Estimation in Regression with Overlapping Observations," *Journal of Business Finance & Accounting*, 38(5–6), 657–683.

Curiale, Salvatore R. (1994), *Regulation No. 126 (11 NYCRR 95) Regulations Governing an Actuarial Opinion and Memorandum*. Albany, NY: New York State Insurance Department.

Fabozzi, Frank J. and T. Dessa Fabozzi, eds. (1995), *The Handbook of Fixed Income Securities*. Burr Ridge, IL: Irwin Professional Publishing.

Friedman, Milton, and Anna J. Schwartz (1982), *Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867–1975*. Chicago: University of Chicago Press.

Harri, Ardian and B. Wade Brorsen (1998), "The Overlapping Data Problem." Available at SSRN: https://ssrn.com/abstract=76460 or http://dx.doi.org/10.2139/ssrn.76460.

Historical Statistics of the United States, 1789–1945 (1949), Washington, DC: Government Printing Office.

———, *Colonial Times to 1970, Bicentennial Edition* (1975), Part I. Washington, DC: U.S. Bureau of the Census.

———, *Millennial Edition* (2002), Susan B. Carter, Scott S. Gartner, Michael Haines, Alan Olmstead, Richard Sutch, and Gavin Wright (eds.). New York: Cambridge University Press.

History of the Federal Reserve, (n.d.). Federal Reserve. https://www.federalreserveeducation.org/about-the-fed/history.

Homer, Sidney and Richard Sylla (1991), *A History of Interest Rates,* 3rd ed. New Brunswick, NJ: Rutgers University Press. Revised 3rd ed., 1996.

——— (2005), *A History of Interest Rates,* 4th ed. Hoboken, NJ: John Wiley & Sons.

"Industrial Revolution" (n.d.). In Wikipedia: https://en.wikipedia.org/wiki/Industrial_Revolution.

Life & Health Valuation Law Manual, 21st ed. (2015), Washington, DC: American Academy of Actuaries.

Macaulay, Frederick R. (1938), *(Some Theoretical Problems Suggested by) The Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856*. New York: National Bureau of Economic Research.

Mitchell, Wesley C. (1903), *A History of the Greenbacks*. Chicago: University of Chicago Press.

National Association of Insurance Commissioners. (2013), *Standard Valuation Law*. Kansas City, MO: NAIC.

——— (2010), *Actuarial Opinion and Memorandum Regulation*. Kansas City, MO: NAIC.

——— (2012), *Actuarial Guideline XLIII, CARVM for Variable Annuities*. Kansas City, MO: NAIC.

——— (2016), *Valuation Manual—NAIC Adoptions Through August 29*. Kansas City, MO: NAIC.

Officer, Lawrence J. (2003), *What Was the Interest Rate Then? A Data Study*. Chicago: MeasuringWorth. http://www.measuringworth.com/interestrates/intstudy.pdf.

——— (2006), *What Was the Consumer Price Index Then? A Data Study*. Chicago: MeasuringWorth. https://www.measuringworth.com/docs/cpistudyrev.pdf.

Shiller, Robert J. (2015), *Irrational Exuberance*. Princeton, NJ: Princeton University Press.

Taleb, Nassim (2010), *The Black Swan—The Impact of the Highly Improbable.* New York: Random House

Taylor, Bryan. (2013), *Seven Centuries of Government Bond Yields.* San Juan Capistrano, CA: Global Financial Data. https://www.globalfinancialdata.com/News/Articles/Seven_Centuries_of_government_bond_yields.pdf.

Williamson, Samuel H. (2016), *The Annual Consumer Price Index for the United States, 1774–2015.* Chicago: MeasuringWorth. https://www.measuringworth.com/uscpi/result.php#.

Appendix A: Definitions

***Absolute Change in Interest Rates*—**As used in *Modern Determinstic Scenarios for Interest Rates,* a change in interest rates expressed as a simple difference that can be either positive or negative in sign, i.e., $\Delta i_t^{abs} = i_{x+t} - i_x$.

***Academy Interest Rate Generator (AIRG)*—**Economic scenario generator maintained by the American Academy of Actuaries and used in the calculation of life insurance reserves computed under VM-20 and variable annuity reserves computed under VM-22 of the Valuation Manual of the National Association of Insurance Commissioners.

***Actuarial Standard of Practice (ASOP*)—Standard of practice binding on members of the American Academy of Actuaries and promulgated by the Actuarial Standards Board.

***Asset Adequacy Analysis (AA Analysis)*—**Annual analysis of the adequacy of assets supporting actuarial reserves, required to be completed by U.S. life insurers under the Standard Valuation Law adopted by the National Association of Insurance Commissioners.

***Conditional Tail Expectation (CTE)*—**A statistical risk measure that measures the conditional mean of a distribution, given that an observation is in the defined tail. When applied to a loss function, CTEx measures the mean loss beyond the xth percentile. As used in *Modern Deterministic Scenarios for Interest Rates,* CTE analysis is not applied to loss functions, but to distributions of interest rates, inflation rates, corporate bond spreads and common stock returns.

