SOCIETY OF ACTUARIES®

CURATED PAST EXAM ITEMS
- Solutions -

GI 301 – Further Topics in General Insurance

Important Information:

- These curated past exam items are intended to allow candidates to focus on past SOA fellowship assessments. These items are organized by topic and learning objective with relevant learning outcomes, source materials, and candidate commentary identified. We have included items that are relevant in the new course structure, and where feasible we have made updates to questions to make them relevant.
- Where an item applies to multiple learning objectives, it has been placed under each applicable learning objective.
- Candidate solutions other than those presented in this material, if appropriate for the context, could receive full marks. For interpretation items, solutions presented in these documents are not necessarily the only valid solutions.
- Learning Outcome Statements and supporting syllabus materials may have changed since each exam was administered. New assessment items are developed from the current Learning Outcome Statements and syllabus materials. The inclusion in these curated past exam questions of material that is no longer current does not bring such material into scope for current assessments.
- Thus, while we have made our best effort and conducted multiple reviews, alignment with the current system or choice of classification may not be perfect. Candidates with questions or ideas for improvement may reach out to education@soa.org. We expect to make updates annually.

GI 301 Learning Objective 1 Curated Past Exam Solutions

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Message to Candidates

While the learning objectives and outcomes for Topic 1 have not materially changed since 2020, starting with the Fall 2025 administration of GI 301 key readings for this topic have changed. The unchanged reading is Clark's LDF curve fitting paper. Questions based on that paper are number 3 in the prior questions presented here.

Question 4 in each prior exam is based on readings from papers by Mack and Venter. Many of the methods and formulas from those papers are also in the required readings for GI 301. For those items we have indicated parts that are no longer applicable (noting that there may be similar concepts in the current readings), but have retained the full item for continuity. The current papers use different notation, but the translation should be obvious.

It is also important to note that the current readings include topics not covered in Mack and Venter and hence the prior exams are not sufficient preparation.

GIADV, Fall 2020, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) through (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) through (d) are for explanatory purposes only. In Excel, the candidate was provided with data from the question in a tabular format. This table also included four columns with missing entries. These columns were labeled and were to be completed for the responses to parts (b) through (d).

Solution:

(a) Explain why this should not be a cause for concern.

Commentary on Question:

There were two reasons cited by Clark. Only one of these reasons was required for full credit. The model solution provides both reasons.

- The scale factor is generally small compared to the mean, so little precision is lost.
- The use of a discrete distribution allows for a mass point at zero, representing the cases in which no change in loss is seen in a given development increment.

(b) Calculate the value of ℓ at its maximum.

Commentary on Question:

The candidate was to complete the column in the Excel table labeled ℓ (cells I9 to I18). The sum of this column is the maximum value.

In order to complete this column, first complete the values in the column labeled "Expected increment", x (cells H9 to H18).

For each row in the table, calculate the expected increment x as onlevel premium times the ELR times the difference between G at the beginning of the interval and G at the end of the interval, where G is the CDF of the loglogistic distribution.

Each row of the ℓ column is the increment (in column E) times the natural logarithm of the expected increment x (calculated in column H) minus the expected increment x.

The sum of column I in the table, labeled ℓ, is equal to 169,574.4397. This the value of ℓ at its maximum.

(c) Estimate the scale factor, σ^2.

Commentary on Question:

The candidate was to complete the column in the Excel table labeled σ^2 (cells J9 to J18). The sum of this column divided by 7 is the scale factor.

In order to complete this column, first complete the values in the column for expected increment x (cells H9 to H18). This was completed for the response to part (b).

For each row in the table, calculate the amount in column J as the square of the difference between the increment (column E) and the expected increment (column H), divided by the expected increment.

The sum of column J, labeled σ^2, divided by 7 is equal to 43.5880387. The value of 7 is the number of rows (10) less the number of estimated parameters (3). This is the value of the scale factor σ^2.

(d) Create a scatter plot in which the x values are the expected incremental losses and the y values are the normalized residuals.

Commentary on Question:

The scatter plot was already set up to plot x, the expected increment, against y, the normalized residual. The expected increment column was completed for part (b). For this part, the candidate needed to complete the column for y, the normalized residual (column K). The scatter plot would be automatically created.

For each row in the table, calculate the normalized residual y as (the increment in column E minus the expected increment in column H) divided by the square root of the scale factor, σ^2, times the expected increment.

The scatter plot was automatically created (below row 40) using the values in columns H and K in the table.

(e) Interpret the scatter plot in part (d) with regard to determining if the model assumptions are correct.

Commentary on Question:
The model solution is an example of a full credit solution. It assumes that the x and y values were calculated correctly.

The residuals should be random about the zero line. That appears to be the case, providing evidence to support the assumptions.

GIADV, Fall 2020, Q4

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) through (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) through (e) are for explanatory purposes only. In Excel, the candidate was provided with data from the question with response cells color coded as indicated in the Excel version of the question, parts (b) through (e).

Solution:

(a) State the three statistical assumptions underlying the chain ladder model.

- The conditional expected accumulated total claims amount at a given development year is the accumulated total claims amount at the previous development year times a development factor that does not vary by accident year.
- The accumulated total claims amounts of different accident years are independent.
- The conditional variance of the accumulated total claims amount at a given development year is the accumulated total claims amount at the previous development year times a proportionality constant that does not vary by accident year.

(b) Complete the triangle of age-to-age factors.

Commentary on Question:

The color-coded cells from C32 to H37 were indicated as the response cells for part (b). Cells C32 to C37 were already prefilled with factors.

The factors are calculated for each accident year as the paid claims at development year *x*+1 divided by the paid claims at development year *x*. The values are as follows:

Age-to-Age Factors

	1	1.650	1.319	1.082	1.147	1.195	1.113
	2	40.425	1.259	1.977	1.292	1.132	
	3	2.637	1.543	1.163	1.161		
(b)	4	2.043	1.364	1.349			
	5	8.759	1.656				
	6	4.260					

(c) Calculate the remaining values of f_k and α_k^2.

Commentary on Question:
The color-coded cells from C39 to H40 were indicated as the response cells for part (c). Cells C39 and C40 were already prefilled with values.

The f_k values are calculated as follows:

- For $k = 2$, it equals the sum of paid claims at development year 3 divided by the sum of paid claims at development year 2, for accident years 1 through 5.
- For $k = 3$, it equals the sum of paid claims at development year 4 divided by the sum of paid claims at development year 3, for accident years 1 through 4.
- The pattern continues for $k = 4$ through 6.

The α_k^2 values are calculated as follows for $k = 2$ to 5:

$$\alpha_k^2 = \frac{1}{7-k-1}\sum_{j=1}^{7-k} c_{j,k}\left(\frac{c_{j,k+1}}{c_{j,k}} - f_k\right)^2$$

where the $c_{j,k}$ are the paid losses in which j is the accident year and k is the development year.

For $k = 6$, $\alpha_6^2 = \dfrac{\left(\alpha_5^2\right)^2}{\alpha_4^2}$.

The values are as follows:

(c)	f_k	2.925	1.448	1.303	1.193	1.163	1.113
	α_k^2	40,350	216	1,094	73	27	10

(d) Square the development triangle by completing the remaining shaded cells, where one calculated value is provided.

Commentary on Question:
The color-coded cells from D25 to I30 were indicated as the response cells for part (d). Cell D30 was already prefilled with the value for $c_{7,2}$.

The remaining c values to square the development triangle are calculated as follows:

$$c_{j,k} = c_{j,k-1} \times f_{k-1}$$

The completed squared triangle is as follows:

	AY	1	2	3	4	5	6	7
	1	5,012	8,269	10,907	11,805	13,539	16,181	18,009
	2	106	4,285	5,396	10,666	13,782	15,599	17,361
	3	3,410	8,992	13,873	16,141	18,735	21,793	24,255
(d)	4	5,655	11,555	15,766	21,266	25,366	29,506	32,839
	5	1,092	9,565	15,836	20,640	24,619	28,637	31,872
	6	1,513	6,445	9,332	12,163	14,508	16,875	18,782
	7	557	1,629	2,359	3,074	3,667	4,265	4,747

(e) Calculate the remaining standard errors of the reserve estimators for the individual accident years.

Commentary on Question:
The color-coded cells from J24 to J30 were indicated as the response cells for part (e). Cells J24, J25, J28, J29 and J30 were already prefilled with the standard error (SE) values. The candidate was required to fill in cells J26 (SE for AY 3) and J27 (SE for AY 4).

$$SE_{AY3} = \sqrt{c_{3,7}^2 \times \left(\frac{\alpha_5^2}{f_5^2} \times \left(\frac{1}{c_{3,5}} + \frac{1}{c_{1,5}+c_{2,5}} \right) + \frac{\alpha_6^2}{f_6^2} \times \left(\frac{1}{c_{3,6}} + \frac{1}{c_{1,6}} \right) \right)} = 1,262$$

$$SE_{AY4} = \sqrt{c_{4,7}^2 \times \left(\frac{\alpha_4^2}{f_4^2} \times \left(\frac{1}{c_{4,4}} + \frac{1}{c_{1,4}+c_{2,4}+c_{3,4}} \right) + \frac{\alpha_5^2}{f_5^2} \times \left(\frac{1}{c_{4,5}} + \frac{1}{c_{1,5}+c_{2,5}} \right) + \frac{\alpha_6^2}{f_6^2} \times \left(\frac{1}{c_{4,6}} + \frac{1}{c_{1,6}} \right) \right)} = 2,562$$

(f) Describe how expected future emergence differs between the two models.

The chain ladder model assumes that expected future emergence for an accident year is proportional to losses emerged to date. The parameterized BF model assumes that expected future emergence for an accident year is proportional to expected ultimate losses.

GIADV, Spring 2021, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Estimate the scale factor, σ^2.

Commentary on Question:

The table of data included two columns with values for the cumulative distribution function, G, at the beginning and ending of the interval for each row (Col J and Col K). CDF values were required to calculate the scale factor. The table of data also included two empty columns, Col H for sigma-squared and Col I for the expected increment. The amounts for the expected increment were required to calculate sigma-squared in Col H.

For each row in the table, the expected increment (Col I) is calculated as the onlevel premium times the ELR times [G at the end of the interval minus G at the beginning of the interval].

Then, for each row, sigma-squared (Col H) is calculated as the square of the difference between the increment and the expected increment divided by the expected increment.

The sum of Col H, sigma-squared, divided by 8 is equal to 105.066236. This is the value of the scale factor. Note that the value of 8 in the formula is the number of rows (10) less the number of estimated parameters (2).

(b) Estimate the process standard deviation of the loss reserve for all accident years combined.

For each accident year, we need to compute the loss reserve at the end of calendar year 2020. The loss reserve is the onlevel premium for the year times the ELR times [1 minus G at the end of the interval].

For accident year 2017, the end of the interval is 48 months, for accident year 2018, the end of the interval is 36 months, and so on. The loss reserve for all accident years combined is 5,730.13.

The process standard deviation of the loss reserve for all accident years combined is 775.91. This is equal to the square root of [the loss reserve for all accident years combined times the scale factor].

(c) Estimate the expected loss for 2022.

Expected loss = expected premium times ELR.
$20{,}000 \times 0.5424 = 10{,}848$.

(d) Estimate the coefficient of variation due to process variance for the 2022 loss.

The coefficient of variation due to process variance for the 2022 loss is the standard deviation due to process variance for the 2022 loss divided by the expected loss for 2022.

The standard deviation due to process variance for the 2022 loss is 1,067.59. This is the square root of [the expected loss for 2022 times the scale factor].

The coefficient of variation due to process variance for the 2022 loss is 1,067.59 / 10,848 = 0.0984.

(e) Estimate the coefficient of variation due to parameter variance for the 2022 loss.

The coefficient of variation due to parameter variance for the 2022 loss is the standard deviation of the ELR divided by the ELR.

The standard deviation of the ELR is the square root of the variance of the ELR which is 0.03834 (= 0.00147^0.5).

The coefficient of variation due to parameter variance for the 2022 loss is 0.07069 (= 0.03834 / 0.5424).

GIADV, Fall 2021, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the maximum likelihood estimate of ultimate losses (*ULT*) for each of the four accident years.

For each accident year, the ultimate is the accident year total loss divided by the cumulative distribution function value at year-end 2020.

ULT2017 = 5,000 / 0.9942028 = 5,029
ULT2018 = 7,000 / 0.9747479 = 7,181
ULT2019 = 6,800 / 0.8900036 = 7,640
ULT2020 = 5,300 / 0.5208633 = 10,175

(b) Estimate the scale factor, σ^2.

For each row in the table, the expected increment (Col H) is the ULT (Col K) times [G(x) at the end of the interval (Col J) minus G(x) at the beginning of the interval (Col I)].

For each row in the table, the sigma-squared value (Col G) is (increment (Col D) minus expected increment (Col H))^2 divided by expected increment (Col H).

The scale factor, σ^2, is the sum of Col G in the table divided by five degrees of freedom which is 138.745.

(c) Estimate the process standard deviation of the loss reserve for all accident years combined.

The process standard deviation of the loss reserve for all accident years is the square root of (the scale factor, σ^2, times the total reserve).

Reserves for each accident year equal the ULT for each accident year minus the accident year total loss at year-end 2020.

Total Reserve =
(5,029 − 5,000) + (7,181 − 7,000) + (7,640 – 6,800) + (10,175 − 5,300) = 5,926

Process standard deviation of the loss reserve for all accident years combined =
(138.745 × 5,926)^0.5 = 906.78

(d) Describe how the graph should appear if the model assumptions are satisfied.

We would expect that the residuals would be randomly scattered around the zero line for all of the ages, and that the amount of variability would be roughly constant.

(e) Determine if the model assumptions are satisfied, based on this graph.

While the variability is fairly constant from age to age, the residuals do not appear to be randomly scattered about zero, with more positive values at 12 months and more negative at 24 months.

