1. Which of the following is not a common feature of universal life insurance?

(A) Premiums that are flexible, subject to minimum account value conditions.

(B) A terminal bonus that is added to the benefit at maturity.

(C) A credited interest rate that is subject to a minimum value $\geq 0\%$.

(D) Surrender charges that apply during the first few years of the policy.

(E) Monthly deductions that cover the cost of insurance and other expenses.
2. Mortality from a certain disease is modelled by a select and ultimate table with a three year select period. An excerpt from the table is given below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(l_x)</th>
<th>(l_{x+1})</th>
<th>(l_{x+2})</th>
<th>(l_{x+3})</th>
<th>(x + 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>130,000</td>
<td>126,000</td>
<td>120,000</td>
<td>112,000</td>
<td>51</td>
</tr>
<tr>
<td>49</td>
<td>120,000</td>
<td>116,000</td>
<td>110,000</td>
<td>102,000</td>
<td>52</td>
</tr>
<tr>
<td>50</td>
<td>110,000</td>
<td>106,000</td>
<td>100,000</td>
<td>92,000</td>
<td>53</td>
</tr>
<tr>
<td>51</td>
<td>100,000</td>
<td>96,000</td>
<td>90,000</td>
<td>82,000</td>
<td>54</td>
</tr>
<tr>
<td>52</td>
<td>90,000</td>
<td>86,000</td>
<td>80,000</td>
<td>72,000</td>
<td>55</td>
</tr>
</tbody>
</table>

A set of patients who are age 50 at the start of the study are assigned to a trial group for a new treatment. If the lives had been selected at age 50, the expected number of lives surviving to age 52 would have been 200.

It is learned that all the lives were actually selected two years before the study began, when each life was age 48.

Calculate the expected number of lives surviving to age 52 from the group.

(A) 187  
(B) 188  
(C) 189  
(D) 190  
(E) 191
3. You are given the following survival function for a newborn:

\[ S_0(x) = (1 - 0.01x)^{0.5}, \quad 0 \leq x \leq 100. \]

Calculate \( 1000 \mu_{25} \).

(A) 2.3
(B) 3.4
(C) 4.5
(D) 5.6
(E) 6.7
4. At time $t = 0$ a mortality study has 100 participants. The table below lists all events during the study.

<table>
<thead>
<tr>
<th>Time, $t$</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>50 participants drop out</td>
</tr>
<tr>
<td>3.6</td>
<td>1 participant dies</td>
</tr>
<tr>
<td>5.5</td>
<td>21 new participants join the study</td>
</tr>
<tr>
<td>5.8</td>
<td>1 participant dies</td>
</tr>
<tr>
<td>6.6</td>
<td>49 participants drop out</td>
</tr>
<tr>
<td>7.4</td>
<td>1 participant dies</td>
</tr>
<tr>
<td>9.0</td>
<td>30 new participants join the study</td>
</tr>
</tbody>
</table>

Calculate the Kaplan-Meier estimator of the survival probability to time 7.

(A) 0.960
(B) 0.962
(C) 0.964
(D) 0.966
(E) 0.968
5. Mortality and morbidity follow the Standard Sickness-Death model illustrated below.

An insurer is interested in the probability that a Healthy life dies within 1 year of the next transition from Healthy to Sick.

Identify which of the following expressions is equal to the required probability.

(A) \[ \int_0^{\infty} t^{01} \mu_{x+t}^{12} \cdot p_{x+t}^{22} \, dt \]

(B) \[ \int_0^{\infty} t^{00} \mu_{x+t}^{01} \cdot p_{x+t}^{12} \, dt \]

(C) \[ \int_0^{\infty} t^{00} \mu_{x+t}^{01} \cdot (1 - p_{x+t}^{11}) \, dt \]

(D) \[ \int_0^{\infty} t^{00} \mu_{x+t}^{01} \cdot p_{x+t}^{12} \, dt \]

(E) \[ \int_0^{\infty} t^{00} \mu_{x+t}^{01} \cdot (1 - p_{x+t}^{11}) \, dt \]
6. You are given the following information about a double decrement model:

<table>
<thead>
<tr>
<th>Age, x</th>
<th>$l^{(r)}_x$</th>
<th>$q^{(1)}_x$</th>
<th>$q^{(2)}_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>10,000</td>
<td>---</td>
<td>0.03</td>
</tr>
<tr>
<td>66</td>
<td>9,600</td>
<td>$q^{(1)}_{65} + 0.02$</td>
<td>---</td>
</tr>
<tr>
<td>67</td>
<td>9,120</td>
<td>$q^{(1)}_{65} + 0.04$</td>
<td>---</td>
</tr>
<tr>
<td>68</td>
<td>8,436</td>
<td>0.07</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Calculate $d^{(2)}_{67}$.

(A) 220
(B) 222
(C) 224
(D) 226
(E) 228
7. You are given:

(i) \( p_{50} = 0.9985 \)

(ii) \( A^{(2)}_{50} = 0.1772 \)

(iii) \( i^{(2)} = 0.06 \)

(iv) Deaths are uniformly distributed between integer ages.

Calculate \( A^{(2)}_{50.5} \).

(A) 0.1783
(B) 0.1795
(C) 0.1807
(D) 0.1819
(E) 0.1831
8. You are given:

(i) The yearly expenses for a long-term care facility, assumed incurred at the end of each year, depend on an individual resident’s level of care at the end of each year, as follows:

- Healthy State Care: 12,000
- Sick State Care: 30,000

(ii) Expenses of 40,000 are incurred at the end of the year of death of the resident.

(iii) There are no other costs or expenses.

(iv) The 1-year transition probabilities for lives ages 70 and 71, respectively, are given in the following matrices. The first row and column represent the Healthy state, the second row and column represent the Sick state, and the third row and column represent the Dead state.

\[
P_{70} = \begin{pmatrix} 0.86 & 0.12 & 0.02 \\ 0.10 & 0.68 & 0.22 \\ 0.00 & 0.00 & 1.00 \end{pmatrix} \quad P_{71} = \begin{pmatrix} 0.84 & 0.13 & 0.03 \\ 0.08 & 0.66 & 0.26 \\ 0.00 & 0.00 & 1.00 \end{pmatrix}
\]

(v) \( i = 0.02 \)

Calculate the expected present value of expenses over the next two years for an individual age 70 who is currently Healthy.

(A) 30,452
(B) 30,573
(C) 30,694
(D) 30,815
(E) 30,936
9. An insurer issues a 10-year last survivor term insurance of 100,000 on (50) and (60). The death benefit is paid at the end of the year of the second death.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Future lifetimes are independent.

(iii) \( i = 0.05 \)

Calculate the single net premium.

(A) 77
(B) 78
(C) 79
(D) 80
(E) 81
10. A fully continuous 10-year joint life insurance on \((x)\) and \((y)\) pays 10,000 at the moment when the first of \((x)\) and \((y)\) dies, if that happens within 10 years, and provided that \((x)\) and \((y)\) do not die at the same time.

You are given the following mortality model for \((x)\) and \((y)\):

\[
\delta = 0.0433
\]

You are also given:

(i) \(\mu_{x+t,y+t}^{01} = 0.06; \ \mu_{x+t,y+t}^{02} = 0.10; \ \mu_{x+t,y+t}^{03} = 0.04\)

(ii) \(\delta = 0.0433\)

Calculate the expected present value of the insurance.

(A) 5,600  
(B) 5,800  
(C) 6,000  
(D) 6,200  
(E) 6,400
11. An insurer issues fully discrete whole life insurance policies to a group of 200 lives all aged 40. Each policy has a death benefit of 150,000.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Lifetimes of the group of 200 lives are independent.

(iii) Gross premiums of 1,140 per policy are paid at the start of each year.

(iv) There are no expenses.

(v) $i = 0.05$

Using the normal approximation, calculate the 95th percentile of the gross future loss random variable.

(A) -194,550

(B) -196,670

(C) -198,790

(D) -200,910

(E) -203,100
12. For a 10-year deferred annuity due issued to (55), you are given:

(i) Premiums are payable at the beginning of each year during the deferral period.

(ii) An annuity-due of 5,000 per month is paid from age 65.