***Empirical CTE$_L$*—**CTE analysis applied to a set of empirical data and measuring CTE for the low values, or left tail, of the distribution. In *Modern Deterministic Scenarios for Interest Rates,* CTE$_L$ as applied to interest rates and other financial values, represents a measure of low values or downward changes in the applicable rate.

***Empirical CTE$_H$*—**CTE analysis applied to a set of empirical data and measuring CTE for the high values, or right tail, of the distribution. In *Modern Deterministic Scenarios for Interest Rates,* CTE$_H$ as applied to interest rates and other financial values, represents a measure of high values or upward changes in the applicable rate.

Interest Rate Group—Grouping of interest rates used in the development and computation of interest rate scenarios in *Modern Deterministic Scenarios for Interest Rates.* The interest rate groupings for Long Interest Rates and Short Interest Rates are detailed in Appendix F.

***MDS AIRG Scenarios*—**Scenarios MDS15 and MDS16, computed using stochastic scenarios generated from the Academy Interest Rate Generator. MDS15 is a high-rate scenario and MDS16 is a low-rate scenario.

***MDS Interest Rate Cycle Scenarios*—**Scenarios MDS13 and MDS14 computed using an assumption of cyclical interest rate patterns. MDS13 uses a 20-year cycle, and MDS14 uses a 40-year cycle.

***MDS Interest Rate Series*—**Historical interest rate series constructed for *Modern Deterministic Scenarios for Interest Rates.* These series consist of two separate series—the MDS Long Interest Rate Series and the MDS Short Interest Rate Series—that were the basis of the empirical interest rate analysis used to develop the parameters for the MDS Scenario Set.

***MDS Interest Rate Scenario Set*—**Set of 16 deterministic interest rate scenarios developed as part of *Modern Deterministic Scenarios for Interest Rates*. The scenario set consists of eight high interest rate scenarios and eight low interest rate scenarios. The scenarios project U.S. Treasury Constant Maturity (CMT) yields using as input the initial CMT rates at the scenario start date. There are four analytic frameworks or scenario types: reversion scenarios, rate change scenarios, cyclical scenarios and AIRG scenarios. The scenarios and their development is described in Section 4. With the exception of the AIRG scenarios, the scenarios are constructed by projecting a long-term rate and a short-term rate, and the full yield curve is developed using regression formulas. For the AIRG scenarios, the full yield curve projection is projected directly.

MDS Long Interest Rate Series—Historical series of long-term interest rates constructed for *Modern Deterministic Scenarios for Interest Rates* and used to develop the long-term interest rate parameters for the MDS Scenario Set.

MDS Rate Change CTE Scenarios—Scenarios MDS9–MDS12 developed using Rate Change CTE Analysis.

MDS Regression Target Scenarios—Scenarios MDS1–MDS8 developed using Reversion Target Analysis.

MDS Short Interest Rate Series—Historical series of short-term interest rates constructed for *Modern Deterministic Scenarios for Interest Rates* and used to develop the short-term interest rate parameters for the MDS Scenario Set.

Modeled Rate Change CTE—Modeled values for the CTEL and CTEH statistics developed as part of the Rate Change CTE analysis performed for *Modern Deterministic Scenarios for Interest Rates.*

Moderately Adverse Conditions—As defined by ASOP 22, "Conditions that include one or more unfavorable, but not extreme, events that have a reasonable probability of occurring during the testing period."

New York 7 Interest Rate Scenarios (NY7)—Deterministic interest rate scenario set required to be used for Asset Adequacy Analysis by New York life insurers under New York Regulation 126. Formerly required to be tested under the Standard Valuation Law of the National Association of Insurance Commissioners, the NY7 continue to be used for Asset Adequacy Analysis by the Appointed Actuaries of most U.S. life insurers.

Primary Interest Rate Group—In the MDS Rate Change Scenarios, the Interest Rate Group at the scenarios start date, which is given the greatest weight in applying the scenario parameters.

Rate Change CTE Analysis—One analytic framework used in *Modern Deterministic Scenarios for Interest Rates* for the development of interest rate scenarios. Under this framework, we analyzed changes in interest rates from an initial value over Rate Change Periods of one to 30 years. The resulting Rate Change Scenarios then project interest rates as a specified change from the initial rate over a t-year Rate Change Period.

Relative Change in Interest Rates—As used in *Modern Determinstic Scenarios for Interest Rates,* a change in interest rates expressed as a ratio to the initial rate: $\Delta i_t^{rel} = (i_{x+t} - i_x)/i_x$.

Reversion Target Analysis—One analytic framework used in *Modern Deterministic Scenarios for Interest Rates* for the development of interest rate scenarios. Under this framework, we analyzed the right and left tails of the interest rate distribution over the whole history of the MDS Interest Rate Series. The resulting Reversion Target Scenarios project interest rates to revert to a moderately adverse reversion target developed through this analysis.

