CARBON CREDIT
PROGRAM DESIGN

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TABLE OF CONTENTS

1. ABBREVIATIONS

CO_2	Carbon Dioxide
EU	European Union
EUA	European Union Allowance
ETS	Emissions Trading System
B	Buildings and Land Use
E	Energy, Manufacturing and Construction
I	Industrial Processes and Product Use
O	Other
T	Transport
W	Waste
GDP	Gross Domestic Product
GDPpc	Gross Domestic Product per capita

2. EXECUTIVE SUMMARY

Realizing global warming as an issue which could cause long-term damage to the nature and the planet, governments of many countries have been promulgating policies to limit the emissions of carbon dioxide and other greenhouse gases to protect the environment. As a developed country, Pullanta aims to reduce the emissions of carbon by 25% over the next decade.

This report represents a comprehensive design of a carbon credits program together with three financial instruments including one intermediate-term, one long-term and one call option to complete the goal of reducing carbon emission with 90% confidence. We also estimate revenues based on the aim for the government to fund more future investments on renewable energy to reduce the pressure of environment. Besides, considering the risks associated with various stakeholders, the report will demonstrate a sensitivity analysis for several assumptions at length.

Analyses based on Pullanta CO2e emissions and related data allowed us to estimate the carbon emission until year 2040. According to social costs of carbon and a Monte Carlo simulation, this report provides projected carbon market prices. All prices in the report are in US dollars (one Pulo equals approximately 0.6 US dollars). The results presented in this report are derived based on our best knowledge subject to main limitations of the growth rate of emission, carbon market price of Pullanta and exact purchase amounts of financial instruments. Therefore, we also provide in the attachment an Excel document containing a scenario analysis with estimations under different circumstances to demonstrate the uncertainties.

3. METHODOLOGIES

Table 1 provides an overview of the models and approaches used to design the program. Technical details of the methods are attached in the appendices.

Table 1: Methodologies

Methodology	Application	Appendix	Justification
Bayesian Estimation	Future carbon emission estimation	Appendix 1	Recursively estimating the unknown carbon emission probability density by a model
Full Credibility $P(\lvert S-\mu_S\rvert \leq 0.9\mu_S) \geq 0.9$	Demonstrating the goal of reduction of carbon emission	Appendix 2	To reach 90% confidence of carbon emission staying with 90% of the annual and ultimate goals
GARCH Model	Forecasting the future yearly and monthly volatilities	Appendix 3	Fitting for modeling time series data when the data exhibits heteroskedasticity and volatility clustering
Monte Carlo Simulation	Simulating carbon credits' market prices	Appendix 4	Utilizing time series models to estimate and simulate, incorporating the randomness
Sensitivity Analysis	Enterprise risk management	Appendix 5	Uncertainty limits the outgrowth of the program and sensitivity analysis helps to deal with the natural intrinsic variability of the program

4. CARBON CREDIT PROGRAM OVERVIEW

To reduce the amount of carbon emissions by the end of year 2030, the program uses cap-and-trade policy tool by placing a cap amount. The overall cap is set to control emissions coming from two parts: auction and free allowance. According to an emissions trading case study in the regional greenhouse gas initiative, 25% of the cap amount is divided into auction while the remaining is for free allowance [Roedner, Katelyn Sutter et al., 2018].

Based on California Emissions Trading System, the reserve price of auction is set at $15 per ton[1] in 2020, which increases at a 5% annual rate plus 5% inflation[2] adjustment. More details are presented in Section 5.

Above the cap amount, there are three financial instruments designed to allow for additional carbon credits under the estimated emissions. Instruments are subdivided into one intermediate-term bond, one long-term bond and one call option, equally distributed in terms of carbon emissions. Companies can choose to invest these instruments to mitigate the risk in case the carbon price will increase.

All carbon credits expire at the end of each year. Section 7 shows detailed descriptions of these instruments. Below are the levels of our carbon credit program (Table 2).

[1] *On Climate Change Policy*. 3 January 2020. Retrieved from https://onclimatechangepolicydotorg.wordpress.com/carbon-pricing/price-floors-and-ceilings/
[2] Retrieved from https://www.c2es.org/content/california-cap-and-trade/

Table 2: Levels of Carbon Credit Program

Level	Name	Class	Feature
5	Intermediate-term bond	Financial instruments	With a maturity of 10 years Coupons are distributed in the form of carbon credits annually at the beginning of each year where the credits come with a validity of one year
4	Long-term bond		With a maturity of 20 years Coupons are distributed in the form of carbon credits annually at the beginning of each year where the credits come with a validity of one year
3	Call Option		Purchasing carbon credits at a prespecified strike price
2	Auction	Cap-and-trade program	Gaining credits share by auctions for individual companies
1	Allowance		Free allowance distributed across various industries

Both carbon tax and carbon credit program aim to limit carbon emissions by creating a price for carbon emissions. Table 3 compares the policy characteristics for carbon tax and carbon credit program.

Table 3: Policy Characteristics Compared to Carbon Taxes

Line	Carbon Tax	Carbon Credit Program
Revenue of the government	Generating stably raising revenues for the government, and enhancing the efficiency of the tax when the revenues are used to reduce other distortionary taxes	Income is unstable depending on market fluctuations
Environment	Not necessarily achieving emission reduction targets solely through taxation May fail to make a substantial impact on pollution	A cap-and-trade system setting limits on emissions ensures the achievement of the goal of reduction to some degree
Awareness of emission reduction costs for companies	Generally considered to reduce emissions at the lowest cost because carbon tax is fixed and legal Companies choose the best emission reduction path	The price of will have greater volatility due to market fluctuations, which will increase compliance costs for companies
Management cost	Increasing the tax item of consumption tax and adjusting the tax rate of the corresponding tax items The management cost is low	Establishing a corresponding emissions trading market and the reporting, monitoring and punishment mechanism The management cost is high but it creates a new industry, bringing huge economic value

Although policy characteristics show that carbon credit program is more likely to achieve emission reduction goals than solely carbon taxation, there are still several risks to be highlighted which may have impacts on various stakeholders and the ultimate goal (Table 4).

Table 4: Risks

Risk	Description	Impact	Recommendation
Potential uncertainty and volatility of carbon market prices	Uncertainty of supply and demand impact volatile prices for allowances	If the issue price of carbon credit is too low, the government will not generate enough income Otherwise, companies may not profit	Enhance the simplicity and transparency of the market
Lack of illiquidity and hedging instruments [King, Michael R, 2018]	Need participants with different abatement costs Otherwise, there is no incentive to trade	Financial participants trading for compliance purposes in the carbon market make the market lack illiquidity and trading motivation	Brokers, dealers and investors such as hedge funds enhance efficiency of market by providing liquidity and promoting price discovery
Hampers of connecting with other countries' cap-and-trade system	Global market price of carbon credits will impact the carbon program	If the global market price is much lower that domestic one, it will impact emissions of carbon and goals of reduction	We suggest the government of pullanta increase international carbon tax to stabilize domestic carbon price

5. CARBON EMISSION

5.1 Annual Estimated Emission and Goals

Combining the data of aggregated and individual companies' CO2e Emissions by sectors from year 2015 to year 2019 (Figure 1), we project future emissions per sector using Non-parametric Bayesian Estimation (Figure 3). The complete steps of carbon emissions estimation are attached in Appendix 1.

Meanwhile, to meet the goal of reducing carbon emissions in 2030 to 25% below the 2018 level, we calculate annual target emissions in different sectors till 2030 (Figure 3) according to the proportions of the actual emissions by each sector within the total annual emissions (Figure 2).

Based on the emissions from 2015 to 2019, figure 3 provides the growth rates across sectors and the average growth rates of emissions. We estimate these average growth rates and use them to predict emissions for future years.

