The Fatal Shot:
An Analysis of Costs and Solutions to Homicides in Chicago

TEAM 9054
MODELING THE FUTURE CHALLENGE 2022

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## Contents

1 Executive Summary 3

2 Background Information 4

3 Data methodology 5

4 Mathematics methodology 7
   4.1 Assumptions 8
   4.2 Variables 8
   4.3 Correlations between various parameters and homicide rates 9
   4.4 Predicting trends for 2022 11
      4.4.1 Opioid Overdoses 12
      4.4.2 Unemployment Rate 12
      4.4.3 Temperature 13
      4.4.4 Police Employment 13
   4.5 Results 14
   4.6 Sensitivity analysis 15
   4.7 Strengths 15
   4.8 Weaknesses 15

5 Risk analysis 16
   5.1 Risk overview 16
   5.2 Expected costs of homicides 16
   5.3 Distributions 17
      5.3.1 Months of highest risk 17
      5.3.2 Locations of highest risk 17
   5.4 Trends 19

6 Recommendations 20
   6.1 Opioids 20
   6.2 Unemployment 21
   6.3 Police 21
   6.4 COVID-19 23

7 Conclusion 24
1 Executive Summary

797 people were murdered in Chicago in 2021 [1]. At a rate of two people being killed on the streets of Chicago every day, the city is currently facing its worst crime crisis in decades. While deadly violence is on the rise in most US cities, Chicago has been hit especially hard [11]. This uptick in murders not only increases costs for the involved parties, but for the city of Chicago as a whole. From emotional trauma to costs in the criminal justice system [15], violent crime has many harmful ramifications, making it an urgent issue in Chicago that needs to be confronted. In our model, we seek to predict the risks associated with homicides in 2022 as well as lay out recommendations for decreasing homicide rates.

First, we designed a model that projected the number of homicides in Chicago in 2022 by taking into account several parameters that we determined to have a strong correlation with homicide rate. In our model, we considered unemployment rates, opioid use, climate, gun sales, and police staffing [68]. Using data for all of these parameters, we used symbolic regression techniques to derive our equation. After finding an equation relating our parameters with homicide rate, we applied multiple regression techniques to predict future values of each parameter. Finally, in a Monte Carlo simulation, we incorporated the stochastic nature of homicide rates in predicting the number of homicides in 2022. We concluded that Chicago homicide levels in 2022 will be lower than they were in 2021, with a mean of 458 homicides and a standard deviation of 11 homicides. These results are in line with observed trends in the first few months of 2022. [3]

With homicide rates predicted, we set out to quantify the risk associated with these homicides in 2022. By employing past studies [15], we found the tangible cost of homicides in 2022 to be $585,518,192 and the total cost (tangible plus intangible costs such as grief) to be $4,451,954,192. We found that the month of August is at the highest risk of homicides due to the combined effect of warm temperatures and low police staffing. In addition, we identified the regions of Chicago with the highest and lowest risk of homicides as being the communities of East Garfield Park and Forest Glen, respectively. Finally, we applied our model to put the risks in 2022 in context with the past and future, finding that the rates in 2022 are similar to those of 2013 and 2014.

Based on our analysis, we outlined policy recommendations for the Chicago city government that will work to ease homicide rates. We centered on policy changes that will resolve issues in three relevant causes of homicides: opioid use, unemployment, and policing. Because we found a strong relationship between high opioid use and high homicide rates, we recommend Chicago spend money on providing opioid treatment resources like safe injection sites. Observing a correlation between high unemployment and homicide rates, we also recommend the city to increase its spending on employment subsidies and welfare programs to reduce economic hardship in crime-ridden areas. Seeing a rapid decline in the number of police officers in the Chicago Police Department (CPD), we recommend that the city government increase the CPD’s budget to invest in higher pay and benefits for police officers, thus reducing the high frequency of resignations seen in the past year. Because all of these parameters are correlated with COVID-19, we also recommend strategies to reduce the higher stress levels and economic difficulties associated with the pandemic. We believe that these strategies will help bring down homicide rates in Chicago.
2 Background Information

On the night of March 4th, 2022, Nyzireya Moore was out celebrating her 12th birthday with her family. Wearing a jeweled crown and a sash reading “Birthday Queen”, she was supposed to be enjoying her special day. But by the next morning, Moore was pronounced dead at Comer’s Children Hospital.

While driving back from a family dinner, Moore was shot in the head just a block away from her home. She was rushed to the hospital but succumbed to her injuries, leaving her parents and the rest of her community in shock. As of writing, no one has been arrested.

At the time of her death, Moore was one of over a dozen children killed in Chicago this year [55]. Violent crime has been a serious issue for Chicagoans for decades. After a surge in crime in the 1990s, though, crime had been steadily decreasing in the city up to the early 2010s. In 2016, however, crime—specifically, homicides—surged, with the number of murders rising by 60% from the previous year [8].

The surge in crime in 2016 can be attributed to the opioid epidemic [38]. That year, the entire nation saw a rise in the abuse of opioids like prescription painkillers, heroin, and fentanyl. These drugs are known to inhibit cognitive function and increase the risk of impulsive decisions, such as violent crime [24]. While we don’t often see opioid statistics in the headlines these days, opioids remain a key contributor to current homicide trends.

Homicide rates modestly declined in the years following 2016 but surged again in 2020. According to Vox [65], widespread protests after the death of George Floyd combined with effects from the COVID-19 pandemic led to Chicago’s highest homicide rate in decades. The pandemic exacerbated mental health issues and opioid use as rising unemployment rates led to increased gang violence [19]. Although the city is slowly moving past the pandemic, its social and economic effects remain.

![Chicago homicides by year](image)

Figure 1: Homicides in Chicago by year, 2014-2021

In 2021, the yearly homicide count further increased from 772 to 797 [10]. As unemployment and
opioid use eased to close-to-normal values, homicides remained at the levels that they were in 2020. Such a result can be attributed to another issue with the city’s crime prevention capability: a police shortage [6]. Like in many other industries amid the COVID pandemic, police officers are quitting the job, citing burnout, terrible working conditions, and low pay. In addition, the recent rise in homicides has been a contributing factor to decreasing, not increasing, police staffing in the CPD. Police officers are beginning to prioritize their own safety amid rising violence, leading to a slew of resignations in the past year.