GIADV, Fall 2021, Q4

Parts (d) to (f) are not on GI 301

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Outstanding Claims Reserves, Version 1.3a, Hardy

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) State the three statistical assumptions underlying the chain ladder model.

- The conditional expected accumulated total claims amount at a given development year (DY) is the accumulated total claims amount at the previous DY times a development factor that does not vary by accident year (AY).
- The accumulated total claims amounts of different AYs are independent.
- The conditional variance of the accumulated total claims amount at a given development year is the accumulated total claims amount at the previous development year times a proportionality constant that does not vary by accident year.

(b) Demonstrate that the test statistic suggested by Mack to test for a calendar year effect is equal to 2.

Step 1: Create the triangle of age-to-age development factors.

	1-2	2-3	3-4	4-5	5-6	6-7
1	1.375	1.440	1.075	1.214	1.106	1.165
2	49.858	1.021	2.162	1.181	1.204	
3	2.039	1.654	1.085	1.223		
4	2.482	1.278	1.440			
5	4.572	1.760				
6	12.563					

Step 2: Create a triangle indicating if the age-to age factors in a column (each stage of development) are smaller (S) or larger (L) than the median (*) factor.

median	3.527	1.440	1.263	1.214	1.155	1.165
	j=1	j=2	j=3	j=4	j=5	j=6
j=1	S	*	S	*	S	*
j=2	L	S	L	S	L	
j=3	S	L	S	L		
j=4	S	S	L			
j=5	L	L				
j=6	L					

Step 3: Count for every diagonal (j > 1) the S's and L's, then determine the Z's (where Z is the minimum of the S_j and L_j for each j).

j	Sj	Lj	Zj
2	0	1	0
3	3	0	0
4	1	2	1
5	4	1	1
6	0	5	0

Step 4: The sum of the Z values is the test statistic.
$Z = 0 + 0 + 1 + 1 + 0 = 2$

(c) A test statistic equal to 2 indicates that there is a calendar year effect and implies that one of the chain ladder assumptions does not hold.

Identify that assumption.

The accumulated total claims amounts of different accident years are independent.

(d) ~~Calculate the development terms *f*(1)-*f*(7) that minimize the sum of squared residuals.~~

~~Create the incremental loss triangle. Let AY = accident year and DY = development year.~~

~~Then for each DY d, *f*(d) is the average down the AYs (i.e., down the column).~~

~~Incremental loss triangle q(AY, DY)~~

	DY						
AY	1	2	3	4	5	6	7
1	6,012	2,257	3,638	898	2,734	1,642	2,828
2	106	5,179	111	6,270	2,116	2,817	
3	4,410	4,582	5,881	1,268	3,594		
4	4,655	6,900	3,211	6,500			
5	2,092	7,473	7,271				
6	513	5,932					
7	1,557						
d	1	2	3	4	5	6	7
f(d)	2,763.57	5,387.17	4,022.40	3,734.00	2,814.67	2,229.50	2,828.00

~~(e) Estimate the loss that will emerge in the next calendar year for accident years 2-7 combined.~~

~~This is the sum of the f(d) values for d = 2 to 7, which is 21,015.73~~

~~(f) Calculate the values of f(1)-f(7) and g(2)-g(7) that minimize the sum of squared residuals by fixing the f-values to estimate the g-values by linear regression, then fixing the g-values to estimate the next iteration of f-values by linear regression, and so on until consecutive g-values agree to two decimal places. Begin the iterative process with the f-values calculated in part (d).~~

~~The starting point for this is the set of f(d) values from part (d) representing iteration 0. The g(d) values given a set of f(d) values are calculated as follows:~~
~~$g(d) = [\sum_{i=1 \text{ to } d-1} f(i) \times q(d-i, i)] / [\sum_{i=1 \text{ to } d-1} f(i)^2]$ for d = 2 to 7 and $g(8) = 1$.~~

~~Given a set of g(d) values, the f(d) values for the next iteration are calculated as follows:~~
~~$f(d) = [\sum_{i=d \text{ to } 7} g(i+1) \times q(i-d+1,d)] / [\sum_{i=d \text{ to } 7} g(i+1)^2]$ for d = 1 to 7.~~

~~Values of f and g, with g values agreeing to two decimal places, are achieved on the thirteenth iteration:~~

D	1	2	3	4	5	6	7	8
f	2,878.41	7,514.33	5,738.47	5,402.98	3,207.24	2,727.52	2,828.00	
g		2.09	0.27	0.74	0.42	0.98	0.65	1.00

GIADV, Spring 2022, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (c), (d) and (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c), (d) and (e) is for explanatory purposes only.

Solution:

(a) State three advantages of using a parametric curve to model development.

- It only requires estimation of a limited number of parameters.
- It does not require evenly spaced dates.
- It provides a smooth development curve.

(b) State one reason the Cape Cod method is generally preferred over the LDF method.

There are fewer parameters to estimate.

(c) Calculate the MLE of *ELR*.

- MLE of *ELR* is the sum of the incremental payments divided by the sum of Onlevel Premium (OLP) times [G(at the end of the interval) minus G(at the beginning of the interval)].
- The end of the interval is "To" months minus 6 if "To" months >=12; otherwise it is "To" months / 2.
- The beginning of the interval is "From" months minus 6 if "From" months >=12; otherwise it is "From" months / 2.

MLE of *ELR*

$= [2{,}500 + 1{,}800 + \ldots + 5{,}300] / [10{,}000 \times (0.50568 - 0) + 10{,}000 \times (0.78509 - 0.50568) + \ldots + 18{,}000 \times (0.43694 - 0)] = 24{,}100 / 38{,}738.09$

$= 0.622127$

(d) Calculate the value of the loglikelihood function at its maximum.

This is the value of $\ell = \sum_i \left[c_i \ln(\mu_i) - \mu_i \right]$.

- c_i is the incremental payment.
- μ_i is the expected increment, which is the MLE of *ELR* times OLP times [G(at the end of the interval) minus G(at the beginning of the interval)].

The value of the loglikelihood function at its maximum is 169,550.97.

(e) Calculate the total reserve for the four accident years combined.

- The total reserve is the total estimated ultimate minus the total payments.
- The total estimated ultimate is the total OLP times the MLE of *ELR*.

Total reserve = 0.622127 × (10,000 + 12,000 + 15,000 + 18,000) – 24,100
= 10,116.97

GIADV, Fall 2022, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the MLEs of *ULT* for each accident year.

1. For each row of data provided, calculate G(at the end of the interval) and G(at the beginning of the interval).
   - The end of the interval is "To" months minus 6.
   - The beginning of the interval is "From" months minus 6 if "From" months >=12 and 0 if "From" months is 0.
2. The MLE_ULT for each accident year (AY) is the AY total divided by G(at the end of the interval in which the AY is as of year-end 2021).

(b) Calculate the value of the loglikelihood function at its maximum.

1. For each row of data provided, calculate the expected increment which is equal to the MLE_ULT of the accident year times [G(at the end of the interval) minus G(at the beginning of the interval)].
2. Then, for each row of data provided, calculate the incremental payment times the natural log of the expected increment minus the expected increment.
3. The value of the loglikelihood function at its maximum is the sum of the amounts in 2 which is equal to 169,391.727.

(c) Estimate σ^2, the scale factor.

1. For each row of data provided, calculate [the square of the increment minus the expected increment] divided by the expected increment.
2. Then, the scale factor equals the sum of the amounts in 1 divided by [the number of data points minus the number of parameters estimated] which is equal to 138.7457.

(d) Explain why an accident year with a single incremental value will always contribute zero to the estimate.

Commentary on Question:
The model solution is an example of a full credit solution.

Let c be the incremental loss and G be the value of the distribution function at the end of the interval. The estimate of ULT is c/G. The estimated expected increment, mu, is ULT × G = c. Hence, the estimate always matches the observed increment and the difference must be zero.

(e) Estimate the process variance of the reserve for accident year 2020.

Process variance of the reserve for accident year 2020 equals
scale factor from part (c) times MLE_ULT for AY 2020 times
[1 − G(at the end of the interval in which AY 2020 is as of year-end 2021, i.e., at 18 months)] which equals 116,603.341.

(f) Estimate the parameter variance of the reserve for accident year 2020.

- The formula for the reserve is MLE_ULT times (1− G). The derivative with respect to MLE_ULT is (1 − G) and the derivative with respect to theta is − MLE_ULT times G′. G′ = − (1 − G(at the end of the interval)) times ("To" months − 6) divided by the square of the MLE of theta.
- Let COV(x,y) be the covariance matrix (*provided in the Excel file*).
- For AY 2020:
  - (1 − G(AY 2020 at 18 months)) = 0.11.
  - − MLE_ULT2020 times G′ = − 7,640.411 × − 0.02977 = 227.4825.
  - Parameter variance of the reserve for AY 2020 equals
    $[(1 - G)^2 \times COV(ULT2020, ULT2020)]$
    $+ [2 \times (1 - G) \times (- MLE\_ULT2020 \times G') \times COV(theta, ULT2020)]$
    $+ [(- MLE\_ULT2020 \times G')^2 \times COV(theta, theta)]$
    = 80,094.703.

GIADV, Fall 2022, Q4

Parts (c) to (f) are not on GI 301

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Outstanding Claims Reserves, Version 1.3a, Hardy

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Demonstrate that the reserve and standard error for accident year 5 have been correctly calculated.

Reserve for accident year (AY) 5:

- Given as 7,221.
- Need to estimate the projected payments for AY5. Use the appropriate value of f_k and the preceding cumulative payment to get a projected payment. We then get the cumulative projected payments at development years 4 through 7 as follows: 12,962, 14,649, 16,167 and 16,879. The reserve for AY5 is 16,879 minus 9,658 which equals 7,221.

Standard error for AY5:

- Given as 1,592.
- Estimate as the square root of:

$$c(5,7)^2 \times [\,\sigma_3^2 / f_3^2 \times (1 / c(5,3)+1 / (c(1,3)+c(2,3)+c(3,3)+c(4,3)))$$
$$+ \sigma_4^2 / f_4^2 \times (1 / c(5,4)+1 / (c(1,4)+c(2,4)+c(3,4)))$$
$$+ \sigma_5^2 / f_5^2 \times (1 / c(5,5)+1 / (c(1,5)+c(2,5)))$$
$$+ \sigma_6^2 / f_6^2 \times (1 / c(5,6)+1/ c(1,6))]$$

- This equals 1,592.

(b) Estimate the coefficient of variation of the unpaid claims for each of accident years 2-7 and overall.

The coefficient of variation (CoV) estimate is given by the standard error estimate divided by the reserve estimate. This is done for AYs 2 to 7 and overall.

(c) ~~Estimate the upper 90% confidence limit of the overall unpaid claims using a lognormal distribution. (Note: The 90th percentile of a standard normal distribution is 1.28.)~~

~~This is equal to the total reserve estimate times the exponential of 1.28 times sigma for the total minus sigma-squared for the total divided by 2. Sigma-squared for the total is the natural log of (1 plus the square of the CoV for the total). The upper 90% confidence limit of the overall unpaid claims using a lognormal distribution is 57,402.~~

~~(d) Allocate the overall amount from part (c) to accident years 2-7 in such a way to reach the same level of confidence for each accident year. (Note: Using Excel's Goal Seek function is an acceptable approach.)~~

~~**Commentary on Question:**~~
~~*The model solution in Excel used Excel's Goal Seek function.*~~

~~First, one needs to calculate the upper 90% confidence limit of the unpaid claims for each AY. These are calculated by using the approach from part (c), except using AY values instead of total values. The Goal Seek "set cell" has the adjusted confidence limit sum minus the amount from (c). The adjusted confidence limit for each AY is the confidence limit formula with 1.28 replaced by a cell that changes in Goal Seek (using any value close to 1.28 for the initial value in the change cell works). This gives the following:~~

~~Accident Year~~	~~Adjusted Confidence Limit~~
~~2~~	~~1,047~~
~~3~~	~~3,133~~
~~4~~	~~6,105~~
~~5~~	~~8,476~~
~~6~~	~~13,573~~
~~7~~	~~25,067~~
~~Overall~~	~~57,402~~

~~(e) Explain the empirical approach to establishing confidence limits as described by Mack.~~

~~**Commentary on Question:**~~
~~*The model solution is an example of a full credit solution.*~~

~~Use the minimum and maximum individual age-to-age factors in each column to establish minimum and maximum limits.~~

~~(f) Explain why this empirical approach to establishing confidence limits does not seem to be reasonable.~~

~~**Commentary on Question:**~~
~~*The model solution is an example of a full credit solution.*~~

~~The different number of age-to-age factors in each column results in inconsistent confidence limits among accident years.~~

(g) Create a scatter plot to check the assumption that the expected losses at age 2 are proportional to the losses at age 1.

For AYs 1 to 6, select the x-values to be the AY payments at development year 1 and the y-values to be the AY payments at development year 2.

(h) Interpret the scatter plot in part (g) with regard to determining whether the assumption is correct. If it is not, recommend an alternative model.

Commentary on Question:
The model solution is an example of a full credit solution.

The intercept of a line through the points is well above zero, so the assumption does not appear to be correct. A constant term should be added to the model.

GIADV, Spring 2023, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (d) to (f). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (d) to (f) is for explanatory purposes only.

Solution:

(a) State two advantages of using the overdispersed Poisson distribution as opposed to the Poisson distribution.

- Can match two moments instead of one.
- Maximum likelihood estimation matches the usual Cape Cod estimate.

(b) Describe a situation where incremental losses may not be independent.

Commentary on Question:

There are two situations provided in the source reading. Only one was required for full credit. The model solution is an example of a full credit solution.

All periods are equally affected by a change in loss inflation.

(c) Describe a situation where incremental losses may not be identically distributed.

There are different risks and mix of business in each period.

(d) Demonstrate that the MLE of *ELR* is 0.5251.

Commentary on Question:

Refer to the Excel solutions spreadsheet.