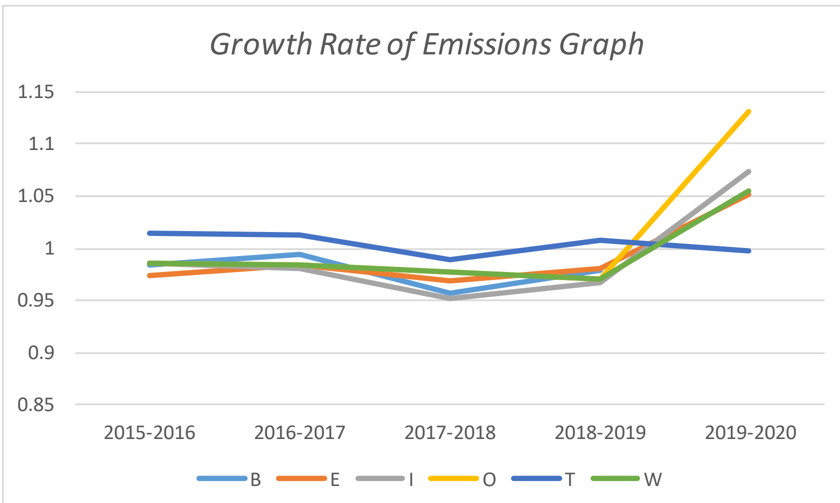
Figure 1: 2015-2019 Actual Aggregated Emissions per Sector

2015-2019 Actual Aggregated Emissions per Sector					
Sector / Year	2015	2016	2017	2018	2019
B	143,795,486	141,370,402	140,517,423	134,355,788	131,591,888
E	526,049,236	511,996,270	503,908,141	488,302,481	478,690,473
I	99,513,056	98,031,719	96,094,290	91,471,269	88,495,505
O	3,212,218	3,163,452	3,113,164	3,044,300	2,952,456
T	163,070,863	165,287,402	167,428,574	165,689,566	167,069,539
W	41,760,703	41,126,714	40,472,937	39,577,662	38,383,635

Source: Appendix 1

Figure 2: Growth Rates of Emissions Across Sectors

Growth Rates of Emissions Across Sectors						
Sector / Year	Growth Rate/i					Average Growth Rate
	2015-2016	2016-2017	2017-2018	2018-2019	2019-2020	
B	0.983135186	0.993966356	0.956150383	0.979428499	-	0.978170106
E	0.973285835	0.984202758	0.969030744	0.980315464	1.050857052	0.99153837
I	0.985114143	0.980236717	0.951890784	0.967467772	1.073857131	0.991713309
O	0.984818523	0.984103347	0.977879656	0.969830806	1.131841892	1.009694845
T	1.01359249	1.012954235	0.98961343	1.008328666	0.998055597	1.004508884
W	0.984818523	0.984103347	0.977879656	0.969830806	1.05553503	0.994433473



Source: Appendix 1

Figure 3: Projected Emissions and Goals per Sector

Projected Emissions and Goals per Sector												
Sector	Year	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
B	Total Emission	128719251	125909323	123160736	120472150	117842256	115269772	112753445	110292049	107884386	105529281	103225588
	Goal	**130887543**	**127508826**	**124217327**	**121010795**	**117887035**	**114843912**	**111879344**	**108991303**	**106177814**	**103436951**	**100766841**
E	Total Emission	503035259	498778761	494558280	490373511	486224152	482109903	478030467	473985551	469974860	465998107	462055004
	Goal	**475697494**	**463417891**	**451455273**	**439801456**	**428448469**	**417388548**	**406614126**	**396117833**	**385892491**	**375931105**	**366226860**
I	Total Emission	95031529	94244032	93463061	92688561	91920480	91158763	90403359	89654214	88911277	88174497	87443822
	Goal	**89110040**	**86809763**	**84568865**	**82385814**	**80259115**	**78187315**	**76168997**	**74202779**	**72287316**	**70421299**	**68603452**
O	Total Emission	3341713	3374110	3406822	3439850	3473199	3506871	3540870	3575198	3609859	3644856	3680192
	Goal	**2965714**	**2889158**	**2814577**	**2741922**	**2671142**	**2602190**	**2535017**	**2469579**	**2405829**	**2343725**	**2283225**
T	Total Emission	166744688	167496521	168251743	169010370	169772419	170537903	171306838	172079241	172855126	173634510	174417408
	Goal	**161412473**	**157245789**	**153186664**	**149232320**	**145380053**	**141627229**	**137971279**	**134409704**	**130940066**	**127559994**	**124267174**
W	Total Emission	40515272	40289742	40065468	39842443	39620659	39400109	39180788	38962687	38745800	38530120	38315641
	Goal	**38556008**	**37560728**	**36591139**	**35646579**	**34726403**	**33829979**	**32956696**	**32105955**	**31277176**	**30469790**	**29683246**

Source: Appendix 1

5.2 A Cap-and-Trade Program

5.2.1 Cap of Emissions

Let us denote A, the actual aggregated emissions in a given year in Pullanta. We can then divide A into three components according to the sources of carbon credits: Free Permit, Auctions and Financial Instruments. Figure 4 below illustrates a conceptualized constitution of A.

μ_A, on the other hand, represents the goal emissions set by the government according to the objective of the project: the level of emission should be reduced by 25% compared to that in 2018. Specifically, we can determine $\mu_{A,h}$, the targeted emission amount in the h^{th} year ahead of 2019 by

$$\mu_{A,h} = (0.75)^{\frac{h}{11}} \cdot Emission_{2018}, \quad h = 1, 2, \ldots 11.$$

Hence, the annual emission goals are a geometric sequence allowing slower reduction at the earlier phase of the program. Since the emission amount in 2018 is given, $\mu_{A,h}$ take fixed values. Figure 3 above displays targeted amount of emissions across sectors in the next decade using above computations.

Then we define x as the amount of emissions allowed excluding the amount of emissions from the three financial instruments, i.e., the sum of emissions from Auctions and Free Permit.

If we use Y to denote the actual amount of emissions coming from the financial instruments, then we have

$$Y = A - x. \qquad (1)$$

We assume that Y conforms to a continuous uniform distribution on the interval $[0, y_{max}]$, then

$$E(A) = \mu_A = x + \frac{y_{max}}{2}. \qquad (2)$$

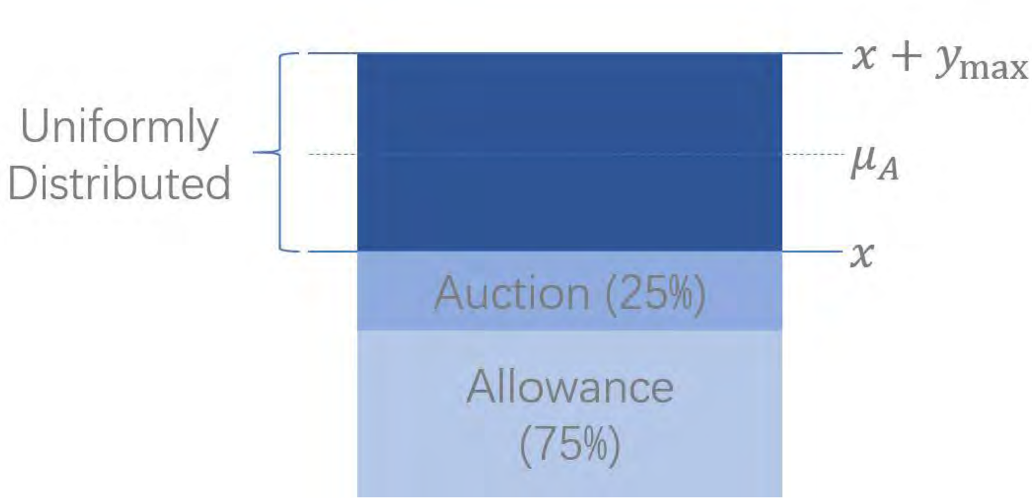
And $x + y_{max}$ is the fixed amount of emission, i.e., a threshold the total emission would not exceed. However, based on our definition of cap, which is x, so y_{max} is distributed to three financial instruments for companies to buy. In our design, we use the previously projected future emissions for this value, because those emissions represent the levels at which companies would have conducted if this carbon credit program was not put in place. That is to say,

$$x + y_{max} = Estimated\ Future\ Emissions.$$

Estimated future emissions have been calculated in Section 5.1.

Combining Equation (1) and (2) yields exact values for x and y_{max}. Figure 5 presents the cap of emissions, x, for each industry in the following years.

Figure 4: Allocation of the Carbon Program



Source: Appendix 2

Figure 5: 2020-2030 Cap of Emissions

2020-2030 Cap of Emissions						
Year	B	E	I	O	T	W
2020	125242281	455180365	85266668	2837801	154450653	36893064
2021	119547415	434482951	81389525	2708764	147427659	35215507
2022	114011131	414361889	77620347	2583320	140600231	33584664
2023	108629172	394801706	73956235	2461373	133963119	31999281
2024	103397393	375787336	70394368	2342829	127511210	30458137
2025	98311757	357304105	66931996	2227596	121239527	28960043
2026	93368331	339337727	63566444	2115586	115143221	27503841
2027	88563289	321874287	60295105	2006711	109217571	26088402
2028	83892901	304900238	57115441	1900887	103457979	24712630
2029	79353538	288402384	54024981	1798032	97859969	23375454
2030	74941665	272367880	51021317	1698065	92419182	22075832

Source: Appendix 1

5.2.2 Allowance and Auction Allocation

According to a “single-round, sealed-bid uniform-price” format in *The Regional Greenhouse Gas Initiative* [Roedner, Katelyn Sutter et al., 2018], we choose 75% of the whole cap of emission as the allocation for the free allowance and the remaining for auctions (Figure 6), seeing in Figure 4.