This decrease in the number of police officers has led to an increased homicide rate in Chicago. According to multiple sources [28] [29] [50] [66], a more well-staffed police force can help lower homicide rates by responding more quickly to instances of violence, managing illegal opioid and gun use, and providing first aid to the injured.

As a result, a multitude of causes have contributed to Chicago’s homicide trends over the past years. Some of these causes themselves—opioid use and unemployment—are also problems that should be addressed by the city government. Because of this, it is ever more critical for high homicide rates to be addressed; in reducing homicide rates, we in turn resolve many other pressing issues that the city of Chicago faces today.

Akin to the numerous causes of homicide rates are the numerous costs associated with them. Homicides are disastrous not only on an emotional level to victims’ loved ones but also on a monetary level in terms of the costs involved. Rough estimates set the total cost of a homicide to be about $10,000,000 [32], which includes emotional pain, criminal justice system costs, and a decrease in a community’s economic productivity. While those directly related to a homicide victim are most impacted by homicides, we cannot neglect the cost that a homicide can have to the rest of the community. Thus, homicides are not a localized problem for only the most crime-ridden areas of Chicago—they are a burden that everyone in the city must take on and confront.

In consideration of all the potential costs of high homicide rates, reducing murders should be a priority for Chicago and the CPD. With homicides increasing each year, our paper seeks to find empirical correlations between each of the parameters noted above to recommend solutions on how to stabilize and reduce the homicide rate in Chicago. For the sake of kids like Moore and many more victims of homicides, we must address this issue with both urgency and austerity.

3 Data methodology

We draw data from 6 main sources: past trends for homicide rates from the CPD, data for opioid use in Illinois, the number of officers employed in the CPD, the unemployment rate in the Chicago region, gun purchase data, and temperature data. These sources are relevant to both finding the relationship between homicides and other parameters and for calculating future trends for such parameters to predict the number of homicides that will occur in the future.
CPD Crime Data [22]

- **Motivation**: Because this paper focuses on homicides, we derive homicide data from the CPD crime database.

- **Parameters**: This dataset provides us with individual records for each incidence of crime in Chicago with the type of crime, time, and place indicated. The dataset includes over 7 million records for crime from 2001 to present.

- **Purpose**: We will use the data from the CPD in order to measure the number of homicides each year and compare that to the frequencies of other parameters which are listed below. Because the data consisted of millions of individual records, it was necessary to sort and compile the data into the number of homicides overall in each year. To accomplish this, we sorted the data in MATLAB as seen here. We used this information to find the parameters that contribute most to Chicago’s homicide count and to provide an informed recommendation on which parameters should be limited in order to minimize the number of homicides.

Illinois Opioid Data Dashboard [40]

- **Motivation**: According to the US Department of Justice [24], drug use is directly correlated with an increase in crime, including homicides. Most notably, a spike in the homicide rate in 2016 [8] has been attributed to a spike in opioid use by multiple sources.

- **Parameters**: This dataset provides us with information on the number of non-fatal and fatal opioid overdoses in Illinois from 2013 to 2021. The data also includes data for Cook County (the Chicago area) specifically.

- **Purpose**: We will use this data to see exactly how opioid use in Chicago impacts the number of homicides in our regression model.

CPD Employment Data [2]

- **Motivation**: According to a study published by the National Bureau of Economic Research [42], police force population has a negative correlation with homicide count. As a result, we incorporated in our model data from the CPD [2] on Chicago’s police force population in the years it was available.

- **Parameters**: This dataset gives provides us with the yearly police population in Chicago from the years 2007-2010 and 2017-2020. We were able to supplement for missing years, with the exception of 2015, with the FBI’s crime database [26].

- **Purpose**: We will use this data to see exactly how the number of police officers in Chicago impacts the number of homicides in our regression model.

Chicago-Naperville-Elgin Unemployment Rate [51]

- **Motivation**: According to a study conducted by the University of Chicago [59], unemployment rate has a positive correlation with homicide count. Thus, by studying the pattern of
unemployment rates in Chicago over the past decade, it can be used to provide us a good grasp of determining trends in the homicide data.

- **Parameters**: This data set provides us with the unemployment rate in the Chicago area by month from 1990-2022.

- **Purpose**: We will use this data to see exactly how the unemployment rate in Chicago impacts the number of homicides in our regression model.

**SafeHome Monthly Gun Purchases in Illinois [34]**

- **Motivation**: According to the Harvard Injury Control Research Center [31], more guns is strongly associated with more homicide, especially in high-income nations such as the United States. While this data regarding the purchasing of firearms is for Illinois, rather than for just Chicago, it can still provide valuable insight into fluctuations in homicide rates for Chicago.

- **Parameters**: This dataset provides us with the number of gun purchases in Illinois every month from January 2000 to October 2020.

- **Purpose**: We will use this data to see exactly how the number of gun sales impacts the number of homicides in our regression model.

**Chicago Monthly Temperature Data [9]**

- **Motivation**: The temperature outdoors has a significant impact on human activity; warmer temperatures mean more time spent outside. Since homicides often occur when people are most active outdoors, the temperature in a given month has a significant correlation with the number of homicides we expect to see. As a result, temperature is an important parameter to be included in our model.

- **Parameters**: This data provides us the average monthly temperatures in Chicago going back to 1950.

- **Purpose**: We will use this data to account for seasonal variations in our regression model. While temperature is ultimately not a parameter that the city government can control, its inclusion is necessary in the model to set a baseline so that the influence of other parameters can be clearly identified.

4 **Mathematics methodology**

In our model, we seek to derive the cost to the city government due to homicides in 2022. We first use a symbolic regression algorithm to develop a mathematical function that correlates various parameters to the number of homicides that we can expect in a given month. Using this function, we then use regression techniques to predict future trends for our associated parameters. Finally, we perform a Monte Carlo simulation to predict the distribution of homicides in 2022.
4.1 Assumptions

1. The parameters that influence the number of homicides are opioid use, unemployment rate, police staffing, gun ownership, and environment temperature. The first four are the leading causes of homicide in most cases [68]. In addition, temperature plays a major role in the variation of homicides between winter and summer months, as homicide rates should increase when the majority of the population is outside and interacting and decrease when people stay inside.

