
# Session 61: Latest Thoughts on Use of Economic Scenario Generators

# Session 061 – Latest Thoughts on Use of Economic Scenario Generators

## A closer look at the AAA ESG & VM-20 / VM-21


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# Agenda

- AAA ESG
- VM-20
- VM-21
- Takeaways

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- **AAA ESG**
- VM-20
- VM-21
- Takeaways

# What’s included in the AAA ESG?

- USD Treasury Rates and Spot Rates
- USD Equity Indices
  - Large cap
  - Small cap
  - International
  - Aggressive
- USD Bond Funds
  - Money market
  - Intermediate-term bonds
  - Long-term bonds
- Blended Indices
  - Fixed Income
  - Balanced

# AAA Interest Rate Model

The SLV model simulates the following three (3) correlated stochastic processes in discrete monthly time:

| | | |
|---|---|---|
| 1. | ${}_1i_t$ | The natural logarithm of the long maturity interest rate |
| 2. | $\alpha_t$ | The nominal spread between the long and short maturity rates |
| 3. | $\nu_t$ | The natural logarithm of the volatility of the long maturity rate process |

The SLV discrete time equations are:

$${}_1i_t = Max\left\langle {}_1\lambda_L, Min\left[ {}_1\lambda_U, (1-\beta_1)\cdot {}_1i_{t-1} + \beta_1 \cdot \ln \tau_1 + \psi \cdot ({}_2\tau_t - \alpha_{t-1}) \right] \right\rangle + {}_1\sigma_t \cdot {}_1Z_t$$

$$\alpha_t = (1-\beta_2)\cdot \alpha_{t-1} + \beta_2 \cdot {}_2\tau_t + \phi \cdot ({}_1i_{t-1} - \ln \tau_1) + \sigma_2 \cdot {}_2Z_t \cdot ({}_1r_{t-1})^{\theta}$$

$$\nu_t = (1-\beta_3)\cdot \nu_{t-1} + \beta_3 \cdot \ln \tau_3 + \sigma_3 \cdot {}_3Z_t$$

where

$${}_1i_t = \ln({}_1r_t)$$

$${}_1\lambda_U = \ln({}_1r_{Max})$$

$${}_1\lambda_L = \ln({}_1r_{Min})$$

$${}_2r_t = \exp({}_1i_t) - \alpha_t$$

If ${}_2r_t < {}_2r_{Min}$, then ${}_2r_t = \kappa \cdot {}_1r_t$

$${}_1\sigma_t = \exp(\nu_t)$$

${}_1Z_t, {}_2Z_t, {}_3Z_t \sim N(0,1)$ with constant correlation matrix $\rho$

Mean Reversion Point Input!

Source: https://www.actuary.org/content/economic-scenario-generators

# AAA Bond Fund Model

The nominal monthly returns on money market and fixed income (e.g., bond) investments are generated according to:

$$r_t = \beta_0 \times \left( i_{t-1}^m + \kappa \right) - \beta_1 \times \left( i_t^m - i_{t-1}^m \right) + \sigma \cdot \sqrt{i_{t-1}^m} \cdot Z_t$$

where $Z_t$ is a standard normal variate and $i_t^m$ is the $m$-year Treasury yield in period $t$. The money market process is a function of the 3-month yield, while the ITGVT and LTCORP bond index returns are respectively modeled from the 7-year and 10-year maturities. The model parameters are provided in Table 4.