When allocating the free allowance, the carbon credits are distributed to each sector according to the proportion of histrical carbon emissions. Within each sector, the carbon credits for every company are distributed equally [An, Jaehyung and Lee, Jinho, 2020].

As for the auction component, the carbon credits are sold publicly to all companies starting at the reserve price without an upper limit.

Figure 6: 2020-2030 Allowance Allocation

2020-2030 Allowance Allocation						
Year	B	E	I	O	T	W
2020	93931711	341385274	63950001	2128351	115837990	27669798
2021	89660561	325862213	61042144	2031573	110570744	26411630
2022	85508348	310771417	58215260	1937490	105450173	25188498
2023	81471879	296101280	55467176	1846030	100472339	23999461
2024	77548045	281840502	52795776	1757122	95633408	22843603
2025	73733817	267978079	50198997	1670697	90929645	21720032
2026	70026248	254503295	47674833	1586689	86357416	20627881
2027	66422466	241405715	45221329	1505033	81913178	19566302
2028	62919676	228675178	42836581	1425665	77593484	18534472
2029	59515154	216301788	40518735	1348524	73394977	17531590
2030	56206249	204275910	38265988	1273549	69314386	16556874

Source: Appendix 1

5.2.3 Expected Revenue of Non-financial Instruments

Because the allowance portion is free to each company, the government will gain no revenue from this part. Therefore, we only consider the revenues coming from auctions.

Multiply the reserve price of auction by the carbon credits allocated to the auction, we obtain the minimum auction revenues (Figure 7).

Figure 7: Auction Revenue in Year 2020 to 2030

2020-2030 Auction Allocation		
Year	**Auction Reserve Price**	**Minimum Auction Revenue**
2020	15	3224515623
2021	15.75	3231789042
2022	16.5375	3236229912
2023	17.364375	3237634978
2024	18.23259375	3235789799
2025	19.14422344	3230468173
2026	20.10143461	3221431538
2027	21.10650634	3208428338
2028	22.16183166	3191193367
2029	23.26992324	3169447070
2030	24.4334194	3142894812

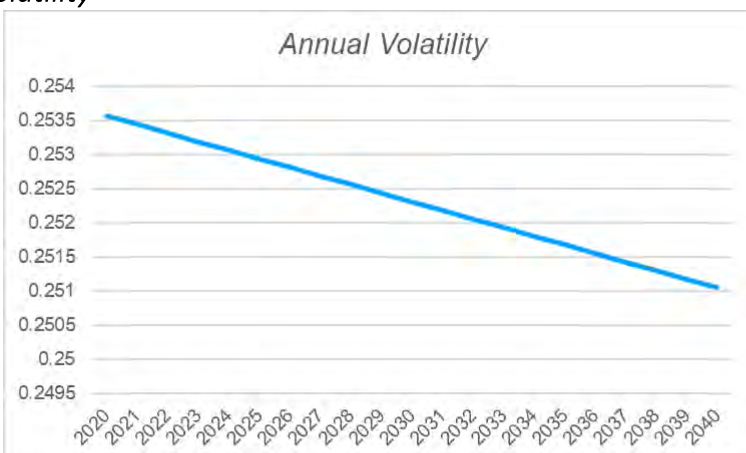
Source: Appendix 1

6. MARKET PRICE PREDICTIONS

6.1 Volatility

To get the carbon market prices, we first provide the estimated volatilities based on the European Union Emissions Trading System[3] carbon market price using a GARCH model. Figure 8 provides predicted annual volatilities.

Figure 8: Annual Volatility



Annual Volatility			
Year	Volatility	Year	Volatility
2020	0.2535732	2031	0.2521816
2021	0.2534463	2032	0.2520555
2022	0.2533196	2033	0.2519295
2023	0.2531929	2034	0.2518035
2024	0.2530663	2035	0.2516775
2025	0.2529397	2036	0.2515517
2026	0.2528132	2037	0.2514258
2027	0.2526868	2038	0.2513001
2028	0.2525604	2039	0.2511744
2029	0.2524341	2040	0.2510488
2030	0.2523078		

Source: Appendix 3

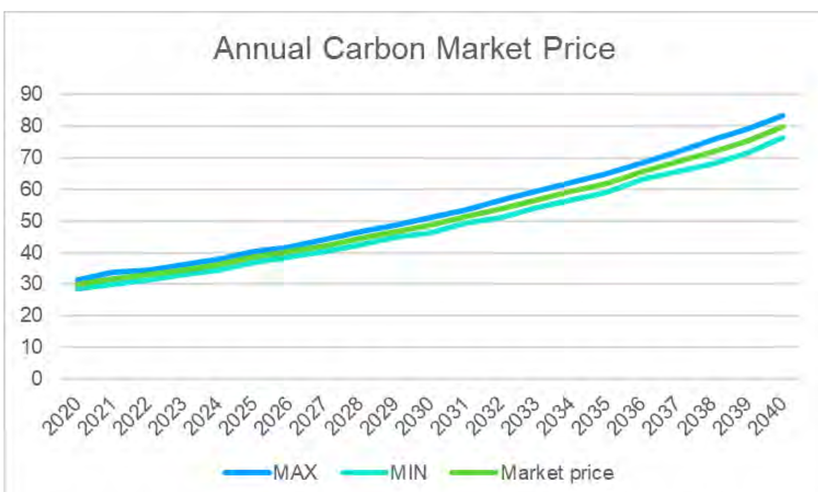
[3] EUA Price. Retrieved from https://sandbag.org.uk/carbon-price-viewer/

6.2 Simulation

According to Yale University economics professor William Nordhaus, we set the social cost of carbon (SSC) is $30 per ton[4], which is the economic losses caused by one ton of carbon emissions into the atmosphere. We will use it as the starting value for market prices in our Monte Carlo Simulation.

Assuming an annual inflation rate of 5% annually which is then used as the drift in a geometric brownian motion, together with the simulated volatilities presented above, we obtain the range of carbon prices through simulation. Then, we calculate the average of the maximum and minimum of the price to be the projected carbon market prices. More descriptions on the simulations can be found in the Appendix 4. Figure 9 providesa trajectory of the annual mean market prices.

Figure 9: Annual Carbon Market Price



[4] Retrieved from https://en.wikipedia.org/wiki/Carbon_credit#Setting_a_market_price_for_carbon

Annual Carbon Market Price			
Year	MAX	MIN	Market price
2020	31.35472892	28.43264673	30.0754572
2021	33.37112221	30.40144384	31.5139193
2022	34.41401416	31.27005028	33.17407458
2023	36.25853597	33.19368155	34.73339494
2024	38.33698753	34.91147964	36.94339445
2025	39.87889546	36.81412213	38.40410936
2026	42.31274222	38.70237496	40.38441097
2027	44.10779177	40.18784186	41.87935562
2028	46.29065088	41.86496891	43.97534673
2029	48.85285058	44.74872888	46.25875712
2030	50.79944006	46.40810983	49.04045633
2031	53.15606775	49.36880771	51.00382544
2032	56.06017555	51.40686547	53.80975671
2033	58.99136269	54.13518118	56.5119049
2034	61.62284695	56.5525338	59.27889681
2035	64.82078181	59.79984086	62.68817194
2036	68.50073536	62.41743011	65.34428088
2037	72.1415176	65.02228445	68.52663528
2038	75.15540771	68.83679712	71.92490586
2039	78.94279605	72.61535573	76.44969448
2040	83.81071381	76.05024147	79.73958102

Source: Appendix 4

7. CARBON CREDIT FINANCIAL INSTRUMENTS

7.1 Bonds

While reducing emissions, in order to raise more funds to invest in developing of renewable energy in the future, we designed two bonds. One is a 10-year bond as an intermediate-term investment and the other is 20-year bond as a long-term investment.

Both intermediate-term (Figure 10) and long-term bonds (Figure 11) are issued before year 2020, and annual carbon credits are paid to each bond holder as coupons at the beginning of each year. Finally, the government repays the principal at the end of the maturity year. Note that the validity of the coupon credits is the same as the usual carbon credits and will expire at the end of each issuance year.

We assume that the discount rate of an intermediate-term bond is 1% while that for a long-term one is 2% [Foster, Joanna M, 2012]. Using the idea of present values,

$$\text{PV} = \sum_i \frac{Coupon\ Value\ in\ the\ ith\ year}{Rate\ to\ Year\ 2020},$$

we calculate the total revenue, R, through

$$R = PV + \frac{R}{1.01^{11}}$$

for intermediate-term bond and

$$R = PV + \frac{R}{1.01^{21}}$$

long-term bond respectively.

Based on the assumption that each coupon contains 0.01 carbon emission in every intermediate-bond and 0.02 per long-term bond, we can derive that the maximum purchased number of bonds is the total present carbon credits in year 2020 divided by the carbon emissions per coupon per bond. Besides, we assume that the expected purchase amount is half of the maximum purchase amount for each bond. More suggestions and analyses are presented in Section 8.