2. The effects of climate change on temperature variation can be neglected. While having a major impact on the environment, the worst-case scenario for climate change is an increase in average temperature by 1.5 degrees Celsius [13] in the next 10 years, which is negligible for its impact on urban human activity. Even so, when recommending funding to reduce homicide rates, the cost of reversing climate change is something that Chicago alone cannot afford, making climate change out of the scope of this paper.

3. The number of opioid users in Chicago at a given month can be modeled by the number of opioid overdoses recorded in Illinois. Unfortunately, we were unable to find data corresponding specifically to Chicago’s opioid overdoses. However, the opioid epidemic is known to be a nationwide issue [35] so rates in Illinois are reflective of trends in Chicago. Likewise, the number of opioid overdoses recorded is directly proportional to the number of active opioid abusers.

4. The number of opioid overdoses in a given month can be derived from dividing the number of overdoses in the corresponding quarter by 3. Due to limitations in data, we assume that there are no fluctuations in the number of overdoses between months within a quarter, i.e. all three months in a quarter have the exact same number of opioid overdoses.

5. Police officers in the CPD are sworn in and serve under a full year contract. Because the CPD has only released police employment by year, we assume that the police department is staffed at a constant level throughout the year. While police officers entering and leaving the program is variable, we neglect these changes in the derivation of our function. Note: this is a generalization we will use for our symbolic regression ONLY.

4.2 Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_H$</td>
<td>Number of homicides in a given month</td>
</tr>
<tr>
<td>$N_O$</td>
<td>Number of opioid overdoses in a given month</td>
</tr>
<tr>
<td>$r$</td>
<td>Unemployment rate in a given month</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature in a given month (°C)</td>
</tr>
<tr>
<td>$N_P$</td>
<td>Police employment in the CPD</td>
</tr>
<tr>
<td>$N_G$</td>
<td>Number of gun sales in a given month</td>
</tr>
<tr>
<td>$m$</td>
<td>Months since December 2012 (Jan. 2013 corresponds to $m = 1$)</td>
</tr>
</tbody>
</table>

Table 1
4.3 Correlations between various parameters and homicide rates

Our model is derived through the method of *symbolic regression* as explained in [56]. In short, symbolic regression is a regression method used to derive complex correlations between numerous input values and an output value. An algorithm receives data involving the relevant and uses decision trees to evaluate the best fitting function to the data.

Symbolic regression is often slow, since a program will need to evaluate both possible structures for the function and actual parameters or constants involved. However, when comparing our data on the number of homicides and the other parameters outlined in *Assumption 1*, there is no easily visible correlation between the parameters. Our situation is further complicated by the fact that many parameters are involved in the number of homicides.

Figure 2: A graphical view of our data

From qualitative inspection of the graphs, we can formulate a few preliminary observations:
• Homicide trends do appear to be seasonal, oscillating between local minima in the winter and local maxima in the summer.

• We observe a spike in homicides in 2016 and 2020 that match to corresponding spikes in opioid use and unemployment.

• The spikes for opioid overdoses correspond to the opioid epidemic in 2016 [38], and unemployment is associated with the COVID-19 pandemic [17]

• We observe a decrease in $N_P$ in the years 2016 and 2020.

• An unusually large, but short-lived spike in the number of gun sales in 2016 is likely an outlier and will not be included in the data used for regression.

With our data, we then began running a symbolic regression in Heuristic Lab, a top environment for heuristic and evolutionary algorithms. As an end-user software, we needed to provide a ratio for training and testing sets and the number of decision tree generations. We used the ideal 80 : 20 testing to training ratio [52] and 10 decision tree generations to ensure that our model would be relatively simple while still being highly accurate. In addition, before running the symbolic regression, we shuffled the data to ensure that there would be no biases due to the order in which the data was considered. Below is the decision tree which maps how our symbolic regression achieved the final equation.

![Regression decision tree](image)

Figure 3: Regression decision tree

Below is a plot of our raw data (highlighted in red) and our regression (highlighted in blue), including the testing and training data sets.
With this data, we returned the following multi-variable regression equation:

\[
N_H = \left( 0.0695N_O - 0.0707 \right) (1.033)^T - 0.00952N_P + 144.21
\]  

(1)

The program returned an $R^2$ value of 0.64. Given the highly chaotic nature of the number of homicides, this model illustrates the correlations between our parameters to a high degree of accuracy.

Our regression matches qualitative trends indicated in past studies: higher homicide rates are associated with an increased use in opioids [24], higher unemployment rate [61], warmer temperatures [57], and fewer police officers [47]. As a result, we conclude that our model matches logical correlations between the given parameters.

Notably, the parameter $-0.00952N_P$ in our model predicts that for every 105 additional police officers in the CPD, the number of homicides will be reduced by 1 per month or 12 per year. Equivalently, this means adding $105/12 \approx 9$ police officers will reduce 1 homicide per year. This finding matches nearly exactly with Chaflin et al., who also found a linear dependence with 10 officers reducing 1 homicide per year.

An additional result was that gun sales were not regarded as a necessary parameter in our regression. Looking back at the data, this is expected since gun sales fluctuate without a clear pattern in the given years. While some studies have associated gun ownership with increasing homicide rates, our regression model concludes that the role of gun ownership can be neglected in the presence of our other parameters. We recommend that more detailed research be conducted to create a more holistic view of the role of gun ownership in the context of other parameters that affect homicide rates.

### 4.4 Predicting trends for 2022

After creating Eq. (1), we continue by projecting each of the four relevant parameters across the year of 2022. We use Eq. (1) and a Monte Carlo analysis to arrive at monthly homicide projections for 2022.
4.4.1 Opioid Overdoses

In 2016, opioid overdoses became an area of national attention [37], resulting in new policy changes being enacted in the US to reduce the frequency of these overdoses [39]. As a result, post-2016, the number of opioid overdoses has stayed relatively constant with some variation. Therefore, for opioid use, we can neglect all data before 2016 as opioid overdoses were increasing in frequency during these years. In addition, in 2020, opioid overdoses spike in May 2020, likely due to the COVID-19 pandemic [20]. However, as Chicago moves out of the pandemic, overdose numbers are returning to pre-pandemic levels, giving reason to neglect all 2020 data. With the data already provided, we manually find the mean and standard deviation of the data, giving a distribution of opioid overdoses per month with mean and standard deviation stated below.