1. For each row of data provided, calculate *G*(at the end of the interval) and *G*(at the beginning of the interval).
   - The end of the interval is "To" months minus 6.
   - The beginning of the interval is "From" months minus 6 if "From" months >=12 and 0 if "From" months is 0.

2. The MLE of *ELR* is the total of losses divided by the total of subject premium times [*G*(at the end of the interval) minus *G*(at the beginning of the interval)].

(e) Estimate the scale factor, σ^2.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

1. For each row of data provided, calculate the expected increment as subject premium times [*G*(at the end of the interval) minus *G*(at the beginning of the interval)] times the MLE of *ELR*.
2. For each row of data provided, calculate the square of [the increment minus the expected increment] divided by the expected increment.
3. Then, the scale factor equals the sum of the amounts in 2 divided by [the number of data points minus the number of parameters estimated] which is equal to 294.0381.

(f) Estimate the process standard deviation of the loss reserve for all accident years combined.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

The ultimate loss estimate for all accident years (AYs) combined is the total subject premium times the MLE of *ELR*. The loss reserve estimate for all AYs combined is the ultimate loss estimate for all AYs combined minus the sum of incremental payments. The estimate of the process standard deviation of the loss reserve for all AYs combined is the square root of [loss reserve estimate for all AYs combined times the estimate of the scale factor] which is equal to 1,569.93.

GIADV, Spring 2023, Q4

Part (f) is not on GI 301

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Outstanding Claims Reserves, Version 1.3a, Hardy

Considerations Regarding the Chain Ladder Model, SOA

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) to (h). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c) to (h) is for explanatory purposes only.

Solution:

(a) State whether or not this implies that the errors in reserve estimates for different accident periods are independent. Justify your answer.

No. This is because the estimators for the different periods are all influenced by the same development factors, so the errors are positively correlated.

(b) State the other two statistical assumptions underlying the chain ladder model.

- The conditional expected accumulated total claim amount at a given development period is the accumulated total claim amount at the previous development period times a development factor that does not vary by accident period.
- The conditional variance of the accumulated total claim amount at a given development period is the accumulated total claim amount at the previous development period times a proportionality constant that does not vary by accident period.

(c) Demonstrate that the standard error for the first half of 2021 has been correctly calculated.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

For the following, let j = accident period (1st Half 2019 is 1, 2nd Half 2019 is 2, 1st Half 2020 is 3, …) and k = development period. Then to square the development triangle, the incremental payments are calculated as $c_{j,k} = c_{j,k-1} \times f_{k-1}$.

$$SE_{2021.1} = \sqrt{c_{5,8}^2\left(\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{c_{5,4}}+\frac{1}{c_{1,4}+c_{2,4}+c_{3,4}+c_{4,4}}\right)+\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{c_{5,5}}+\frac{1}{c_{1,5}+c_{2,5}+c_{3,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{c_{5,6}}+\frac{1}{c_{1,6}+c_{2,6}}\right)+\frac{\alpha_7^2}{f_7^2}\left(\frac{1}{c_{5,7}}+\frac{1}{c_{1,7}}\right)\right)} = 4,862$$

(d) Calculate the standard error for the full year of 2021.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

$$SE_{2021} = \sqrt{SE_{2021.1}^2 + SE_{2021.2}^2 + 2c_{5,8}c_{6,8}\left(\frac{\left(\alpha_4^2/f_4^2\right)}{\left(c_{1,4}+c_{2,4}+c_{3,4}+c_{4,4}\right)}+\frac{\left(\alpha_5^2/f_5^2\right)}{\left(c_{1,5}+c_{2,5}+c_{3,5}\right)}+\frac{\left(\alpha_6^2/f_6^2\right)}{\left(c_{1,6}+c_{2,6}\right)}+\frac{\left(\alpha_7^2/f_7^2\right)}{\left(c_{1,7}\right)}\right)} = 8,773$$

(e) Describe Mack’s nonparametric test for correlations between development factors.

Rank the development factors in each column. Calculate Spearman’s rank correlation coefficient for each pair of adjacent columns. The test statistic is a weighted average of those coefficients, which is compared to the distribution of the test statistic under the null hypothesis of no correlation.

(f) ~~Describe the adjustment that Venter suggests to correct for correlation between adjacent development factors.~~

~~Add the covariance to the product of adjacent development factors.~~

(g) Describe Mack’s nonparametric test for calendar year effects.

Identify within each column of development factors those with rank larger than the mean rank and those with rank smaller than the mean rank. For each diagonal, record the number of ranks larger than the mean rank or the number of ranks smaller than the mean rank, whichever is smaller. The test statistic is the sum of these values over all diagonals, which is compared to the distribution of the test statistic under the null hypothesis of no calendar year effects.

(h) Describe a model that Venter suggests could account for calendar year effects.

Use a multiplicative model with factors for accident period, development period and diagonal.

GIADV, Fall 2023, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) State two reasons why this is the case.

The Cape Cod method requires that fewer parameters be estimated.
The Cape Cod method uses the information in the exposure base.

(b) Calculate the maximum likelihood estimates of *ULT* for each of the four accident years.

For each row of data provided, calculate *G*(at the end of the interval). The end of the interval is "To" months minus 6. The MLE for each accident year is the accident year total divided by *G*(at the end of the interval) from the row showing the accident year valued at the end of 2022.

- ULT_{2019} = 5,000 / $G(42)$ = 5,843.894
- ULT_{2020} = 7,000 / $G(30)$ = 8,643.159
- ULT_{2021} = 6,800 / $G(18)$ = 9,433.850
- ULT_{2022} = 5,300 / $G(6)$ = 11,327.367

(c) Calculate $\hat{\sigma}^2$, the estimate of the scale factor.

Commentary on Question:

AY = Accident Year

1. For each row of data provided, calculate *G*(at the start of the interval) and *G*(at the end of the interval).
2. For each row of data provided, calculate the expected increment as the MLE of ULT_{AY} times [*G*(at the end of the interval) minus *G*(at the start of the interval)].

3. For each row of data provided, calculate the square of [the increment minus the expected increment] divided by the expected increment.
4. Then, the scale factor equals the sum of the amounts in step 3 divided by [the number of data points minus the number of parameters estimated]. This equals 42.198.

(d) Estimate the process standard deviation of the loss reserve for all accident years combined.

1. The total ultimate loss estimate equals $ULT_{2019} + ULT_{2020} + ULT_{2021} + ULT_{2022}$.
2. The total loss reserve estimate equals the total ultimate estimate minus the total paid.
3. The estimate of the process standard deviation of the loss reserve for all AYs combined is the square root of [the total loss reserve estimate times the estimate of the scale factor]. This equals 685.883.

(e) A likelihood ratio test indicates that $\omega = 1$ is a plausible value. Using this value and re-estimating the other parameters leads to a significant reduction in the estimated scale factor.

Explain why this reduction is to be expected.

The slight change in this parameter will lead to small changes in the MLEs of the other parameters. As a result, the numerator of the scale factor will be similar. However, the denominator will change from 4 to 5, leading to a significant reduction.

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GIADV, Spring 2024, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question tested a candidate's understanding of the Clark's stochastic LDF model. This question included data and results from a completed analysis (using Clark's stochastic reserving model) in Excel. Candidates were not required to perform any calculations. Candidates were to respond in Word. Each question part presented a report excerpt. Candidates were expected to critique the excerpt based on the data and analysis in Excel. The model solution for each part provides an example of a full credit solution but does not represent the only acceptable full credit response.

Solution:

(a) Critique Part 1.

Part 1: *I have been provided with a loss triangle representing accident years 2016-2023. All possible twelve-month development periods are represented. I noticed that one of the increments is negative. While the algorithm provides results for this case, Clark's procedure should not be used when there are negative increments.*

The model will still work if some actual points show decreasing losses. However, if there is real expected negative development then a different model should be used.

The report is thus incorrect. Negative increments are acceptable, though not in all situations. It should have said that the procedure can be used with negative observed increments provided expected increments are positive.

(b) Critique Part 2.

Part 2: *I have performed maximum likelihood estimation of the two parameters. The calculations are in cells G7:K43 and the maximizing values are in cells H44 and H45. The values that maximize the likelihood function are $\theta = 37.44$ and ELR = 0.5506.*

The calculations are incorrect. When obtaining the likelihood value, the increments to be used should be 0, 6, 18, 30, and so on. The report used 0, 12, 24, 36, and so on.

(c) Critique Part 3.

Part 3: *I have calculated the estimate of the scale factor in cells M7:M44 as* $\hat{\sigma}^2 = 309.1$.

The individual numbers are correct as is the sum. However, the sum should be divided by 34 not 33. This is because the number of observations(36) minus the number of estimated parameters (2) is 34.

(d) Critique Part 4.

Part 4: *I have calculated the loss reserve estimate for all years in cell F60 as 31,103. I have further calculated the process standard deviation of the loss reserve in cell F62 as 3,101. Assuming a normal distribution, two standard deviations provide 95% confidence. Therefore, we can be 95% confident that the actual development will be between 24,901 and 37,304.*

The calculations in the report are correct, as is the assumption. However, the process standard deviation only accounts for one aspect of the potential error. An additional source is the estimation error (parameter variance). Hence, the 95% confidence interval should be wider than the one indicated.

(e) Critique Part 5.

Part 5: *Clark recommends several graphs that can be used to verify if the assumptions of the model hold. Two such graphs are on the worksheet. One graph plots normalized residuals against increment age and the other plots them against the expected increments. The second graph shows a curved, rather than horizontal pattern. Hence the underlying assumptions may not hold.*

This part of the report is correct.

GIADV, Spring 2024, Q4

Parts (d) to (g) are not on GI 301

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Outstanding Claims Reserves, Version 1.3a, Hardy

Considerations Regarding the Chain Ladder Model, SOA

Stochastic Loss Reserving Using Generalized Linear Models, CAS Taylor and McGuire

Commentary on Question:

This question tested a candidate's understanding of modeling the chain ladder method. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the following amounts using the chain ladder (CL) method:

(i) Age-to-age factors

(ii) Expected values for each DY (starting with DY 2), based on the previous DY, for which there are observed values

Part (i): The average development for each interval between development years. They are calculated as follows: For the period 1-2 it is the (sum of DY 2) divided by the (sum of DY1 excluding AY10), for the period 2-3 it is the (sum of DY 3) divided by the (sum of DY2 excluding AY9), and so on.

Part (ii): Each amount is the prior observed amount multiplied by the appropriate age-to-age factor. They are calculated as follows: the DY2 column is the 1-2 factor multiplied by the DY1 observed column from AY1 to AY9, the DY3 column is the 2-3 factor multiplied by the DY2 observed column from AY1 to AY8, and so on.

(b) Construct the following scatterplots for each of DYs 1 to 3:

(i) Development along with a fitted regression line

(ii) Weighted residuals versus values from the previous DY

For part (i):
DY1: the x-axis is DY1 and the y-axis is DY2, for AY1 to AY9
DY2: the x-axis is DY2 and the y-axis is DY3, for AY1 to AY8
DY3: the x-axis is DY3 and the y-axis is DY4, for AY1 to AY7

For part (ii):
The same x-axis is used as in part (i). The y-axis residuals are calculated as the estimated values from part (i) for DYb less the data for DYb divided by the square root of the data from the DYa. In which Yb =Ya + 1.

The scatterplots are completed automatically when these amounts are input in the appropriate cells.

(c) Determine the validity of the assumptions underlying the CL estimates based upon part (b). Justify your determination.

Commentary on Question:
The model solution is an example of a full credit solution.

The first set of charts from part (b)(i) looks good, but that is mostly due to scale. However, the line should be through the origin which is not the case here.

The second set from part (b)(ii) shows a pattern of residuals that are negative, then positive. Therefore, the assumptions are not valid.

(d) ~~Assume that the scatterplots indicate that the assumptions are not valid. Mack provides two other formulas for calculating age-to-age factors that could be tested.~~

~~State one of these alternatives. Do not do any calculations.~~

~~**Commentary on Question:**~~
~~*The model solution is an example of a full credit solution.*~~

~~f_{k0} is the C(i,k) squared weighted average of the individual development factors~~

~~(e) Input the appropriate values to perform this regression analysis.~~

~~AY1: the x1 column is DY1, the increment column is DY2 minus DY1, for AY1 to AY9~~
~~AY2: the x2 column is DY2, the increment column is DY3 minus DY2, for AY1 to AY8~~

~~AY3: the x3 column is DY3, the increment column is DY4 minus DY2, for AY1 to AY7~~

~~The regressions are completed automatically when these amounts are input in the appropriate cells.~~

~~(f) Determine the validity of the assumptions underlying the CL estimates based upon the results of the regression analysis in part (e). Justify your determination.~~

~~**Commentary on Question:**~~
~~*The model solution is an example of a full credit solution.*~~

~~The intercept is not zero for DY2 because it is more than double the standard error. This violates Mack's assumption because it is significant. For DY1, the intercept is nearly significant at 1.6 standard deviations. This is a borderline violation of Mack's assumption.~~

~~The slopes are clearly not zero for D1 to D3, which supports Mack's assumption.~~

~~(g) Propose an alternative model that is more consistent with the results of the regression analysis in part (e).~~

~~**Commentary on Question:**~~
~~*The model solution is an example of a full credit solution.*~~

~~The "linear model plus constant" model is more appropriate when both terms are significant.~~

GIADV, Fall 2024, Q3

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question tested a candidate's understanding of the Clark's stochastic LDF model. This question included data and results from a completed analysis (using Clark's stochastic reserving model) in Excel. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only..

Solution:

(a) Explain why the gamma distribution is also appropriate for use in Clark's model.

Any distribution that places probability from zero to infinity can be used as it will meet the requirement of increasing development

(b) Explain why the gamma distribution may not be the most reasonable choice.