Source: https://www.actuary.org/content/economic-scenario-generators

# AAA Equity Model

**Table 5: SLV Model Processes**

$$\tilde{v}(t) = Min\left[v^{+}, (1-\phi)\times v(t-1) + \phi \times \ln(\tau)\right] + \sigma_{v} \times {}_{v}Z_{t}$$

$$v(t) = Max\left\{v^{-}, Min\left[v^{*}, \tilde{v}(t)\right]\right\}$$

$$\mu(t) = A + B\times\sigma(t) + C\times\sigma^{2}(t)$$

$$\ln\left[\frac{S(t)}{S(t-1)}\right] = \frac{\mu(t)}{12} + \frac{\sigma(t)}{\sqrt{12}}\times {}_{s}Z_{t}$$

$S(t)$ = stock index level at time $t$

$v(t)$ = natural logarithm of annualized volatility in month $t$

$\sigma(t)$ = annualized volatility of stock return process in month $t = \exp\left[v(t)\right]$

$\mu(t)$ = mean annualized log return ("drift") in month $t$

$v^{-}$ = lower bound for log volatility = $\ln\sigma^{-}$

$v^{+}$ = upper bound for log volatility (before random component) = $\ln\sigma^{+}$

$v^{*}$ = absolute upper bound for log volatility = $\ln\sigma^{*}$

Source: https://www.actuary.org/content/economic-scenario-generators

# AAA Calibration

- All AAA models calibrated based on historical data using constrained maximum likelihood estimation (MLE):

| Asset Class | Market Proxies | Historic Period |
|---|---|---|
| Money Market | 3 Month Treasury returns | 1955.12 – 2003.12 |
| U.S. ITGVT | U.S. Intermediate Term Government Bonds | 1955.12 – 2003.12 |
| U.S. LTCORP | U.S. Long Term Corporate Bonds | 1955.12 – 2003.12 |
| Fixed Income | 65% ITGVT + 35% LTCORP | n/a |
| Balanced Allocation | 60% Diversified Equity + 40% Fixed Income | n/a |
| Diversified Large Cap U.S. Equity | S&P500 Total Return Index | 1955.12 – 2003.12 |
| Diversified International Equity | MSCI-EAFE $USD Total Return Index | 1969.12 – 2003.12 |
| Intermediate Risk Equity | U.S. Small Capitalization Index | 1955.12 – 2003.12 |
| Aggressive Equity | Emerging Markets, NASDAQ, Hang Seng | 1984.12 – 2003.12 |

Source: https://www.actuary.org/content/economic-scenario-generators

# AAA Calibration

- Calibrations are provided by the AAA and rarely change
- Practitioners need not worry about calibration of this ESG except when other indices and risk factors must be modeled beyond what the AAA supports (e.g. foreign economies, inflation, etc.)
  - May require use of alternative ESGs and statistical techniques to complement the AAA ESG!

Source: https://www.actuary.org/content/economic-scenario-generators

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# VM-20 Deterministic vs. Stochastic Reserve

- Deterministic Reserve
  - Based on a single prescribed scenario:

> Scenario 12 – Deterministic scenario for valuation
>
> There are uniform downward shocks each month for 20 years, sufficient to get down to the one standard deviation point (84%) on the distribution of 20-year shocks. After 20 years, shocks are zero.

- Stochastic Reserve
  - Based on a set of scenarios from the AAA Interest Rate and Equity generators
  - Conditional Tail Expectation (70) metric

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-20 AAA ESG Mean Reversion Formula

- Mean Reversion Formula is a weighted sum of three parts:
  - Median 20-year treasury rate from last 50 years * 20%
  - Average 20-year treasury rate from last 10 years * 30%
  - Average 20-year treasury rate from last 3 years * 50%
- Result of the sum is then rounded to the nearest 25 bps

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-20 Guidance on ESGs

- Choosing reduced scenario sets
  - Allowed as long as it can be demonstrated that it does not materially reduce the reserve that would otherwise be calculated and the sets are chosen by utilizing the AAA picker
  - Demonstration need **not** be brute force. Theoretical, statistical, or mathematical arguments sufficient for the insurance commissioner are allowed.
  - Should periodically test different numbers of scenarios on representative sample policies
- AAA ESG Scenario Picker
  - Ranks full 10,000 scenario set according to some significance criteria (e.g. rates or a specific return)
  - Creates percentile buckets based on subset size (e.g. pick 100 -> segment into 1%, 2%, …, 99%, 100%-iles)
  - Pick the scenario represented by the mid-point of each bucket