<table>
<thead>
<tr>
<th>Mean</th>
<th>178 opioid overdoses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>22.816 opioid overdoses</td>
</tr>
</tbody>
</table>

We can use this distribution to generate projections of $N_O$. As $N_O$ stayed relatively constant with some variation, these projections will be randomly generated using the distribution, independent of month.

4.4.2 Unemployment Rate

The unemployment rate can be modeled with an exponential decay regression since it will be harder and harder to find jobs as the unemployment rate decreases. When processing the data, it strongly fits an exponential decay model, with unemployment gradually decreasing over the years. However, we notice that there is a large spike originating in 2020, but this spike was likely caused by the COVID-19 pandemic [62]. As Chicago adjusts to the post-pandemic life, we expect the overall employment rate to return to the same decay exhibited in earlier years. Hence, in our regression, we neglect the spike in unemployment due to COVID-19 and solely use past years’ data.

After performing an exponential fit on the monthly unemployment rates in MATLAB, we obtain the following graph.

![Figure 5: Monthly unemployment rate plotted against the curve fit](image)
From the exponential fit, we find that unemployment rate can be modeled by the equation

\[ r = a \cdot e^{bm}, \]  

with \( a = 0.09504 \) and \( b = -0.01243 \). We also obtain the 95% confidence intervals to be \( a \in (0.09215, 0.09794) \) and \( b \in (-0.01327, -0.01159) \).

### 4.4.3 Temperature

The temperature can be modeled with a sine curve [53]. Since we neglect climate change in Assumption 2, the parameters of this curve do not vary in time.

![Temperature data in Chicago](image)

Figure 6: Temperature regression curve

We obtain our equation

\[ T = 13.54 \sin(0.52m - 2.10) + 7.33 \]  

with \( R^2 = 0.99 \).

### 4.4.4 Police Employment

**Note:** In predicting next year’s police employment trends, we have more information available to us than for past years. Hence we will no longer assume police recruitment is by year as we did in our symbolic regression.

Retaining its employees has been a major issue for the CPD recently. In 2021, the department lost 900 officers while recruiting only 247 officers [67]. Resignations and retirements are owed to a lack of motivation for the job, so we predict about the same number of officers will resign in 2022—a reduction of 75 per month. We further see that in October 2021, about 12,100 officers were employed with a target of 13,176 officers to be reached under the CPD’s 2022 budget [5]. A new recruitment technique, employed this year, is hiring 60-70 new police officers approximately every 6 weeks, which translates to roughly 46 new recruits per month. Based on this information, we find that the CPD will lose about \( 75 - 46 = 29 \) police officers per month this year. Hence, we can set up an equation in point-slope form. Our point is (106,12100) since October 2021 is the 106th month after December 2012 and 12100 is the number of officers employed in this October
2021. Our slope is \(-29\) as the CPD is losing approximately 29 officers per month, as explained earlier. Thus, we have that \(N_P - 12100 = -29(m - 106)\), which gives

\[
N_P = 15174 - 29m
\]  
(4)

We can assume a standard deviation of 2.5 officers in the monthly decrease in \(N_P\), as the actual monthly decrease in \(N_P\) will rarely be exactly 29 officers. Thus, accounting for such variation in the expected monthly decrease in \(N_P\) allows for our model to consider a more realistic distribution of potential values of \(N_P\).

### 4.5 Results

Combining our regressions for all of the parameters listed in Assumption 1, we ran a Monte Carlo simulation with 10,000 trials that normally distributed our coefficients in each equation. Using a Monte Carlo simulation accounts for the stochastic nature of homicide rates, and also allows us to find estimates for the range of homicides we may expect in a given year.

The results we returned from the simulation are shown in the following graph:

![Figure 7: Results of our Monte Carlo simulation](image)

The mean number of predicted homicides is indicated in green, while estimated upper and lower bounds of two standard deviations away from the mean are indicated in red. Numerical results are shown in this table:

<table>
<thead>
<tr>
<th>Projection</th>
<th>Number of homicides</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu - 2\sigma)</td>
<td>436</td>
</tr>
<tr>
<td>(\mu)</td>
<td>458</td>
</tr>
<tr>
<td>(\mu + 2\sigma)</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 2: Predicted number of homicides for 2022
4.6 Sensitivity analysis

We now conduct a sensitivity analysis on the $N_O$, $r$, $T$, and $N_P$ used in our model by varying each by $\pm 10\%$. The effects on our value of $N_H$ for 2022 are shown below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>+10%</th>
<th>−10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_O$</td>
<td>+4.29%</td>
<td>−4.32%</td>
</tr>
<tr>
<td>$r$</td>
<td>+1.39%</td>
<td>−0.089%</td>
</tr>
<tr>
<td>$T$</td>
<td>+0.085%</td>
<td>−0.085%</td>
</tr>
<tr>
<td>$N_P$</td>
<td>−31.33%</td>
<td>+31.26%</td>
</tr>
</tbody>
</table>

Table 3: Effect of adjusting on $N_H$ in 2022

To find the impact that each parameter had on $N_H$, we reran the Monte Carlo simulation. In each of these simulations, we kept all parameters the same except for the given parameters above, which we individually modified. We repeated this process eight times, varying each of the four parameters by $\pm 10\%$.

We found that variations in $N_O$, $r$, and $T$ had minimal impact on our model, making our model resilient to changes in those variables. However, an abnormality occurs with variations in $N_P$ having a significant impact on our model’s projection. This can be explained by the police force having a direct, negative relation to monthly homicides in Eq. (1). In addition, because we only have access to yearly—not monthly—CPD employment data, our model may be prone to error in this regard.

4.7 Strengths

An evident strength of our model is its ability to take into account a complex relationship between relevant parameters to homicide rates. Unlike predicting climate change or crop growth, homicide rates are governed by human behavior and often impulsive decisions, not the laws of nature. As a result, our model is highly robust in its ability to predict a significant degree of accuracy ($R^2 = 0.64$) in a highly chaotic situation.