Figure 10: Intermediate-term Bond

Intermediate-term Bond (Discount rate = 1%)					
Year	Total Carbon Credit	Predicted Carbon Value	Rate to year 2020	Present Carbon Credit	Present Carbon Value
2020	25838959.57	777118522.6	1	25838959.57	777118522.6
2021	36440222.99	1148374246	1.01	36079428.7	1137004204
2022	46714842.85	1549721681	1.0201	45794375.89	1519186041
2023	56671999.83	1968410952	1.030301	55005284.7	1910520277
2024	66320630.18	2450109201	1.04060401	63732822.04	2354506784
2025	75669432.17	2906017148	1.05101005	71996868.31	2764975604
2026	84726872.32	3421644832	1.061520151	79816546.37	3223344211
2027	93501191.53	3915769651	1.072135352	87210249.48	3652309051
2028	102000411	4485503439	1.082856706	94195668.22	4142287170
2029	110232337.9	5099210947	1.093685273	100789816.5	4662411641
2030	118204571.3	5796806116	1.104622125	107009056.4	5247772956
Total Present Carbon Credut	767469076.1	Unit Price	3.945216926	Carbon Emission per Bond	0.01
Total Present Value	31391436462	Maximum Purchase Amount	76746907612	Expected Revenue	1.51392E+11
Total Issue Revenue	3.02783E+11	Expected Purchase Amount	38373453806	Expected Carbon Emission	383734538.1

Figure 11: Long-term Bond

Long-term Bond (Discount rate = 2%)					
Year	**Total Carbon Credit**	**Predicted Carbon Value**	**Rate to year 2020**	**Present Carbon Credit**	**Present Carbon Value**
2020	25838959.57	777118522.6	1	25838959.57	777118522.6
2021	36440222.99	1148374246	1.02	35725708.81	1125857104
2022	46714842.85	1549721681	1.0404	44900848.57	1489544099
2023	56671999.83	1968410952	1.061208	53403291.18	1854877604
2024	66320630.18	2450109201	1.08243216	61270010.84	2263522178
2025	75669432.17	2906017148	1.104080803	68536136.08	2632069265
2026	84726872.32	3421644832	1.126162419	75235037.92	3038322691
2027	93501191.53	3915769651	1.148685668	81398414	3408913127
2028	102000411	4485503439	1.171659381	87056368.63	3828333995
2029	110232337.9	5099210947	1.195092569	92237489.22	4266791611
2030	118204571.3	5796806116	1.21899442	96968919.1	4755400042
2031	125924507.3	6422631586	1.243374308	101276426.9	5165485199
2032	133399344.4	7178186269	1.268241795	105184472.7	5659950886
2033	140636089.2	7947613302	1.29360663	108716271.2	6143763579
2034	147641560.8	8752028848	1.319478763	111893851.5	6632944078
2035	154422396	9680457710	1.345868338	114738114.8	7192722672
2036	160985054.1	10519452595	1.372785705	117268888.8	7662851205
2037	167335821.8	11466960832	1.400241419	119504979.3	8189274132
2038	173480817.4	12477591461	1.428246248	121464220.7	8736302638
2039	179425995.3	13717062522	1.456811173	123163522.3	9415813649
2040	185177150.5	14765948391	1.485947396	124618913.8	9937059974
Total Present Carbon Credut	1870400846	**Unit Price**	3.274173064	**Carbon Emission per Bond**	0.02
Total Present Value	1.04177E+11	**Maximum Purchase Amount**	93520042297	**Expected Revenue**	1.531E+11
Total Issue Revenue	3.06201E+11	**Expected Purchase Amount**	46760021148	**Expected Carbon Emission**	935200423

7.2 Call Option (Short-term Investments)

Pullanta can issue call options which provide buyers with the right to buy carbon credits at a specified strike price one month later.

Below is an example of trade details for a call option bought on February 1st, 2020 (Figure 12).

The current carbon credit price, S, is estimated by the social cost of carbon for the first month. The strike price of option, E, is our projected carbon market price in the next month via Monte Carlo Simulation and volatilities estimated according to EU carbon market prices. The risk-free interest rate, r, is a three-month U.S. treasury bill [Chen, James, 2020].

Then, get the current price of call option, C,

$$C = S\mathrm{N}(d_1) - E\mathrm{N}(d_2)\,,$$

where N is a standard cumulative normal distribution function,

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + rT + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln\left(\frac{S}{E}\right) + rT - \sigma^2 T/2}{\sigma\sqrt{T}},$$

where σ is the monthly volatility and T is the time to expire.

Finally, according to the same method as the maximum purchase amount of bond, we assume carbon emissions per option is 0.0002 and expected purchase amount is half of the maximum of purchase amount. Therefore, the ultimate monthly expected revenue and emissions can be predicted as follows.

Figure 12: Call Option

Call Option	
Month	2020/3/1
Market price	30.24510088
Current Stock Price	30
Strike Price of Option	30.24510088
Time to Exercise	0.083333333
Volatility	0.253573139
Risk-free Interest Rate	0.0285
d1	-0.042113188
d2	-0.115313448
Current Price of Call Option	0.76187691
Y/month	2153246.631
Maximum Purchase Amount/month	10766233153
Expected Purchase Amount/month	5383116577
Revenue/month	1640508.889
Unit Price	0.000152375
Expected Revenue	820254.4443
Carbon Emission Per Option	0.0002
Expected Carbon Emission	1076623.315

Risk-free Interest Rate is 2.85%[5]

[5] *Current Analysis & Forecast: 3 Month Treasury Bill Yield*. Retrieved from https://www.forecast-chart.com/year-treasury-3mo.html

7.3 The Trade of Carbon Credits in the Secondary Market

In the primary market, governments raise fiscal revenue through issuing bonds and call options to the public. In the secondary market, enterprises whose marginal benefit per unit of carbon is greater than the carbon market price, tend to buy carbon credits. Entities whose marginal benefit per unit of carbon is less than the carbon market price, will likely sell carbon credits. In our design, we decided not to set any limit on buying the credits, but purchases will stop and remain stable due to the increasing marginal cost. The limit on sales is automatically set by the carbon credit held by entities.

The carbon credit price should be determined by the equilibrium between the quantity of demand and the quantity of supply. The government can charge carbon credit transaction tax in the secondary market.

8. RECOMMENDATION

8.1 Data Limitations & Assumptions

Although data of CO2e Emissions by Sector are adequate to estimate the annual and ultimate emissions of carbon to achieve the goal of reduction, several assumptions have an influence on various stakeholders and likelihood of achieving goals. Table 5 lists all of the assumptions used for designing this program and their corresponding impacts.

Table 5: Data Limitations and Corresponding Assumptions

Data Limitation	Corresponding Assumption	Justification
No data of future emissions growth rates	Future growth rates remain similar to historical ones from year 2015 to 2019. The mean value of those rates was used to project future ones.	Estimated data following the growth trend of the number of companies. We also provide sensitivity analysis for this rate.
Missing values for some companies; No information about the number of companies per sector in the future	Count the number of companies including the ones with missing values regardless of problems in data recording Project future numbers of companies by taking five times the number of companies in year 2015	Combining the data of aggregation and companies to estimate the total amount of companies. From the sensitivity analysis below, the number of estimated companies is reasonable
No information about the distribution of actual emissions	Assuming that the amount of actual emissions is uniformly distributed on the interval $[x, x + y_{max}]$, where x is the limit set by the cap-and-trade program, and y_{max} is the amount of total emission of three carbon credit financial instruments	Due to the lack of required statistic data, we assume that the amount of actual emissions is represented by a symmetrical probability distribution, such as a continuous uniform distribution

No exact information about annual reduction goal until year 2030	Assuming annual goal of emission is $(0.75)^{\frac{h}{11}} \cdot Emission_{2018}, \quad h = 1, 2, \ldots 11$ where h is the h^{th} year ahead of 2019	The geometric growth of carbon emission reduction enables the institutions to smoothly adapt to the program
No data for inflation rate	Assuming annual inflation rate is 5%	Based on the California cap-and-trade program, launched in 2013
No exact data of drift as a reference	Assuming inflation of 5% annually as the annual drift	Based on California Cap-and-Trade System, we refer to the 5% inflation rate as the trend of market prices
No data about Pullanta carbon market price to analyze volatility	Using European weekly carbon market prices from year 2018 to 2020 as a reference to analyze volatility	Selecting carbon market price of EU countries with comparable economic development levels as Pullanta
No data for current social cost of carbon	Assuming the social cost of carbon $30 per ton as the current carbon credit price [Nordhaus, William]	Based on the suggestion of Yale University economics professor William Nordhaus, we set up $30 per ton social cost of carbon with inflation assumed above as carbon credit price
No exact information to get the market price	Utilizing the Monte Carlo Simulation to get the maximin and minimum price and take their average as current market price	Utilizing simulation to gain unpredictable market prices understand the impact of risk and uncertainty caused by price fluctuations
Risk-free Interest rate	Assuming the interest rate on a three-month U.S. Treasury bill as the risk-free rate	Investments must be accompanied by small risks, so the risk-free rate does not exist in fact
No information about setting strike price of option	Assuming the estimated carbon market price of next month as the strike price of option	Whether consumers decide to exercise an option is based on a comparison of the actual market price next month with the strike price of the option

No information about the exact purchase amounts of instruments	Assuming half of the issuance amount has been sold for each instrument allows us to calculate the minimum revenue of the government	No matter how much carbon credit is traded, the ultimate emission reduction target will be reached. We provide more analysis in exact purchase amounts recommendation

8.2 Sensitivity Analysis

The aim of our carbon program is to reduce emissions amount to 75% of the 2018 level at the end of 2030; meanwhile, we need to be 90% confident to achieve the 90% of the annual and ultimate reduction goals. Therefore, based on the assumptions we mentioned before, Figure 13 provides specific details and data to complete a proper sensitivity analysis. Then, Figure 14, 15 and 16 are legible sensitivity analyses.