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-20 Guidance on ESGs (continued)

- Discount Rates for Stochastic Reserve
  - Uses the 1-year treasury rate from AAA ESG at the start of each projection year * 1.05 (different from deterministic reserve!)
- Fund Mapping
  - Companies recommended to map funds into one of the indices supported by the AAA ESG
  - Proxy funds may be necessary
    - Be prepared to explain proxy fund construction
- Derivatives
  - May reflect derivative programs in deterministic and stochastic reserves
    - “Clearly Defined Hedging Strategy” (CDHS)
    - Derivatives for non-hedging purposes (e.g. replication) that support the investment strategy of the underlying policies
    - Largely aligned with VM-21 treatment of derivatives

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

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# VM-21 Standard Scenario vs. CTE Amount

- Standard Scenario
  - Based on a single prescribed scenario
  - Discount rate chosen as issue year basis, Plan Type A, 10-20 year duration from Model #820 regulation
  - Derivatives allowed, but must be approved (i.e. supporting the underlying policies)
    - Only credited for hedges held as of the valuation date
- CTE Amount
  - Based on a set of scenarios from the AAA Interest Rate and Equity generators
  - CTE (70) metric
  - Discount rates are a bit complicated:
    - Rate at which cash flows are invested adjusted for credit losses
  - With derivatives, formula depends on hedge effectiveness:
    - E * Best Efforts + (1-E) * Adjusted

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-21 Differences and Similarities to C-3 Phase II

- Similarities:
  - Accumulated deficiency (VM-21) is similar to additional asset requirement (C-3 Phase II)
  - CTE Amount (VM-21) is similar to TAR (C-3 Phase II)
- Differences:
  - C-3 Phase II utilizes a CTE (90) metric
  - VM-21 ignores federal income tax in accumulated deficiencies and discount rates, whereas C-3 Phase II does account for federal income tax in projections and in discount rates

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-21 Guidance on ESGs

- Alternative ESGs are allowed when:
  - Funds cannot be mapped into suitable indices from the AAA ESG
    - Alternative ESGs must respect the principle that funds with higher expected returns should exhibit higher volatility (i.e. greater risk with greater reward)
    - Alternative ESG scenarios should be readily available to share if requested for any regulatory review
  - Company can demonstrate on an annual basis no material change in the Total Asset Requirement (TAR) that would normally be calculated
- Choosing reduced scenario sets
  - Must be able to demonstrate no material change in reserve from running more scenarios

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-21 Derivatives and Extending the AAA ESG

- Calibration of Equity (i.e. for unsupported indices)
  - Alternative calibrations are recommended to have a distribution of gross wealth ratios at least as wide as published calibration points
    - Caveat: All points need not be satisfied as long as there is no material impact to the computed reserve
- Derivative Challenges
  - Requires risk-neutral valuation techniques to compute values of derivatives along RW paths
    - Analytic
    - PDE
    - Monte Carlo
  - Future Markets
    - Evolving implied volatility surfaces and maintaining arbitrage free traits

Source: https://www.naic.org/documents/cmte_a_latf_related_val_2019_edition.pdf

# VM-21 Example Techniques to define Future Markets

- SABR
  - Stochastic volatility model, common on interest rate derivatives
- Carr-Pelts
  - Technique to generate arbitrage free surfaces
- Model Implied surface / state variables
  - Utilizing surface implied by parameterized model
  - Simple level changes via variance/volatility state variable
  - Complex changes involving skew of the surface (e.g. may be based on regression analysis or other similar techniques)
  - Tracking State Variables (e.g. Bates):

Useful states in Bates!