According to our sensitivity analysis, our model is resilient to changes in values for opioid use, unemployment rate, and temperature. In addition, our findings for the relationship between $N_H$ and $N_P$ align with previous findings [50].

Our results themselves also prove to be highly accurate. As of April 21, 2022, 158 people had been killed in Chicago this year [3], while we projected 132 homicides for the same time period (see Table 6 below). This means that we accurately predicted a decline in homicides in comparison to 2021, so our predictions for the remainder of 2022 are likely to be accurate as well.

4.8 Weaknesses

A weakness of our model was that we did not have sufficient data for our simulation. We could not find data for the specific distributions of data for $N_O$, $T$, and $N_P$, so we assumed a normally
distributed model for all of them with a standard deviation of 10% of the mean: $\sigma = 0.1\mu$. Consequently, our results for the upper and lower bounds ($\pm 2\sigma$) for the number of homicides may differ from reality.

The biggest weakness of our model is that a lack of data forced us to make assumptions about trends in $N_P$. As with many other police departments, the CPD limits the data it releases to the public, including specific police staffing data. Because we found that $N_H \propto N_P$, any error in this variable will have a significant impact on our results. While this estimation proves to be accurate for the first third of 2022—as demonstrated in the accuracy of our predictions—it may be prone to error as the year goes on and new policies are implemented by the CPD.

5 Risk analysis

5.1 Risk overview

Homicides are disastrous to the communities that they affect. The potential costs associated with a homicide not only include monetary costs like managing hospital fees, the criminal justice system, and economic productivity losses, but also intangible costs such as the pain and suffering imparted on loved ones. We seek to find the potential risk due to a multitude of effects of homicides.

We also seek to analyze the distribution of homicides to identify the times and places in which homicides occur most frequently, rank the months in order of homicide risk for a given year, and rank the districts of the city in order of homicide risk in a given year. To extend our analysis, we will compare the risks associated with homicides in 2022 with past years. We also extend our regression analysis to risks associated with homicides for future years (after 2022).

Our specific risk parameters for analysis remain the same: opioid overdoses, unemployment rate, temperature, and police employment in the CPD. Our goal for this section is to determine which of these four risk factors are the most severe towards causing homicide and therefore warrant the most resources. We use this ultimately to guide our recommendations in the next section.

5.2 Expected costs of homicides

To establish the severity of projected Chicago homicides in 2022, we associate monetary costs to each homicide. According to McCollister et al., the tangible costs of one homicide (including criminal justice system costs and economic productivity losses) is $1,278,424. Using our predictions from Section 4.5, we project the tangible cost of Chicago homicides in 2022 to be:

<table>
<thead>
<tr>
<th>Projection</th>
<th>Tangible Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu - 2\sigma$</td>
<td>$557,392,864$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$585,518,192$</td>
</tr>
<tr>
<td>$\mu + 2\sigma$</td>
<td>$613,543,520$</td>
</tr>
</tbody>
</table>

Table 4: Predicted tangible cost of homicides for 2022
However, homicides also entail intangible costs in the form of the distress and decreased quality of life that loved ones experience. McCollister et al. found these intangible costs to total $8,442,000 per homicide [15], so the total cost of a homicide can be estimated as $9,720,424. Thus, we project the total cost of homicides in 2022 to be:

<table>
<thead>
<tr>
<th>Projection</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu - 2\sigma$</td>
<td>$4,238,104,864$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$4,451,954,192$</td>
</tr>
<tr>
<td>$\mu + 2\sigma$</td>
<td>$4,665,803,520$</td>
</tr>
</tbody>
</table>

Table 5: Predicted total cost of homicides for 2022

5.3 Distributions

We characterize the distribution of risk based on time and location.

5.3.1 Months of highest risk

We first determine which months of the year are most severe and least severe in terms of homicide risk. This is obtained by modifying our Monte Carlo simulation to output monthly projections for homicide instead of yearly projections.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homicides</td>
<td>32</td>
<td>32</td>
<td>33</td>
<td>35</td>
<td>38</td>
<td>42</td>
<td>44</td>
<td>45</td>
<td>43</td>
<td>40</td>
<td>38</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 6: 2022 monthly homicide projections

We find that homicide in Chicago increases from January onward, peaks during August, and decreases from August onward. Thus, our model suggests that month has an effect on total homicides. Homicides in months such as January or December are noticeably lower than homicide in months such as July or August. This suggests that perhaps temperature is a driving factor behind homicide, and is also consistent with the idea that colder weather is a deterrent for crime [14]. However, there still is a general increase in homicide as the year goes on regardless of month or temperature. The projected homicides in December 2022 is higher than the projected homicides of all months in 2022 up to April. This confirms that along with temperature, the development of other factors are also important towards homicide risk.

5.3.2 Locations of highest risk

Although we have found the characteristic risk for Chicago as a whole, homicide rates differ between regions of the city. For example, the most affluent of neighborhoods are expected to have low opioid overdose levels and low unemployment, while those on the outskirts of the city see drastically higher levels of opioid overdoses and unemployment. As a result, we seek to find the regions of Chicago that are most at risk of homicides.

In our analysis, we focus on the term $\alpha = 0.0695N_O - 0.0707/r$ in Eq. (1). Temperature changes within Chicago are negligible, and the distribution of police stations is approximately uniform.
according to population density [46]. Hence, we rank regions of Chicago according to the value of \( \alpha \) in each region; higher levels of \( \alpha \) indicate greater risk.