The gamma distribution is light-tailed and may not fit a typical development pattern

(c) Recommend which of the three distributions should be used based upon fit to the data. Justify your recommendation including one numerical and one graphical argument.

Commentary on Question:

The model was completed for the three distributions and two model fit statistics (loglikelihood and scale) were included in the model output. A review of only one of these statistics was required to earn full credit for the numerical argument. As for the graphical argument for model fit, one should look at graphs of normalized residuals against months of development for each of the three distributions. The model solution is an example of a full credit solution.

The following table shows the scale fit statistics:

Model	Scale
Weibull	5,788
Gamma	4,432
Loglogistic	4,093

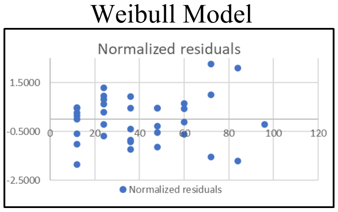
The lower the scale value, the better a model fits the dataset. The scale value of the Weibull model is considerably larger than the scale value for both the gamma and loglogistic models.

A graph of the normalized residuals against the development period can give a visual check for model fit. The plot for a good model fit should not have a rising or decreasing pattern.

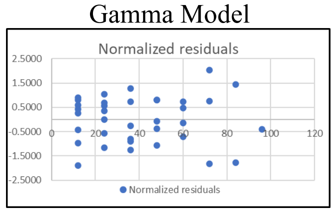
The following graphs plot the normalized residuals for each model against months of development.

The normalized residuals are calculated as the difference between the paid loss data increment and the mu-hat estimate all divided by the square root of sigma-square estimate for the data point times the mu-hat estimate.

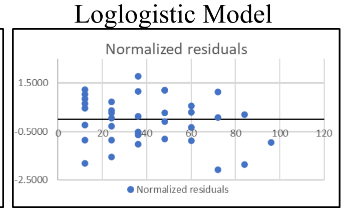
Weibull Model



Gamma Model



Loglogistic Model



Reviewing the three graphs of residuals, the Weibull graph indicates a slightly increasing pattern. The gamma and loglogistic graphs appear to have a more horizontal pattern. This would indicate that Weibull model is a poor fit, and the gamma and loglogistic models fit the data reasonably well.

Overall, the Weibull model is easily rejected by both the test statistic and the graph of residuals. Additionally, both the gamma and loglogistic models are reasonable by both the test statistic and the graph of residuals. However, the loglogistic model scale factor test statistic indicates the best fit. For these reasons, I recommend the loglogistic model.

GIADV, Fall 2024, Q4

Parts (d), (g) and (h) are not on GI 301

Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Outstanding Claims Reserves, Version 1.3a, Hardy

Considerations Regarding the Chain Ladder Model, SOA

Stochastic Loss Reserving Using Generalized Linear Models, CAS Taylor and McGuire

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the development factors (f_k) and complete the triangle using Mack's chain ladder approach.

The development factors (f_k) are calculated as follows:

- For k = 1, it equals the sum of paid claims at development year 2 divided by the sum of paid claims at development year 1, for accident years 1 through 9.
- For k = 2, it equals the sum of paid claims at development year 3 divided by the sum of paid claims at development year 2, for accident years 1 through 8.
- For k = 3, it equals the sum of paid claims at development year 4 divided by the sum of paid claims at development year 3, for accident years 1 through 7.

The pattern continues for k = 4 through 9.

The values in the triangle are represented by $c_{j,k}$ where j is the AY. To complete the triangle, the remaining $c_{j,k}$ values are calculated as follows:

$$c_{j,k} = c_{j,k-1} \times f_{k-1}$$

(b) Calculate the values of α_8^2 and α_9^2.

$$\alpha_8^2 = \left(\frac{1}{10-8-1}\right)\sum_{j=1}^{2} c_{j,8}\left(\frac{c_{j,9}}{c_{j,8}} - f_8\right)^2 \text{ and } \alpha_9^2 = \frac{(\alpha_8^2)^2}{\alpha_7^2}.$$

(c) Calculate the standard error of the reserve estimator for AYs 2 and 3.

VAR_k = variance of the AY k reserve estimator
SE_k = standard error of the AY k reserve estimator

$$VAR_2 = c_{2,10}^2 \times \frac{\alpha_9^2}{f_9^2} \times \left(\frac{1}{c_{2,9}} + \frac{1}{c_{1,9}}\right) \text{ and}$$

$$VAR_3 = c_{3,10}^2 \times \left[\frac{\alpha_8^2}{f_8^2} \times \left(\frac{1}{c_{3,8}} + \frac{1}{c_{1,8} + c_{2,8}}\right) + \frac{\alpha_9^2}{f_9^2} \times \left(\frac{1}{c_{3,9}} + \frac{1}{c_{1,9}}\right)\right].$$

So we get $SE_2 = \sqrt{VAR_2}$ and $SE_3 = \sqrt{VAR_3}$.

(d) ~~Calculate a 95% confidence interval for the AY 8 reserve estimate using Mack's approach based on the lognormal distribution. (The 97.5 percentile of the normal distribution is 1.96.)~~

The confidence interval is calculated as $\mu \pm z\sqrt{\sigma^2}$.
The AY 8 reserve estimate is $R_8 = c_{8,10} - c_{8,3}$ with variance, Var_8.
Var_8 is provided with the data.
$\sigma^2 = \ln(1 + Var_8/R_8^2)$, $\mu = \ln(R_8 - \sigma^2/2)$, and $z = 1.96$.

(e) Explain why your assistant's approach is incorrect.

The variance of the sum of random variables is only the sum of the variances when the variables are uncorrelated. The individual estimators are correlated because they rely on the same estimates of the age-to-age factors.

(f) Explain why the correct value is larger than that obtained via your assistant's approach.

The correct value is larger because the correlations are positive, the variance of the sum will exceed the sum of the variances.

(g) ~~Venter restates one of Mack's assumptions as~~ $\cancel{E[q(w, d + 1) \mid \text{data to } w + d] = f(d)c(w, d)}$~~.~~

~~State the assumption in words.~~

~~The expected value of the incremental losses to emerge in the next period is proportional to the total losses emerged to date, by accident year.~~

~~(h) State a formula for each of the three alternative expressions including a verbal description of what they represent.~~

~~Formula 1:~~ $\cancel{E[q(w, d + 1) \mid \text{data to } w + d] =}$

~~Formula 2:~~ $\cancel{E[q(w, d + 1) \mid \text{data to } w + d] =}$

~~Formula 3:~~ $\cancel{E[q(w, d + 1) \mid \text{data to } w + d] =}$

$\cancel{E[q(w, d + 1) \mid \text{data to } w + d]}$	~~Formula as a function of~~ $\cancel{f(d)}$	~~Verbal Description of Formula~~
~~Formula 1~~	$\cancel{f(d)c(w, d) + g(d)}$	~~linear with constant~~
~~Formula 2~~	$\cancel{f(d)h(d)}$	~~factor times parameter~~
~~Formula 3~~	$\cancel{f(d)h(w)\, g(w + d)}$	~~includes a calendar year effect~~

GI 301 Learning Objective 2 Curated Past Exam Solutions

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GIADV, Fall 2024, Q10 9

GIADV, Spring 2023, Q10

Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total IBNR for claims excess of 400,000 as of December 31, 2022 using each of the following approaches:

(i) Development factors calculated using a simple average

(ii) Theoretically-derived development factors based on Siewert's formula

(i) Development factors calculated using a simple average

- Create development triangle of reported claims excess of 400,000 by subtracting the development triangle of reported claims at 400,000 limit from the development triangle of reported claims at total limits.
- Create a triangle of age-to age development factors with the created triangle.
- Average age-to-age development factors and then calculate cumulative development factors (CDFs) with them.
- Calculate IBNR with the CDFs minus one times the reported excess claims on the diagonal of the triangle.

(ii) Theoretically-derived development factors based on Siewert's formula

- Create a triangle of age-to age development factors with the development triangle of reported claims at total limits.
- Select age-to-age development factors and then calculate CDFs with them.
- Calculate the CDFs for claims excess of 400,000 using the formula $CDF_t \times (1 - R_{72}) \div (1 - R_t)$ for t = 12 to 72.
- Calculate IBNR with the CDFs minus one times the reported excess claims on the diagonal of the triangle.

(b) Describe two considerations in the calculation of R_t values.

Commentary on Question:
There are more than two considerations. The model solution is an example of a full credit solution.

- The different trend rates that are associated with claims at differing limits
- Whether to use actual historical data, industry data, or a combination

(c) Explain why alternative methods should be considered based on the results from part (a).

Commentary on Question:
The model solution is an example of a full credit solution.

There are large differences in the estimated IBNR between the two methods.

(d) Identify two considerations when applying the increased limits factors approach.

Commentary on Question:
There are more than two considerations. The model solution is an example of a full credit solution.

- Treatment of ALAE
- Whether the factors are applicable to claims or premiums

GIADV, Fall 2023, Q10

Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) Understand the differences in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate total IBNR for the layer as of December 31, 2022 using Siewert's formula.

Step 1: Calculate estimated CDFs for 250,000 and 750,000 limits.
At each age of development, the estimated CDF for a limit is the total limits CDF times the severity relativity for a limit at 84 months of development divided by the severity relativity for a limit at the age of development.

Step 2: Project ultimate claims for 250,000 and 750,000 limits.
For each accident year, the projected ultimate claims for a limit is the reported claims for the limit times the estimated CDF for that limit at its age of development.

Step 3: Estimate the ultimate claims for the layer 500,000 excess of 250,000.
For each accident year, this is the projected ultimate claims for 750,000 limits minus the projected ultimate claims for 250,000 limits.

Step 4: Estimate the IBNR for the layer 500,000 excess of 250,000.
For each accident year, this is the projected ultimate claims for the layer minus the reported claims for the layer. The reported claims for the layer is the reported claims for 750,000 limits minus the reported claims for 250,000 limits. These amounts summed by accident year gives the total layer IBNR.

(b) Describe a peculiarity with the CDFs derived from Siewert's formula in part (a).

Commentary on Question:

The model solution is an example of a full credit solution.

The estimated CDFs for 250,000 limits are higher than the CDFs for 750,000 limits for accident year 2021 and accident year 2022. It is unusual for lower limits to have a higher CDF.

(c) Calculate the layer IBNR for AY 2022 as of December 31, 2022 using the ILF method.

Commentary on Question:
Claim amounts are shown in thousands.

The estimated ILF is at the Jan. 1, 2020 cost level. Therefore, 2.5 years of trend is required to take the factor to an AY 2022 level (i.e., assumed to be the average date of the year, July 1, 2022).

Residual trend factor for the 750,000 limit is (1.022)/(1.01) = 1.0119.
ILF trended is $1.19 \times 1.0119^{2.5} = 1.2257$.
AY 2022 Ultimate claims at 750,000 limits is 5,019 × 1.2257 = 6,152.
AY 2022 Ultimate claims for the layer is 6,152 – 5,019 = 1,133.
AY 2022 Layer IBNR is AY 2022 ultimate claims for the layer minus AY 2022 reported claims for the layer = 1,133 – (3,978 – 3,721) = 876.

GIADV, Spring 2024, Q10

Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) understand the difference in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question tested a candidate's knowledge regarding the development of excess limits and layers. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total IBNR for the layer of 150,000 excess of 50,000 using volume-weighted average loss development factors.

Commentary on Question:

DY = development year, AY = Accident Year

Step 1: Calculate IBNR for the 50,000 limit.
Using the data triangle at the 50,000 limit: For each stage of development, DYx to DYx+1, the age-to-age factor is the sum of the DYx+1 column divided by [the sum of the DYx column excluding the amount from the current valuation date (i.e., the last data point in the column)]

The cumulative development factors (CDFs) are calculated as follows: 72 months to ultimate is set to 1.0. The 60 months to ultimate factor is the 60-72 months age-to-age factor times the 72 months to ultimate factor. The 48 months to ultimate factor is the 48-60 months age-to-age factor times the 72 to ultimate factor. This relation continues for all development ages.

IBNR by AY is calculated as follows: For AY2023 it the DY1 amount times the (12 months to ultimate factor minus 1.0), for AY 2022 it is the DY2 amount times the (24 months to ultimate factor minus 1.0), and so on.

Step 2: Calculate total IBNR for the 200,000 limit.

Subtract the total reported amount at 200,000 limit (sum of the diagonal from the 200,000 limit triangle) from the provided total ultimate at 200,000 limit.

Step 3: Calculate IBNR for the layer 150,000 excess 50,000.
This is the total IBNR at 200,000 limit minus the total IBNR at 50,000 limit.

(b) Explain why the expected method would be a viable alternative to the development method.

Commentary on Question:
There are several possible acceptable responses to this question. The model solution is an example of a full credit solution.

The expected method is often used for immature experience periods when excess claims data may be too sparse, which is the case here. Furthermore, there is no history to calculate the development factors.

(c) Describe a limitation of using the expected method.

For more mature experience periods, the lack of sensitivity to change in reported claims makes it less accurate.

(d) Calculate the AY 2023 IBNR for the layer using the expected method.

Step 1: For each of 50,000 and 200,000 limit, calculate trended earned premium. All the claims have been trended to December 31, 2023. The premium is provided for calendar year 2023 – it is reasonable to assume that the average earned date is the middle of the year. Therefore, we trend the premiums by one-half year given an annual premium inflation trend of 3%.

Step 2: For each of 50,000 and 200,000 limit, calculate the ultimate claims using the appropriate expected loss ratios for the corresponding limit.

Step 3: Calculate the AY 2023 IBNR for the layer 150,000 excess 50,000.
The AY 2023 IBNR for the layer is the ultimate for the layer minus the reported for the layer in which:

- The layer ultimate is the ultimate for the 200,000 limit from step 2 minus the ultimate for the 50,000 limit from step 2.
- The reported for the layer is the 200,000 limit reported amount for AY 2023 at 12 development months minus the 50,000 limit reported amount for AY 2023 at 12 development months.

