

Scenario Modeling Module

MODULE INTRODUCTION

Welcome to the Scenario Modeling Module.

Throughout your actuarial studies you have learned about the importance of modeling. Entire modules of Fundamentals of Actuarial Practice demonstrate how modeling, which James C. H. Anderson called "a delightful mixture of science and art," is used to help actuaries complete crucial activities throughout the Actuarial Control Cycle.

With that as a backdrop, it is not a surprise that modeling economic scenarios is crucial to those on the Quantitative Finance and Investment (QFI) track. Well-trained candidates are able to define, build, and maintain models. The better they are able to account for external forces and model constraints, the more effective they will be as actuaries.

The focus of this module is to provide you with extensive practice building and running the types of economic scenarios you may be required to create as an actuary. You will deploy RStudio throughout the module to build simulations, calibrate and validate stochastic models, and apply variance reduction techniques. Throughout this module you will see many R functions and you will be asked to try to create them first before they are revealed to you.

The first five sections will explain the purpose of an Economic Scenario Generator (ESG), i.e. to generate stochastic scenarios. In these scenario simulations you will see how the random numbers generated from Monte Carlo and Lattice methods give rise to the modeled economic components, e.g. equity indices and interest rates. You will see that you need a "model" to turn a given random number sequence into a useful path of economic values stretching into the future. These types of models consist of dynamic formulas and parameters, which you will learn how to determine in Section 6: Fittings and Section 7: Calibrations.

Section 8: Validations will help you to understand whether the generated scenarios serve their intended purpose. Section 9: Variance Reduction will help you reduce the number of scenarios to run while still serving your purposes. Section 10: Nested Simulations will explain how to simulate scenarios within scenarios.

We hope you find this module interesting and applicable as you continue toward your actuarial fellowship.



Module Learning Objectives

After completing this module, you should be able to:

- Explain the difference between risk-neutral and real-world scenarios and deterministic and stochastic scenarios.
- Translate mathematic formulas into stochastic simulation functions in R.
- Build simulation models for equity returns, interest rate movements, and inflation.
- Construct a regime-switching lognormal model for equity returns.
- Construct Vasicek and Cox-Ingersoll-Ross (CIR) models for interest rates and inflation.
- Use analytic plotting functions for visualization in fitting distributions.
- Calibrate simulation models using Maximum Likelihood Estimation and Non-Linear Minimization techniques.
- Validate simulated scenarios for both risk-neutral and real-world measures.
- Apply variance reduction techniques in Monte Carlo simulations to reduce runtimes and improve accuracy.

Module Sections

The Scenario Modeling module consists of eleven sections:

Section 1: Overview

- Section 2: Scenarios
- Section 3: Simulations
- Section 4: Lattices
- Section 5: Deflators
- Section 6: Fittings
- Section 7: Calibrations
- Section 8: Validations
- Section 9: Variance Reduction
- Section 10: Nested Simulations
- Section 11: Applications

In addition, this module contains an End-of-Module Test and an End-of-Module Exercise.



SECTION 1: OVERVIEW

In this section, you will learn about how \underline{R} or $\underline{RStudio}$ required to complete this module. The readings and organization of the module is also listed.



SECTION 2: SCENARIOS

Introduction

In this section you will learn about the basic features of scenarios and the differences between risk-neutral scenarios and real-world scenarios.

Learning Objectives

- Learn the basic features of the scenarios that you will build.
- Understand the big picture differences between real-world and risk-neutral scenarios.
- Practice simple hands-on activities to future enhance your understanding in the differences between riskneutral and real-world scenarios.
- Establish the groundwork to facilitate building scenarios in the rest of the module.



SECTION 3: SIMULATIONS

Introduction

Analytical solutions used to be the standard methodology for actuarial and risk management analysis. With the increased complexity of models, analytical approaches are now less effective. Simulation models have gained popularity, enabled by advancing of technology and computing power.

Learning Objectives

- Learn the key building blocks of Monte Carlo simulation.
- Learn popular simulation models of lognormal and regime-switching lognormal design for equity return simulations and Vasicek and Cox-Ingersoll-Ross (CIR) for interest rate simulations.
- Gain experience in translating theoretic formulas of these models into R codes.
- Learn how to create markdown reports in R.
- Practice hands-on activities in writing R code for building these models, running them, and having their results validated.
- Gain familiarity with these simulation models for both equity returns and interest rates.



SECTION 4: LATTICES

Introduction

While Monte Carlo simulation is a powerful technique in financial modeling there are certain times it struggles, such as in the valuation of exotic options that involve path dependency or early exercise. In these situations an alternative technique is used called lattices.

Learning Objectives

- Learn the fundamental theories about lattice trees.
- Practice hands-on activities in writing R code for building binomial trees and plotting them for visualization purposes.
- Practice hands-on activities to gain familiarity with available R functions for building binomial trees.
- Gain understanding of the connection between Monte Carlo simulated scenarios and lattice's binomial trees.
- Practice hands-on activities in building the connection between Monte Carlo simulations and lattice's binomial trees.