$$\frac{dS_t}{S_t} = (r_t - u_t - \lambda_t m_J)\, dt + \sigma_t \sqrt{z_t} dW + J dN_t,$$

$$dz_t = \overline{\kappa}_t (1 - z_t) dt + \overline{\xi}_t \sqrt{z_t} dV,$$

$$\langle dW\, dV \rangle = \rho_t dt,$$

$$z_t\big|_{t=0} = z_0, \qquad S_t\big|_{t=0} = S_0.$$

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# Takeaways

- **AAA ESG** is heavily **prescribed** and is based on **historical data calibrations** utilizing MLE, but has important shortcomings in the coverage of risk factors
- Current practice highlights significant overlap between **VM-20** and **VM-21**, particularly with regard to the **AAA ESG.**
- Treatment of **derivatives** inside both **VM-20** and **VM-21** require significant care around defining future markets upon which to value the derivatives

# Thank You

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# SESSION 61: LATEST THOUGHTS ON USE OF ECONOMIC SCENARIO GENERATORS

## INTRODUCTION TO THE ACADEMY INTEREST RATE GENERATOR ("AIRG")

MAY 21, 2019

Matthew Zhang
FSA, MAAA, CERA

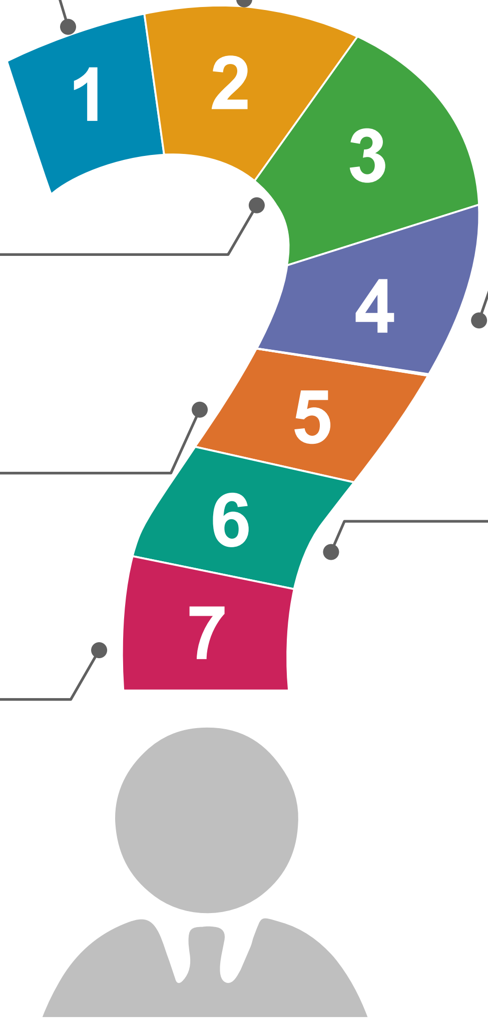
# Economic scenario generators factors

**Risk factors**

Which risk factors do I need to consider?

**Scenario type**

Do I need real world or risk neutral scenarios?




**Process and parameterization**

Which methods are most appropriate?

How do I parameterize those methods?

**Calibration**

How do I evaluate the appropriateness of my scenarios?

**Number**

How many scenarios do I need?

**Time**

What time-scale is appropriate?

What frequency do I need my scenarios for?

**Most important question**

**Are the scenarios appropriate?**

# 1 Risk factors

## 1 US TREASURY RATES

- US Treasuries yield curves from 3-month to 30-year terms
- Rates are graded from starting date US Treasury rates to provide a smooth projection
- Non-US economies are not considered

## 2 BOND AND EQUITY FUNDS

- A series of indices representing bond, equity, and blended portfolios
- Bond and blended portfolios are derived to be consistent with projected US Treasuries
- Factors such as defaults, credit spreads, forex volatility, are baked into fund level parameters, **but are not explicitly modeled**

**Bond funds**

- US money market (3m)
- US intermediate government (7yr)
- US long corporate (10yr)

**Equity funds**

- US diversified (S&P 500)
- US small cap (Russel 2000)
- International equity (MSCI-EAFE)
- Aggressive / exotic

**Composite funds**

- Fixed income (65% US intermediate government, 35% US long corporate)
- Balanced (60% US diversified, 40% fixed income)