(e) Calculate the AY 2023 IBNR for the layer using the ILF method.

Step 1: Convert the premium ILF to a claim ILF.
This is the premium ILF time the expected loss ratio for the 200,000 limit divided by the expected loss ratio for the 50,000 limit.

Step 2: Calculate the AY 2023 layer ultimate claims at the 200,000 limit.
This is the ultimate claims from the 50,000 limit from part (a) times the premium ILF from step 1.

Step 3: Calculate the AY 2023 IBNR for the layer 150,000 excess 50,000.
The AY 2023 IBNR for the layer is the ultimate for the layer minus the reported for the layer in which:

- The layer ultimate is the ultimate for the 200,000 limit from step 2 minus the ultimate for the 50,000 limit from part (a).
- The reported for the layer is the 200,000 limit reported amount for AY 2023 at 12 development months minus the 50,000 limit reported amount for AY 2023 at 12 development months.

GIADV, Fall 2024, Q10

Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) understand the difference in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question tested a candidate's knowledge regarding the development of excess limits and layers. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the IBNR as of December 31, 2023 by AY for the 200,000 limit using Siewert's formula.

Commentary on Question:

CDF = Cumulative Development Factor

Step 1: Calculate the first three CDFs for unlimited amounts.

For t = 12, 24 and 36,

$CDF_t(\text{Unlimited}) = CDF_t(100k) \times R_t(100k) / R_{72}(100k)$

Step 2: Calculate the first three CDFs for 200k limits.

For t = 12, 24 and 36,

$CDF_t(200k) = CDF_t(\text{Unlimited}) \times R_{72}(200k) / R_t(200k)$

Step 3: Calculate IBNR by AY for 200k limit

AY2023: $[CDF_{12}(200k) – 1] \times$ Reported 200k limit (AY2023, 12 months)

AY2022: $[CDF_{24}(200k) – 1] \times$ Reported 200k limit (AY2022, 24 months)

AY2021: $[CDF_{36}(200k) – 1] \times$ Reported 200k limit (AY2021, 36 months)

(b) Explain why actuarial judgement is needed when using Siewert's formula based on the results in part (a).

Commentary on Question:

The model solution is an example of a full credit solution.

Actuarial judgement is needed when using Siewert's formulas because the estimated relativities at alternative limits can result in unusual cumulative development factors. We can see the following unusual results from part (a):

- The calculated CDF at 200k limit for 24 months is lower than that for 36 months.
- The calculated CDF at 200k limit for 24 months is lower than that CDF at 100k limit for 24 months.

(c) Calculate the IBNR as of December 31, 2023 by AY for the 200,000 limit using the ILF method.

Step 1: Calculate the ILF trend for 200k limit as
(claims trend 200k limit + 1)/ (claims trend 100k limit + 1)

Step 2: Calculate the ILF trend period in years for each AY as the period from the average date of loss for the AY to the date of the cost level for the ILF.
→ 0.5 for AY2021, 1.5 for AY2022 and 2.5 for AY2023.

Step 3: Calculate the trended ILF factor for each AY as the
ILF × [(1 + ILF trend *from step 1*)^(AY trend period *from step 2*)]

Step 4: Calculate the ultimate claims at 200k limit for each AY as
AY trended ILF *from step 3* × AY Ultimate 100k *provided in the 2nd table*

Step 5: Calculate the IBNR at 200k limit for each AY as
AY Ultimate 200k *from step 4* – AY Reported as of December 31, 2023

GI 301 Learning Objective 3 Curated Past Exam Solutions

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GIRR, Fall 2020, Q8

Learning Objectives:

3. The candidate will understand the procedure for estimating premium liabilities.

Learning Outcomes:

(3a) Understand the purpose of general insurance premium liabilities.

(3b) Calculate the premium liabilities for a general insurance company.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a) Describe the difference between claim liabilities and premium liabilities.

Claim liabilities relate to claims that occurred on or before the accounting/valuation date, regardless of whether it has been reported or not.

Premium liabilities relate to claims that occurred after the accounting/valuation date and are associated with the unexpired portion of policies effective on or before the accounting/valuation date.

(b) Describe each of these approaches.

Premium approach: the premium liability equals the net unearned premium liability from financial statements (or net UPR) less anticipated profit margin.

Claim approach: the premium liability is evaluated directly from actuarial analysis of claims experience.

(c) Provide one challenge with the premium approach.

Any of the following is acceptable:

- This approach relies on an up to date pricing basis
- This approach relies on a relatively stable claims and exposure environment (since you are not using claim experience directly in the calculation)
- The unearned premium reserve may not reflect most current view of future claims experience (particularly due to lag between setting rates and effective date)
- It may be difficult to quantify profit margin (particularly for commercial lines)
- Rates and profit margins tend to vary with the underwriting cycle

(d) Calculate the equity in unearned premiums as of June 30, 2020, net of reinsurance.

Line of Business	Gross Written Premium (000)	Gross Expected Claim Ratio incl. ALAE	Gross Unearned Premium (000) as of Jun 30, 2020	Gross Expected Claims (000)
Property	1,305	82%	870.00	713.40
General Liability	1,539	56%	1,026.00	574.60
Automobile	1,244	79%	829.30	655.10
Total	4,088		2,725.00	1,943.00

Remaining time of policies as of Jun. 30, 2020: 0.6667 (8 months)

Expected ULAE for premium liabilities gross of reinsurance = total gross expected claims × 12.9% = 1,943 × 12.9% = 250.65

Expected ULAE for premium liabilities net of reinsurance = expected ULAE for premium liabilities gross of reinsurance = 250.65

	Gross of Reinsurance	Net of Reinsurance
Unearned Premiums (000)	2,725.00	2,043.75
Expected Claims (000)	1,943.00	1,457.25
Expected ULAE (000)	250.65	250.65
Total Expected Claims and LAE		1,707.90
Maintenance Expenses = 2,725 × 16% × 30%		130.80
Profit-sharing Commissions = 2,725 × 3.2%		87.20
Total Premium Liabilities		1,925.90
Profits (Equity) in the unexpired policy		118.00

GIRR, Spring 2021, Q7

Learning Objectives:

3. The candidate will understand the procedure for estimating premium liabilities.

Learning Outcomes:

(3a) Understand the purpose of general insurance premium liabilities.

(3b) Calculate the premium liabilities for a general insurance company.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a) Verify the calculation of ultimate claim ratios.

Earned Premiums

	Property		Liability	
Calendar Year	Gross	Net	Gross	Net
2017	1,025	760	1,950	1,803
2018	1,050	774	2,550	2,274
2019	1,150	849	4,000	3,446
2020	1,250	922	5,450	4,543

Earned premiums $(EP)_y$ = Unearned premiums $(UEP)_{y-1}$
+ Written premiums $(WP)_y - UEP_y$

e.g., Property, Gross: $EP_{2018} = UEP_{2017} + WP_{2018} - UEP_{2018}$
$= 500 + 1{,}100 - 550 = 1{,}050$

Ultimate Claim Ratios including ALAE

	Property		Liability	
Accident Year	Gross	Net	Gross	Net
2017	45%	39%	55%	46%
2018	46%	40%	60%	52%
2019	44%	39%	65%	59%
2020	47%	41%	70%	66%

e.g., Property, Gross, AY2018 = 46% = 480 / 1,050

(b) Recommend expected claim ratios for each line of business, gross and net of reinsurance, that will be used in the determination of premium liabilities as of December 31, 2020. Justify each recommendation.

	Property		Liability	
	Gross	Net	Gross	Net
Recommended claim ratios	45.5%	39.9%	70.1%	65.6%

Justification:

- Property gross and net are stable with little discernible trend, so the average used.
- Liability has rising trend, so recommend using the latest year. [Could even project out a year].

(c) Calculate the premium liabilities, both gross and net of reinsurance.

		Property		Liability		Total	
		Gross	Net	Gross	Net	Gross	Net
(1)	Unearned premiums	650.00	514.00	3,000.00	2,460.00	3,650.00	2,974.00
(2)	Selected claim ratios	46%	40%	70%	66%		
(4)	Expected claims = (1)(2)	295.98	205.10	2,102.75	1,613.65	2,398.73	1,818.75
(5)	ULAE = 2,398.73×10%					239.87	239.87
(6)	General expenses = 3,650.00×15%×25%					136.88	136.88
(7)	Incentive commissions = 3,650.00×3%					109.50	109.50
(8)	Premium liabilities = sum[(4),(5),(6),(7)]					2,884.98	2,304.99

(d) Determine the equity in unearned premiums.

Equity in unearned premiums = UEP_{net} – Premium $liabilities_{net}$

= 2,974.00 – 2,304.99 = 669.01.

GIRR, Fall 2021, Q8

Learning Objectives:

3. The candidate will understand the procedure for estimating premium liabilities.

Learning Outcomes:

(3a) Understand the purpose of general insurance premium liabilities.

(3b) Calculate the premium liabilities for a general insurance company.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

Commentary on Question:

This question tests the candidate's understanding of unearned premiums and premium liabilities.

Solution:

(a) Calculate unearned premium by line of business as of December 31, 2020.

	(1)	(2) = (1)/12	(3)	(4)
			Unearned Premiums (UEP) by Quarter	
Underwriting Quarter	Months Remaining at Dec. 31, 2020	Unearned Proportion	Auto	Homeowners
2020Q1	1.5	0.125	26,250.00	40,000.00
2020Q2	4.5	0.375	75,187.50	121,875.00
2020Q3	7.5	0.625	123,437.50	206,250.00
2020Q4	10.5	0.875	179,462.50	281,750.00
Unearned Premiums at Dec. 31, 2020			404,337.50	649,875.00

Notes: (3) = (2)×(Auto Written Premiums)
(4) = (2)×(Homeowners Written Premiums)

(b) Calculate the equity in unearned premiums as of December 31, 2020 by line of business.
Homeowners expected claims need to be calculated by quarter; auto does not since expected claim ratios are the same for each quarter.

2021 Unexpired Months Allocated to Accident Quarter

Underwriting Quarter	Q1	Q2	Q3	Q4	Total Months Unexpired
2020Q1	1.5				1.5
2020Q2	3	*1.5*			*4.5*
2020Q3	3	3	1.5		7.5
2020Q4	3	3	3	1.5	10.5

2021 Unearned Premiums Allocated to Accident Quarter

Underwriting Quarter	Q1	Q2	Q3	Q4	Total
2020Q1	40,000				40,000
2020Q2	81,250	*40,625*			121,875
2020Q3	82,500	82,500	41,250		206,250
2020Q4	80,500	80,500	80,500	40,250	281,750
Total UEP	284,250	203,625	121,750	40,250	649,875
Gross Expected Claim Ratios	70%	70%	80%	70%	
Expected Claims	198,975	142,538	97,400	28,175	467,088

e.g., 2020Q2 @ 2021 Q2: 40,625 = 121,875×1.5/4.5
Expected claims for Q2: 142,538 = 203,625×70%

Auto expected claims = 72%×404,337.50 = 291,123

		Auto	Homeowners
(1)	Unearned premiums	404,337.50	649,875.00
(2)	Expected claims	291,123.00	467,087.50
(3)	ULAE (7.5%×(2) for Auto, 10%×(2) for Homeowners)	21,834.23	46,708.75
(4)	Maintenance expenses (15% ÷ 3 × (1))	20,216.88	32,493.75
(5)	Net premium liabilities ((2) + (3) + (4))	333,174.10	546,290.00
(6)	Equity/(Deficiency) ((1) – (5))	71,163.40	103,585.00

(c) Describe two potential implications of this result.

- A premium deficiency reserve may be required for the company.
- General Liability rates appear to be inadequate and should be reviewed.

GIRR, Spring 2022, Q14

Learning Objectives:

3. The candidate will understand the procedure for estimating premium liabilities.

Learning Outcomes:

(3a) Understand the purpose of general insurance premium liabilities.

(3b) Calculate the premium liabilities for a general insurance company.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a) Verify that the following amounts are consistent with the written premiums provided:

(i) Calendar half-year 2021-1 gross earned premium of 510,927

(ii) Year-end 2021 gross unearned premiums of 515,716

(i) Calendar half-year 2021-1 gross earned premium

Calendar/ Accident Half Year	Written Premiums	% Earned in 2021-1	Earned Premiums 2021-1
2020-1	500,255	25.0%	125,064
2020-2	518,366	50.0%	259,183
2021-1	506,720	25.0%	126,680
Total			510,927

This value is consistent.

(ii) Year-end 2021 gross unearned premiums:

Written Premiums 2021-1	Written Premiums 2021-2	Total
126,680	389,036	515,716

This value is consistent.

(b) Recommend the expected claim ratio to be used in the determination of premium liabilities as of December 31, 2021. Justify your recommendation.

	(1)	(2)	(3) = (2)/(1)	(4)	(5) = $1.01^{(4)}$	(6) = (3)(5)	(7) = (3)(5)
						Trended to 2021 Cost Level Claim Ratio	
Calendar/ Accident Half Year	Earned Premiums	Ultimate Claims including ALAE	Claim Ratio	# of Years for Past Trend (years)	Claim Trend	Jan-Jun	July-Dec
2019-1	518,804	364,784	70.31%	2	1.0201	71.73%	
2019-2	520,827	232,393	44.62%	2	1.0201		45.52%
2020-1	514,671	365,518	71.02%	1	1.0100	71.73%	
2020-2	509,071	229,396	45.06%	1	1.0100		45.51%
2021-1	510,927	366,542	71.74%	0	1.0000	71.74%	
2021-2	512,630	233,315	45.51%	0	1.0000		45.51%
Total	3,086,930	1,791,948	58.05%			71.73%	45.51%

Unearned premiums at Dec. 31, 2021	Policies Written in 2021-1	Policies Written in 2021-2	Total
Earned in 2022-1	126,680	259,357	
Earned in 2022-2		129,679	
Total	126,680	389,036	515,716
Average accident dates in 2021:	2021-04-01	2021-10-01	
Average accident dates in 2022:			
Earned in 2022-1	2022-02-15	2022-04-01	
Earned in 2022-2		2022-08-15	
Claim trend factors:			
Earned in 2022-1	1.00878	1.00497	
Earned in 2022-2		1.00872	
Expected claim ratio			
Earned in 2022-1	72.36%	72.089%	
Earned in 2022-2		45.911%	
Weighted average expected claim ratio:		65.573%	

(b) Calculate the premium liabilities as of December 31, 2021, both gross and net of reinsurance.

