Session 4: From What If to How To: Reserving for GMMB with Open Source Research, Collaboration, R Codes and High Performance Computing

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A SCENARIO REDUCTION ALGORITHM FOR A DYNAMIC HEDGING MODEL OF GMMB GUARANTEES

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Sponsor:	Autorité des marchés financiers (AMF)

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0.1 Structure of the presentation

1. Variable Annuities (VA)

2. Scenario Reduction

3. Implementation in CUDA and R

4. Conclusion

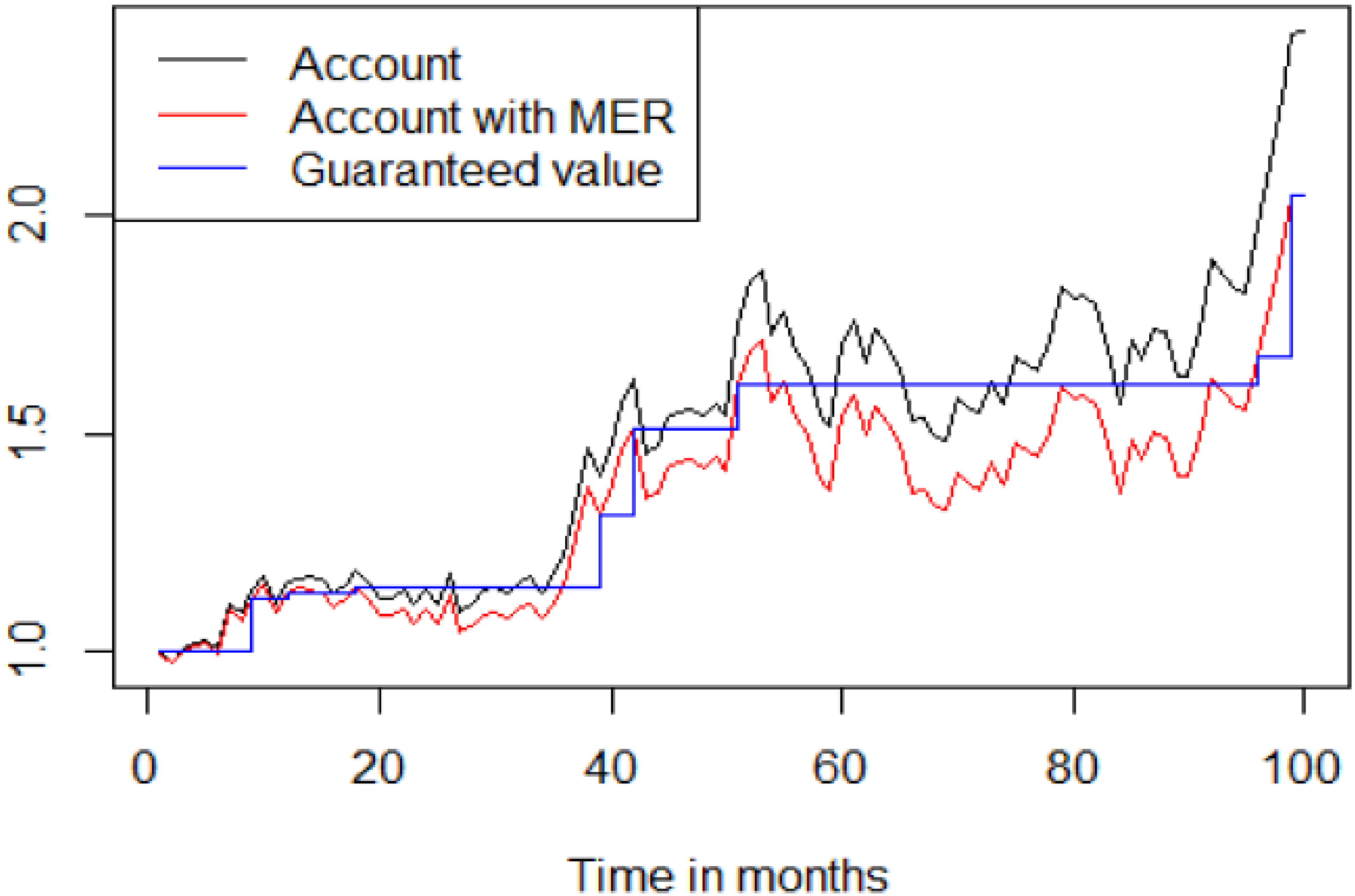
1.1 VA Benefits over mutual funds

Benefit	Variable Annuities	Mutual Funds
Guarantee of the principal at Maturity	Yes	No
Guarantee of the principal at Death	Yes	No
Transfer to beneficiary without will	Yes	No
Lock in market gains using resets	Yes	No

1.2 VA GMMB in relation to a put option

- Variable annuities are similar to derivatives.
- Variable annuities often have resets (e.g. GMMB is similar to reset put).
- The main differences between variable annuities and derivatives are:
    - o Premium paid periodically and can be proportional to account value;
    - o Payments are related to mortality;
    - o Maturity of variables annuities is longer than put option.

1.3 VA account value projected over time



Hedging errors

1.4 VA Lognormal pricing model for GMMB

- We let S_t be the price of a stock market index.
- We let F_t be the price of the variable annuity funds.
- We let $F_t = S_t \frac{F_0}{S_0}\left(1 - w_{tot}\right)^t$, where w_{tot} is the fee charged to the

variable annuity fund.

- In the physical probability measure, we suppose that S_t follows a regime-switching Log Normal model with two regimes.
- We have $S_{t+1} = S_t \exp\left(\mu_{h_t} + \sigma_{h_t} z_{t+1}\right)$, where h_t is the state of the Markov chain and $z_{t+1} \sim N(0,1)$.

1.5 Calculating VA hedging errors

- For sake of simplicity, we suppose a constant survival rate to lapse and mortality θ.

- For sake of simplicity, we also suppose that there are no resets.

- To calculate the hedging errors, we use the Black-Scholes framework. Thus, we obtain:

$$\begin{aligned} P_t &= \theta^{T-t} E^Q\left[\exp\left(-r\left(T-t\right)\right)\max\left(K-F_T,0\right)\right] \\ &= \theta^{T-t}\left(Ke^{-r\left(T-t\right)}N\left(-d_2\right)-S_t e^{-\tilde{w}_{Tot}\left(T-t\right)}N\left(-d_1\right)\right). \end{aligned}$$

1.6 Calculating VA hedging errors (cont'd)

Where:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \tilde{w}_{Tot} + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \tilde{w}_{Tot} - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t},$$

$$\tilde{w}_{Tot} = 12\ln\left(1 - w_{Tot}\right),$$

r is the risk free rate and σ is the volatility.

1.7 Calculating VA hedging errors (cont'd)

- In this presentation, we are only considering delta hedging.

- The delta is given by:

$$\Delta_{P_t} = \frac{\partial P_t}{\partial S_t} = -\theta^{T-t} \exp\left(-\tilde{w}_{Tot}\left(T-t\right)\right) N\left(-d_1\right).$$

- The position in the risk-free asset is given by:

$$P_t - \Delta_{P_t} S_t = B_t.$$

1.8 Calculating VA hedging errors (cont'd)

- The hedging portfolio is rebalanced at each time: $\{t_0, t_1, t_2, \ldots, t_n\}$.

- Just before the rebalancing occurs, the value of the replicating portfolio is given by

$$\Pi_{t_{i+1}^-} = \Delta_{P_{t_i}} S_{t_{i+1}^-} + B_{t_i} \exp\left[\left(t_{i+1}^- - t_i\right) r\right].$$

- Just after the rebalancing occurs, the value of the replicating portfolio is given by

$$\Pi_{t_{i+1}} = \Delta_{P_{t_{i+1}}} S_{t_{i+1}} + B_{t_{i+1}}, 0 \le i \le n.$$

1.9 Calculating VA hedging errors (cont'd)

- The hedging errors are given by:

$$HE_{t_{i+1}} = P_{t_{i+1}} - \Pi_{t_{i+1}^-} = \Pi_{t_{i+1}} - \Pi_{t_{i+1}^-}$$

$$= S_{t_{i+1}}\left(\Delta_{P_{t_{i+1}}} - \Delta_{P_{t_i}}\right) + B_{t_{i+1}} - B_{t_i}\exp\left[\left(t_{i+1} - t_i\right)r\right].$$

- The discounted value of the hedging errors is given by:

$$HE_{Tot} = \sum_{i=1}^{n} \exp\left[-\delta\, t_i\right] HE_{t_i}.$$

where δ is a constant force of interest.