# 2 Scenario type

| | Real world (AIRG) | Risk neutral |
|---|---|---|
| Objectives | Realistic projection of assets and liabilities informed by historical market movements | Market consistent valuation |
| Output characteristics | Riskier assets typically have higher expected risk-adjusted returns<br><br>Scenarios may exhibit "realistic" behavior (volatility clustering, realistic inversions to the yield curve, etc.) | All assets expect to earn the risk free rate, regardless of risk profile<br><br>Scenarios appear "realistic", but do not have meaning on a stand-alone basis |
| Usage | Assessing a range of possible outcomes<br><br>The AIRG is strongly linked to US capital and reserve standards | Monte Carlo simulation replicates market prices |
| Calibration | Past data used to inform model parameters<br><br>Subjective – reflect assumptions about market<br><br>Stylized facts relevant to the product need to be considered | Projected scenarios must reproduce today's market-observed prices<br><br>The past is not important |

# 3 Process and parameterization

### Generic rate process

$$dr = \overbrace{\mu(r,t)}^{\text{Drift Component}}\, dt + \underbrace{\sigma(r,t)}_{\text{Volatility Component}}\, dW$$

### Example: Vasicek model – features mean reversion

$$\mu(r,t) = a * (\mathrm{b} - r_t)$$
$$\sigma(r,t) = \sigma$$

### Key considerations

- Constant vs. non-constant parameters
- Single vs. multiple stochastic factors

### Scenario property considerations

- Mean reversion
- Allowance of negative rates
- Fat tail
- ... Many more

# ❸ Process and parameterization
## Stochastic Log Volatility interest rate model

**Long maturity (20-year) Treasury rate**

$$\ln({}_{long}r_t) = (1-\beta_{long}) * \ln({}_{long}r_{t-1}) + \beta_{long} * \ln(\tau_{long}) + \psi * (\tau_{spread} - \alpha_{t-1}) + {}_{long}\sigma_t * {}_{long}Z_t$$

Limited by parameterized "soft" max/min (rate between 1.15%-18%)

$\beta$: mean reversion strength

$\psi$: steepness adjustment

$\tau$: mean reversion target

**Logarithm of volatility of long maturity rate**

$$\ln({}_{long}\sigma_t) = (1-\beta_{longvol}) * \ln({}_{long}\sigma_{t-1}) + \beta_{longvol} * \ln(\tau_{longvol}) + \sigma_{longvol} * {}_{longvol}Z_t$$

**Spread between long maturity and short maturity (1-year) Treasury rates**

$$\alpha_t = (1-\beta_{spread}) * \alpha_{t-1} + \beta_{spread} * \tau_{spread} + \phi * [\ln({}_{\text{long}}r_{t-1}) - \ln(\tau_{long})] + \sigma_{spread} * {}_{spread}Z_t * {}_{long}r_{t-1}^{\theta}$$

$\phi$: spread tilting parameter

$\theta$: exponent of spread volatility – set to 1

## 3 Process and parameterization

# AIRG fund models

### Bond fund process

$$r_t = \frac{1}{12} * (i_{t-1} + \kappa_{spread}) + \beta * (i_{t-1} - i_t) + \sigma * \sqrt{i_{t-1}} * Z_t$$

$\beta$: bond fund duration

$i_t$: The yield of a projected Treasury curve at time t, for an appropriate reference maturity

### Equity fund process

**Logarithm of annualized volatility**

$$\ln({}_{return}\sigma_t) = (1-\phi) * \ln({}_{return}\sigma_{t-1}) + \phi * \ln(\tau) + \sigma_{vol} * {}_{vol}Z_t$$