Secondary Interest Rate Group—In the MDS Rate Change Scenarios, the next closest Interest Rate Group, which is given lesser weight in applying the scenario parameters.

Sample CTE_L—See Empirical CTE_L.

Sample CTE_H—See Empirical CTE_H.

Appendix B: Data Sources Considered

Source	Description	Start	End	Link	Notes
American Academy of Actuaries	Life Reserve Working Group Report, July 22, 2010: Supplemental Information for Discussion of VM-20 Reinvestment Spread Proposal			https://www.actuary.org/files/publications/Sup plemental%20Information%20on%20Spreads%2 0Final.pdf	Discusses their use of JP Morgan's JULI index, with comparison to Moody's averages.
BOA/Merrill Lynch Report: The Longest Pictures, a Picture Guide to Global Markets since 1800				http://www.merrilledge.com/Publish/Content/a pplication/pdf/GWMOL/GlobalStrategyApicture guidetofinancialmarketssince1800.pdf	
Board of Governors of the Federal Reserve System (U.S.)	One-Year Treasury Bill: Secondary Market Rate Discount Basis (daily)	1959	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Board of Governors of the Federal Reserve System (U.S.)	One-Year Treasury Bill: Secondary Market Rate (monthly)	1959	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to monthly rate series. Daily and yearly average rate series also available, with different start dates.
Board of Governors of the Federal Reserve System (U.S.)	20-Year Treasury Constant Maturity Rate	1953 1993	1987 Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to monthly rate series. Daily and yearly average rate series also available, with different start dates.
Board of Governors of the Federal Reserve System (U.S.)	Three-Month Treasury Bill: Secondary Market Rate (monthly)	1934	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to monthly rate series. Daily and yearly average rate series also available, with different start dates.
Board of Governors of the Federal Reserve System (U.S.)	Three-Year Treasury Constant Maturity Rate (monthly)	1953	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to monthly rate series. Daily and yearly average rate series also available, with different start dates.
Bureau of Labor Statistics	Various consumer price index reports	January 1, 1913	Current	http://data.bls.gov/pdq/querytool.jsp?survey=c u	Interactive tool allowing selection of various CPI reports
Federal Reserve Board Selected Interest Rates (Daily) H.15	One-year Treasury Bill: Secondary Market Rate Discount Basis	July 15, 1959	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Federal Funds Effective Rate	July 1, 1954	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Market yield on U.S. Treasury securities at 10-year constant maturity, quoted on investment basis	January 1, 1962	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Market yield on U.S. Treasury securities at one-year constant maturity, quoted on investment basis	January 1, 1962	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Market yield on U.S. Treasury securities at 30-year constant maturity, quoted on investment basis	February 15, 1977	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.

Source	Description	Start	End	Link	Notes
Federal Reserve Board Selected Interest Rates (Daily) H.15	Market yield on U.S. Treasury securities at three-month constant maturity, quoted on investment basis	January 1, 1982	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Market yield on U.S. Treasury securities at five-year constant maturity, quoted on investment basis	January 1, 1962	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Moody’s yield on seasoned corporate bonds, all industries, AAA	January 1, 1983	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
Federal Reserve Board Selected Interest Rates (Daily) H.15	Moody’s yield on seasoned corporate bonds, all industries, BAA	January 1, 1986	Current	https://www.federalreserve.gov/releases/h15/d ata.htm	Start date relates to daily rate series. Monthly and yearly average rate series also available, with different start dates.
FRASER Federal Reserve Archive	*Business Booms & Depressions 1775–1943*; includes bond yields since 1857			https://fraser.stlouisfed.org/docs/publications/ 1943chart_busibooms.pdf	
FRASER Federal Reserve Archive	G.13 Selected interest rates, monthly file 1929–1995; includes short-term/long-term Treasuries, commercial paper, bankers’ acceptance rates			https://fraser.stlouisfed.org/title/1238	
FRASER Federal Reserve Archive	G.14 U.S. government security yields and prices			https://fraser.stlouisfed.org/theme/21#!1259	
Global Financial Data	Various historical databases of historical financial and economic data	TBD	TBD	https://www.globalfinancialdata.com/index.ht ml	This site has constructed long-term data series on interest rates, equity returns, asset values and other financial and economic data for research purposes. It requires a subscription and appears to include a wealth of data, including some data sets going back centuries.
Global Financial Data Research paper				https://www.globalfinancialdata.com/News/Art icles/Seven_Centuries_of_government_bond_yi elds.pdf	
Historical Statistics of the United States 1789–1945	N185–187 Short-term open market rates in NYC 1890–1945 N196–200 Basic yields of corporate bonds by term to maturity 1900–1945 N201–202 Yields on railroad bonds 1857–1936 N203–204 Yield on U.S. govt. bonds and high-grade municipal bonds 1919–1945 N206–211 Indexes of yields on common stocks 1871–1937 N215–220 Indexes of common stock prices 1871–1937			https://www2.census.gov/prod2/statcomp/doc uments/HistoricalStatisticsoftheUnitedStates17 89-1945.pdf	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical interest rate data in section X of Part 2: “hist_stats_colonial-1970p2chX.pdf”	X444–455 Money market yields 1890—1970 X474–486 Bond and stock yields 1857—1970 X487–491 Basic yields of corporate bonds by term to maturity 1900–1970			https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.