SECTION 5: DEFLATORS

Introduction

Deflators are the bridge between risk-neutral and real-world valuations. For real-world valuation, the scenarios must represent where the world is heading based on some historical data and subject-matter expert's opinions. For risk-neutral valuation, the scenarios must be capable of reproducing the real current prices.

Specifically and practically, deflators are introduced to make the real-world scenarios market consistent.

Learning Objectives

- Learn the mathematical definition of deflators.
- Practice hands-on activities in writing R code for implementing deflators for a binomial tree situation and validating the implementation.
- Practice hands-on activities in writing R code for implementing deflators for a Monte Carlo situation and validating the implementation.



SECTION 6: FITTINGS

Introduction

Fitting distributions consists in finding a mathematical function that represents in a good way a statistical variable. You learned about distributions of uniform, normal, gamma, etc. during your undergraduate studies. What you are doing in the real world is that you are trying to fit observed data into one of these distributions, so that you can estimate the likelihood the data is actually distributed that way.

Specifically, fitting is the technique that helps to reveal analytic details of observed data for choosing a model for simulations.

Learning Objectives

- Learn the fundamental theories and approaches for fitting distributions to observed data.
- Practice hands-on activities in writing R code with analytic plotting functions available in R for rendering visualization of observed data.
- Gain understanding and experiences in translating logarithmic likelihood functions for the use of maximum likelihood functions in estimating model parameters.
- Practice hands-on activities in writing R code for approaches such as matching of moments and maximum likelihood estimation in estimating model parameters.
- Apply the maximum likelihood estimation function you will build in this section to actual historical data.



SECTION 7: CALIBRATIONS

Introduction

Calibrations goes beyond the fitting of distributions to focus on solving for the parameters of models chosen through fitting analyses. The techniques are materially different between risk-neutral and real-world valuations because one is to designed to reproduce market prices and the other is to create hypothetical evolutions of history.

Learning Objectives

- Learn the fundamental theories and practical approaches in solving for model parameters.
- Practice hands-on activities in writing R code for constructing logarithmic functions of various simulation models, i.e. lognormal and regime-switching lognormal models for equity returns and Vasicek and CIR models for interest rates.
- Learn how to use the Maximum Likelihood Estimation (MLE) function and the Non-Linear Minimalization (NLM) function in R to solve for model parameters in both risk-neutral and real-world measures.
- Apply these functions to real observed data for both equity returns and interest rates.
- Understand how to implement a process of bootstrapping to calibrate interest rate models.



SECTION 8: VALIDATIONS

Introduction

Validation is used to verify that the model is behaving in the intended manner. Effective validation of an ESG requires comparing simulated output data with some predefined benchmark of acceptance criteria, which are completely different between risk-neutral and real-world measures.

Learning Objectives

- Learn the fundamental theories and practical approaches in validating simulated data for both riskneutral and real-world measures.
- Practice hands-on activities in writing R code for validating simulated scenarios under the real-world measure on both ex-post and ex-ante data for both lognormal and regime-switching lognormal models.
- Practice hands-on activities in writing R code for validating simulated scenarios under risk-neutral measure with a Martingale testing approach.



SECTION 9: VARIANCE REDUCTIONS

Introduction

Although Monte-Carlo simulation is a powerful tool, even with modern computers, it can become cumbersome when there is a need to run millions or hundreds of millions of scenarios to fully realize a result. Variance reduction techniques are meant to help to improve model convergence speed, reduce runtime and associated costs, and lower estimation variance.

Learning Objectives

- Learn the theories of four popular variance reduction techniques: antithetic variates, control variates, stratified sampling, and importance sampling.
- Practice hands-on activities in writing R code for building these variance reduction techniques and experience possible issues in implementing them.
- Understand what each of these techniques can do to improve model convergence speed, reduce runtime, and lower estimation variance when compared to a pure Monte Carlo simulation.



SECTION 10: NESTED SIMULATIONS

Introduction

Stochastic simulation is a complex mathematical process that uses a combination of probability and random variables to forecast financial performance, or in the case of reserve setting, to forecast financial requirements. "Nested" simulations, as the name implies, are stochastic simulations inside of other simulations.

Learning Objectives

- Learn fundamental theories about nested simulations, mainly stochastic simulations in stochastic simulations.
- Practice hands-on activities in writing R code to construct nested simulations with practical approaches in combining inner and outer scenarios.
- Understand and experience ways of validating simulated nested stochastic scenarios.



SECTION 11: APPLICATIONS

Introduction

Given your new understanding of the fundamental theories and your experience with the hands-on activities from Section 2 to 10 of this module, this section offers opportunities for you to further familiarize your understanding and further enhance your experience through several case studies.

Learning Objectives

- Apply the understanding and the experience you gained from Section 2 to 10 of this module to the case studies in this section.
- Utilize the R code you have built from Section 2 to 10 of this module to create a markdown report for each of the case studies.