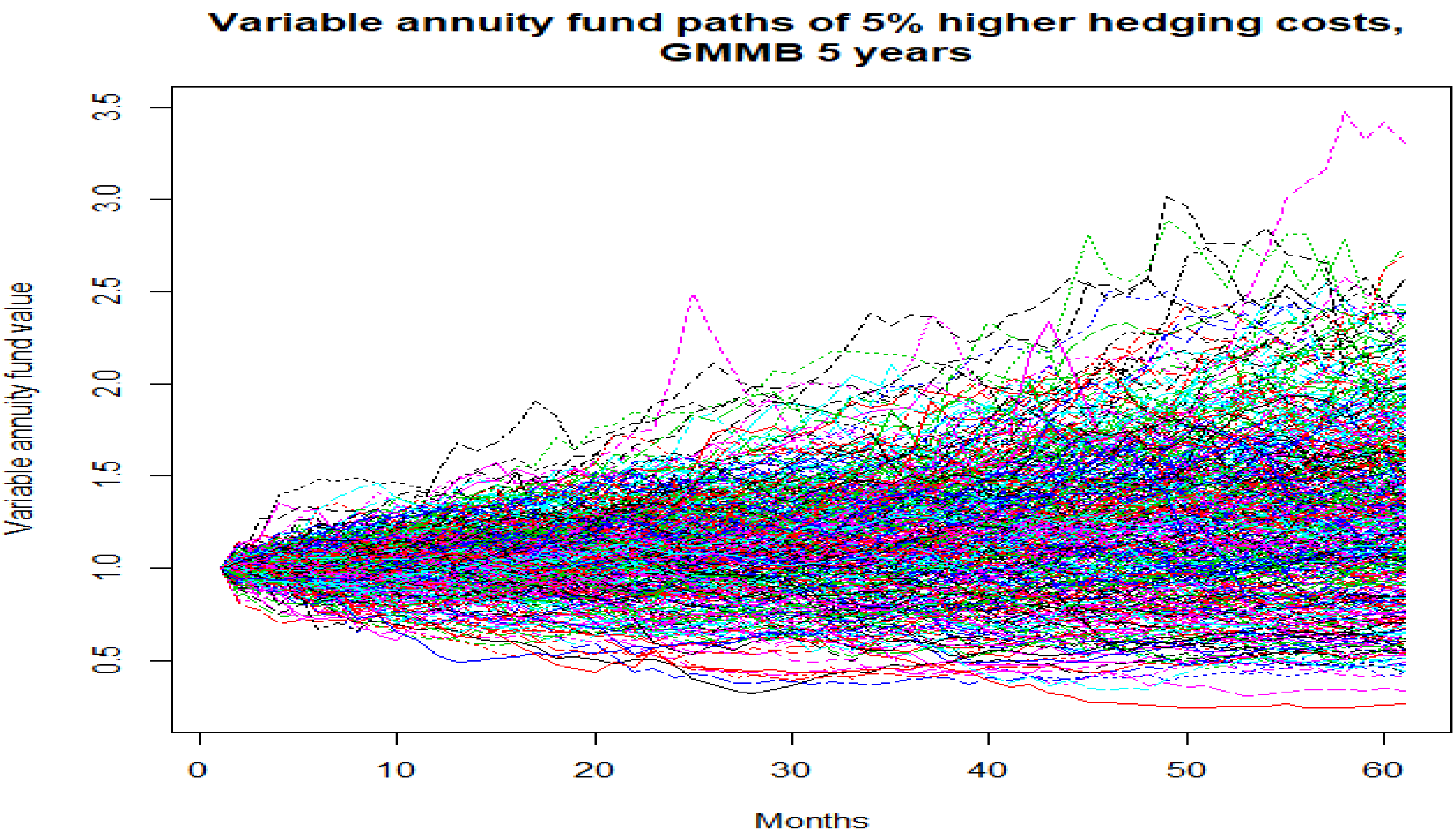
1.10 Calculating VA hedging errors (cont'd)

- The parameters that we consider are:
  - Risk neutral: $K = 1,\ r = 0.02,\ \sigma = 0.157,\ S_0 = 1$
  - Actuarial: $w_{tot} = 1 - \left(1 - 0.02\right)^{\frac{1}{12}},\ \theta = 0.96$
  - Regime switching: $\mu_1 = 0.0084, \sigma_1 = 0.0330,$

    $\mu_2 = -0.0080,\ \sigma_2 = 0.0734, p_{21} = 0.0850,$

    $p_{11} = 0.9767$
  - Discount rate: $\delta = 0.02$

1.11 Mapping VA fund value path to hedging errors



At first sight, there is no clear connection between variable annuity fund paths and the hedging costs.

1.12 Mapping VA fund value path to hedging errors (cont'd)

- It turns out that we can define some variables that are highly correlated with the hedging errors as will be shown.

- We define the following variables:
  - $\varphi_j = \frac{1}{N-1}\sum_{i=1}^{N-1} \ln\left(S_{j,t_{i+1}} / S_{j,t_i}\right)$ (mean of log-returns);
  - $\psi_j = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N-1}\left(\ln\left(S_{j,t_{i+1}} / S_{j,t_i}\right) - \varphi_j\right)^2}$ (standard deviation of log-returns);

1.13 Mapping VA fund value path to hedging errors (cont'd)

- $\psi_j = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N-1}\left(\ln\left(S_{j,t_{i+1}}/S_{j,t_i}\right)-\varphi_j\right)^2}$ (standard deviation of log-returns);
- $\tau_j = \frac{1}{(N-1)\psi_j^3}\sum_{i=1}^{N-1}\left(\ln\left(S_{j,t_{i+1}}/S_{j,t_i}\right)-\varphi_j\right)^3$ (skewness of log-returns);
- $\kappa_j = \frac{1}{(N-1)\psi_j^4}\sum_{i=1}^{N-1}\left(\ln\left(S_{j,t_{i+1}}/S_{j,t_i}\right)-\varphi_j\right)^4$ (kurtosis of log-returns);
- $\beta_j = \max_{1\le i\le N}\left(\ln S_{j,t_i}\right)-\min_{1\le i\le N}\left(\ln S_{j,t_i}\right)$ (cumsum range);

1.14 Mapping VA fund value path to hedging errors (cont'd)

- $\lambda_j = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N-1}\left(S_{j,t_{i+1}} - S_{j,t_i} - \alpha_j\right)^2}, \quad \alpha_j = \frac{1}{N-1}\sum_{i=1}^{N-1} S_{j,t_{i+1}} - S_{j,t_i}$

  (SD Order 1);

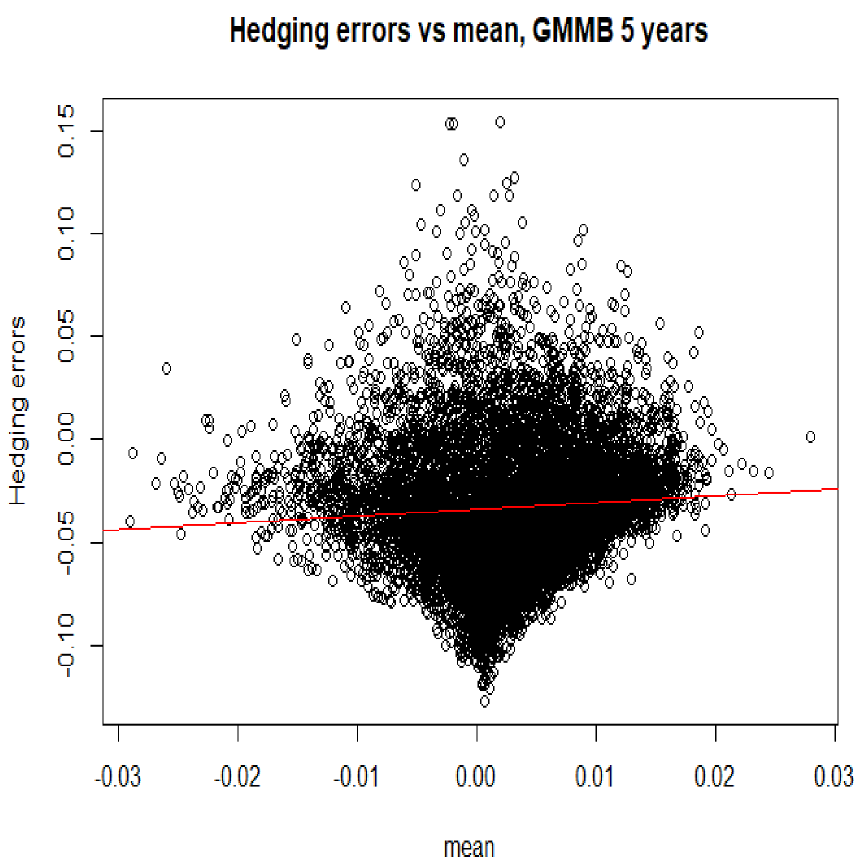
- $HE_{j,Tot}$ is the total discounted hedging errors for scenario j .