Figure 13: Sensitivity Analysis Assumptions

A Cap-and-Trade System	
Allocation	25% of cap for auction and 75% for allowance
Auction Reserve Price	$15 per ton in 2020 with increasing rate of 5% and inflation of 5% annually
Two Bonds	
EUA Price	1 Unit
Drift	5%
Volatility	1 Unit
Number of Companies	1 Unit
2020 Average Emission	1 Unit
Carbon Emission Growth Rate	1 Unit
Market Price	1 Unit
Discount Rate	1% for intermediate-term bond and 2% for long-term bond

Carbon Emission Per Bond	0.01 ton per bond for intermediate-term bond and 0.02 ton per bond for long-term bond
Call Option	
EUA Price	1 Unit
Drift	5%
Volatility	1 Unit
Number of Companies	1 Unit
2020 Average Emission	1 Unit
Carbon Emission Growth Rate	1 Unit
Current Carbon credit Price/S	1 Unit
Strike Price of Option/E	1 Unit
Risk-free Interest rate/r	2.85%
Carbon Emission Per Bond	0.0002 ton per call option

For the two carbon bonds, based on the uncertainty of estimated carbon market prices, drift, volatility, discount rate and expected purchase amount, Figure 14 and 15 provide respective sensitivity analyses for these parameters to as an illustration for the feasibility and accuracy of our proposal.

For each scenario in our sensitivity analysis, we provide several different magnitudes of changes in the parameters: 20% and 40% for two bonds and 1% and 2% for call option respectively.

The characteristic of uncertainties in the method of simulation leading to the randomness of carbon market price and thus the emissions is are presented in Scenario 1 to 3, It can be seen that the emissions would have a favorable change from the increases of prices and volatilities in general, yet the changes are not too sensitive to any of the parameters.

From scenario 4 to 6, the numbers of companies, average emission in year 2020 and the growth rate of carbon emission are critical factors of our program design, the in crease in which would raise the estimated carbon emissions substantially. However, in practice, it would probably be unlikely to have such dramatic changes in the number of companies and the emission growth rates. In fact, the government could monitor these data easily and use them as indicators for potential abnormal movements in the actual emissions. Immediate actions can be carried out towards unexpected changes to avoid future over-emissions.

The remaining factors would would also to some extent influence the expected revenue and carbon emissions, but those impacts are not as significant. More analyses and recommendations are in Section 8.

Figure 14: Intermediate-term Bond Sensitivity Analysis

Intermediate-term Bond Sensitivity Analysis					
	Variable Values	Expected Revenue in the Beginning of 2020	Percentage Change in Expected Revenue	Expected Carbon Emission	Percentage Change in Carbon Emission
	Base Case	1.51392E+11	-	383734538.1	-
Scenario 1	EUA Price Increases 20% and 40%	1.51522E+11	0.000864037	383402402.4	-0.000865535
		1.51427E+11	0.000230861	383645907.9	-0.000230967
Scenario 2	Drift Increases 20% and 40%	1.51463E+11	0.000469678	383554136.8	-0.00047012
		1.51461E+11	0.000457923	383558655.9	-0.000458343
Scenario 3	Volatility Increases 20% and 40%	1.51504E+11	0.000744105	383448573	-0.000745216
		1.51647E+11	0.001684363	383085997.7	-0.001690075
Scenario 4	Company Number Increases 20% and 40%	6.67839E+11	3.411331552	1694925176	3.416921095
		1.38908E+12	8.175439648	3492464860	8.101252334
Scenario 5	2020 Average Emission Increases 20% and 40%	6.67839E+11	3.411331552	1694925176	3.416921095
		1.38908E+12	8.175439648	3492464860	8.101252334
Scenario 6	Carbon Emission Growth Rate Increases 20% and 40%	2.59141E+12	16.11726898	6210454808	15.18424768
		1.04608E+13	68.09755864	24375595937	62.52202765
Scenario 7	Market Price Increases 20% and 40%	1.63503E+11	0.08	345361084.3	-0.1
		1.69559E+11	0.12	306987630.4	-0.2
Scenario 8	Discount Rate Increases 20% and 40%	1.48199E+11	-0.021085115	376203633.3	-0.019625298
		1.47645E+11	-0.024746111	374818220	-0.023235641
Scenario 9	Expected Purchase Amount Increases 20% and 40%	1.09002E+11	-0.28	460481445.7	0.2
		42389647849	-0.72	537228353.3	0.4

Source: Intermediate-term Bond and Appendix 5

Figure 15: Long-term Bond Sensitivity Analysis

Long-term Bond Sensitivity Analysis					
	Variable Values	Expected Revenue in the Beginning of 2020	Percentage Change in Expected Revenue	Expected Carbon Emission	Percentage Change in Carbon Emission
	Base Case	1.531E+11	-	935200423	-
Scenario 1	EUA Price Increases 20% and 40%	1.53314E+11	0.001394022	933893077.4	-0.001397931
		1.53182E+11	0.000531375	934702951.4	-0.000531941
Scenario 2	Drift Increases 20% and 40%	1.53062E+11	-0.000251187	935435215.4	0.000251061
		1.53294E+11	0.001265345	934014061.4	-0.001268564
Scenario 3	Volatility Increases 20% and 40%	1.5323E+11	0.000844723	934409098.5	-0.000846155
		1.53193E+11	0.000606382	934632645.1	-0.000607119
Scenario 4	Company Number Increases 20% and 40%	7.40535E+11	3.836920843	4438021551	3.74552988
		1.84994E+12	11.08319482	10668691853	10.40792026
Scenario 5	2020 Average Emission Increases 20% and 40%	7.40535E+11	3.836920843	4438021551	3.74552988
		1.84994E+12	11.08319482	10668691853	10.40792026
Scenario 6	Carbon Emission Growth Rate Increases 20% and 40%	7.6147E+12	48.73663376	39735340477	41.48858266
		1.06134E+14	692.2302967	5.10951E+11	545.3543993
Scenario 7	Market Price Increases 20% and 40%	1.65348E+11	0.08	841680380.7	-0.1
		1.71472E+11	0.12	748160338.4	-0.2
Scenario 8	Discount Rate Increases 20% and 40%	1.45723E+11	-0.048184816	895033819.7	-0.042949727
		1.38786E+11	-0.093497728	857137887.4	-0.08347145
Scenario 9	Expected Purchase Amount Increases 20% and 40%	1.10232E+11	-0.28	1122240508	0.2
		42868112480	-0.72	1309280592	0.4

Source: Long-term Bond and Appendix 5

In addition, we consider the uncertainties in current carbon credit prices, strike prices of a call option and risk-free interest rate and their impact on amount of emissions (Figure 16).