Volatility is bound

**Mean drift**

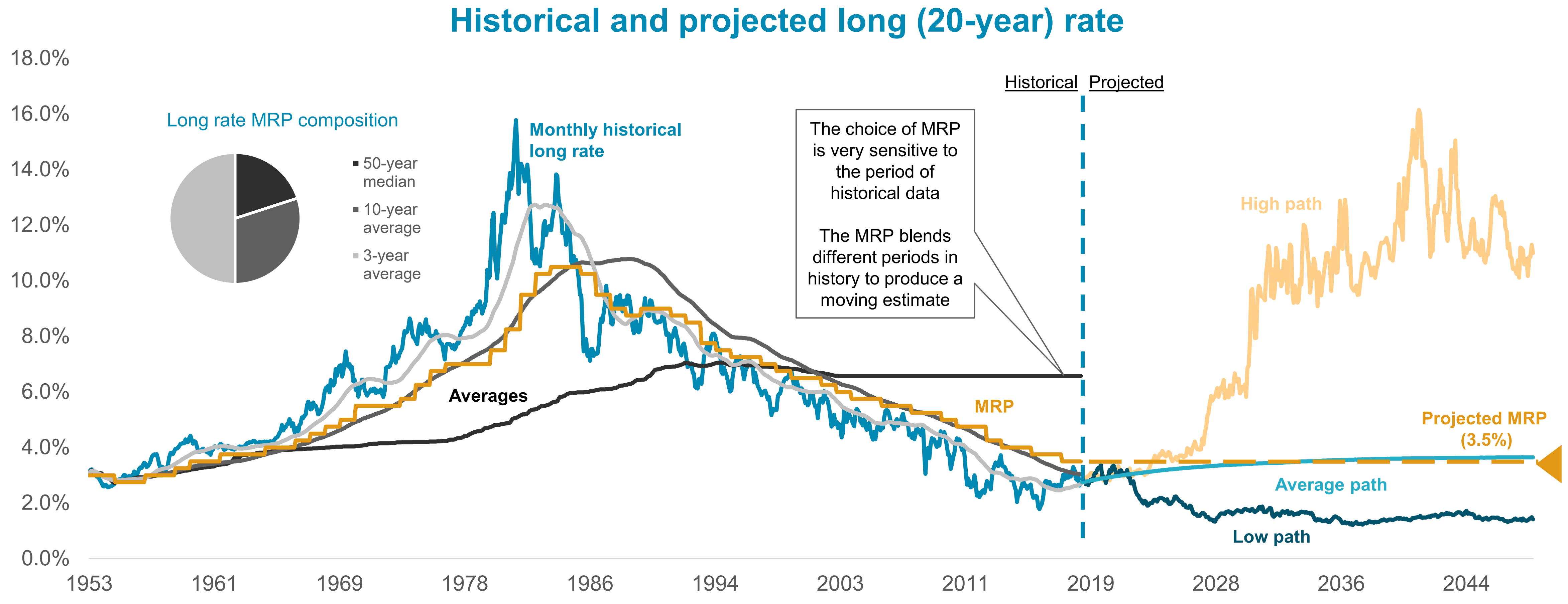
$$\mu_t = A + B * {}_{return}\sigma_t + C * {}_{\mathrm{return}}\sigma_t^2$$

**Monthly logarithmic return**

$$\ln\left(\frac{S(t)}{S(t-1)}\right) = \frac{\mu_t}{12} + \frac{{}_{return}\sigma_t}{\sqrt{12}} * {}_{equity}Z_t$$