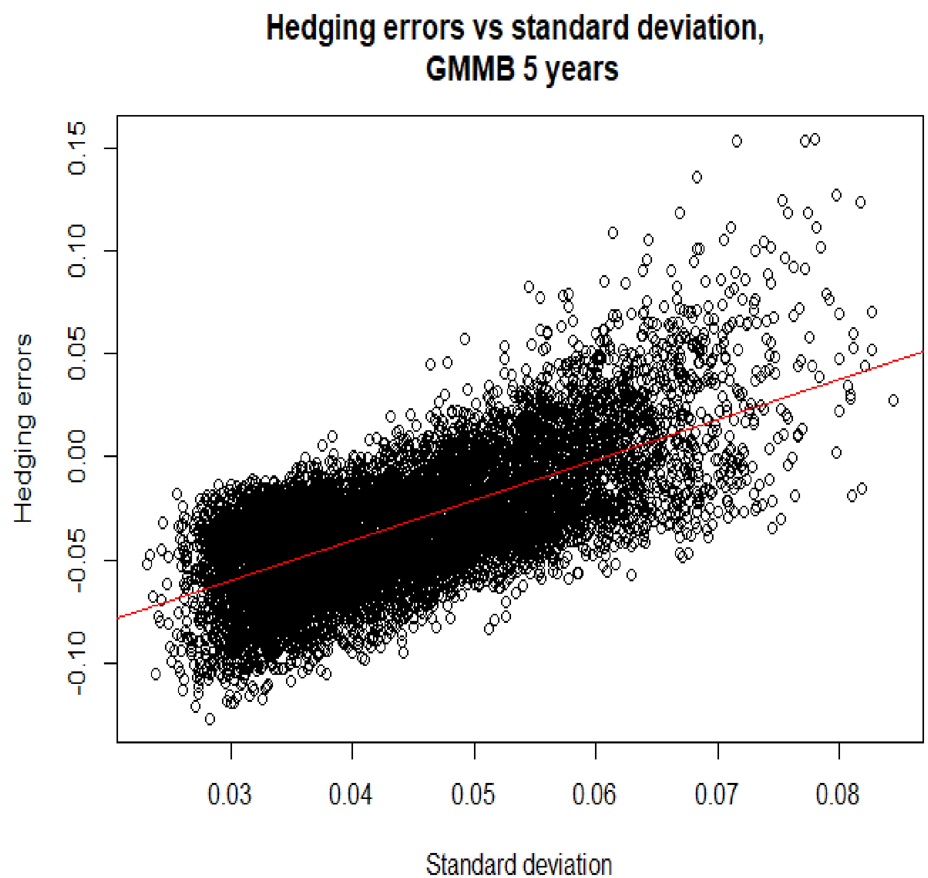
1.15 Mapping VA fund value path to hedging errors (cont'd)

Correlation matrix GMMB 5 years

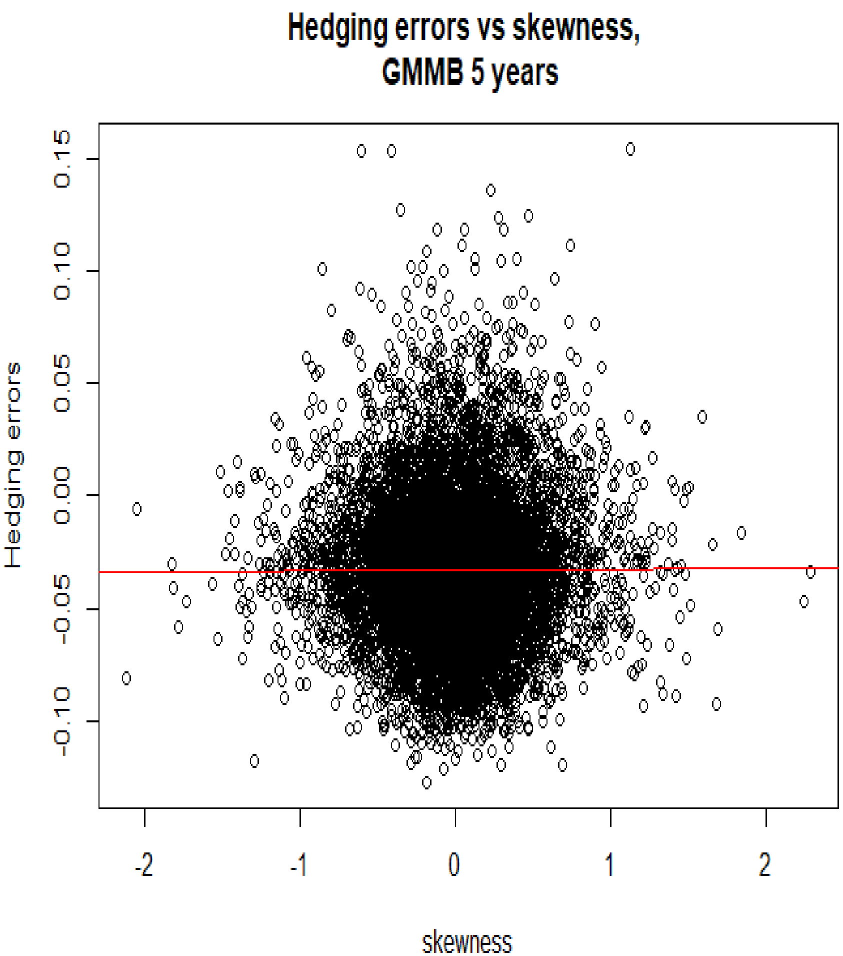
	HE_{Tot}	φ	ψ	τ	κ	β	λ
HE_{Tot}	1	0.0652	0.6768	-0.009	0.1897	0.4479	0.7466
φ	0.0652	1	-0.481	0.0034	-0.071	0.0336	0.3277
ψ	0.6768	-0.481	1	-0.006	0.2895	0.3743	0.5460
τ	-0.009	0.0034	-0.006	1	0.0391	0.0006	-0.051
κ	0.1897	-0.071	0.2895	0.0391	1	0.0486	0.2233
β	0.4479	0.0336	0.3743	0.0006	0.0486	1	0.4411
λ	0.7466	0.3277	0.5460	-0.051	0.2233	0.4411	1

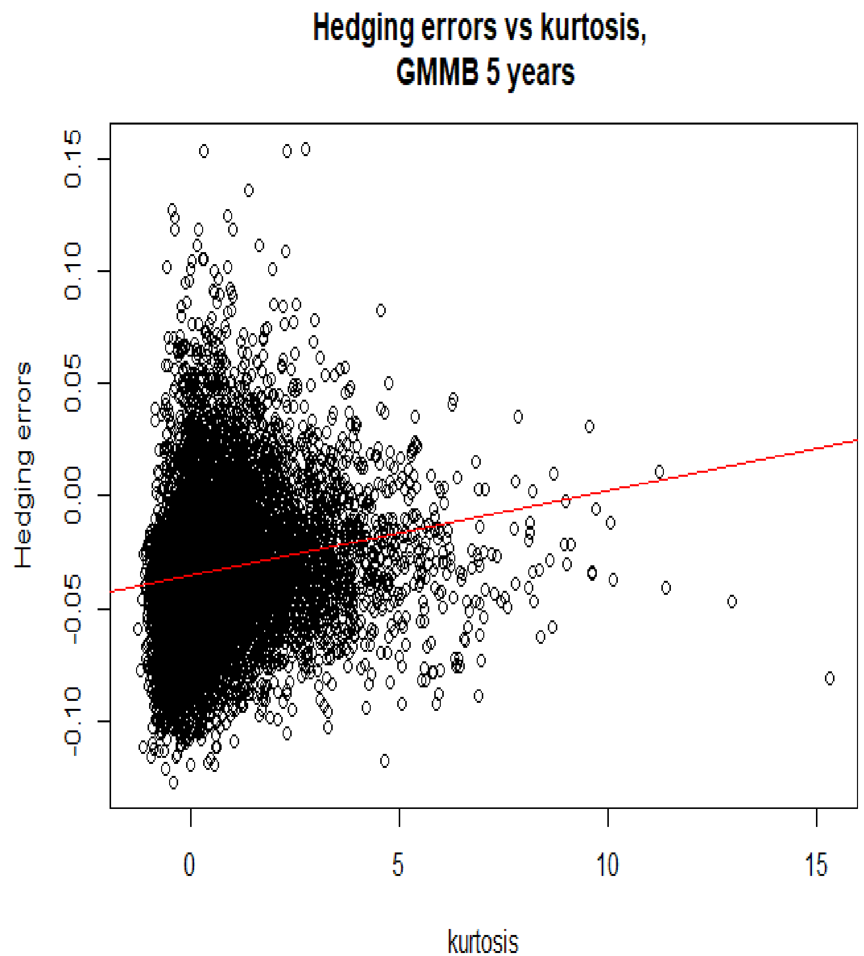
1.16 Mapping VA fund value path to hedging errors (cont'd)