Source	Description	Start	End	Link	Notes
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	E135–E166: Consumer Price Indexes, all items 1800–1970, by group 1913–1970	1800	1970	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	E1–E22: Implicit price deflators for GNP 1929–1970	1929	1970	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	E23–E39: Wholesale prices by major product group 1890–1950	1890	1970	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	E40–E51: Wholesale prices by major product group 1890–1950	1890	1950	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	E52–E63: Wholesale prices by major product group 1749–1890	1749	1890	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Colonial Times to 1970; saved in various PDFs. Historical price data in sections E and F of Part 1: “hstat1970_cen_1975_v1.pdf”	F1–F5: GNP 1869–1970, in real and 1958 dollars with price deflators	1869	1970	https://fraser.stlouisfed.org/title/237	Various series of primarily historical interest. Millenial online edition provided more up-to-date source of HSUS data.
Historical Statistics of the United States Millenial Online Edition	Basic yields of corporate bonds, by term to maturity 1900–1975	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Common stock dividend yields 1802–1999	1802	1999	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Common stock prices 1802–1999	1802	1999	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Consumer price indexes, for all items 1774–2003	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Implicit price deflators for GDP and major components 1929–2002	1929	2002	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Long-term bond yields 1798–1997	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Money market rates 1831–1997	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Money market rates 1831–1997	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	“Prices and Price Indexes” (essay)	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Historical Statistics of the United States Millenial Online Edition	Wholesale price indexes for historical comparisons, by commodity group 1860–1990 [Hanes]	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates

Source	Description	Start	End	Link	Notes
Historical Statistics of the United States Millenial Online Edition	Yields of government bonds, by term to maturity 1950–1998	Varies	Varies	http://hsus.cambridge.org/HSUSWeb/index.do	Includes multiple series, with various start and end dates
Measuring Worth inflation studies				https://www.measuringworth.com/inflation/	
Measuring Worth interest rate studies	First link includes details of the compilation. Second link provides access to the data series. Data are available annually back to 1798 or thereabouts. Third link provides a brief guide.			https://www.measuringworth.com/interestrate s/intstudy.pdf https://www.measuringworth.com/interestrate s/ https://www.measuringworth.com/interestrate s/intguide.php	Lawrence Officer interest rate study ultimately used as major source for interest rate data.
National Bureau of Economic Research	"This Time Is Different," Working Paper			http://www.nber.org/papers/w13882.pdf	
National Bureau of Economic Research	Various	Varies	Varies	http://www.nber.org/releases/#navDiv=4	The organization maintains links to data published by other organizations (including some of the items listed above), as well as some of its own data.
Robert Shiller, *Irrational Exuberance*, stock data	S&P 500 price, dividend, earning; CPI; long-term interest rate	January 1, 1871	Current	http://www.econ.yale.edu/~shiller/data.htm	File name "ie_data.xls". Downloaded from web site. Monthly data.
Robert Shiller, *Market Volatility*, chapter 26 data	S&P 500 price, dividend, earning; CPI; one-year and 10-year interest rate; per capita consumption	January 1, 1871	Current	http://www.econ.yale.edu/~shiller/data.htm	File name "chapter 26.xls". Downloaded from web site. Annual data.
WSJ blog article on BOA Merrill Lynch Report on U.S. govt. bond yields since 1800	Shows a graph of rates compiled by BOA			http://s.wsj.net/public/resources/images/OBPF145_BofAlo_K_20110818111723.jpg http://blogs.wsj.com/marketbeat/2011/08/18/so-exactly-what-kind-of-record-low-are-we-at-for-bond-yields/	
Yahoo Finance	CBOE 10-year interest rate 1953–	January 1, 1962	Current	http://finance.yahoo.com/q?s=^tnx	Yahoo Finance data redundant with Fed data.
Yahoo Finance	CBOE five-year interest rate 1953–	January 1, 1962	Current	http://finance.yahoo.com/q?s=%5Efvx&ql=1	Yahoo Finance data redundant with Fed data.
Yahoo Finance	CBOE 30-year interest rate 1977–	February 15, 1977	Current	http://finance.yahoo.com/q?s=%5ETYX&ql=0	Yahoo Finance data redundant with Fed data.
Yahoo Finance	S&P 500 prices	January 1, 1950	Current	http://finance.yahoo.com/q?s=^gspc	Yahoo Finance data redundant with Fed data.