		Gross	Net
(1)	Unearned premium reserve	515,716	386,787
(2)	Expected claim ratio	65.573%	65.573%
(3)	Expected claims = (1)(2)	338,173	253,629
(4)	Expected ULAE = $5.7\% \times (3)_{gross}$	19,276	19,276
(5)	Maintenance expenses = $30\% \times 18\% \times (1)_{gross}$	27,849	27,849
(6)	Total premium liabilities = (3) + (4) + (5)	385,297	300,754

GIRR, Fall 2022, Q9

Learning Objectives:

3. The candidate will understand the procedure for estimating premium liabilities.

Learning Outcomes:

(3a) Understand the purpose of general insurance premium liabilities.

(3b) Calculate the premium liabilities for a general insurance company.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a) Calculate the premium deficiency reserve or equity in the unearned premium as of December 31, 2021.

	Gross	Net
Unearned Premium	5,000,000	4,000,000
Expected Claims		
(unearned premium × expected claims ratio):	3,000,000	2,400,000
ULAE = 3,000,000 × 10%	300,000	300,000
Reinsurance cost = 4,000,000 × 5% (net only)		200,000
General expenses = 5,000,000 × 20% × 25%	250,000	250,000
Premium Liabilities	3,550,000	3,150,000

Equity/(deficiency)= 4,000,000 – 3,150,000 = 850,000

Therefore, there is equity in the unearned premium.

(b) Recalculate the premium deficiency reserve or equity in the unearned premium as of December 31, 2021, incorporating this legislative change.

	Net
Unearned Premium	4,000,000
Expected Claims	
(unearned premium × expected claims ratio × 1.5):	3,600,000
ULAE = 3,000,000 × 10% × 1.5	450,000
Reinsurance cost = 4,000,000 × 5% (net only)	200,000
General expenses = 5,000,000 × 20% × 25%	250,000
Premium Liabilities	4,500,000

Equity/(deficiency)= 4,000,000 – 4,500,000 = (500,000)

Therefore, there is a premium deficiency reserve.

GI 301 Learning Objective 4 Curated Past Exam Solutions

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GIADV, Fall 2020, Q6

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) and (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (c) and (d) are for explanatory purposes only.

Solution:

(a) There are two sources of systemic risk: internal risk and external risk.

Define each source.

Internal risk is the extent to which the adopted actuarial valuation approach is an imperfect representation of the real-life process. External risk is all risks that are outside the modeling process.

(b) There are two sources of independent risk: parameter risk and process risk.

Define each source.

Parameter risk is the extent to which randomness prevents accurate selection of parameters. Process risk is the pure randomness associated with the insurance process.

(c) Calculate the coefficient of variation for each risk source for both lines combined.

The solution below uses the following abbreviations:
M = line of business Motor, H = line of business Home
CoV = coefficient of variation
p_x *= percentage of liabilities for line of business x*
IND_x *= independent risk CoV for line of business x*
ISR_x *= internal systemic risk CoV for line of business x*
ESR_x *= external systemic risk CoV for line of business x*
$\rho_{IND(MH)}$ *= correlation between motor and home liabilities for IND*
$\rho_{ISR(MH)}$ *= correlation between motor and home liabilities for ISR*
$\rho_{ESR(MH)}$ *= correlation between motor and home liabilities for ESR*

IND CoV
$= [p_M^2 \times IND_M^2 + p_H^2 \times IND_H^2 + 2 \times p_M \times p_H \times IND_M \times IND_H \times \rho_{IND(MH)}]^{0.5}$
$= 5.200\%$

ISR CoV =
$= [p_M^2 \times ISR_M^2 + p_H^2 \times ISR_H^2 + 2 \times p_M \times p_H \times ISR_M \times ISR_H \times \rho_{ISR(MH)}]^{0.5}$
$= 4.508\%$

ESR CoV =
$= [p_M^2 \times ESR_M^2 + p_H^2 \times ESR_H^2 + 2 \times p_M \times p_H \times ESR_M \times ESR_H \times \rho_{ESR(MH)}]^{0.5}$
$= 2.988\%$

(d) Calculate the consolidated coefficient of variation from the three sources of uncertainty. Assume independence between each of the sources of uncertainty.

Consolidated CoV = $[\text{IND CoV}^2 + \text{ISR CoV}^2 + \text{ESR CoV}^2]^{0.5} = 7.503\%$

GIADV, Spring 2021, Q6

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) through (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) through (c) are for explanatory purposes only.

Solution:

(a) Verify that the internal systemic risk coefficient of variation is 5.0% (rounded to one decimal place).

Commentary on Question:

In the solution that follows, claim liabilities are shown in thousands.

$[(0.05^2)(8 / 12)^2 + (0.09^2)(4 / 12)^2 + (2)(0.05)(0.09)(0.25)(8)(4) / (12^2)]^{0.5}$
$= 0.05011$

(b) Calculate the aggregate coefficient of variation for both lines combined.

$[0.052^2 + 0.050^2 + 0.033^2]^{0.5} = 0.07933$

(c) Calculate the amount of the risk margin at the 80% adequacy level.

$(0.07933)(12{,}000)(0.8416) = 801$

(d) Provide one argument in favor of and one argument against assuming the lognormal distribution for claims in this situation.

In favor is the fact that the distribution is likely to be positively skewed and this is better represented by the lognormal distribution.

Against is the fact that at lower adequacy levels the lognormal distribution provides smaller margins and/or lognormal decreases (relative to the CoV) as the CoV increases.

GIADV, Fall 2021, Q6

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) and (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (c) and (d) are for explanatory purposes only.

Solution:

(a) Identify the source of uncertainty to which each of the following belongs:

(i) uncertainty from changes to the process of setting up case reserves

(ii) insurance process too complex for any model to fully capture

(iii) unavailability of data required to conduct a credible valuation

(iv) randomness associated with the insurance process compromising the ability to select appropriate parameters

(v) uncertainty of claim costs arising from catastrophes

(vi) pure effect of the randomness associated with the insurance process

(i) III (external systemic risk)
(ii) II (internal systemic risk)
(iii) II (internal systemic risk)
(iv) I (independent risk)
(v) III (external systemic risk)
(vi) I (independent risk).

(b) Provide two reasons why stochastic modeling techniques do not enable a complete analysis of all sources of uncertainty.

Commentary on Question:

There are several reasons that can be provided. Only two reasons were required for full credit. The model solution is an example of a full credit solution providing two reasons.

A good stochastic model will fit the past data well and, in doing so, fit away most past systemic episodes of risk external to the valuation process, leaving behind largely random sources of uncertainty.

A stochastic model is highly unlikely to incorporate uncertainty arising from sources internal to the actuarial valuation process, i.e., internal systemic risk.

(c) Calculate the coefficient of variation for each risk source for the total insurance liabilities.

Commentary on Question:
The following abbreviations are used here: IND = independent risk, INT = internal systemic risk, EXT = external systemic risk, CoV = coefficient of variation, ρ(X) is the correlation between CL and PL for risk source X, A(CL) and A(PL) represent the amount of liabilities for CL and PL respectively, and A = A(CL) + A(PL).

For each risk source, CoV for CL and PL combined is given by:
$[\text{CoV}(CL)^2 \times (A(CL)/A)^2 + \text{CoV}(PL)^2 \times (A(PL)/A)^2$
$+ 2 \times \rho(X) \times \text{CoV}(CL) \times \text{CoV}(PL) \times (A(CL)/A) \times (A(PL)/A)]$^0.5

	IND	INT	EXT
CoV by risk source	3.94%	3.75%	4.04%

(d) Calculate the amount of the risk margin for the total insurance liabilities at the 75% adequacy level.

Total CoV = [3.94%^2 + 3.75%^2 + 4.04%^2]^0.5 = 6.78%

Risk Margin = A × Total CoV × z-value at 75^{th} percentile of normal distribution
= 15,000 × 6.78% × 0.674 = 685.1

GIADV, Spring 2022, Q6

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Define internal systemic risk.

Internal systemic risk refers to the uncertainty arising from the actuarial valuation models used being an imperfect representation of the insurance process as it pertains to insurance liabilities.

(b) Describe how internal systemic risk contributes to correlation effects in an assessment of insurance liability risk margins.

It contributes through the correlation between valuation classes and the correlation between outstanding claim liabilities and premium liabilities.

(c) Describe the three main sources of internal systemic risk:

(i) Specification error

(ii) Parameter selection error

(iii) Data error

(i) The error that can arise from an inability to build a model that is fully representative of the underlying insurance process.

(ii) The error that can arise because the model is unable to adequately measure all predictors of claim cost outcomes or trends in these predictors.

(iii) The error that can arise due to poor data or unavailability of data required to conduct a credible valuation.

(d) Identify which main source of internal systemic risk corresponds to each of the following potential risk indicators:

(i) Best predictors have been identified

(ii) Extent, timeliness, consistency and reliability of information

(iii) Knowledge of past processes affecting predictors

(iv) Number and importance of subjective adjustments to factors

(v) Ability to detect trends in key claim cost indicators

(vi) Value of predictors used

(i) Parameter selection error
(ii) Data error
(iii) Data error
(iv) Specification error
(v) Specification error
(vi) Parameter selection error

(e) Provide the reasoning behind using a CoV scale with these two characteristics.

Commentary on Question:
The model solution provides reasoning for each. Alternative correct responses were acceptable. For example, an alternative correct response for the characteristic "higher for long-tail lines" is that "it is generally more difficult to develop a modelling approach that is representative of the underlying insurance process for long-tail LoBs."

- Nonlinearity: The marginal improvement in outcomes between fair and good modelling infrastructures is less than the marginal improvement between poor and fair modelling infrastructures.
- Higher for long-tail lines: Key predictors are often less stable for long-tail LoBs and past episodes of systemic risk more likely to impair the ability to fit a good model.

GIADV, Fall 2022, Q6

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the internal systemic risk CoV for each of the following:

(i) Motor CL

(ii) Home CL

(iii) Total CL

For (i) and (ii), the first step is to know which source of risk is associated with each of the 6 potential risk indicators. 1 is parameter risk, 2 is data risk, 3 is specification risk, 4 is specification risk, 5 is data risk and 6 is parameter risk. Given the weights of 25% for specification risk, 55% for parameter risk and 20% for data risk and the fact that risk indicator weights are equal within each source of internal systemic risk (ISR), we have the following weights by potential risk indicator: 27.5% for each of 1 and 6, 10% for each of 2 and 5 and 12.5% for each of 3 and 4. This gives a weighted average score of 4.6 for Motor CL and 3.35 for Home CL. Using the scorecard table, this produces an ISR_CoV of 7% for Motor CL and 7.5% for Home CL. For (iii), we note that the percentage of CL in Motor is 5,351 / (5,351 + 2,486) = 68.3% (P_M) and percentage of CL in Home is 1 – 68.3% = 31.7% (P_H). Then the Total CL internal systemic risk CoV is given by: $[\text{ISR\_CoV}_M^2\, P_M^2 + \text{ISR\_CoV}_H^2\, P_H^2 + 2 \times \text{Correlation(Motor CL, Home CL)} \times \text{ISR\_CoV}_M\, \text{ISR\_CoV}_H\, P_M\, P_H]^{0.5}$ = 6.04%.

(b) Calculate the internal systemic risk CoV for total insurance liabilities, both lines combined.

This is the square root of the sum of the product of the following 3 matrices:

1. internal systemic risk correlation matrix of i and j
2. the CoV_i times CoV_j matrix
3. liability weight (W) matrix of W_i time W_j

Matrix 1 is provided.
Matrix 2 is created from the ISR CoVs.
Matrix 3 is created from the weights by liability.

This results in the ISR_CoV for total insurance liabilities, both lines combined, being equal to 5.01%.

(c) Calculate the risk margin for the total insurance liabilities at the 75% adequacy level.

Commentary on Question:
The model solution uses the exact value of the z-value (0.67448975..). It was also acceptable to use the approximate value of 0.674.

This is equal to:
z-value × the total insurance liabilities
× [CoV Independent Risk squared + CoV External Systemic Risk squared + CoV Internal Systemic Risk squared + 2 × 20% × CoV External Systemic Risk × CoV Internal Systemic Risk]$^{0.5}$
= *z*-value × 16,045,000 × 0.112089
= 1,213,047

GIADV, Spring 2023, Q8

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4a) Describe a risk margin analysis framework.
(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.
(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for part (c). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (c) is for explanatory purposes only.

Solution:

(a) Describe two of these components, other than the three listed above.

Commentary on Question:

There are many components. Only two were required for full credit. The model solution is an example of a full credit solution.

- Portfolio preparation: Determine valuation portfolio, claim groups and techniques to deploy for each claim group.
- Analysis of correlation effects: Select correlation coefficients between valuation classes and between outstanding claim and premium liabilities for internal systemic risk and for each external systemic risk category.

(b) Identify four subjective decisions that are required in this approach.