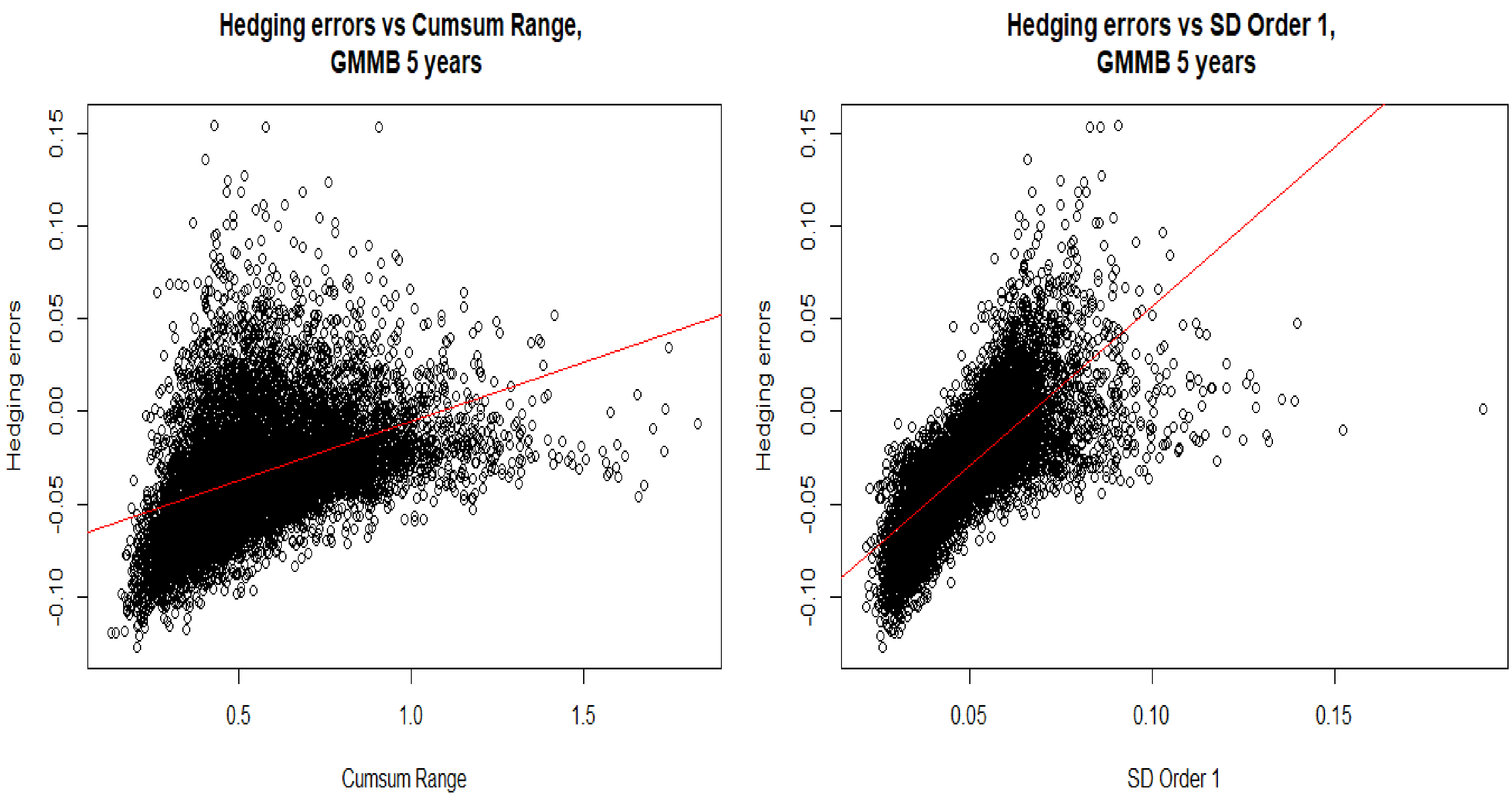


1.17 Mapping VA fund value path to hedging errors (cont'd)





1.18 Mapping VA fund value path to hedging errors (cont'd)



1.19 Mapping VA fund value path to hedging errors (cont'd)

- We have seen that there are some variables that are highly correlated with hedging errors.
- These highly correlated variables do not use information about delta positions.
- These highly correlated variables can be used as input to a scenario reduction algorithm.

1.20 VA valuation: Literature review

- Many methods were developpped to reduce calculation time in the actuarial literature:
  - Representative contracts approach (see Gan (2013) et Gan & Li (2015, 2017) for example);
  - Scenario reduction approach (see Longley-Cook (2003));
  - Variance reduction approach (see Hsieh et al. (2018));
  - Replicating portfolio approach (see Vadiveloo (2012)).

- **To the best of our knowledge, scenario reduction has not been considered for dynamic hedging models for GMMB in the literature.**

1.21 VA valuation: Reserve and Capital

- In Canada, the reserve is approximately given by

$$\text{Reserve} = P_0 + CTE_{80\%}\left(HE_{Tot}\right)$$

- In Canada, the capital is approximately given by

$$Capital = CTE_{95\%}\left(HE_{Tot}\right) - CTE_{80\%}\left(HE_{Tot}\right)$$

2.1 General scenario reduction steps

In this presentation, we consider scenario reduction algorithms that can be described as follows:

- **Step 1:** Generate random process of interest (e. g. regime-switching model for equity).

- **Step 2:** Use a dimension reduction algorithm to obtain a matrix D as follows:

$$D_j = g\left(S_j\right) = g\left(S_{j,t_0}, S_{j,t_1}, \ldots, S_{j,t_n}\right)$$

  where $g(\cdot)$ is a bounded function (e. g. standard deviation) and D is the row D_j of matrix j.

2.2 General scenario reduction steps (cont'd)

- **Step 3:** Use an algorithm to identify the representative scenarios which are pivots based on the matrix D calculated in **Step 2** (e. g. Chueh's (2002) scenario reduction algorithm, etc.). The vector index of the chosen scenarios is defined as:

$$I = \begin{bmatrix} I_1 & \cdots & I_L \end{bmatrix}, 1 \le L \le N.$$

- **Step 4:** Determine a probability $p_k, 1 \le k \le L$ for each identified pivots $S_{I_k}, 1 \le k \le L$ (e. g. Chueh's scenario reduction algorithm, etc.).

2.3 General scenario reduction steps (cont'd)

- **Step 5:** Calculate the hedging errors $HE_{I_k,Tot}, 1 \le k \le L$ using the pivots identified in Step 4 (i. e. $S_{I_k}, 1 \le k \le L$). With the hedging errors and their associated probability $p_k, 1 \le k \le L$ calculated in Step 4, estimate risk measures such as $VaR_\alpha\left(HE_{Tot}\right)$ and $CTE_\alpha\left(HE_{Tot}\right)$.

- Standardization: Suppose we have a vector $x = \left(x_1, \ldots, x_n\right)$. Then, we can standardize this vector using the following relationship:

$$\tilde{x}_j = \left(x_j - \min_{1 \le j \le n} x_j\right)\left(\max_{1 \le j \le n} x_j - \min_{1 \le j \le n} x_j\right)^{-1}.$$

2.4 Scenario reduction: Chueh's algorithm

The distance formula is

$$\sqrt{\sum_{t=1}^{n} (i_t - i_t^P)^2 \cdot V^t}$$

where:

$i_t, t = \{1,2, \dots, n\}$ - economic scenario path
$i_t^P, P = \{1,2, \dots, m\}$ - pivot
m - number of pivots
V - weights factor

Image of what the algorithm does

2.5 Three scenario reduction methods

- In our algorithm, we consider the following distance matrix:

$$D = \begin{bmatrix} \tilde{\psi} & \tilde{\beta} & \tilde{\lambda} \end{bmatrix} = \begin{bmatrix} \tilde{\psi}_1 & \tilde{\beta}_1 & \tilde{\lambda}_1 \\ \vdots & \vdots & \vdots \\ \tilde{\psi}_N & \tilde{\beta}_N & \tilde{\lambda}_N \end{bmatrix}.$$

- We consider three versions of our general scenario reduction algorithm (i. e. different **Step 3**):