# 4 Calibrating economic scenarios

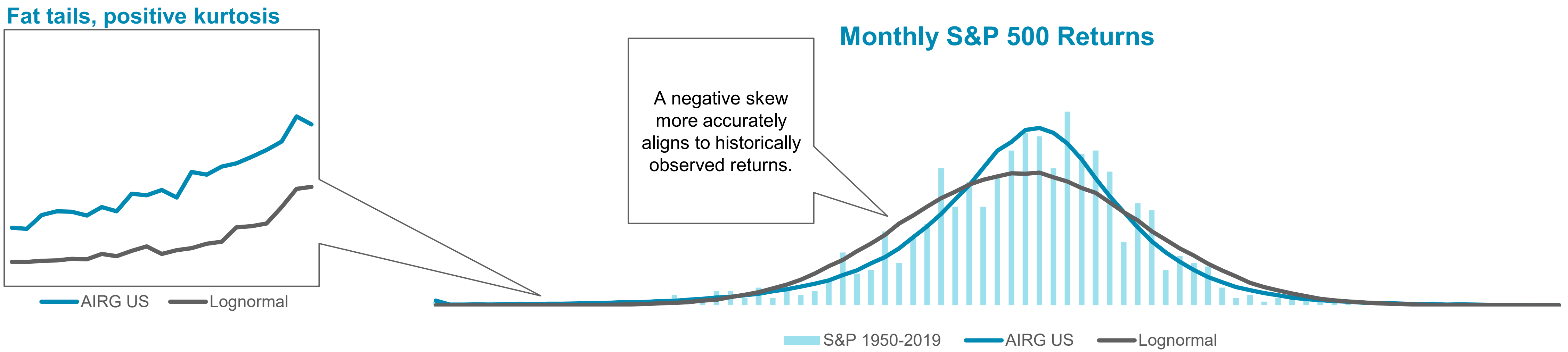
## Interest rate calibration




The dynamic MRP is a compromise between balancing historical data and current market realities

# ❹ Calibrating economic scenarios
## Equity calibration

- Dynamics (volatility, correlations, etc.) are representative of possible outcomes
- Subjective judgement is applied
  – E.g. The US diversified equity fund fits volatility parameters from S&P500 data, while overall return expectations are subjectively set to produce a “reasonable” 8.75% annual return
- The AAA periodically reviews the suitability of parameters against emerging market trends

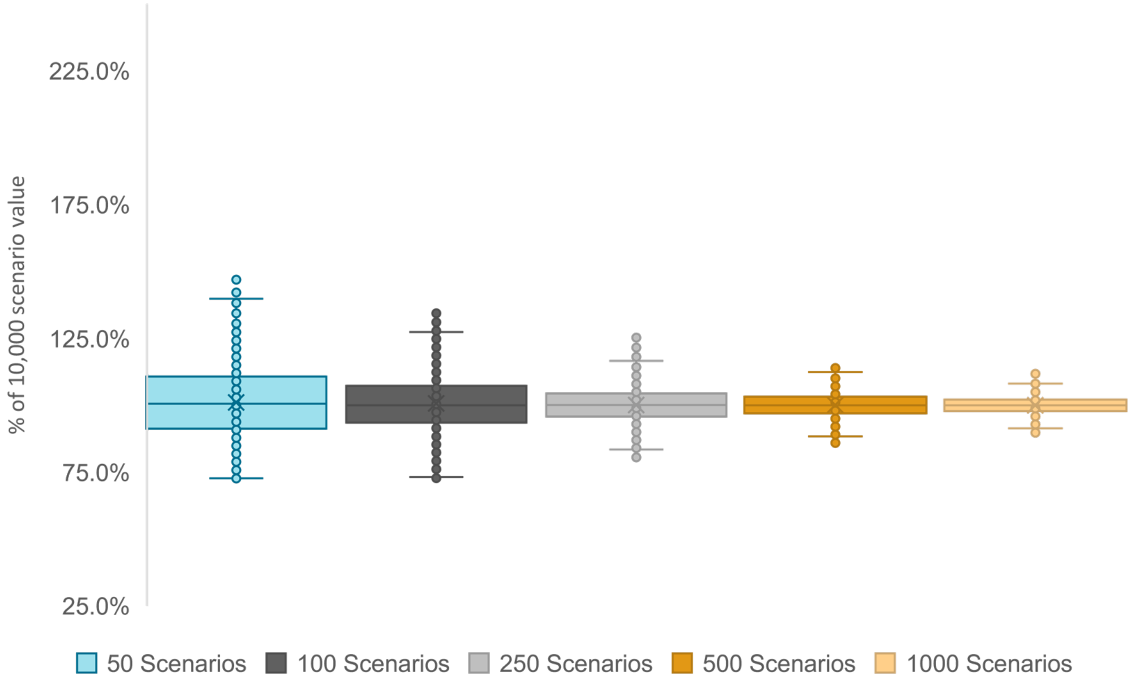



A simple lognormal model with constant parameters fails to exhibit the observed fat-tailed characteristics of historical S&P 500 returns