(iii) Mortality follows the Standard Ultimate Life Table

(iv) $i = 0.05$

(v) Monthly annuity values are calculated using the two-term Woolhouse formula.

Calculate the annual net premium for this policy.

(A) 56,091

(B) 58,126

(C) 60,161

(D) 62,196

(E) 64,231
13. An insurer issues a 10-year sickness income policy to (55). Transitions follow the Standard Sickness-Death Model:

You are given:

(i) The sickness benefit is 5,000 per month, payable at the start of each month that the policyholder is Sick.
(ii) The policyholder pays premiums at the start of each month conditional on being Healthy.
(iii) Woolhouse two-term approximations are used for evaluating the monthly annuities.
(iv) The policyholder is Healthy at the start of the policy.
(v) There are no other benefits.
(vi) \( \ddot{a}_{550}^{00(12)} = 7.1253 \)
(vii) \( i = 0.05 \)

Calculate the monthly net premium.

(A) 251.5
(B) 253.6
(C) 255.7
(D) 257.8
(E) 259.9
14. For a special fully discrete whole life insurance on (60) and (70), you are given:

(i) \( P \) is the annual net premium payable until the first death.

(ii) The insurance pays 100,000 at the end of the year of the first death, and 2\( P \) at the end of the year of the second death.

(iii) Mortality of each life follows the Standard Ultimate Life Table.

(iv) The future lifetimes of (60) and (70) are independent.

(v) \( i = 0.05 \)

Calculate \( P \).

(A) 3,982

(B) 4,103

(C) 4,224

(D) 4,345

(E) 4,466
For a fully discrete whole life insurance on (35), you are given:

(i) The benefit is 250,000 in the first 30 years and 125,000 thereafter.
(ii) Level gross premiums of 1,000 are payable annually for the first 30 years.
(iii) Per policy expenses are 550 in the first year and 25 in all other years.
(iv) Commissions are 5% of premiums.
(v) Mortality follows the Standard Ultimate Life Table.
(vi) \( i = 0.05 \)

Calculate the gross premium reserve at the end of the 20th year.

(A) 24,505
(B) 24,698
(C) 24,891
(D) 25,084
(E) 25,277
16. For a fully discrete whole life insurance of 100,000 on (45), you are given:

(i) The full preliminary term reserve at the end of year 20 is 26,526.57
(ii) $\ddot{a}_{45} = 20.45115$
(iii) $1000 \, q_{45} = 0.77$
(iv) $1000 \, q_{65} = 5.91$
(v) $i = 0.04$

Calculate the full preliminary term reserve at the end of year 21.

(A) 27,900
(B) 28,100
(C) 28,300
(D) 28,500
(E) 28,700
17. An insurer has issued a fully continuous whole life insurance policy of 500,000 to (50). Reserves are evaluated using the following model:

You are given:

(i) The premium of 6,000 per year is payable continuously in State 0.

(ii) Claims expenses of 1,000 are incurred at the time of death or lapsation.

(iii) Maintenance expenses are 150 per year, payable continuously.

(iv) A benefit of 60% of the reserve at exit is paid immediately on lapsation.

(v) \( \delta = 0.05 \)

(vi) The following values:

<table>
<thead>
<tr>
<th>( \mu_{75}^{01} )</th>
<th>( \mu_{75}^{02} )</th>
<th>( 25 , V^{(0)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.018</td>
<td>200,000</td>
</tr>
</tbody>
</table>

Calculate \( \frac{d}{dt} V^{(0)} \) at \( t = 25 \).

(A) 12,800
(B) 13,100
(C) 13,400
(D) 13,700
(E) 14,000
18. For a fully discrete insurance the profit signature is:

\[ \Pi_0 = -220; \quad \Pi_1 = 76; \quad \Pi_k = 45 \text{ for } k = 2,3,\ldots \]

Calculate the discounted payback period (DPP), at a risk discount rate of 20%.

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11
19. Barry is a member of a final salary defined benefit pension plan. You are given:

(i) All retirements occur at exact age 65. There are no exits other than death and retirement.

(ii) There are no death benefits.

(iii) The traditional unit credit method is used to fund the benefits.