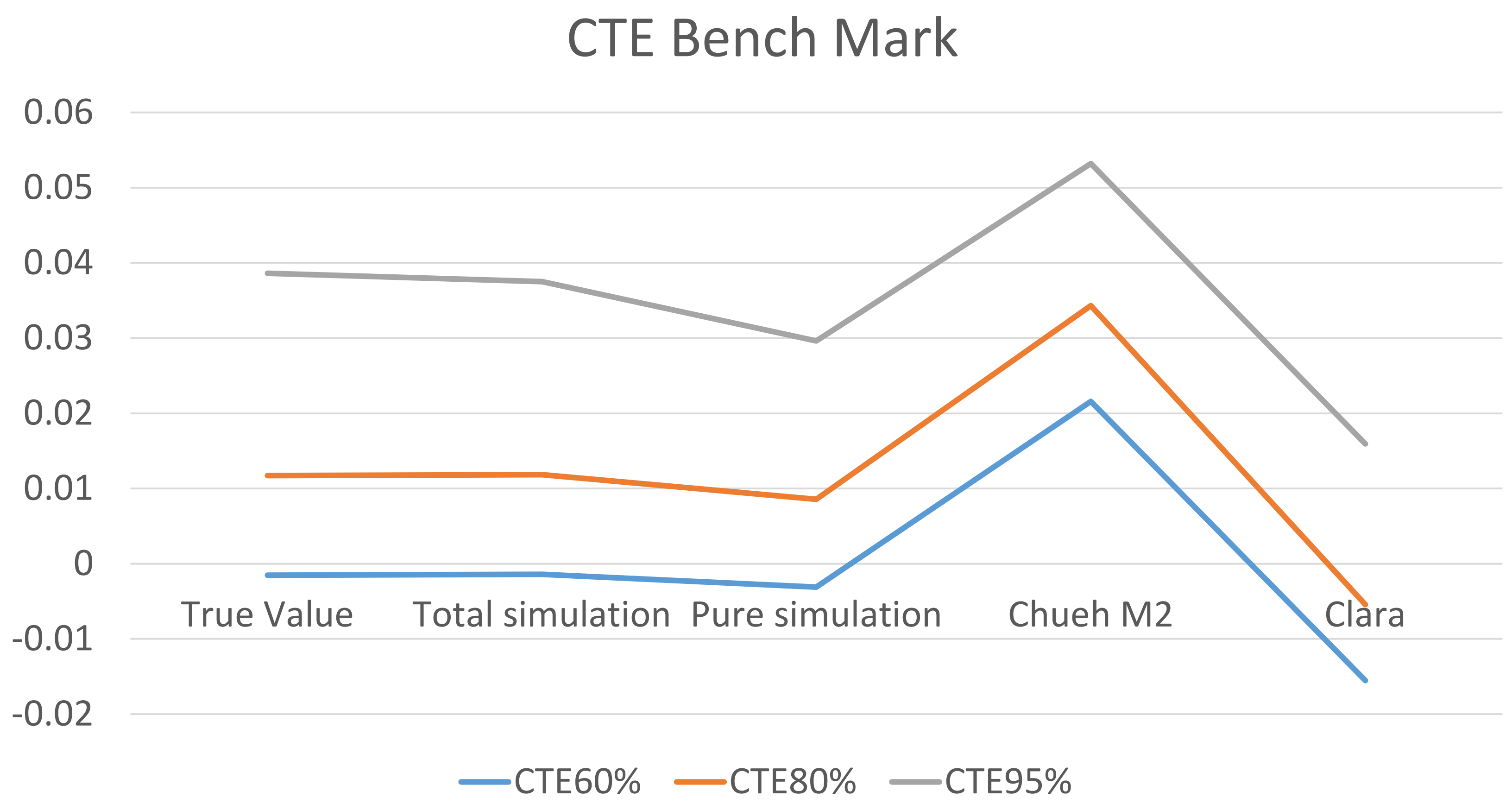
  - Pure simulations;
  - Chueh scenario;
  - Clara's clustering algorithm.

2.6 Scenario reduction: Naïve approach

- As a first approach, we might want to consider the variable annuity fund path value as input to the scenario reduction algorithm.
- For 10,000,000 simulated paths as True Value.
- For 10,000 simulated paths and 500 pivots, for a 5 year GMMB, we obtain the following table:

Approach	CTE60%	CTE80%	CTE95%
True Value	-0.001540	0.011710	0.038597
Total simulation	-0.001410	0.011850	0.037505
Pure simulation	-0.003112	0.008580	0.029640
Chueh M2	0.0215670	0.034330	0.053204
Clara	-0.015520	-0.005420	0.015944

2.7 Scenario reduction: Naïve approach

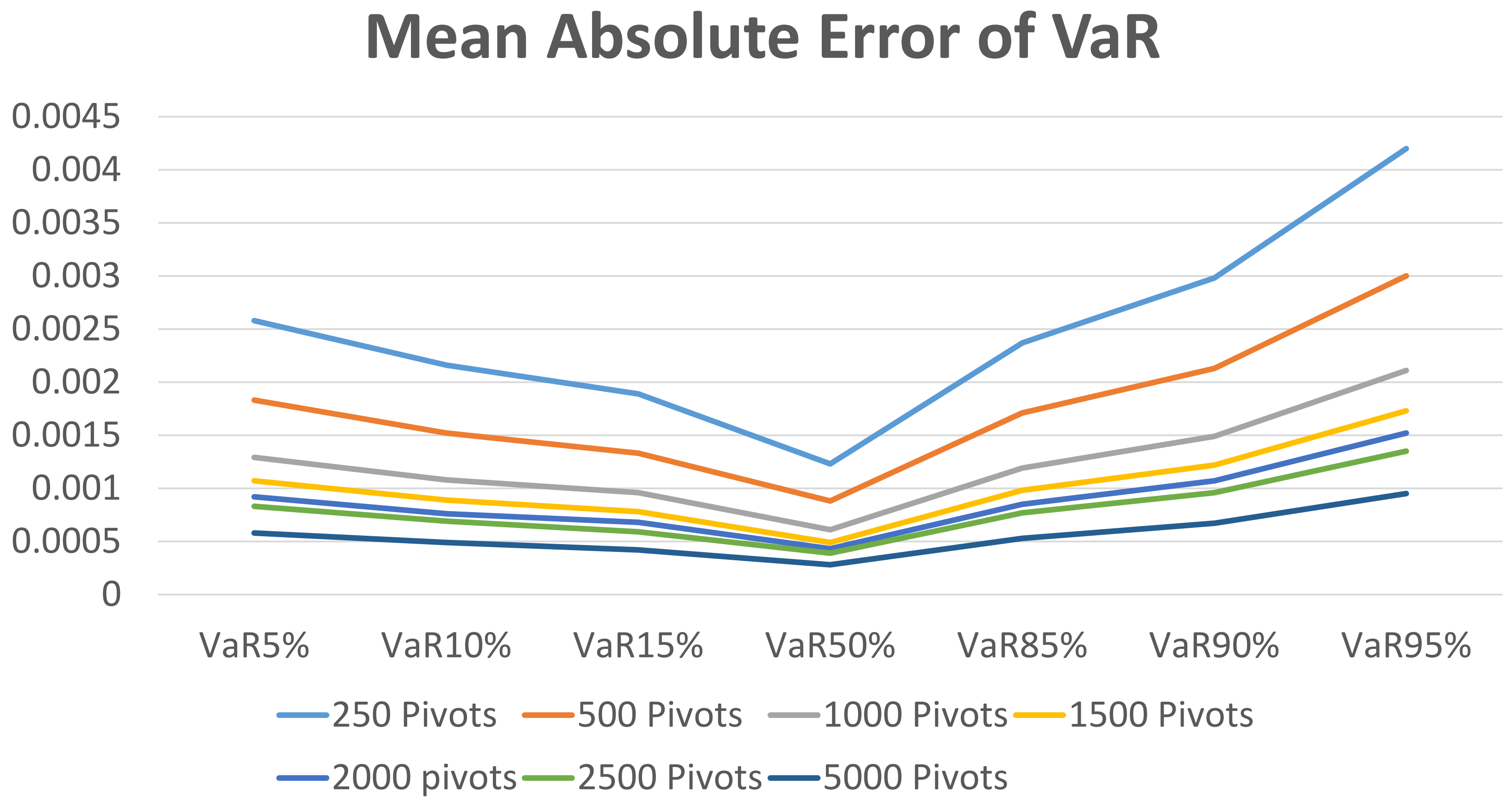


2.8 Scenario reduction: Pure simulation Benchmark, 10,000 repetitions

Mean absolute deviation

Number of Pivots	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
250	0.00258	0.00216	0.00189	0.00123	0.00237	0.00298	0.00420
500	0.00183	0.00152	0.00133	0.00088	0.00171	0.00213	0.00300
1,000	0.00129	0.00108	0.00096	0.00061	0.00119	0.00149	0.00211
1,500	0.00107	0.00089	0.00078	0.00049	0.00098	0.00122	0.00173
2,000	0.00092	0.00076	0.00068	0.00043	0.00085	0.00107	0.00152
2,500	0.00083	0.00069	0.00059	0.00039	0.00077	0.00096	0.00135
5,000	0.00058	0.00049	0.00042	0.00028	0.00053	0.00067	0.00095