We will need to adjust \( \alpha \), however, because the population in each region varies—a high value of \( N_O \) may reflect a greater population in a given area, but not a high frequency of opioid overdoses. Hence, we assign a new value \( n_O \) to indicate the number of overdose deaths per 100,000 residents. Given our value for \( N_O \) reflects the entirety of Illinois with population \( 12.6 \times 10^6 \), this yields

\[
\alpha = 0.0695n_O \frac{12.6 \times 10^6}{10^5} - \frac{0.0707}{r} = \frac{8.757n_O - 0.0707}{r} \tag{5}
\]

We derived \( n_O \) from opioid data in 2019 [36]. As in Section 4.4.1, opioid trends have been relatively constant since 2016, so we can use this database for present values. Current values of \( r \) can be found from [60]. Calculating \( \alpha \) for each community, we return the following map:

![Map of the 77 communities with value of \( \alpha \) indicated](image)

Figure 8: Map of the 77 communities with value of \( \alpha \) indicated

In the following table we note the 3 communities with the highest and lowest risk:

<table>
<thead>
<tr>
<th>Community</th>
<th>( \alpha )</th>
<th>Risk ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>East Garfield Park</td>
<td>81.9</td>
<td>1</td>
</tr>
<tr>
<td>North Lawndale</td>
<td>68.2</td>
<td>2</td>
</tr>
<tr>
<td>Fuller Park</td>
<td>67.4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7: Highest risk communities

<table>
<thead>
<tr>
<th>Community</th>
<th>( \alpha )</th>
<th>Risk ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyde Park</td>
<td>-0.842</td>
<td>75</td>
</tr>
<tr>
<td>Beverly Hills</td>
<td>-0.884</td>
<td>76</td>
</tr>
<tr>
<td>Forest Glen</td>
<td>-1.04</td>
<td>77</td>
</tr>
</tbody>
</table>
Table 8: Lowest risk communities

From qualitative examination of the map, we also see that the areas of lowest risk tend to be in the north side and downtown, while the areas of highest risk are in the west side and the south side.

5.4 Trends

We refer back to our initial models to determine how each of the four risk factors develop over time. To begin with, our model assumes the overall change in temperature from year to year to be negligible (temperature still fluctuates within a year). Accordingly, our trend analysis only extends to the three risk factors of opioid overdoses, unemployment rate, and police employment.

Across all three of these risk factors, we first look to explain any data points that don’t fit in with the general projections of our model. Both opioid overdoses and unemployment rate sharply spiked during COVID (2020) while police employment decreased correspondingly. However, apart from the 2020 region, all other data points seem to generally fit in with the predictions of their associated model. Now, it remains to project the future trends of the three risk factors. As mentioned earlier, opioid overdoses were declared to be a national public health emergency in 2017 and a five-point strategy was announced to reduce opioid overdoses nationwide [27]. These efforts are reflected in the graph of Chicago opioid overdoses; overdoses steadily increased up to around 2017 and then remained relatively constant with the exception of 2020 (COVID). From this, it seems that the national policies in combating opioid overdoses were effective and our model therefore predicts opioid overdoses to continue remaining constant.

Next, our model predicts unemployment rate to continue to decrease in an exponentially decaying fashion. This trend is clearly evident in Chicago’s unemployment rate graph. Unemployment has been steadily decreasing since 2013, again with the exception of 2020. Therefore, when we developed our exponential decay model for unemployment, we completely ignored the 2020 data. Although it is reasonable to expect unemployment rate to continue to decay exponentially in the long term, our model likely underpredicted the true unemployment rate during the years immediately following COVID. Thus, we realistically can expect unemployment to decrease, but at a less pronounced rate than our model predicts.

Finally, our model predicts police employment to decrease at a small, constant rate during 2022. Although the assumption of constant decline within the timespan of a year is reasonable, it is less reasonable to use the same linear equation to predict police employment in years past 2022 because the starting police employment varies from year to year. In addition, the starting employment seems relatively random based on the corresponding graph of CPD employees by year. We were only able to obtain our estimate for the starting police employment in 2022 by referencing the employment in October 2021. Using the same statistic to predict starting police employment beyond 2022 would likely result in an unrealistic prediction. In the end, although police employment is a significant factor towards reducing Chicago homicide risk, predicting its trend into further years is difficult without more time-relevant data.
Combining all the factors, our projection that opioid overdose remains constant and that unemployment rate decreases implies that homicides should gradually decrease in the future. However, the unpredictability of total police employment combined with the direct, negative relation police employment has with total homicide means that our projection of total homicide is also unpredictable past 2022.

It is worth noting, though, that the number of homicides in 2022 mirrors those of years just before COVID. Some of these years are noted below:

<table>
<thead>
<tr>
<th>Year</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2019</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homicides</td>
<td>431</td>
<td>429</td>
<td>502</td>
<td>506</td>
<td>458</td>
</tr>
</tbody>
</table>

Table 9: Past years with similar homicide rates to 2022 prediction

Thus, although the spike in homicides due to COVID will have diminished this year, reform is still necessary to continue lowering homicide rates—notably in continuing to combat the opioid epidemic.

6 Recommendations

We now propose recommendations for action that can be taken by the Chicago city government to help reduce homicide rates. These recommendations will focus on three relevant factors found in Eq. (1): opioid use, unemployment, and police staffing. All of these factors can be connected back to the effects of the COVID-19 pandemic; hence we also list out recommendations for how to strategically manage the pandemic to ease homicide rates. We ultimately found that effective policing is the most powerful method in reducing homicide rates.

Although we found temperature to be a relevant factor in Eq. (1), adjusting the climate to cooler levels is not a feasible nor economical strategy in terms of reducing homicides; estimates cite that it will cost over $50 trillion in global spending to prevent further global warming [30].

6.1 Opioids

With the exception of 2020, opioid overdoses in Chicago have remained relatively constant since 2016, when opioid misuse was declared to be a national priority. The five priorities in place were: improving access to prevention, treatment, and recovery services, targeting the availability and distribution of overdose-reversing drugs, strengthening public health data and collection, supporting research on addiction and pain, and advancing the practice of pain management [27].

Out of these five priorities, we recommend that Chicago invest resources particularly into improving access to prevention, treatment, and recovery services. Currently, Chicago has no sites for people to perform safe medical injections [33]. These sites are locations where individuals can enter and have their drugs tested for safety before receiving clean needles to administer injections. Supervisors are also present to assist in case of an overdose. We recommend Chicago to establish safe injection sites across the city, particularly in the regions of highest risk from Section 5.3.2 because they have
been found to be successful in mitigating opioid overdoses [54]. With a reduction in overdoses, we can mitigate the impulsive decision-making that results from opioid abuse and hence decrease the homicide rate.