(iv) The salary scale is $s_x = 1.025^x$

(v) $i = 0.045$

(vi) Barry entered the plan at age 30.

(vii) Barry’s Actuarial Liability at age 45 is 151,665.

Calculate Barry’s Normal Contribution at age 45.

(A) 10,925

(B) 12,540

(C) 14,155

(D) 15,755

(E) 17,390
20. Wei, (35), is employed at a firm which provides a post-retirement medical benefit to its retirees that covers the first 2 years following retirement. You are given:

(i) Retirement is assumed to occur at age 64.

(ii) Wei is assumed to remain employed with the firm until retirement or earlier death.

(iii) Health insurance premium inflation is assumed to be 2.69% per year.

(iv) Currently, annual health insurance premiums are 4500 at age 64 and increase by 2.50% with each year of age.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) \( i = 0.05 \)

Calculate the expected present value of Wei’s benefits under this plan.

(A) 3640

(B) 4040

(C) 4460

(D) 4500

(E) 5280
1. 
(10 points) A fully discrete 30-year endowment insurance policy of 400,000 is sold to Jenna who is 50 years old.

You are given that:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) Expenses for the policy are:

- 15% of premium the first year and 5% thereafter;
- Issue expenses of 500 incurred at time 0;
- Maintenance expenses of 40 per policy in every year including the first; and
- Termination expenses of 1000 incurred at the time that the benefit is paid.

(a) (2 points) Show that the gross premium calculated using the equivalence principle is 7,250 to the nearest 10. You should calculate the value to the nearest 1.

(b) (5 points) The company decides to charge a gross premium of 7,500.

(i) The gross loss-at-issue random variable, $L_0^g$ can be written in the form $L_0^g = A v^{\Delta (K_{50}, 1, 30)} + B$, where $A$ and $B$ are constants.

Calculate $A$ and $B$.

(ii) Calculate the expected value of $L_0^g$.

(iii) Calculate the standard deviation of $L_0^g$. 

Exam LTAM: Spring 2022
1. Continued

The endowment policy contains an option where the endowment benefit at the end of 30 years can be taken as a whole life annuity due, with a 10-year guarantee, payable monthly from age 80, based on the same pricing assumptions as for the endowment insurance. The policyholder can choose whether or not to exercise this option at the maturity date.

You are given that:

(i) The actuarial present value of the monthly payments at age 80 is equal to the amount of the endowment.

(ii) Monthly annuities are calculated using the two-term Woolhouse formula.

(iii) There are no further expenses.

(c) (1 point) Calculate the amount of the monthly payment.

(d) (1 point) Describe briefly two additional risks that the company has assumed by offering this option.

(e) (1 point) Describe briefly two key factors that Jenna should take into consideration in deciding whether or not to select the annuity option.
2. (10 points) An insurer issues a 20-year term insurance to (50) which pays a death benefit of 500,000, and an additional 100,000 on diagnosis of a critical illness (CI).

The insurer uses the Markov model illustrated below to value the CI benefit.

![Markov model diagram]

You are given the following information:

(i) Premiums are payable continuously whilst the policyholder is healthy.

(ii) The annual net premium rate is 4,850.

(iii) Death and CI benefits are payable immediately on transition.

(iv) \( i = 0.05 \).

(v) \( V^{(j)} \) denotes the net premium reserve at time \( t \), conditional on being in State \( j \) at that time.

(vi) The following values from the model illustrated above:

<table>
<thead>
<tr>
<th>( x )</th>
<th>10( a_x^{00} )</th>
<th>10( A_x^{01} )</th>
<th>10( A_x^{02} )</th>
<th>10( A_x^{12} )</th>
<th>10( p_x^{00} )</th>
<th>10( p_x^{01} )</th>
<th>10( p_x^{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>10.1729</td>
<td>0.34249</td>
<td>0.39077</td>
<td>0.47904</td>
<td>0.75055</td>
<td>0.13135</td>
<td>0.75283</td>
</tr>
<tr>
<td>70</td>
<td>6.5690</td>
<td>0.49594</td>
<td>0.54335</td>
<td>0.62237</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Continued

(a) (3 points)

(i) Show that $A_{60/70}^{02} = 0.0902$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Show that $v^{(0)}_{10} = 21,850$ to the nearest 50. You should calculate the value to the nearest 1.