Figure 16: Option Sensitivity Analysis

Call Option Sensitivity Analysis						
	Variable Values	Expected Revenue in the Beginning of 2020	Percentage Change in Expected Revenue	Expected Carbon Emission	Percentage Change in Carbon Emission
	Base Case	820254.4443	-	1076623.315	-
Scenario 1	EUA Price Increases 1% and 2%	826252.058	0.007311894	1068632.543	-0.007422068
		842624.6977	0.027272334	1045456.849	-0.028948348
Scenario 2	Drift Increases 1% and 2%	880892.4903	0.073925897	979515.2989	-0.090196836
		817724.4819	-0.003084363	1079923.777	0.003065568
Scenario 3	Volatility Increases 1% and 2%	824828.476	0.005576357	1070551.186	-0.005639976
		842872.4192	0.02757434	1045088.838	-0.029290168
Scenario 4	Company Number Increases 1% and 2%	991394.952	0.208643195	1301253.443	0.208643195
		1162535.46	0.417286389	1525883.571	0.417286389
Scenario 5	2020 Average Emission Increases 1% and 2%	991394.952	0.208643195	1301253.443	0.208643195
		1162535.46	0.417286389	1525883.571	0.417286389
Scenario 6	Carbon Emission Growth Rate Increases 1% and 2%	847495.618	0.033210638	1112378.663	0.033210638
		874736.7916	0.066421276	1148134.01	0.066421276
Scenario 7	Current Carbon Credit Price Increases 1% and 2%	827750.4342	0.009138615	1066597.752	-0.009312044
		835043.1611	0.018029426	1056456.937	-0.018731137
Scenario 8	Strike Price of Option Increases 1% and 2%	741642.7871	-0.095838136	1165222.575	0.082293647
		654621.6101	-0.201928603	1242754.031	0.154307187
Scenario 9	Risk-free Interest rate Increases 1% and 2%	820249.4534	-6.08469E-06	1076629.866	6.08462E-06
		820244.4124	-1.22302E-05	1076636.482	1.22299E-05
Scenario 10	Expected Purchase Amount Increases 1% and 2%	811887.849	-0.0102	1087389.548	0.01
		803193.1519	-0.0208	1098155.782	0.02

8.3 Recommendations

According to the above sensitivity analysis, ruling out the first 6 scenarios, we find that expected purchase amount is the most influential factor on the ultimate carbon emissions as well as expected revenue on bonds and strike price on call options. Specifically, we provide the following recommendations.

Expected Purchase Amount Recommendation

Expected purchase amount is directly related to the revenue. Because the elasticity of demand is -0.50 [Goldstein, David B, 2015], methods of reducing issuance price such as coupons cannot increase total profit. Therefore, based on the confidence of premise that a given emission quota is sufficient to meet the emission reduction target, more transactions of three financial instruments could generate more revenues for the government, thus benefiting the program in both short and long run.

Market Price Recommendation

Based on a series of assumptions and uncertainties related to projected carbon market prices, Pullanta should pay more attention to the influence of market prices while pricing bonds, determining carbon credit issuance prices and the strike price of the call option. Therefore, we suggest that the government should take actions to control carbon market prices as stable as possible and mitigate substantial price fluctuations, so that carbon emissions can be predictable and manageable.

Discount Rate or Risk-free Interest Rate Recommendation

Pullanta needs to create a mutually beneficial environment both ecologically and economically. As a result, we recommend that the government should adopt macroeconomic regulation and control to stabilize economic development so as to keep the discount rate and risk-free interest rate within a reasonable range of change. In this way, it could reduce the risks of excessive emissions and abnormal revenues.

Internalizing Negative Externalities by Penalties Recommendation

We recommend a policy that will make companies to internalize negative externalities of their carbon emissions where “externalities” are events due to climate change driven by carbon emissions. If the entities exceed carbon emissions limits because they do not have enough carbon credits to cover their emissions, the government will fail to achieve the emission goal. Therefore, we recommend that the government will enforce tough legal sanctions and high penalties up to €100 (approximately 181.82 Pulo)[6]. Meanwhile, the earnings of the company will be regulated.

[6] https://www.carbon-cap.com/emissions-trading

9. CONCLUSION

We recommend this comprehensive implementation plan for our carbon credits program. Based on the cap-and trade system and three financial instruments, Pullanta government could gain economic income while cutting down emissions. Our carbon credits program and analyses demonstrate that under the premise of 90% certainty of achieving the reduction goal, the government can raise funds through the financial instruments to increase investments in additional climate change mitigation efforts in the future.

10. APPENDICES

APPENDIX 1: Carbon Emissions Estimation

From the table of Pullanta Companies CO2e Emissions data from 2015 to 2019, we first classify the emissions by companies according to their industrial types. There are six categories of sectors present in the data: Buildings and Land Use, Energy, Manufacturing and Construction, Industrial Processes and Product Use, Transport, Waste and Other. Because CompanyData contains many missing values, we combine the average emission for each sector in CompanyData with the average numbers of companies from AggregateData to obtain the ultimate estimated carbon emissions for each sector in year 2020 according to the following steps. Besides, due to the missing data of Buildings and Land Use sector, we directly use the aggregated data to estimate subsequent emissions for this sector.

Initially, according to the company's emissions from 2015 to 2019 provided in CompanyData, annual total carbon emissions of each sector are calculated according to the sector classification. Next we count the numbers of each type of companies in CompanyData. Then, we calculate the annual average emission for each sector (Figure 1-1) according to

$$The\ Annual\ Average\ Emission\ for\ Each\ Sector = \frac{The\ Total\ Carbon\ Emission\ Counted\ per\ Year\ for\ Each\ Sector}{The\ Number\ of\ Company\ Counted\ Per\ Year\ for\ Each\ Sector}.$$

There are many zeros in CompanyData which are not completely regarded as missing values. Considering possible company failures, new construction, and problems in data recording, we assume the counts of companies remain constant, for simplicity.

Secondly, based on aggregate carbon emissions data from 2015 to 2019 provided in AggregateData, we can roughly predict the number of companies per year for each sector (Figure 1-2) by

$$The\ \ Number\ of\ Companies\ per\ Year\ for\ Each\ Sector$$
$$= \frac{The\ Total\ Carbon\ Emission\ per\ Year\ for\ Each\ Sector}{The\ Average\ Emission\ \ per\ Year\ for\ Each\ Sector}.$$

Finally, we determine the number of companies in year 2020 and subsequent years as the average predict number of companies (Figure 1-2).

Figure 1-1: Average Emission and company number for each Sector from CompanyData

Bayesian Estimation						
Sector / Year		2015	2016	2017	2018	2019
E	Average Emission	1610506.5	1570881.775	1576489.705	1548035.97	1322642.897
	Company Number	302	302	302	302	302
I	Average Emission	64646.75964	68688.03788	66997.35415	62212.51212	61874.61058
	Company Number	1361	1361	1361	1361	1361
O	Average Emission	112096.8809	110077.5893	106712.5862	106005.069	101149.931
	Company Number	29	29	29	29	29
T	Average Emission	1659489.34	1680966.455	1904510.487	2037678.5	2119404.936
	Company Number	78	78	78	78	78
W	Average Emission	249579.3963	240046.0711	232433.1491	219111.9006	230313.7578
	Company Number	161	161	161	161	161

Bayesian Estimation

Based on the annual average emissions of each of the remaining five sectors in CompanyData, $\overline{x_i} = \frac{\sum_{j=1}^{5} m_{ij} x_{ij}}{m_i}, i = year\ 1,2 \ldots 5$, where $m_{\{ij\}}$ is the number of companies in the j[th] sector in the i[th] year, we estimate expected process variance (EPV),

$$\hat{v} = \frac{\sum_{i=1}^{5}\sum_{j=1}^{5} m_{ij}(x_{ij} - \overline{x_i})^2}{\sum_{i=1}^{5}(n_i - 1)},$$

where n is the number of total number of exposures.

Then, according to variance of hypothetical mean (VHM),

$$\hat{a} = \frac{[\sum_i^5 m_i (x_i - \bar{X})^2] - \hat{v}(6-1)}{N - N^{-1}(\sum_{i=1}^{5} n_i{}^2)},$$

where $\bar{x}$ is the sum of $\overline{x_i}$, and N is total number of the estimated companies above, we can obtain $k = \frac{\hat{v}}{\hat{a}}$ and $Z_i = \frac{n_i}{n_i + k}$.

Figure 1-2

n_1	1510	Estimating EPV, $\hat{v}$	1.48214E+12
n_2	6805	Estimating VHM, $\hat{a}$	7.84072E+11
n_3	145	k	1.890305972
n_4	390	$\hat{\mu}$	763208.6324
n_5	805	Total Company Number	9655
Z_1	0.998749707	$\overline{x_1}$	1525711.37
Z_2	0.999722295	$\overline{x_2}$	64883.85488
Z_3	0.987131173	$\overline{x_3}$	107208.4113
Z_4	0.995176441	$\overline{x_4}$	1880409.944
Z_5	0.997657295	$\overline{x_5}$	234296.855
		$\bar{x}$	381447.2156

Finally, expected hypothetical mean (EHM) is estimated by is

$$\hat{\mu} = \frac{\sum_{i=1}^{5} \bar{x}_i Z_i}{\sum_{i=1}^{5} Z_i},$$

and we note $Z_i \cdot \bar{x}_i + (1 - Z_i) \cdot \hat{\mu}$ as the carbon emissions for one company in a certain sector.