6.2 Unemployment

When considering recommendations for reducing unemployment, it is important to note its role in our derived model. In Eq. (1), unemployment rate contributes to homicide rates with a factor $-0.0707/r$. Since $r$ hovers around the 5% mark, this factor can be approximated to have an average value of $-1.41$. However, this term is seen to add to the opioid term $0.0695N_O$, which, for values of $N_O$ in the 200 range, has an approximate average value of 13.9. The opioid term is seen to be an order of magnitude greater than that of unemployment. As a result, we can conclude that the effect of unemployment on homicide rates is not as relevant in the context of opioid use and our other factors. Nonetheless, steps can still be taken to reduce the unemployment rates (as outlined below) to facilitate a reduction in homicide rates.

The most direct method of dealing with staggering unemployment rates is employment subsidies. Encouraging companies to hire more employees gets rid of the risk of long-term unemployment for many residents and connects them with employers. Subsidized job programs can provide almost a 3000 dollar increase in annual wages for the workers within 4 years [64], which further incentivizes the unemployed to look for jobs. Though costly for the government, this is a definite way of providing welfare to communities hit hard by the pandemic in Chicago, which in turn can decrease the homicide rate.

Another way to increase employment is through increasing the standard of education within the city. Education has a direct relationship with one’s ability to find a job [25]. By investing in the Chicago Public Schools system, future generations will be better prepared for their jobs. Those who struggle with unemployment due to their education can now find an outlet to improve their conditions. This plan may be beneficial, but it is a long-term solution, so unemployment rates may drop years after this is enacted.

Lastly, the government can take a more proactive approach to unemployment by reducing the benefits available for those without jobs in Chicago. During the pandemic, the government provided over $804 billion dollars of aid in the form of [41] stimulus checks for the unemployed in order to lessen the financial impacts of COVID-19. However, as life starts to go back to normal, the government can potentially lessen some of the benefits to encourage the unemployed to actively seek out jobs.

6.3 Police

While measures must be taken to reduce Chicago’s unemployment rate, this must be done in tandem with increasing the size of the CPD in order to effectively bring down Chicago’s homicide rate. Furthermore, since increasing the size of Chicago’s police force would also reduce the number of opioid overdoses due to increased surveillance [48], this strategy can simultaneously have an
impact on other relevant factors in our model. As a result, police staffing proves to be the most overarching and impactful approach to resolving homicide rates.

In recent years, the total membership of the CPD has remained relatively constant, but in 2021, the burnout and fatigue that came with the COVID pandemic caused many officers to reconsider their employment in the CPD. As a result, the department’s membership saw a staggering drop of about 8.5% from 13,000 to 11,900 as of January 2022 [45] due to resignation or retirement. In a time when crime is spiking due to unemployment caused by the pandemic, such a drastic fall in policing has only made matters worse by further increasing violent crime, resulting in a vicious cycle that will continue to wreak havoc on vulnerable Chicagoland. To increase the number of police officers in Chicago, we recommend that the government provide incentives to join the CPD through higher wages for police officers, increase public appreciation of the CPD to increase police officers’ job satisfaction, and take measures to make officers feel safer in their jobs.

The most straightforward method of increasing employment in any job is to increase pay, and this practice holds true for police employment. As violent crime surges due to the pandemic, being a police officer is not nearly as attractive of a career as it once was. Every day, police officers are exposed to an array of hazards such as overexertion, violence, vehicle accidents, and hazardous chemicals, which can lead to injuries that are very expensive to cover and can result in long-term damage [49]. Evidently, officers face more risks in their job than almost any other profession, and these risks have only been magnified due to a rise in criminal activity. The only way to make this profession attractive to the public is by increasing the salary to at least partially compensate for the immeasurable risks that it entails.

In addition to increasing wages, the government must increase public appreciation for Chicago police officers for police employment to rise. According to a study conducted by the Pew Research Center, 86% of police officers think that the public doesn’t understand the risks they face and 67% of officers have been verbally abused in the past month [43]. To make matters worse, calls to defund the police have further demoralized officers by making them feel less wanted and necessary in the community. To combat this issue, the government should advertise the CPD similarly to how the army and navy are advertised. The latter two are portrayed as heroic and patriotic professions that display great bravery and devotion to one’s nation, while the former is stigmatized by high risk and little reward. The government should launch a media campaign to shift the public’s opinion on the courage and loyalty displayed by an individual in the CPD so that officers are more satisfied with their jobs and other citizens are more inclined to join the CPD.

Finally, in order for the number of CPD officers to rise, officers need to feel safer in their jobs. While attention has been paid to ensuring that the public feels less threatened by police officers through the implementation of body cameras and similar devices, little has been done to make officers feel safe when they are on duty. If officers are taught that their body cameras are not only there to protect the public but also to protect officers, then they will feel less antagonized and thus be less inclined to resort to violence at the first sign of danger. When officers feel that their safety is not a priority, they will be made to feel like criminals, and resignation rates will rise.
Following the above recommendations would result in a larger police force which would in turn cause a significant reduction in homicides in Chicago, which would overall be beneficial to the community. It now As per Eq. (1), we found that on average, an addition of 9 police officers to the CPD would reduce the yearly homicide count by 1. Given that the average cost of employing a police officer is roughly $149,362 [44], the cost of employing 9 police officers is approximately $1,344,258, and as we found earlier, the cost of 1 homicide is $1,278,424. Thus, the net tangible cost of reducing the homicide count by 1 is $65,834 per year. Although we evaluated the cost of employing 9 police officers to be higher than the tangible savings of 1 homicide, the relatively small difference of $65,834 indicates that homicide reduction directly neutralizes the cost of employing more police officers. Not to mention, police officers also work to reduce opioid use [48], an effect that may decrease or even eliminate the net tangible cost of $65,834. It is also important to note that police officers help combat other forms of crime that are costly to the city [12]. As a result, we recommend the City of Chicago to increase the CPD’s budget because it is not only a strong mechanism to ensure peace in the city but is also cost-effective for the government.

The data from the previous paragraph also allows us to find the amount of money that must be spent to reduce homicides by a given amount. If the city aspires to reduce homicide rates to pre-COVID and pre-opioid epidemic levels, we evaluate the cost needed to reduce homicide levels to roughly those in 2014: 429. So, to get the homicide counts down from 458, our projection for 2022, to 429, the government would have to spend roughly $38,983,482 employing police officers (assuming other factors are constant) per year.