- risk indicators
- measurement and scoring criteria
- importance afforded to each risk indicator
- CoVs that map to each score from the balanced scorecard

(c) Calculate the following:

(i) Total independent risk CoV for both valuation classes combined (***X***)

(ii) Correlation between the valuation classes for outstanding claims for internal systemic risk

(iii) Internal systemic risk CoV for premium liabilities for both valuation classes combined (***Y***)

(iv) Total external systemic risk CoV for both valuation classes combined (***Z***)

Commentary on Question:

OC = Outstanding Claims, PL = Premium Liabilities, VC = Valuation Class,
AUT = Auto VC, LIA= Liability VC
IND = Independent Risk, ISR = Internal Systemic Risk,
ESR = External Systemic Risk

(i) ***X*** = square root of the sum of the squares (over VC and type of liability) of the % of total liabilities × independent risk CoV

(ii) Correlation between the VCs for OC for ISR = [(square of total ISR CoV for OC × % of OC in total liabilities) − (sum over VC of the squares of OC ISR CoV × % of OC in total liabilities)] ÷ [2 × (OC ISR CoV for AUT × % AUT OC in total liabilities) × (OC ISR CoV for LIA × % LIA OC in total liabilities)]

(iii) ***Y*** = square root of [sum over VC of the squares of (ISR CoV for PL × % PL in total liabilities) + 2 × (ISR CoV for AUT PL × % AUT PL in total liabilities) × (ISR CoV for LIA PL × % LIA PL in total liabilities) × amount from part (ii)] ÷ % PL in total liabilities

(iv) ***Z*** = square root of [square of (ESR CoV for AUT × % AUT in total liabilities) + square of (ESR CoV for LIA × % LIA in total liabilities) + 2 × ESR correlation × (AUT ESR CoV × % AUT in total liabilities) × (LIA ESR CoV times % LIA in total liabilities)]

GIADV, Fall 2023, Q8

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4a) Describe a risk margin analysis framework.
(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.
(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Identify two reasons that quantitative methods should not be used to assess these correlation effects.

Commentary on Question:

There are more than two reasons. The model solution is an example of a full credit solution.

- It's difficult to separate the past correlation effects between independent risk and systemic risk.
- Correlations associated with external systemic risk sources may differ materially from correlations associated with past episodes of systemic risk.

(b) Complete the following internal systemic risk balanced scorecard:

Complete in the Excel spreadsheet.

Risk Source	Risk Indicator	Score (1 or 5)	Reason for receiving the score	Weight
Parameter selection error	Ability to identify and use best predictors	5		30%
	Best predictors are stable over time	1		
Specification error				40%
Data error				30%

Commentary on Question:
There are many ways to complete this table and have it represent a proper solution that earns full credit, The model solution is an example of a full credit solution.

Risk Source	Risk Indicator	Score (1 or 5)	Reason for receiving the score	Weight
Parameter selection error	Ability to identify and use best predictors	5		30%
	Best predictors are stable over time	1		
Specification error	Range of results produced by models	1	Large variance between the two model results suggests great uncertainty in our ability to model. More modeling approaches may need to be considered.	40%
	Ability to model using more granular data	5	Claim level data is available and can help better understand key predictors.	
Data error	Timeliness, consistency and reliability of information from business	5	Regular communication between actuaries and the portfolio managers can ensure timeliness and reliability of information.	30%
	Data subject to appropriate reconciliation	5	The data is reconciled against another source with differences well understood.	

(c) Select the internal systemic risk CoV using the completed internal systemic risk balanced scorecard from part (b).

First calculate the weighted average score using part (b):

$$5\times(0.3/2) + 1\times(0.3/2) + 1\times(0.4/2) + 5\times(0.4/2) + 5\times(0.3/2) + 5\times(0.3/2) = 3.6$$

Looking up the weighted average score of 3.6 in the CoV scale table gives an Internal Systemic Risk CoV of 6.5%.

GIADV, Spring 2024, Q8

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4a) Describe a risk margin analysis framework.
(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.
(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question tested a candidate's understanding of risk margins as set out in Marshall. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe two considerations why correlation effects exist within internal systemic risk.

Commentary on Question:

The model solution is an example of a full credit solution.

The correlation between valuation classes may be caused by the "same actuary effect" and the use of the same template or valuation models across different valuation classes.

There may be linkages between the premium liability methodology and outcomes from the outstanding claim valuation, which contributes to the correlation effects between outstanding claim and premium liabilities.

(b) Calculate the following for the company:

(i) Total internal systemic risk CoV

(ii) Total external systemic risk CoV

(iii) Total consolidated CoV for all sources of risk

(iv) Risk margin at the 80% adequacy level

Commentary on Question:

ISR = Internal Systemic Risk, ESR = External Systemic Risk,
ISRρ(A, B)= ISR Correlation between A and B, CE = Central Estimate

(i)

Calculate the ISR variances for both OSC and PL, for both Auto and Home as $(\text{CE} \times \text{ISR CoV})^2$

ISR covariance for (Auto OSC, Auto PL) is calculated as follows

$ISR\rho(\text{Auto OSC, Auto PL}) \times ISR\ CoV(\text{Auto OSC}) \times ISR\ CoV(\text{Auto PL}) \times CE(Auto\ OSC) \times CE(Auto\ PL)$

ISR covariances for (Auto OSC, Home OSC) and (Auto PL, Home OSC) are calculated similarly. Note that Home PL has no correlation with the other Line-Liability combinations, so it may be excluded from the ISR covariance calculations.

$\text{Total ISR CoV} = [\sum \text{ISR variances} + 2 \times \sum \text{ISR covariances}]^{1/2} / \sum \text{CE}$

(ii)

$\text{Total ESR CoV} = [\sum[\text{CE(i)ESR CoV(i)}]^{1/2} / \sum \text{CE}$

(iii)

Total Consolidated CoV

$= [\text{Independent risk CoV}^2 + \text{Total ISR CoV}^2 + \text{Total ESR CoV}^2]^{1/2}$

(iv)

$\text{Risk Margin 80\% adequacy} = \text{Total Consolidated CoV} \times z\ value_{0.8} \times \sum \text{CE}$

GIADV, Fall 2024, Q8

Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(4a) Describe a risk margin analysis framework.
(4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.
(4c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question tested a candidate's understanding of the framework for assessing risk margins as presented by Marshall et al.

Solution:

(a) Complete the following table using the information provided above.

Internal Systemic Risk			
Risk	**Risk Component**	**Risk Indicator**	**Which of I - VI are considered when scoring this risk indicator against best practice**
1			
2			
3			

Commentary on Question:

The model solution is an example of a full credit solution.

Internal Systemic Risk			
Risk	**Risk Component**	**Risk Indicator**	**Which of I - VI are considered when scoring this risk indicator against best practice**
1	Specification Error	Extent of monitoring and review of model and assumption performance	IV
2	Parameter Selection Error	Ability to identify and use best predictors	III
3	Data Error	Timeliness, consistency and reliability of information from business	V

(b) Identify two of these categories of ESR sources that are created from the information provided. Identify which of I through VI creates each.

Commentary on Question:
The model solution is an example of a full credit solution.

- Claim management process change risk from I
- Legislative risk from VI

(c) Select the appropriate CoV to be used for each line of business. Justify your selections.

Commentary on Question:
The model solution is an example of a full credit solution.

- Scales 1 and 2 are not reasonable because a higher score should be associated with a lower CoV.
- Scale 5 is not reasonable as the scale should not be linear.
- Scales 3 and 4 appear reasonable.
- CoVs for a long-tail line are generally higher than those for a short-tail line. Therefore, Scale 3 should be used for property and Scale 4 should be used for liability.
- With a weighted score of 3.9 for each line of business, we have:
  - Property CoV = 5.5%
  - Liability CoV = 9.5%

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GIRR, Fall 2020, Q14

Learning Objectives:

5. The candidate will understand the methods to monitor actual versus expected experience.

Learning Outcomes:

(5a) Identify and describe approaches for monitoring results.

(5b) Prepare a comparison of actual to expected claims.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 37.

Commentary on Question:

This question tests the candidate's understanding of various diagnostic tests on development triangles. This question also tests the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

(a) Calculate the difference between the actual and expected reported claims from December 31, 2019 through September 30, 2020 for all accident years, using a linear interpolation of the development pattern.

Accident Year	(1) Ultimate	(2) Reported Claims @ Dec. 31, 2019	(3) Reported Claims @ Sep. 30, 2020	(4) = (2)/(1) Expected % Reported @ Dec. 31, 2019	(5) Expected % Reported @ Sep. 30, 2020
2014	6,557	6,557	6,557	100.0%	100.0%
2015	7,293	7,242	7,283	99.3%	99.8%
2016	8,087	7,544	7,923	93.3%	97.8%
2017	7,150	5,988	6,572	83.7%	90.9%
2018	7,572	5,018	6,335	66.3%	79.4%
2019	7,875	3,537	5,129	44.9%	60.9%
Total	44,534	35,886	39,799		

e.g., (5) for AY2019: 60.9% = (3/12)×44.9% + (9/12)×66.3%

AY	(6) Expected Reported at 9 months	(7) Actual Reported at 9 months	(8) = (7) – (6) Difference
2014	0	0	0
2015	38	41	3
2016	365	379	14
2017	511	584	73
2018	993	1,317	324
2019	1,261	1,592	331
Total	3,168	3,913	745

e.g., (6) for AY2019:

$$\frac{[(1)-(2)]\times[(5)-(4)]}{1-(4)}=\frac{(7,875-3,537)\times(.609-.449)}{1-.449}=1,261$$

(b) Provide an interpretation of the results for the actual versus expected analysis derived in part (a).

The actual versus expected differences are significant in recent years. This means that the development factor assumptions were not appropriate for this projection.

(c) Evaluate if the data indicates a possible change in case reserve adequacy using two different diagnostic tests.

Commentary on Question:
Note: The reported claims for AY 2018 at 12 months was given as 3.292, but it should have been 3,292. Some candidates noticed the error, and some did not. However, credit was given regardless of the value used, provided the work was done correctly.

Accident	Ratios of Paid Claims to Reported Claims					
Year	12	24	36	48	60	72
2014	0.46	0.59	0.71	0.78	0.87	0.92
2015	0.46	0.60	0.72	0.78	0.86	
2016	0.48	0.60	0.72	0.78		
2017	0.45	0.60	0.69			
2018	454.43	0.57				
2019	0.41					

The latest diagonal shows a decrease in ratios which could mean a decrease in settlement (numerator) or increase in case reserve adequacy (denominator).

Accident	Average Case Estimates					
Year	12	24	36	48	60	72
2014	5.508	10.142	12.651	13.500	11.583	9.596
2015	5.775	10.613	13.258	14.154	12.155	
2016	6.042	11.183	13.853	14.928		
2017	6.292	11.815	16.541			
2018	–5.488	13.728				
2019	7.824					

The latest diagonal shows a decrease in ratios which could mean a decrease in settlement (numerator) or increase in case reserve adequacy (denominator).

(d) Evaluate if the data indicates a possible change in case settlement rates using a diagnostic test different than either of the two tests from part (c).

Accident Year	Reported Counts 12	24	36	48	60	72
2014	774	842	853	853	853	853
2015	807	883	890	890	890	
2016	830	927	938	938		
2017	734	797	808			
2018	724	799				
2019	714					

Accident Year	Ratios of Closed Counts to Reported Counts 12	24	36	48	60	72
2014	0.62	0.79	0.85	0.88	0.92	0.93
2015	0.60	0.79	0.86	0.88	0.91	
2016	0.63	0.79	0.86	0.88		
2017	0.62	0.80	0.86			
2018	0.62	0.80				
2019	0.63					

Closed to reported counts are relatively stable which means a change in settlement rate is not likely.

GIRR, Fall 2021, Q9

Parts (a) to (c) are not in GI 301

Learning Objectives:

5. The candidate will understand the methods to monitor actual versus expected experience.

Learning Outcomes:

(5a) Identify and describe approaches for monitoring results.

(5b) Prepare a comparison of actual to expected claims.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 37.

Commentary on Question:

This question tests the calculation of ultimate claims using the Cape Cod method. This question also tests the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

~~*(a) Describe why an exposure base that is not inflation-sensitive is preferred over an exposure base that is inflation-sensitive.*~~

Part (a) not in GI 301

~~*The exposure base that requires the least adjustment is preferred because additional adjustments add imprecision to the projection process.*~~

~~*(b) Derive a selected adjusted expected pure premium.*~~

Part (b) not in GI 301

-	~~(1)~~	~~(2)~~	~~(3)~~	~~(4) = 1/(3)~~
~~Accident Year (AY)~~	~~Earned Exposures~~	~~Reported Claims as of Dec. 31, 2020~~	~~Cumulative Development Factors~~	~~Expected % Developed~~
~~2012~~	~~8,391~~	~~1,002~~	~~1.008~~	~~99.2%~~
~~2013~~	~~8,402~~	~~1,045~~	~~1.020~~	~~98.0%~~
~~2014~~	~~8,788~~	~~1,216~~	~~1.038~~	~~96.3%~~
~~2015~~	~~9,088~~	~~664~~	~~1.063~~	~~94.0%~~
~~2016~~	~~9,325~~	~~710~~	~~1.097~~	~~91.1%~~
~~2017~~	~~9,704~~	~~593~~	~~1.146~~	~~87.3%~~
~~2018~~	~~10,073~~	~~739~~	~~1.227~~	~~81.5%~~
~~2019~~	~~10,339~~	~~632~~	~~1.432~~	~~69.8%~~
~~2020~~	~~10,591~~	~~448~~	~~2.148~~	~~46.6%~~
~~Total~~	~~84,701~~	~~7,049~~		