Figure 1-3: Estimated Number of Companies from AggregateData and Ultimate Estimated Total Emission in Year 2020

Bayesian Estimation Result			
Sector	Estimating Company Number/N	2020 Average Emission	2020 Total Emission
E	329.9115355	1524758.018	503035259
I	1460.276064	65077.783	95031528.83
O	28.89496171	115650.3648	3341712.864
T	88.92950161	1875021.057	166744688.1
W	172.01312	235535.9392	40515271.77

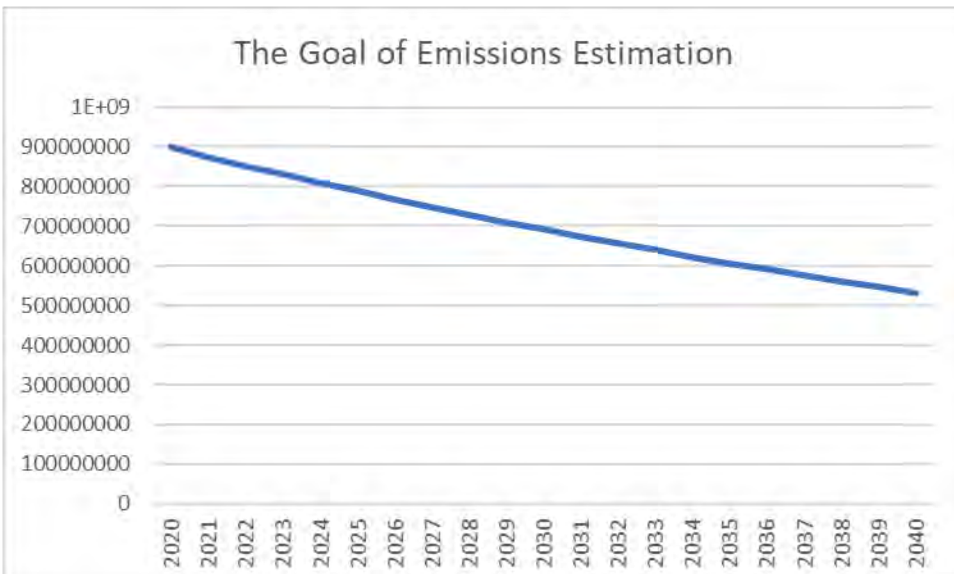
APPENDIX 2: Proof of Goal

To reach the goal of reducing carbon emissions to 25% of 2018 level by the end of the year 2030, we estimate the goal of emissions, μ_A, of each year from 2020 to 2030 using

$$\mu_A = (0.75^{\frac{1}{11}})^x \times Emission_{2018},$$

where x is the number of years ahead of year 2019.

Figure 2-1: The Goal Emissions Estimation, μ_A



To demonstrate that program objective mathematically: with 90% certainty to have aggregated carbon emissions within 90% of the annual and ultimate goals,

$$P(|A - \mu_A| \leq 0.9\mu_A) \geq 0.9,$$

where A means actual emissions in the current year. In order to have the above inequality hold, we need to show that

$$n_e \geq (\frac{1.645}{0.9})^2 \frac{{\sigma_A}^2}{{\mu_A}^2},$$

where n_e is the number of exposures in a certain year. Based on the hypothesized amount of emissions, we assume that A is uniformly distributed on the interval $[x, x + y_{max}]$, where x is the capped amount of both issued carbon free allowance and paid carbon credit by the government, and y_{max} is the amount of total emissions resulting from the three carbon credit financial instruments.

Figure 2-2:

As a result, $\mu_A = \frac{(x+y_{max})+x}{2}$, and ${\sigma_A}^2 = \frac{{y_{max}}^2}{12}$. In this case, we can get $n_e \geq (\frac{1.645}{0.9})^2 \frac{{y_{max}}^2}{12\mu_A^2}$. Because the number of exposures is one in a certain year, then $n_e = 1$, we simplify the above-mentioned inequality to $y_{max} \leq 1.895\mu_A$, which is always true from year 2020 to 2030 under our hypothesis. Here we use the data of year 2020 as an example to prove the inequity. Throughout Bayesian Estimation, we obtain $(x + y_{max})$ = 898629272 for year 2020. Besides, $\mu_A = 937699055$. Hence, $x = 859559489$ and $y_{max} = 78139566$. We find that $y_{max} \leq 1776939709$. Therefore, $n_e \geq (\frac{1.645}{0.9})^2 \frac{{\sigma_A}^2}{{\mu_A}^2}$ always holds for year 2020 and years after.

APPENDIX 3: R Code

```
install.packages("rugarch")
library(rugarch)

# set working directory
setwd("C:/Users/11/Desktop/R")

# load data
dfweek <- read.csv('carbonweek.csv')

# calculate weekly return
dfweek$Return <- c(diff(log(dfweek$Price)), NA)
# visualize return
hist(dfweek$Return, breaks = 20)


# default specification of the model
ug_spec <- ugarchspec()
ug_spec
# specify a standard GARCH(1,1), mean using ARMA(1,0), normal distribution
ug_spec <- ugarchspec(variance.model=list(model="sGARCH",garchOrder=c(1,1)),
                      mean.model=list(armaOrder=c(1,0),include.mean=TRUE),
                      distribution.model="norm",fixed.pars=list(omega=0))
# fit model using weekly returns
ugfit <- ugarchfit(spec=ug_spec, data=dfweek$Return[-nrow(dfweek)])
ugfit

# calculate annual volatility
annualvol <- sqrt(52)*ugfit@fit$sigma
# visualize annual volatility
plot(annualvol, type='l')

# forecast 21 periods (weeks) ahead
ugfore <- ugarchforecast(ugfit, n.ahead=21)
# transform to annual volatility
annualfore <- sqrt(52)*ugfore@forecast$sigmaFor
```

APPENDIX 4: Monte Carlo Simulation

A Geometric Brownian Motion was adopted to simulate carbon market prices whose stochastic differential equation (SDE) is

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

Where W_t is a Brownian motion and μ and σ are the two parameters referred to as drift and volatility, respectively. S_t is a stochastic process that describes the price trajectory.

First the two parameters in the model need to be estimated. We assume that a 5% inflation rate can be used as the annual drift and estimated annual volatilities by fitting a GARCH model to the EU carbon market prices[7]. In addition, we set $30 per ton which is the social cost as the starting value of price in year 2020.

Subsequently, according to the well-known result for the analytic solution to the SDE shown above,

$$S_t = S_0 \cdot \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right),$$

where S_0 is the initial price. When we run the simulation, first we randomly generate daily prices, for which we need to derive expected daily drift by

$$\mu_{\{daily\}} - \frac{\sigma^2_{\{daily\}}}{2}$$

where daily drift is converted by dividing annual drift by 252 trading days and daily volatility is converted by dividing annual volatility by the square root of $252^{\frac{1}{2}}$.

[7] https://sandbag.org.uk/carbon-price-viewer/

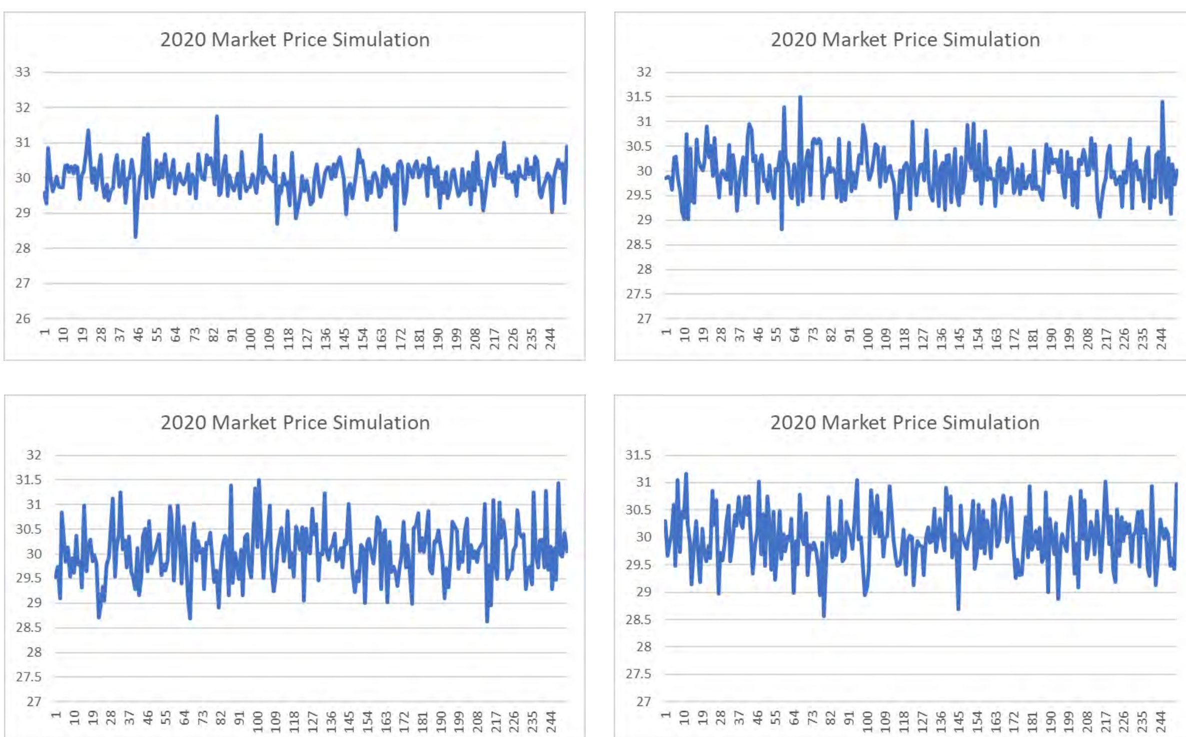
To get the daily market price, we can obtain the daily $log\ return$ by, for instance, on the first day,