(iii) Show that $v^{(1)}_{10} = 95,700$ to the nearest 10. You should calculate the value to the nearest 1.

(b) (3 points) You are given that $\mu_{60}^{01} = 0.00818; \mu_{60}^{02} = 0.00724; \mu_{60}^{12} = 0.01811$.

(i) Calculate $\frac{d}{dt}v^{(0)}$ at $t = 10$.

(ii) Calculate $\frac{d}{dt}v^{(1)}$ at $t = 10$.

(iii) Explain why $v^{(1)}$ is decreasing while $v^{(0)}$ is increasing at $t = 10$.

The policy terms are revised for new policies such that the CI benefit is paid six months after diagnosis, conditional on the policyholder surviving to that time, and conditional on the diagnosis occurring within the 20-year term.

(c) (2 points) Write down an integral expression for the expected present value at issue of the revised CI benefit.

(d) (2 points) One reason for the change is that mortality is very high in the first year after a CI diagnosis.

(i) Define the Markov property.

(ii) Explain whether high mortality in the period immediately after a CI diagnosis is consistent with the Markov property.
3. (9 points) You are an actuary conducting a valuation of an annuity portfolio. In previous valuations, you have considered simple, deterministic longevity models to capture mortality improvements. You are now exploring stochastic longevity models, and in particular, you have estimated the Cairns-Blake-Dowd (CBD) model with the following results:

\[
l_q(x,t) = \log \left( \frac{q(x,t)}{1-q(x,t)} \right) = K_t^{(1)} + K_t^{(2)}(x-73),
\]

where \( K_t^{(1)} = K_{t-1}^{(1)} - 0.01 + 0.05Z_t^{(1)} \) and \( K_t^{(2)} = K_{t-1}^{(2)} + 0.015Z_t^{(2)} \),

and where for any year \( t \):

- \( Z_t^{(1)} \) and \( Z_t^{(2)} \) are correlated standard normal (i.e. N(0,1)) random variables;
- \( \mathbb{E}[Z_t^{(1)}Z_t^{(2)}] = 0.15 \);
- for any \( s \neq t \), \( (Z_s^{(1)}, Z_s^{(2)}) \) is independent of \( (Z_t^{(1)}, Z_t^{(2)}) \).

(a) (1 point) Explain why you might want to consider stochastic longevity models.

(b) (1 point) One advantage of the CBD model over other stochastic longevity models is better smoothing of the “year effects”.

Describe what is meant by “year effects” in a longevity model. You should include an example of a phenomenon that could create a year effect.

(c) (4 points) You are given that \( K_{2021}^{(1)} = -4.50 \) and \( K_{2021}^{(2)} = 0.02 \).

(i) Show that the mean of \( l_q(70,2022) \) is \(-4.6\) to the nearest 0.1. You should calculate the value to the nearest 0.001.

(ii) Show that the standard deviation of \( l_q(70,2022) \) is \(0.06\) to the nearest 0.01. You should calculate the value to the nearest 0.001.

(d) (3 points)

(i) By differentiating the function, show that \( \log \left( \frac{q}{1-q} \right) \) is an increasing function of \( q \), for \( 0 < q < 1 \).

(ii) Hence, calculate \( \Pr[q(70,2022) \leq 0.01] \).
4. (8 points) ABC Insurance uses the following model for calculating premiums and reserves for joint life insurance policies.

The forces of transition are as follows, where $\mu_z^x$ is the force of mortality at age $z$ under the Standard Ultimate Life Table (SULT).

\[
\mu_{01}^{xy} = \mu_y^x - 0.005; \quad \mu_{02}^{xy} = \mu_z^x; \quad \mu_{13}^x = \mu_z^x + 0.0025; \quad \mu_{23}^y = \mu_y^x + 0.005
\]

(a) (1 point) State with reasons whether the future lifetimes of $(x)$ and $(y)$ are independent under this model.

(b) (1 point) Show that $\lambda = p_x^0 p_y^0$, where $p_x^0$ and $p_y^0$ are from the SULT, and where $\lambda > 0$ is a constant which you should specify.