Appendix C: Data Sources Used

Source	Web Link	Time step	Data Series Description	Time Period Start	Time Period End	How Used
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Monthly average	20-year constant maturity Treasury yield	1953	2015	MDS Long Interest Rate Series
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Daily	20-year constant maturity Treasury yield	1962	2015	Daily to yearly average interest rate transition statistics
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Monthly average	30-year constant maturity Treasury yield			MDS Long Interest Rate Series
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Daily	30-year constant maturity Treasury yield	1978	2015	Daily to yearly average interest rate transition statistics
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Daily	Three-Month Treasury Bill: Secondary Market Rate	1954	2015	Daily to yearly average interest rate transition statistics
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Monthly	Moody's AAA Seasoned Corporate Bond Yields	1919	2015	Corporate bond spread modeling considerations
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Monthly	Moody's Baa Seasoned Corporate Bond Yields	1919	2015	Corporate bond spread modeling considerations
Federal Reserve Board Selected Interest Rates: H.15	https://www.federalreserve.gov/releases/h15/data.htm	Calendar year average	Moody's AAA Corporate Bond Yield	1923	1941	MDS Long Interest Rate Series
Historical Statistics of the United States Millenial Online Edition	http://hsus.cambridge.org/HSUSWeb/index.do	Calendar year	Table Ca149, U.S. Gross Domestic Product implicit price deflators	1929	2002	Inflation rate modeling considerations
Historical Statistics of the United States Millenial Online Edition	http://hsus.cambridge.org/HSUSWeb/index.do	Calendar year average	Table Cj1192, Yield on fully taxable U.S. government bonds due or callable after 15 years	1942	1952	MDS Long Interest Rate Series
Historical Statistics of the United States Millenial Online Edition	http://hsus.cambridge.org/HSUSWeb/index.do	Calendar year average	Table Cj1230, prime bankers' acceptances, 90 days	1918	1930	MDS Short Interest Rate Series
Measuring Worth study: "The Annual Consumer Price Index for the United States, 1774–2015"	https://www.measuringworth.com/inflation/	Calendar year	U.S. Consumer Price Index for Urban Consumers	1919	2015	Inflation rate modeling considerations
Measuring Worth study: "What Was the Interest Rate Then? A Data Study"	https://www.measuringworth.com/interestrates/	Calendar year average	Three-month Treasury Bill: New Issues Rate	1931	1933	MDS Short Interest Rate Series
Measuring Worth study: "What Was the Interest Rate Then? A Data Study"	https://www.measuringworth.com/interestrates/	Calendar year average	Three-month Treasury Bill: Secondary Market Rate	1934	2015	MDS Short Interest Rate Series
Measuring Worth study: "What Was the Interest Rate Then? A Data Study"	https://www.measuringworth.com/interestrates/	Calendar year average	London market discount rate on bills of exchange	1825	1912	MDS Short Interest Rate Series

Source	Web Link	Time step	Data Series Description	Start	End	How Used
Measuring Worth study: "What Was the Interest Rate Then? A Data Study"	https://www.measuringworth.com/interestrates/	Calendar year aaverage	Yield on Bank of England consolidated annuities (consuls), compiled as Measuring Worth UK Long, contemporary series	1729	1921	MDS Long Interest Rate Series
Robert Shiller, *Irrational Exuberance,* stock data	http://www.econ.yale.edu/~shiller/data.htm	Monthly	Standard & Poor's 500 price and dividend data	1871	2015	Common stock modeling considerations

Appendix D.1: MDS Long Interest Rate Series Construction

This appendix contains the MDS Long Interest Rate Series description and construction.

Appendix D.2: MDS Short Interest Rate Series Construction

This appendix contains the MDS Short Interest Rate Series description and construction.

Appendix E: MDS Interest Rate Series Reversion Target Statistics

	MDS Short Interest Rate			MDS Long Interest Rate		
Statistic	1729–2015	1825–2015	1920–2015	1729–2015	1825–2015	1920–2015
Mean	NA	3.44	3.54	4.11	4.22	5.27
Standard Deviation	NA	2.26	2.96	1.81	2.14	2.59
10th Percentile	NA	0.38	0.15	2.69	2.60	2.51
15th Percentile	NA	1.16	0.30	2.86	2.69	2.69
20th Percentile	NA	1.73	0.38	2.98	2.83	2.93
30th Percentile	NA	2.34	1.37	3.17	3.09	3.73
40th Percentile	NA	2.86	2.48	3.28	3.23	4.19
50th Percentile	NA	3.16	3.33	3.44	3.31	4.61
60th Percentile	NA	3.59	4.09	3.84	3.58	5.01
70th Percentile	NA	4.25	4.92	4.31	4.27	6.06
80th Percentile	NA	4.97	5.77	4.88	5.01	7.43
85th Percentile	NA	5.32	6.31	5.09	6.06	8.00
90th Percentile	NA	5.97	7.33	6.05	7.43	8.53
CTEL90	NA	0.18	0.07	2.48	2.40	2.38
CTEL85	**NA**	**0.40**	**0.12**	**2.59**	**2.48**	**2.46**
CTEL80	NA	0.69	0.18	2.67	2.56	2.55
CTEL70	NA	1.15	0.46	2.80	2.69	2.78
CTEH70	NA	4.73	5.37	4.89	5.12	6.70
CTEH80	NA	5.03	5.88	5.22	5.52	7.20
CTEH85	**NA**	**5.50**	**6.40**	**5.63**	**6.04**	**7.75**
CTEH90	NA	6.00	7.08	6.15	6.72	8.52