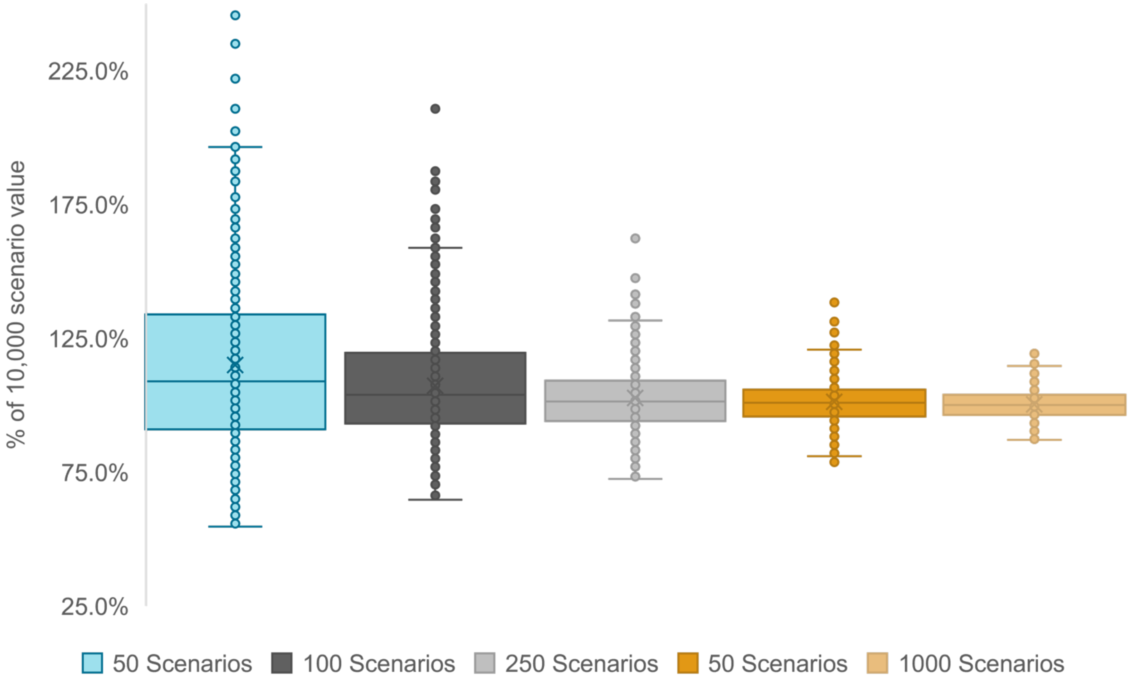
# 5 Number of scenarios

## Sample total asset requirements for a GMAB rider

### CTE 70




### CTE 98




The AIRG can generate from 50 to 10,000 scenarios, but the user must make a decision on the appropriate number. Using too few scenarios will not produce sufficient convergence

# 6 Time

## Time Horizon

- The AIRG caps the projection length at 150 years
- Must be at least as long as the modeled liabilities
- Yield curves in the first projection year are graded from the Treasury curve as of the valuation date

## Frequency

- The AIRG produces scenarios of monthly, quarterly, semi-annual, or annual frequency
- Scenarios should be frequent enough to satisfy product level modeling needs
  - E.g. a VA rider with monthly ratchets should be modeled monthly and requires monthly scenarios

Increased time horizon and timestep frequency can dramatically increase the runtime requirements

## 7 Most important question

**Are the scenarios appropriate**

OLIVER WYMAN