2.9 Scenario reduction: Pure simulation Benchmark, 10,000 repetitions

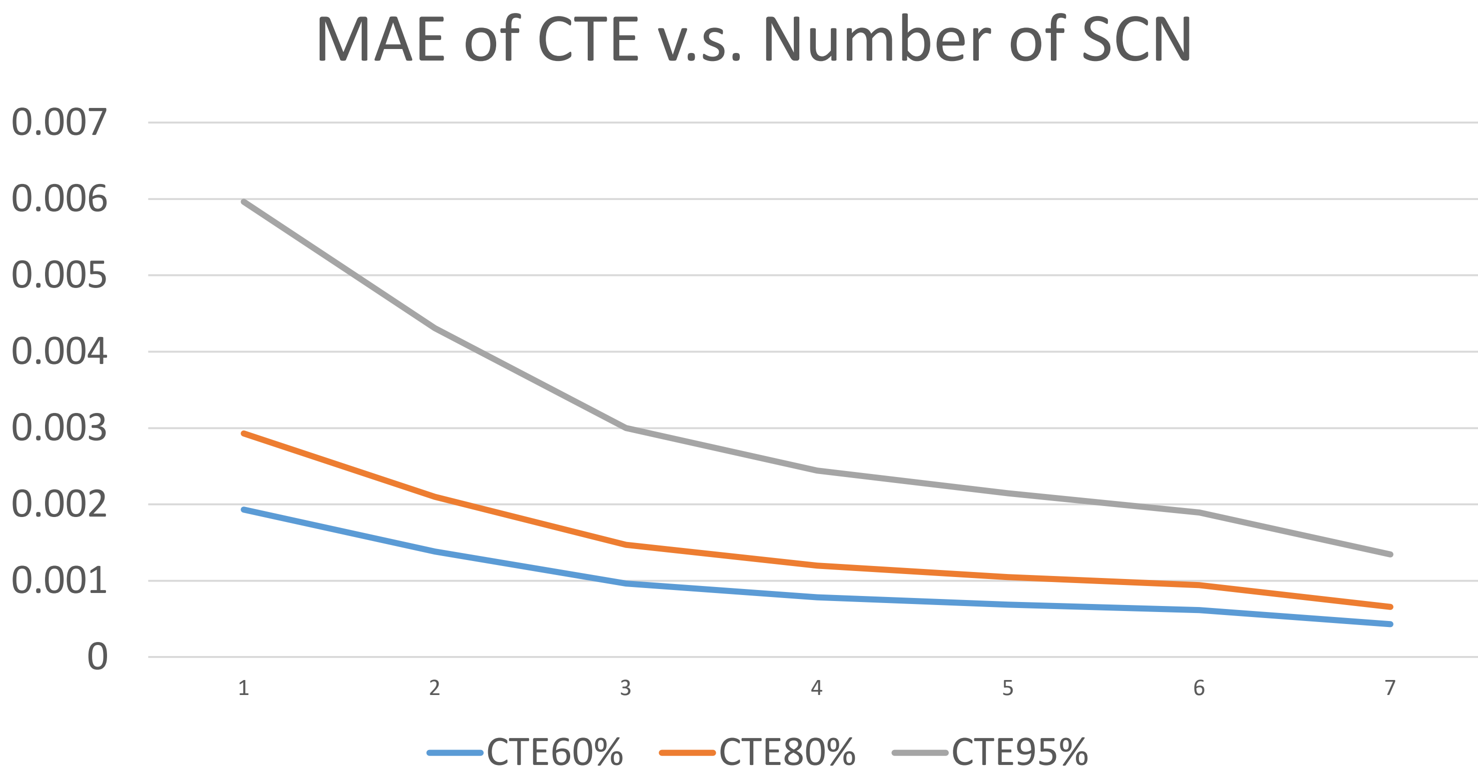


2.10 Scenario reduction: Pure simulation benchmark, 10,000 repetitions

Mean absolute deviation

Number of pivots	CTE60%	CTE80%	CTE95%
250	0.001930	0.002929	0.005962
500	0.001381	0.002098	0.004307
1,000	0.000965	0.001472	0.003001
1,500	0.000784	0.001197	0.002442
2,000	0.000685	0.001046	0.002144
2,500	0.000616	0.000941	0.001893
5,000	0.000430	0.000659	0.001343

2.11 Scenario reduction: Pure simulation benchmark, 10,000 repetitions



2.12 Scenario reduction: 10,000 simulated paths yield 500 pivots, repeated 10,000 times

Mean absolute deviation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Pure Simulations	0.00182	0.00151	0.00133	0.00088	0.00171	0.00213	0.00302
Chueh M2	0.00313	0.00240	0.00201	0.00123	0.00154	0.00180	0.00230
Clara	0.00179	0.00134	0.00113	0.00071	0.00113	0.00139	0.00190

Proportion of time closer to true value than simulation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Chueh M2	0.3325	0.3542	0.3713	0.3906	0.5371	0.5499	0.5885
Clara	0.5079	0.5404	0.5529	0.5682	0.6276	0.6391	0.6387

2.13 Scenario reduction: 10,000 simulated paths yield 500 pivots, repeated 10,000 times

Mean absolute deviation

Approach	CTE60%	CTE80%	CTE95%
Pure simulation	0.001381	0.002098	0.004307
Chueh M2	0.000920	0.001318	0.002445
Clara	0.000732	0.001104	0.002393

Proportion of time closer to true value than simulation

Approach	CTE60%	CTE80%	CTE95%
Chueh M2	0.631900	0.645100	0.671900
Clara	0.693600	0.693500	0.679600

2.14 Scenario reduction: 10,000 simulated paths yield 1000 pivots, repeated 10,000 times

Mean absolute deviation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Pure Simulations	0.00132	0.00109	0.00096	0.00061	0.00119	0.00150	0.00209
Chueh M2	0.00189	0.00146	0.00123	0.00075	0.00099	0.00116	0.00151
Clara	0.00127	0.00098	0.00083	0.00052	0.00084	0.00103	0.00136

Proportion of time closer to true value than simulation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Chueh M2	0.3803	0.4165	0.4267	0.4276	0.5587	0.5751	0.6020
Clara	0.5132	0.5303	0.5501	0.5514	0.6132	0.6183	0.6251

2.15 Scenario reduction: 10,000 simulated paths yield 1000 pivots, repeated 10,000 times

Mean absolute deviation

Approach	CTE60%	CTE80%	CTE95%
Pure simulation	0.000959	0.001465	0.003014
Chueh M2	0.000612	0.000880	0.001685
Clara	0.000546	0.000810	0.001726

Proportion of time closer to true value than simulation

Approach	CTE60%	CTE80%	CTE95%
Chueh M2	0.6432	0.6586	0.6843
Clara	0.6748	0.6864	0.6741

2.16 Scenario reduction: 15,000 simulated paths yield 500 pivots repeated 10,000 times

Mean absolute deviation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Pure Simulations	0.00187	0.00155	0.00135	0.00087	0.00169	0.00215	0.00301
Chueh M2	0.00341	0.00261	0.00216	0.00136	0.00160	0.00187	0.00239
Clara	0.00177	0.00132	0.00111	0.00070	0.00111	0.00133	0.00182

Proportion of time closer to true value than simulation

Approach	VaR5%	VaR10%	VaR15%	VaR50%	VaR85%	VaR90%	VaR95%
Chueh M2	0.3104	0.3367	0.3564	0.3553	0.5177	0.5449	0.5733
Clara	0.5134	0.5511	0.5609	0.5698	0.6298	0.6444	0.6493

2.17 Scenario reduction: 15,000 simulated paths yield 500 pivots repeated 10,000 times

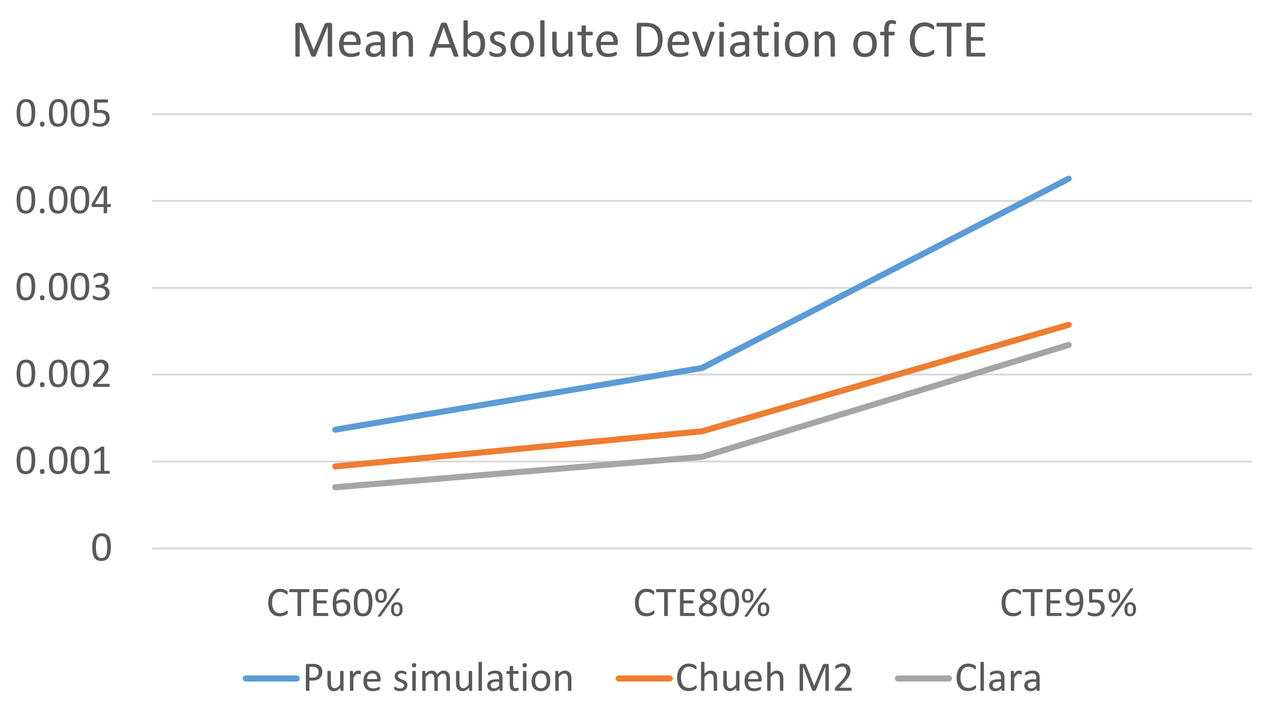
Mean absolute deviation

Approach	CTE60%	CTE80%	CTE95%
Pure simulation	0.001365	0.002077	0.004257
Chueh M2	0.000941	0.001348	0.002574
Clara	0.000704	0.001054	0.002342