Appendix F.1: Rate Change CTE Analysis - MDS Long Interest Rate Series – Absolute Changes

This appendix contains details of the rate change CTE analysis, MDS Long Interest Rate Series absolute percentage point changes in rates.

Appendix F.2: Rate Change CTE Analysis - MDS Short Interest Rate Series – Absolute Changes

This appendix contains details of the rate change CTE analysis, MDS Short Interest Rate Series absolute percentage point changes in rates.

Appendix F.3: Rate Change CTE Analysis - MDS Long Interest Rate Series – Relative Changes

This appendix contains details of the rate change CTE analysis and MDS Long Interest Rate Series relative changes in rates.

Appendix F.4: Rate Change CTE Analysis - MDS Short Interest Rate Series – Relative Changes

This appendix contains details of the rate change CTE analysis and MDS Short Interest Rate Series relative changes in rates.

Appendix G.1: Tables of all Rate Change CTE Parameter Development

This appendix provides parameter fitting details and contains the tables of all rate change CTE parameter development.

Appendix G.2: Graphs of Rate Change CTE_L85 Parameter Development, Long Rates

This appendix provides parameter fitting details and contains the graphs of rate change CTE_L parameter development, Long Rates.

Appendix G.3: Graphs of Rate Change CTE_H85 Parameter Development, Long Rates

This appendix provides parameter fitting details and contains the graphs of rate change CTE_H 85 parameter development, Long Rates.

Appendix G.4: Graphs of Rate Change CTE_L75 Parameter Development, Short Rates

This appendix provides parameter fitting details and contains the graphs of rate change CTE_L75 parameter development, Short Rates.

Appendix G.5: Graphs of Rate Change CTE_H75 Parameter Development, Short Rates

This appendix provides parameter fitting details and contains the graphs of rate change CTE_H 75 parameter development, Short Rates.

Appendix H: Transitional Rate Change - Empirical Analysis and Parameter Development

	Historical CTE Data				Scenario Parameters After Smoothing			
	Absolute Change		Relative Change		Absolute Change		Relative Change	
Short Rate	CTEL85	CTEH85	CTEL85	CTEH85	CTEL85	CTEH85	CTEL85	CTEH85
Interest Rate Group 1	(0.080)	0.096	(0.528)	5.744	-	1.250	-	
Interest Rate Group 2	(0.627)	1.182	(0.442)	1.114	(0.750)	1.250	(0.500)	
Interest Rate Group 3	(1.063)	1.246	(0.326)	0.373	(1.250)	1.250	(0.400)	
Interest Rate Group 4	(1.153)	1.322	(0.240)	0.267	(1.250)	1.250	(0.300)	
Interest Rate Group 5	(2.190)	2.588	(0.227)	0.321	(2.500)	2.250	(0.200)	

	Historical CTE Data				Scenario Parameters After Smoothing			
	Absolute Change		Relative Change		Absolute Change		Relative Change	
Long Rate	CTEL85	CTEH85	CTEL85	CTEH85	CTEL85	CTEH85	CTEL85	CTEH85
Interest Rate Group 1	0.067	0.800	0.025	0.301	-	0.750	-	
Interest Rate Group 2	(0.504)	0.789	(0.149)	0.247	(0.500)	0.750	(0.150)	
Interest Rate Group 3	(0.526)	0.691	(0.112)	0.133	(0.750)	0.750	(0.125)	
Interest Rate Group 4	(0.700)	1.058	(0.091)	0.140	(1.000)	0.750	(0.100)	
Interest Rate Group 5	(1.952)	1.663	(0.160)	0.151	(2.000)	1.500	(0.150)	

Parameter Fitting Notes:

1. CTL85, Interest Rate Group 1: The left tail CTE is very near zero for Interest Rate Group 1, both for long and short rates. We have set them both to zero, because we think a change of 0.10% or less would be immaterial to scenario results.

2. CTL85, Interest Rate Groups 2-4: For both long and short rates, the absolute magnitude of changes increases with interest rate level, while the magnitude of relative changes decrease with interest rate. These patterns were maintained, but with some smoothing.