In the following table, you are given annuity values calculated using SULT mortality, at different forces of interest, and assuming independent future lifetimes for joint life functions.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>3.50%</th>
<th>3.75%</th>
<th>4.00%</th>
<th>4.25%</th>
<th>4.50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{a}_{40</td>
<td>50}$</td>
<td>19.3199</td>
<td>18.6530</td>
<td>18.0216</td>
<td>17.4235</td>
</tr>
<tr>
<td>$\overline{a}_4$</td>
<td>22.4243</td>
<td>21.4880</td>
<td>20.6129</td>
<td>19.7939</td>
<td>19.0268</td>
</tr>
<tr>
<td>$\overline{a}_5$</td>
<td>20.0391</td>
<td>19.3174</td>
<td>18.6358</td>
<td>17.9917</td>
<td>17.3825</td>
</tr>
</tbody>
</table>

(c) (1 point) Determine $\overline{a}_{xy}^{00}$ at $\delta = 4\%$, for $x = 40$ and $y = 50$, based on the ABC Insurance model.
4. Continued

(d) (3 points)

(i) Write down an integral expression for the expected present value of a payment of 1 immediately on the first death of \( x \) and \( y \), using the multiple state model notation.

(ii) Show that, for \( x = 40 \) and \( y = 50 \), at \( \delta = 4\% \), the expected present value of a payment of 1 immediately on the first death, under the ABC Insurance joint life mortality model, is 0.227 to the nearest 0.001. You should calculate the value to the nearest 0.00001.

(e) (1 point) ABC Insurance issues a fully continuous joint life policy of 500,000 on \( x \), who is 40, and \( y \), who is 50.

Calculate the annual rate of net premium for this policy using the ABC Insurance joint life mortality model, with \( \delta = 4\% \).

(f) (1 point) Exactly 10 years after issue \( y \) dies while \( x \) survives. At that time, \( x \) uses the sum insured to provide a continuous annuity paying \( X \) per year while she survives. The annuity amount is determined using the equivalence principle with \( \delta = 4\% \).

Calculate \( X \).
5.  
(9 points) Barry is the sole member of a final average salary pension plan which is being valued on January 1, 2022. At the valuation date Barry is 63 years old and has 12 years of service. His salary at the valuation date is 72,100. His salary in 2021 was 70,000.

The pension plan benefits are as follows:

- The annual benefit is 1.6% of final average salary times years of service.
- Final average salary is the salary earned during the 12 months preceding retirement.
- Benefits are reduced by 0.4% for each month retirement precedes age 65.

The valuation at January 1, 2022 uses the projected unit cost (PUC) method with the following assumptions:

(i) Salaries increase by 3% each January 1.
(ii) Retirements before 65 occur in the middle of the year of age.
(iii) Decrements before retirement follow the Standard Service Table.
(iv) $i = 0.05$
(v) Under post-retirement mortality $a_{63.5}^{(12)} = 13.5139$.

(a) (3 points)

(i) Show that if Barry retires on July 1, 2022, the expected present value of Barry’s retirement benefits at that time would be 178,200 to the nearest 50. You should calculate the value to the nearest 1.

(ii) Show that the expected cost of mid-year retirement decrements in 2022 is 16,500 to the nearest 50. You should calculate the value to the nearest 1.
5. Continued

You are also given that the projected accrued liability for Barry’s retirement benefits on January 1, 2023, assuming that he is employed then, is 191,309.

(b) (2 points)

(i) Show that the actuarial liability of Barry’s retirement benefits at January 1, 2022 is 167,150 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate the normal cost for Barry’s retirement benefits as of January 1, 2022.

The pension trustees are considering changing to the traditional unit credit (TUC) funding method.

(c) (2 points)

(i) State with reasons whether the actuarial liability as of January 1, 2022, under the TUC funding method will be bigger or smaller than the actuarial liability as of January 1, 2022 under the PUC funding method.

(ii) State with reasons whether the normal cost for 2022 under the TUC funding method will be bigger or smaller than the normal cost for 2022 under the PUC funding method.