Proportion of time closer to true value than simulation

Approach	CTE60%	CTE80%	CTE95%
Chueh M2	0.6204	0.6346	0.6687
Clara	0.6937	0.7006	0.6823

2.18 Scenario reduction: 15,000 simulated paths yield 500 pivots repeated 10,000 times



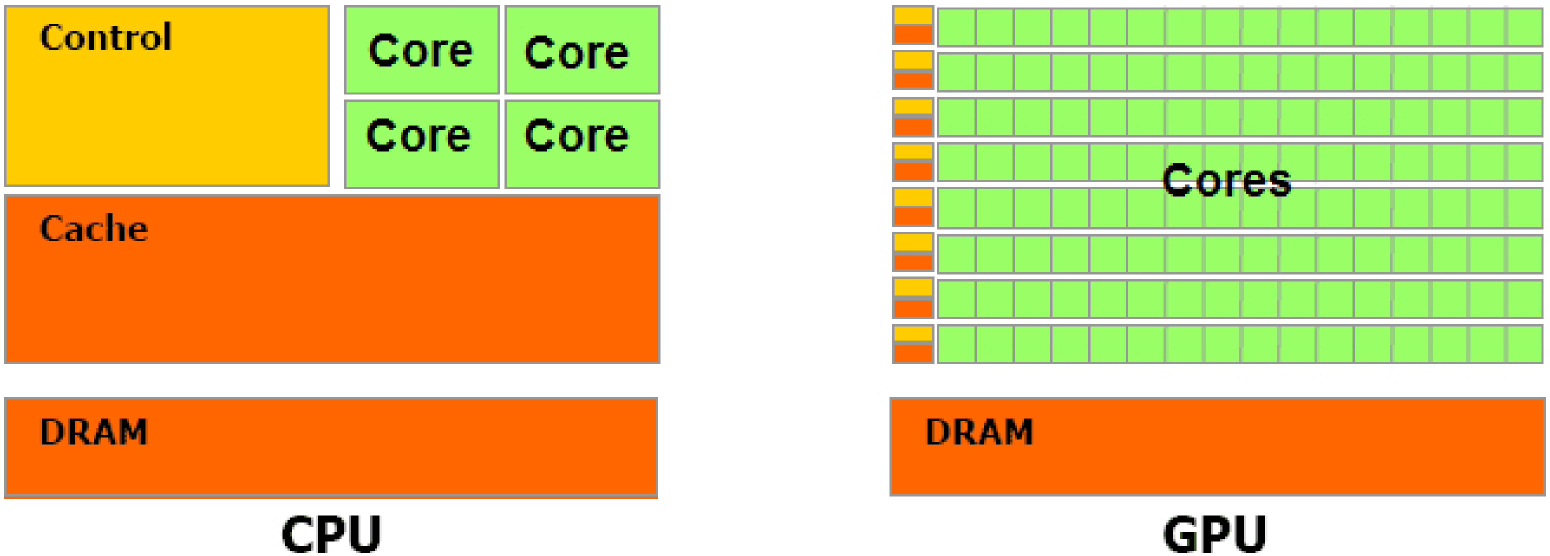
2.19 Scenario reduction: Summary of the results

- We have shown that it is possible to use a scenario reduction algorithm to reduce calculation time for a dynamic hedging model.
- The variability of the CTE95% with the scenario reduction algorithm based on the Clara algorithm with 10,000 simulations and 500 pivots is approximately equal to 1,500-2,000 simulations (i. e. calculation time reduced by a factor of 3-4).
- The scenario reduction algorithm based on the Clara algorithm is less variable than simulations for the VaR and CTE.
- The scenario reduction Chueh algorithm performs well in the right tail.

3.1 Implementation in HPC CUDA and R: Introduction

- We implemented a GPU version of the Chueh scenario reduction algorithm.
- Some parts of Chueh scenario reduction algorithm can be parallelized.
- The GPU version is significantly faster than the CPU version.
- The GPU implementation open opportunities for future research.

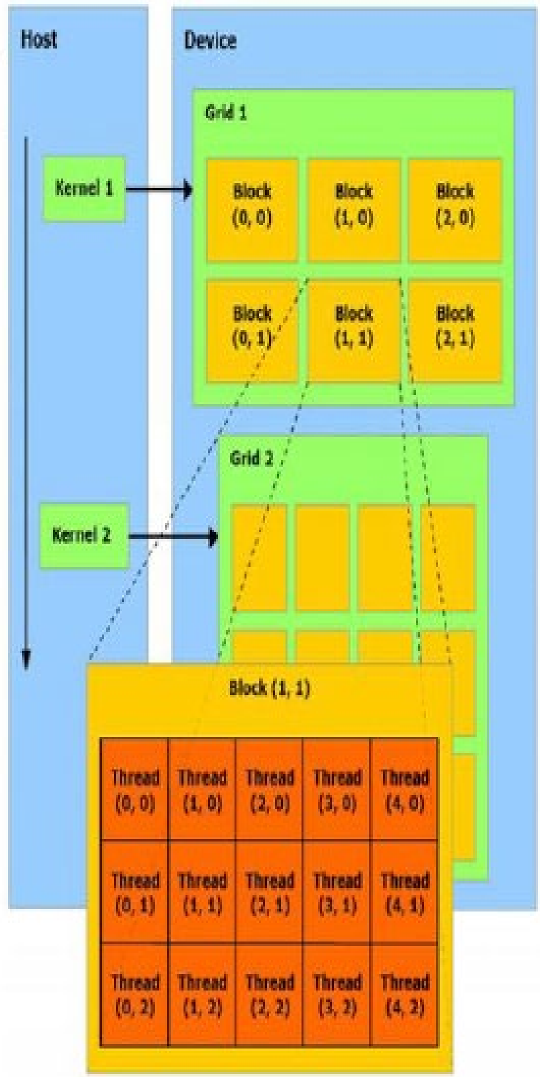
3.2 Implementation in HPC CUDA and R: Introduction

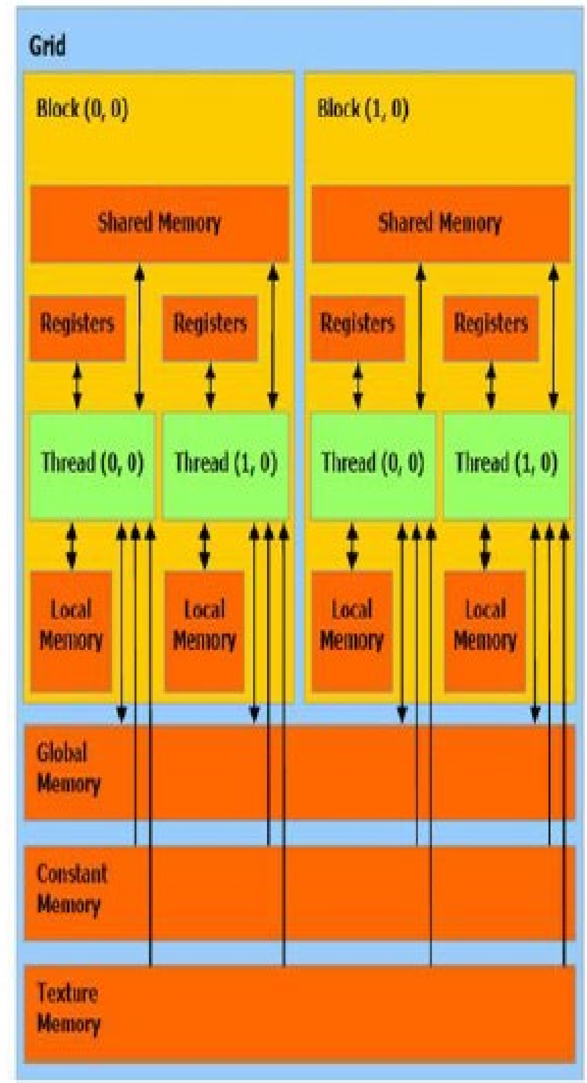


A **CPU** consists of a **few cores** optimized for sequential serial processing while a **GPU** has a massively parallel architecture consisting of **thousands** of smaller, more efficient **cores** designed for handling multiple tasks simultaneously.