3. CTH85, long rates: For long rates, the absolute changes were very similar for Interest Rate Groups 1-4, so the same parameters were used. Only Group 5 was significantly different, and this difference was maintained. Relative change parameters not needed.

Appendix I: Regression Parameters for Full Yield Curve Construction

	Regression Coefficients		Regression	
U.S. Treasury Term	Short Rate	Long Rate	Intercept	R-Square
0.25	1.0000	-	-	1.0000
0.5	1.0191	0.0671	(0.1952)	0.9986
1	0.9431	0.1635	(0.2942)	0.9961
2	0.7128	0.4443	(0.8718)	0.9946
3	0.6934	0.3763	-	0.9965
5	0.4319	0.6733	(0.5166)	0.9897
7	0.2726	0.8268	(0.6064)	0.9918
10	0.1856	0.8693	(0.2936)	0.9947
20	0.0212	0.9868	-	0.9999
30	(0.0265)	1.0167	-	0.9999

Appendix J: Interest Rate Scenario Calculation

This appendix is the workbook to compute scenarios MDS1-MDS14 based on selected user inputs.

Appendix K: AIRG Scenario Construction

This appendix is the workbook to compute the AIRG-based scenarios, MDS15-MDS16.

Appendix L: Historical Common Stock Returns

This appendix contains historical common stock returns.

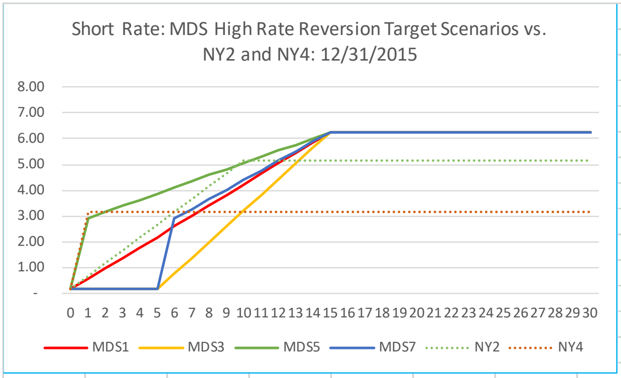
Appendix M: Historical Corporate Bond Spreads

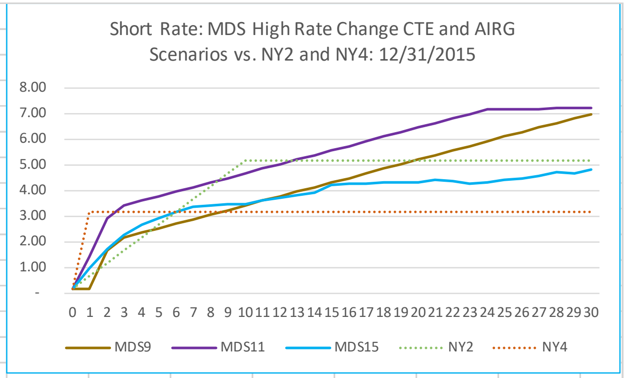
This appendix contains historical corporate bond spreads.

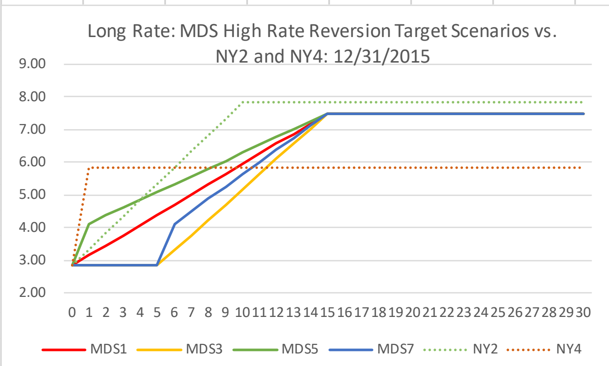
Appendix N: Historical Inflation Rates

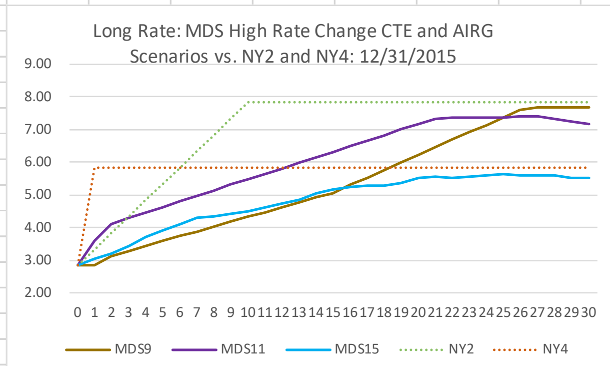
This appendix contains historical inflation rates.