(d) (2 points) You are given the following additional information

- Barry retires on January 1, 2023.
- The earned interest rate during 2022 was 5.1%.
- The valuation as of January 1, 2023 uses the PUC funding method, based on the same assumptions as the previous year.
- The normal cost for 2022 was paid in full on January 1, 2022.
- \( a_{04}^{(12)} = 13.3735 \).

Calculate the gain or loss to the pension plan during 2022.
6. (10 points) An insurer offers “immediate needs annuities” (INAs) to lives age 80 or older who are moving permanently into long term care facilities (LTCFs). The annuity may be purchased as a single premium annuity-due, or as a single premium deferred annuity-due, with a 2-year deferral period. The annuity payments are paid directly to the care home to cover the resident’s costs.

(a) (2 points) You are given the following excerpt from a select and ultimate mortality table used to price the INA. The selection age is the age at which the life moves into an LTCF. The selection period is 1 year.

<table>
<thead>
<tr>
<th>[x]</th>
<th>( l_x )</th>
<th>( l_{x+1} )</th>
<th>( x + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>1200</td>
<td>700</td>
<td>95</td>
</tr>
<tr>
<td>95</td>
<td>1000</td>
<td>550</td>
<td>96</td>
</tr>
<tr>
<td>96</td>
<td>750</td>
<td>385</td>
<td>97</td>
</tr>
<tr>
<td>97</td>
<td>450</td>
<td>195</td>
<td>98</td>
</tr>
<tr>
<td>98</td>
<td>220</td>
<td>50</td>
<td>99</td>
</tr>
<tr>
<td>99</td>
<td>75</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

(i) Calculate the 1-year mortality probability for (95) assuming that she moved into an LTCF at age 95.

(ii) Calculate the 1-year mortality probability for (95) assuming that she moved into an LTCF at age 94.

(iii) Describe how the selection effect in this model differs from that of typical select and ultimate tables used for pricing life insurance.
6. Continued

(b) (4 points) You are given the following additional information for a profit test of an annuity-due contract issued to a 95-year old who is just moving into an LTCF.

- For a single premium of 110,000 the annuity payment is 50,000 at the start of each year including the first, conditional on survival.
- Pre-contract expenses are 300.
- Expenses of 100 are incurred with each annuity payment. There are no other expenses.
- Reserves for the first two years are:
  \( _0V = 0; \quad _1V = 105,000; \quad _2V = 80,000 \)
- The insurer earns 7% per year.
- The hurdle rate is 12% per year.

You are also given the following partially completed profit test table, which is missing the entries for the first two policy years:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( _{t-1}V )</th>
<th>Premium</th>
<th>Annuity</th>
<th>Expenses</th>
<th>Interest</th>
<th>Expected End-year reserve</th>
<th>Pr_( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>80,000</td>
<td>0</td>
<td>50,000</td>
<td>100</td>
<td>2,093</td>
<td>31,656</td>
<td>337</td>
</tr>
<tr>
<td>4</td>
<td>62,500</td>
<td>0</td>
<td>50,000</td>
<td>100</td>
<td>868</td>
<td>12,846</td>
<td>422</td>
</tr>
<tr>
<td>5</td>
<td>50,100</td>
<td>0</td>
<td>50,000</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(i) Determine the missing entries for the first three rows of the table (missing entries are indicated by --).

(ii) Show that the net present value of the contract, is 6,710 to the nearest 10. You should calculate the value to the nearest 1.

(iii) Show that the profit margin is 6.1% to the nearest 1%. You should calculate the profit margin to the nearest 0.001%.
6. Continued

(c) (3 points) The deferred annuity contract is identical to the annuity-due contract, but with no payment in the first two years, and with a lower single premium payable at issue. The time 1 reserve for the deferred annuity contract is $V = 54,900$. All other reserves and expenses are identical to the annuity-due contract.

Calculate the premium required under the deferred annuity contract that generates the same profit margin as the annuity-due.

(d) (1 point) The insurer offers the deferred annuity at a single premium of 28,500. Describe one advantage and one disadvantage of the deferred annuity, compared with the annuity-due, from the perspective of the purchaser of the annuity.

**END OF EXAMINATION**