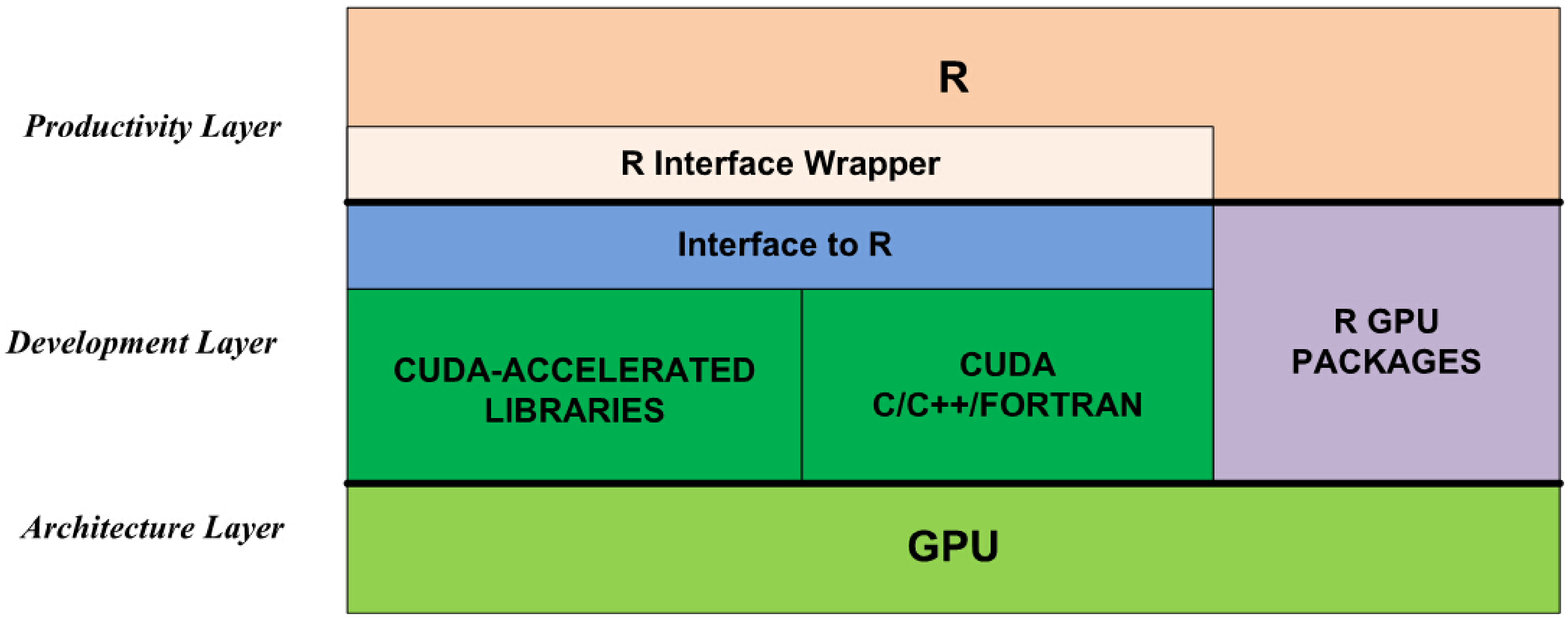
3.3 Implementation in CUDA and R: Introduction

- **Kernels** are executed by threads
  - A **kernel** is a simple C program.
  - Each **thread** has it own ID
  - Thousands of **threads** execute same kernel.
- **Threads** are grouped into **blocks**
  - Threads in a block can synchronize execution.
- **Blocks** are grouped in a **grid**
  - Blocks are **independent**





3.4 Implementation in CUDA and R: Introduction



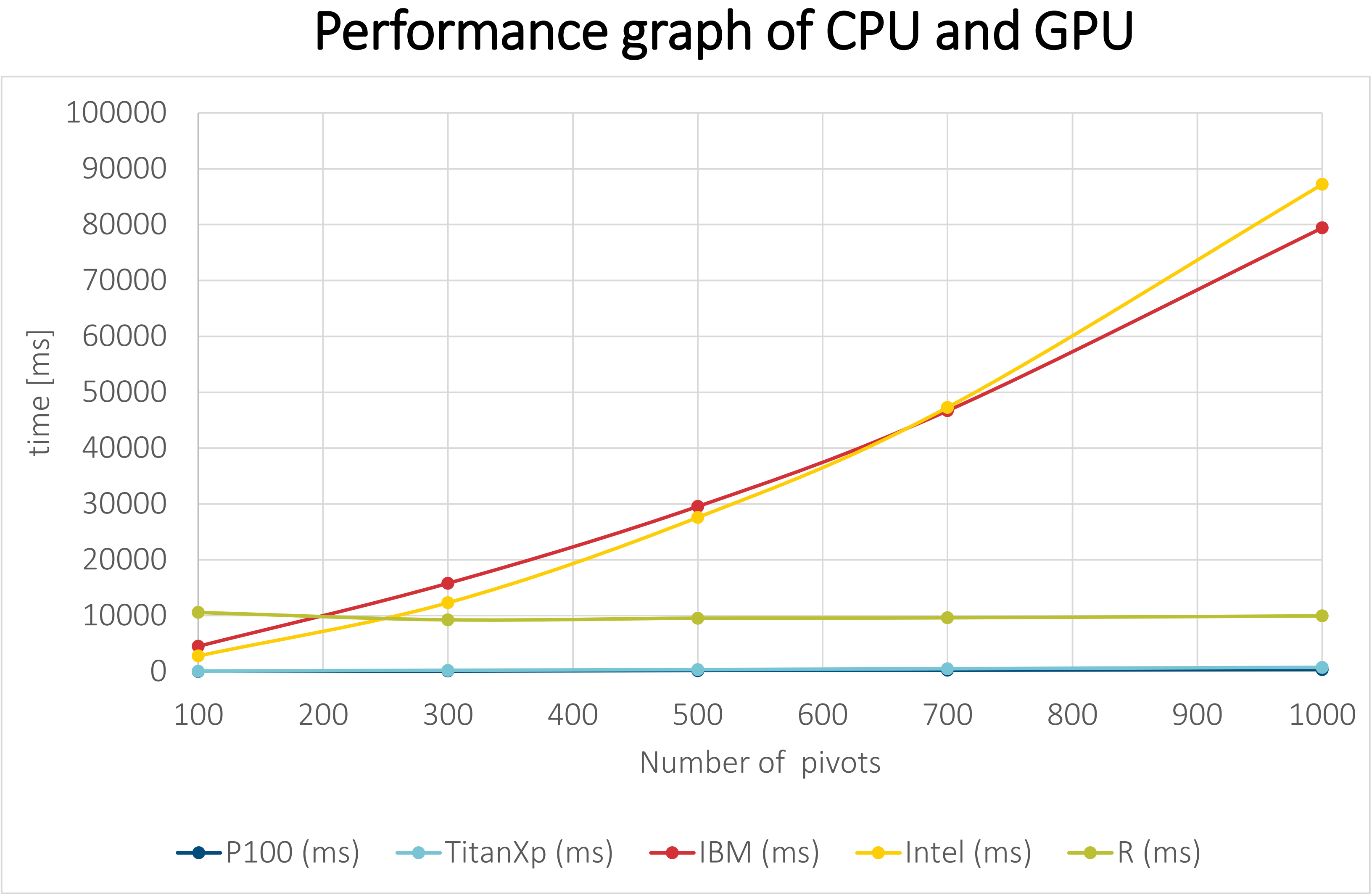
- Use **R** GPU packages from CRAN.
- Access the GPU through CUDA libraries.
- Access the GPU through the CUDA-acceleraged programming languages, including C, C++ and Fortran with Open ACC commands and PGI compiler.

3.5 Implementation in CUDA and R: Results

Pivots	Intel i7-78000X	Nvidia Titan Xp	IBM Power8	Nvidia P100
100	2794	81.45	4526.24	34.73
300	12320	215.92	15803.69	97.16
500	27597	349.85	29562.57	167.87
700	47305	492.07	46698.59	243.60
1000	87233	737.17	79435.62	369.27

†All times in [ms]

3.6 Implementation in CUDA and R: Results



4.1 Conclusion

- Scenario reduction algorithms can be applied to reduce calculation time for dynamic hedging models for GMMB guarantees.

- The calculation time of the Chueh scenario reduction algorithm can significantly be reduced using a GPU implementation.

We could extend our work by incorporating:

- other guarantees;
- stochastic lapses;
- stochastic interest rates;
- basis risk.

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