### 6.4 COVID-19

Mitigating COVID-19 risk reduces the effect of the previous 3 parameters, and thus reduces risk of homicide as well. Unemployment spiked during the COVID-19 pandemic to 14.8% [18]. As such, all the recommendations regarding unemployment also apply to COVID as a means of reducing its effect on homicide rates. Furthermore, people were more susceptible to drug abuse during the COVID-19 pandemic [16] due to pandemic-induced mental health issues. So, policy recommendations regarding COVID-19 in turn can improve the situation regarding opioid abuse, and thus, homicide.

As a result, recovering from the COVID-19 pandemic will prove to accelerate a reduction in homicide rates. Strategies to accomplish this include implementing citywide policies such as universal mask-wearing, social distancing, and accessible testing [7]. However, retaliation against mask-wearing and other COVID prevention mandates can manifest themselves in a violent fashion [21]. Although it is rare such retaliation becomes violent, and even more rare that it results in a homicide, it is worth the consideration of the city government as to whether or not retaliation to mandates can create an unsafe city environment. Overall, we recommend that if cases rise to pandemic levels sometime in the future, the city government should implement policies such as universal mask-wearing not only to reduce homicide rates, but infection rates as well.

In addition, the COVID-19 pandemic increases stress levels and thus criminal motivation [23]. We
recommend that the city of Chicago subsidizes the therapy industry to improve the mental health of Chicagoans. 75% of people who went to therapy found benefit in it [58]. As such, subsidizing therapy would make it more accessible. Furthermore, the city of Chicago can implement various urban design strategies to inhibit the effect of mental health issues induced by COVID. Cities with more bike lanes and better access to public transportation often have higher mental health levels [63]. While redesigning the city of Chicago even slightly will be extremely costly—as a mile of bike line can cost $125,000 per mile [4]—we recommend that the city of Chicago explores urban redesign options that are empirically proven to improve mental health. If this is done, then the people of Chicago will be less susceptible to rash decision-making, lowering homicide rates.

7 Conclusion

Overall, our model predicted the number of homicides in Chicago in 2022, as well as the risk that homicides bring to the well-being of all Chicago residents and to governmental agencies. We analyzed the risks of homicides in specific months and districts of Chicago, and provided recommendations for the Chicago city government to incorporate in order to lower the homicide rate. We ultimately concluded that increasing the size of the CPD is the highest priority for the Chicago government to reduce homicide levels, while measures should also be taken to reduce opioid use and unemployment, two other primary causes of homicide. We decided that increasing the size of the CPD will be the most effective strategy because along with reducing crime in general, it will directly decrease opioid use, which will result in further decreases in homicide rates.
Acknowledgements

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We also thank our math team coach, Paul Kim, for sticking with us throughout our high school journey and throughout this competition.

Lastly, we thank The Actuarial Foundation and The Institute of Competition Sciences for organizing the Modeling the Future Competition and providing essential resources that have helped us throughout the derivation of our model.

References


[40] Opioid data dashboard, . URL http://idph.illinois.gov/opioiddatadashboard/.


[52] Why 70/30 or 80/20 relation between training and testing sets: A pedagogical explanation, . URL https://scholarworks.utep.edu/cs_techrep/1209/.


Code

Sorting CPD Crime Data

```matlab
% reading excel file of 2020 crimes
readtable Crimes_2020.xlsx

% iterates through number of crime datapoints
for i=1:211556
    if (type(i) == 'HOMICIDE')
        if (month(date(i))==1)
            a(1) = a(1)+1;
        end
        if (month(date(i))==2)
            a(2) = a(2)+1;
        end
        if (month(date(i))==3)
            a(3) = a(3)+1;
        end
        if (month(date(i))==4)
            a(4) = a(4)+1;
        end
        if (month(date(i))==5)
            a(5) = a(5)+1;
        end
        if (month(date(i))==6)
            a(6) = a(6)+1;
        end
        if (month(date(i))==7)
            a(7) = a(7)+1;
        end
        if (month(date(i))==8)
            a(8) = a(8)+1;
        end
        if (month(date(i))==9)
            a(9) = a(9)+1;
        end
        if (month(date(i))==10)
            a(10) = a(10)+1;
        end
        if (month(date(i))==11)
            a(11) = a(11)+1;
        end
        if (month(date(i))==12)
            a(12) = a(12)+1;
        end
    end
end

for i=1:12
    disp(a(i))
end
```
Monte Carlo simulation in Section 4.5

```matlab
1 % We first create an empty list to store simulation results
2 chi = []; % Running simulation 10000 times
3 for j = 1:10000
4     % Normally distributing a from Eq. (2)
5     a = normrnd(0.09504, 0.00145);
6     % Normally distributing b from Eq. (2)
7     b = normrnd(-0.01243, 0.00042);
8     homicide = 0;
9     % Simulating in 2022 (months 85 to 96) given month 1 is January 2013
10    for i = 85:96
11        % Deriving values with respective normal distributions
12        projected_unemployment = a*exp(b*i);
13        opioid = normrnd(178, 22.816);
14        temp = 13.54*sin(0.52*i-2.10) + 7.33;
15        sd_p = normrnd(29, 2.5);
16        police = 15174 - sd_p*i;
17        % Add homicide count in each month
18        homicide = homicide + (0.0695*opioid - 0.0707/projected_unemployment)
19            *1.033^temp - 0.00952*police + 144.21;
20    end
21 % Adding simulation results to list
22 chi = [chi, homicide];
23 end
24 % Plotting our results in a histogram
25 histogram(chi)
26 xlabel("Homicides")
27 ylabel("Frequency")
28 title("Homicides in Chicago 2022")
29 mu = mean(chi)
30 sigma = std(chi);
31 worst = mu + 2*sigma
32 best = mu - 2*sigma
33 xline(mu, 'Color', 'g', 'LineWidth', 2);
34 xline(mu - 2*sigma, 'Color', 'r', 'LineWidth', 2);
35 xline(mu + 2*sigma, 'Color', 'r', 'LineWidth', 2);
```