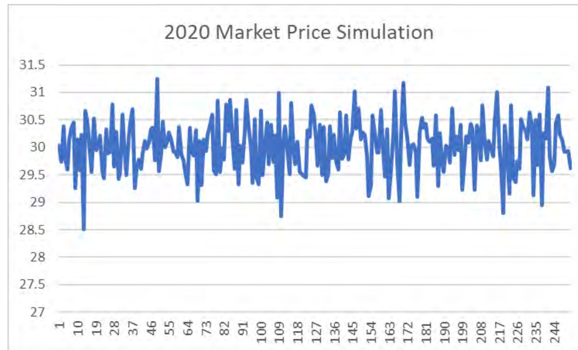
$$\ln\frac{S_1}{S_0} = \mu_{\{daily\}} - \frac{\sigma^2_{\{daily\}}}{2} + \sigma_{\{daily\}} \times W_1$$

where W_t can be computed by the inverse of standard normal cumulative distribution of a random value generated between 0 and 1. Prices in the sequel are then simulated recursively.

Due to the randomness from the Brownian Motion, we can simulate a range of daily market prices. Then, we simply calculate the mean of the maximize price and the minimize price to be daily market prices.

Figure 4-1: Examples of Market Simulation in Year 2020





Similarly, for generating monthly market prices, we first estimate the monthly volatilities via a GARCH model. Then, we transform monthly volatilities to get daily volatilities by dividing $21^{\frac{1}{2}}$. The monthly drift is determined to be $1.05^{\frac{1}{12}}$. The same simulation algorithm can be run as described above.

APPENDIX 5: Explanation of Sensitivity Analysis

Firstly, we analyze the intermediate-term bonds. We let each share of bond contain 0.01-ton carbon emissions as total coupons to be allocated annually prior to maturity. Then, we obtain the total issuance amount according to

$$Maximum\ Purchase\ Amount = \frac{Total\ Present\ Carbon\ Credit\ in\ Year\ 2020}{0.01}$$

As we mentioned before, the expected purchase amount is then half of the maximum purchase amount.

Secondly, we achieve the unit price by

$$Unit\ Price = \frac{Total\ Revenue}{Maximum\ Purchase\ Amount}$$

Thirdly, the expected revenue can be calculated by

$$The\ Expected\ Revenue = The\ Expected\ Purchase\ Amount \times Unit\ Price$$

In the sensitivity analysis, the change in unit price would be affected by changes in our assumptions on the parameters. Thus, the percentage change in price would be altered accordingly.

$$The\ Percentage\ Change\ in\ Price = \frac{The\ Changed\ Unit\ Price - The\ Original\ Unit\ Price}{The\ Original\ Unit\ Price}$$

According to the price elasticity of demand of -0.5, we can further obtain the percentage change in demand by

$$The\ Percentage\ Change\ in\ Demand = -0.5 \times The\ Percentage\ Change\ in\ Price,$$

through which we can calculate the change in expected purchase amount according to

$$\begin{aligned}&\mathit{The\ Change}\ \text{in}\ \mathit{Purchase\ Amount}\\&\qquad = \mathit{The\ Original\ Expected\ Purchase\ Amount} \times (1\\&\qquad + \mathit{The\ percentage\ Change\ in\ Demand})\end{aligned}$$

The change in expected revenue and the expected carbon emission should be given by

$$\begin{aligned}&\mathit{The\ Change\ in\ Expected\ Revenue}\\&\qquad = \mathit{The\ Changed\ Purchase\ Amount} \times \mathit{The\ Changed\ Unit\ Price},\end{aligned}$$

$$\mathit{The\ Change\ in\ Expected\ Carbon\ Emission} = \mathit{The\ Change\ in\ Purchase\ Amount} \times 0.01.$$

Therefore, we obtain the percentage change in expected revenue and the percentage change in expected carbon emission by

$$\begin{aligned}&\mathit{The\ Percentage\ Change\ in\ Expected\ Revenue}\\&\qquad = \frac{\mathit{The\ Changed\ Expected\ Revenue} - \mathit{The\ Original\ Expected\ Revenue}}{\mathit{The\ Original\ Expected\ Revenue}}\end{aligned}$$

$$\begin{aligned}&\mathit{The\ Percentage\ Change\ in\ Expected\ \ Carbon\ Emission}\\&= \frac{\mathit{The\ Changed\ Expected\ \ Carbon\ Emission} - \mathit{The\ Original\ Expected\ \ Carbon\ Emission}}{\mathit{The\ Original\ Expected\ \ Carbon\ Emission}}\end{aligned}$$

Similarly, for the sensitivity analyses of the long-term bonds and the call option, we simply substitute the carbon emissions allocated to each unit of the instrument to 0.02 and 0.0002-ton, respectively, and follow the same steps described above.

APPENDIX 6: GDP

Economic and environmental quality have close correlation, so both GDP and GDPpc are positively related carbon emissions [Archer, Hannah, 2018]. As economic activity increases, so does carbon dioxide emissions. This analysis provides the correlation coefficient of GDP or GDPpc and carbon emissions, which are $0.43^{\frac{1}{2}}$ and $0.3997^{\frac{1}{2}}$ respectively (Figure 6-1 and 6-2).

Figure 6-1: The Correlation Coefficient of GDP and Carbon Emissions

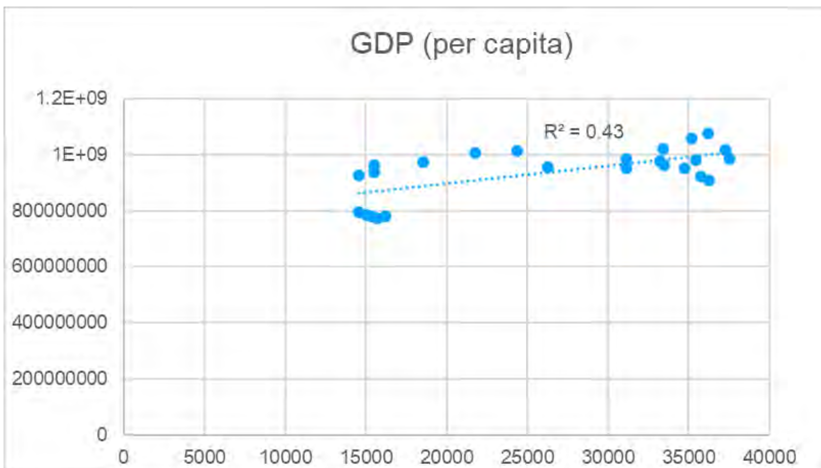
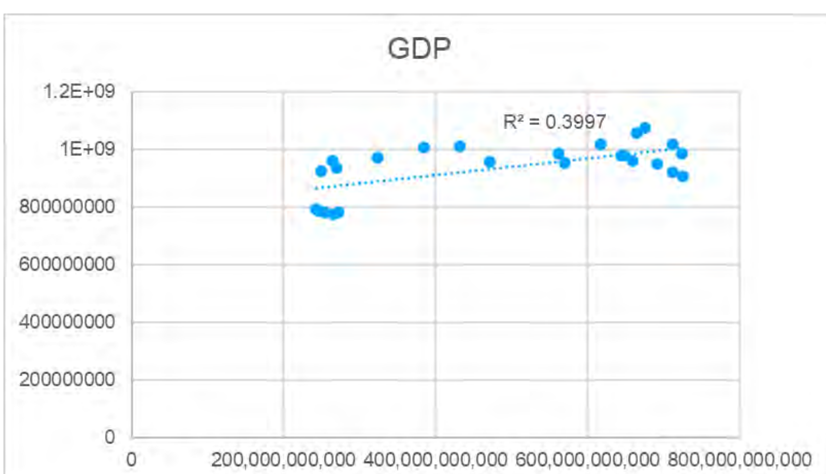


Figure 6-2: The Correlation Coefficient of GDPpc and Carbon Emissions



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