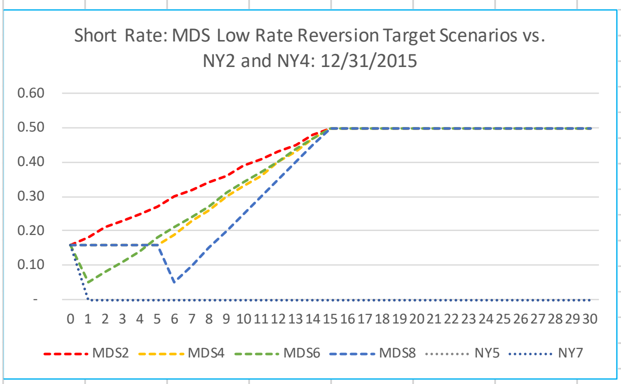
Appendix O: December 31, 2015 MDS and NY7 Scenario Graphs

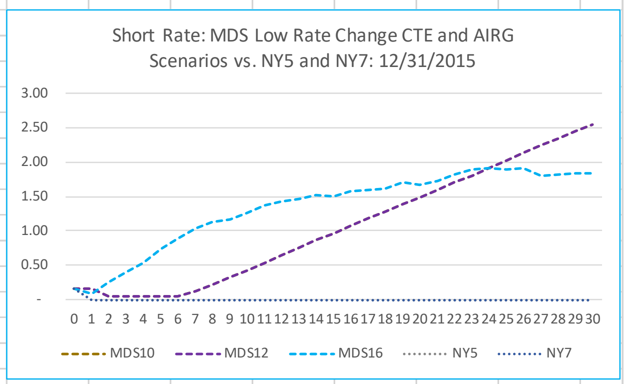


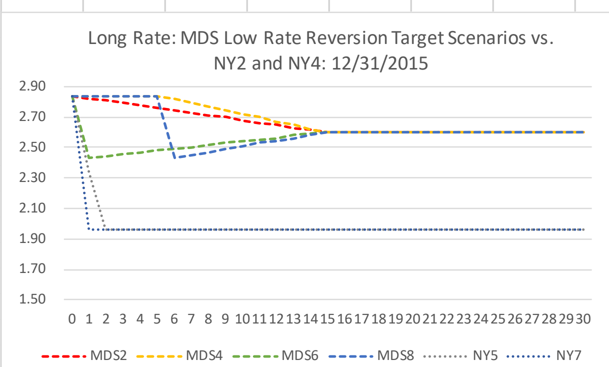


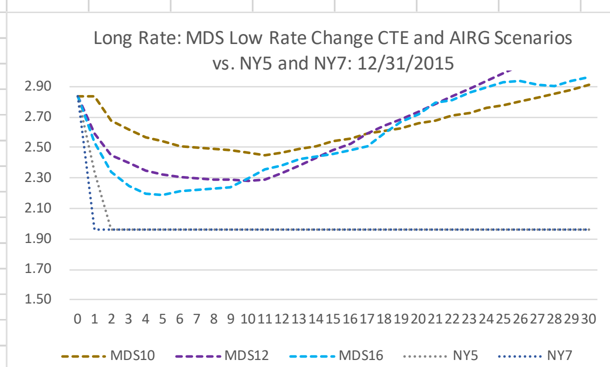


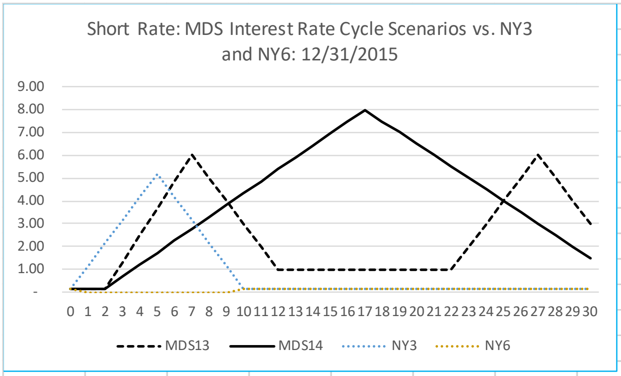


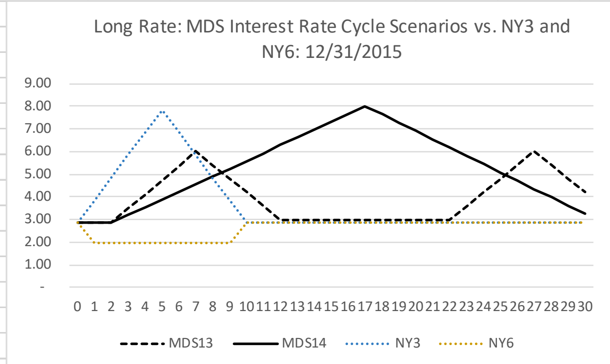









Short Rate: MDS Interest Rate Cycle Scenarios vs. NY3 and NY6: 12/31/2015
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
MDS13
MDS14
NY3
NY6


Long Rate: MDS Interest Rate Cycle Scenarios vs. NY3 and NY6: 12/31/2015
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
MDS13
MDS14
NY3
NY6