~~*e.g., Column (3) for 2015: 1.063 = 1.008×1.012×1.018×1.024*~~

~~AY~~	~~(5) = (1)(4)~~ ~~Used-Up On-Level Exposures~~	~~(6) = 0.99^(2020-AY)~~ ~~Pure Premium Trend~~	~~(7)~~ ~~Tort Reform~~	~~(8) = (2)(6)(7)~~ ~~Adjusted Claims at Dec. 31, 2020~~
~~2012~~	~~8,324~~	~~0.923~~	~~0.950~~	~~878~~
~~2013~~	~~8,236~~	~~0.932~~	~~0.950~~	~~925~~
~~2014~~	~~8,463~~	~~0.941~~	~~0.950~~	~~1,088~~
~~2015~~	~~8,546~~	~~0.951~~	~~0.950~~	~~600~~
~~2016~~	~~8,497~~	~~0.961~~	~~0.950~~	~~648~~
~~2017~~	~~8,470~~	~~0.970~~	~~0.950~~	~~547~~
~~2018~~	~~8,209~~	~~0.980~~	~~1.000~~	~~724~~
~~2019~~	~~7,220~~	~~0.990~~	~~1.000~~	~~626~~
~~2020~~	~~4,931~~	~~1.000~~	~~1.000~~	~~448~~
~~Total~~	~~70,897~~			~~6,484~~

~~Adjusted expected pure premium = 6,484 / 70,897 = 0.0915~~

(c) ~~Derive projected ultimate claims.~~

Part (c) not in GI 301

~~AY~~	~~(9) = 0.0915×(1)/[(6)(7)]~~ ~~Expected Claims (Ultimate)~~	~~(10) = 1 – (4)~~ ~~Expected % Undeveloped~~	~~(11) = (9)(10)~~ ~~Expected Unreported~~	~~(12) = (2) + (11)~~ ~~Projected Ultimate Claims~~
~~2012~~	~~875~~	~~0.8%~~	~~7~~	~~1,009~~
~~2013~~	~~868~~	~~2.0%~~	~~17~~	~~1,062~~
~~2014~~	~~899~~	~~3.7%~~	~~33~~	~~1,249~~
~~2015~~	~~920~~	~~6.0%~~	~~55~~	~~719~~
~~2016~~	~~934~~	~~8.9%~~	~~83~~	~~793~~
~~2017~~	~~963~~	~~12.7%~~	~~122~~	~~715~~
~~2018~~	~~940~~	~~18.5%~~	~~174~~	~~913~~
~~2019~~	~~955~~	~~30.2%~~	~~288~~	~~920~~
~~2020~~	~~969~~	~~53.4%~~	~~518~~	~~966~~
~~Total~~	~~8,322~~		~~1,297~~	~~8,346~~

(d) Calculate the difference between the expected reported claims underlying the Cape Cod calculations in part (c) and actual reported claims as of December 31, 2020.

AY	(2) Reported Claims as of Dec. 31, 2020	(13) = (9) - (11) Expected Reported Claims	(14) = (2) - (13) Difference Actual vs. Expected
2012	1,002	868	134
2013	1,045	851	194
2014	1,216	865	351
2015	664	865	(201)
2016	710	852	(142)
2017	593	840	(247)
2018	739	766	(27)
2019	632	667	(35)
2020	448	451	(3)
Total	7,049	7,025	24

(e) Describe two other possible circumstances that could cause an anomaly as shown above.

Commentary on Question:
Other possible circumstances are possible.

Any two of the following are acceptable:
- Development may be lower in recent years due to operational changes or changes in experience.
- Experience may have improved beginning in AY 2016. Maybe frequency or severity improved due to loss prevention or loss control activities.
- A policy change may have been made in 2016 which reduced claim exposure (e.g., higher deductible).

GIRR, Spring 2022, Q5

Parts (a) to (f) are not in GI 301

Learning Objectives:

5. The candidate will understand the methods to monitor actual versus expected experience.

Learning Outcomes:

(5a) Identify and describe approaches for monitoring results.

(5b) Prepare a comparison of actual to expected claims.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 37.

Commentary on Question:

This question tests the candidate's understanding of estimating ultimate claims using the development method, expected method and the Bornhuetter Ferguson method. This question also tests the estimation of claim liabilities and the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

(a) ~~Calculate the ultimate claims for all accident years using the development method with paid claims. Justify any selections you make.~~

Not in GI 301 LO 5

~~*Paid Claims Age-to-age factors*~~

~~*AY*~~	~~*12-24*~~	~~*24-36*~~	~~*36-48*~~	~~*48-60*~~	~~*60-72*~~	~~*72-84*~~	
~~*2015*~~	~~*1.949*~~	~~*1.344*~~	~~*1.195*~~	~~*1.096*~~	~~*1.050*~~	~~*1.011*~~	
~~*2016*~~	~~*1.691*~~	~~*1.332*~~	~~*1.210*~~	~~*1.059*~~	~~*1.058*~~		
~~*2017*~~	~~*1.828*~~	~~*1.445*~~	~~*1.201*~~	~~*1.067*~~			
~~*2018*~~	~~*1.770*~~	~~*1.359*~~	~~*1.263*~~				
~~*2019*~~	~~*1.749*~~	~~*1.278*~~					
~~*2020*~~	~~*1.528*~~	-	-	-	-	-	
~~*Simple All*~~	~~*1.752*~~	~~*1.352*~~	~~*1.217*~~	~~*1.074*~~	~~*1.054*~~	~~*1.011*~~	
~~*Vol Wtd 5*~~	~~*1.704*~~	~~*1.349*~~					
~~*Vol Wtd All*~~	~~*1.737*~~						
~~*Medial All*~~	~~*1.810*~~	~~*1.361*~~	~~*1.198*~~	-	-	-	~~*Tail factor*~~
~~*Selected:*~~	~~*1.810*~~	~~*1.361*~~	~~*1.217*~~	~~*1.074*~~	~~*1.054*~~	~~*1.011*~~	~~*1.011*~~

~~*Rationale for selections:*~~

- ~~*Medial all selected for 12-24 and 24-36 due to outliers*~~
- ~~*Simple all years average selected thereafter*~~
- ~~*Bondy method selected for tail factor as there was still development at 84 months*~~

~~*AY*~~	~~*Paid Claims*~~	~~*Development Factors*~~ ~~*Age-to-Age*~~	~~*Age-to-Ult.*~~	~~*Ultimate Claims*~~
~~*2015*~~	~~*31,530*~~	~~*1.011*~~	~~*1.0106*~~	~~*31,866*~~
~~*2016*~~	~~*32,966*~~	~~*1.011*~~	~~*1.0214*~~	~~*33,671*~~
~~*2017*~~	~~*32,690*~~	~~*1.054*~~	~~*1.0765*~~	~~*35,189*~~
~~*2018*~~	~~*32,579*~~	~~*1.074*~~	~~*1.1561*~~	~~*37,665*~~
~~*2019*~~	~~*26,519*~~	~~*1.217*~~	~~*1.4071*~~	~~*37,315*~~
~~*2020*~~	~~*19,889*~~	~~*1.361*~~	~~*1.9148*~~	~~*38,083*~~
~~*2021*~~	~~*12,410*~~	~~*1.810*~~	~~*3.4648*~~	~~*42,999*~~
~~*Total*~~	~~*188,583*~~			~~*256,789*~~

~~*e.g., AY2017:*~~

~~*1.0765 = 1.011×1.011×1.054*~~

~~*35,189 = 32,690×1.0765*~~

(b) ~~Calculate the trended on-level claim ratios for all accident years using the ultimate claims calculated in part (a).~~

Not in GI 301 LO 5

~~*Annual claim ratio trend = (1.05)(1 – 0.013) – 1 = 3.635%*~~

~~*AY*~~	~~*Earned Premiums*~~	~~*Premium On-Level Factors*~~	~~*Claim Trend @3.635%*~~	~~*Ultimate Paid Claims (000)*~~	~~*Trended On-Level Claim Ratio based on Paid Claims*~~
~~*2015*~~	~~*49,736,108*~~	~~*1.0722*~~	~~*1.2389*~~	~~*31,866*~~	~~*74.03%*~~
~~*2016*~~	~~*52,114,124*~~	~~*1.0681*~~	~~*1.1955*~~	~~*33,671*~~	~~*72.31%*~~
~~*2017*~~	~~*55,021,088*~~	~~*1.0420*~~	~~*1.1535*~~	~~*35,189*~~	~~*70.80%*~~
~~*2018*~~	~~*56,278,147*~~	~~*1.0265*~~	~~*1.1131*~~	~~*37,665*~~	~~*72.57%*~~
~~*2019*~~	~~*58,829,789*~~	~~*1.0182*~~	~~*1.0740*~~	~~*37,315*~~	~~*66.91%*~~
~~*2020*~~	~~*61,195,354*~~	~~*1.0092*~~	~~*1.0364*~~	~~*38,083*~~	~~*63.91%*~~
~~*2021*~~	~~*60,091,505*~~	~~*1.0000*~~	~~*1.0000*~~	~~*42,999*~~	~~*71.56%*~~

e.g., 2017:

70.80% = (35,189×1.1535×1000)/(55,021,088×1.0420)

(c) ~~Recommend a 2021 cost level expected claim ratio to use for estimating expected claims. Justify your recommendation.~~

Not in GI 301 LO 5

~~AY~~	~~Trended On-Level Claim Ratio based on Paid Claims~~	~~Reported Claims~~
~~2015~~	~~74.03%~~	~~76.80%~~
~~2016~~	~~72.31%~~	~~74.90%~~
~~2017~~	~~70.80%~~	~~73.80%~~
~~2018~~	~~72.57%~~	~~71.20%~~
~~2019~~	~~66.91%~~	~~77.70%~~
~~2020~~	~~63.91%~~	~~73.50%~~
~~2021~~	~~71.56%~~	~~79.40%~~
~~Average all years except 2021:~~	~~70.09%~~	~~74.65%~~
~~Average excluding high-low (except 2021):~~	~~70.65%~~	~~74.18%~~

~~Recommended claim ratio: 74.18%~~
~~Rationale: Recommend reported claim ratios as they seem more consistent. Exclude high and low to smooth fluctuations.~~

~~(d) Calculate expected claims for all accident years based on the recommendation in part (c).~~
Not in GI 301 LO 5

~~AY~~	~~Earned Premiums~~	~~Premium On-Level Factors~~	~~Claim Trend @3.635%~~	~~Claim Ratio at Each AY Level~~	~~Expected Claims~~
~~2015~~	~~49,736,108~~	~~1.0722~~	~~1.2389~~	~~64.19%~~	~~31,927,613~~
~~2016~~	~~52,114,124~~	~~1.0681~~	~~1.1955~~	~~66.27%~~	~~34,537,640~~
~~2017~~	~~55,021,088~~	~~1.0420~~	~~1.1535~~	~~67.00%~~	~~36,866,223~~
~~2018~~	~~56,278,147~~	~~1.0265~~	~~1.1131~~	~~68.41%~~	~~38,497,891~~
~~2019~~	~~58,829,789~~	~~1.0182~~	~~1.0740~~	~~70.32%~~	~~41,369,001~~
~~2020~~	~~61,195,354~~	~~1.0092~~	~~1.0364~~	~~72.23%~~	~~44,202,496~~
~~2021~~	~~60,091,505~~	~~1.0000~~	~~1.0000~~	~~74.18%~~	~~44,572,874~~
~~Total~~					~~271,973,737~~

~~e.g., 2017:~~
~~67.00% = 74.18%×1.0420/1.1535~~
~~36,866,223 = 67.00%×55,021,088~~

~~(e) Calculate ultimate claims for all accident years using the Bornhuetter Ferguson method based on paid claims. Use the expected claims from part (d).~~
Not in GI 301 LO 5

~~AY~~	~~Paid Claims (000)~~	~~Expected Claims~~	~~Age-to-Ultimate Development Factors~~	~~Ultimate Claims~~
~~2015~~	~~31,530~~	~~31,927,613~~	~~1.0106~~	~~31,866,187~~
~~2016~~	~~32,966~~	~~34,537,640~~	~~1.0214~~	~~33,689,509~~
~~2017~~	~~32,690~~	~~36,866,223~~	~~1.0765~~	~~35,308,413~~
~~2018~~	~~32,579~~	~~38,497,891~~	~~1.1561~~	~~37,777,546~~
~~2019~~	~~26,519~~	~~41,369,001~~	~~1.4071~~	~~38,488,222~~
~~2020~~	~~19,889~~	~~44,202,496~~	~~1.9148~~	~~41,006,787~~
~~2021~~	~~12,410~~	~~44,572,874~~	~~3.4648~~	~~44,118,543~~
~~Total~~	~~188,583~~			~~262,255,207~~

~~*e.g., 2017: 35,308,413 = 32,690×1,000 + 36,866,223×(1 – 1/1.0765)*~~

~~(f) Calculate the total unpaid claims for this line of business as of December 31, 3021, showing the case estimate and indicated IBNR separately.~~

Not in GI 301 LO 5

~~Total reported claims: 238,061,000~~

~~*Total unpaid claims = 271,794,051 – 188,583,000 = 83,211,051*~~
~~*Case estimate = 238,061,000 – 188,583,000 = 49,478,000*~~
~~*IBNR = 83,211,051 – 49,478,000 = 33,733,051*~~

(g) Calculate the difference between the actual and expected reported claims for this line of business from December 31, 2021 through March 31, 2022 for all accident years, using linear interpolation.

	(1)	(2)	(3)	(4)	(5)
	As of Dec. 31, 2021				
	Selected Ultimate		Reported Claims at	Expected % Reported at	
AY	Claims	Reported Claims	Mar. 31, 2022	Dec. 31, 2021	Mar. 31, 2022
2015	33,050,822	32,886,000	32,925,000	99.50%	99.63%
2016	34,902,242	34,555,000	34,599,600	99.01%	99.13%
2017	36,660,362	35,972,000	36,055,609	98.12%	98.34%
2018	37,986,078	35,453,000	36,105,780	93.33%	94.53%
2019	41,178,916	33,927,000	35,158,600	82.39%	85.12%
2020	42,698,643	31,041,000	32,342,000	72.70%	75.12%
2021	45,316,988	34,227,000	33,780,455	75.53%	74.82%
Total	271,794,051	238,061,000	240,967,044		

e.g., 2017:

(4): 98.12% = 36,660,362 / 35,972,000

(5): 98.34% = 98.12%×3/4 + 99.01%×1/4

	(6)	(7)	(8)
	Actual versus Expected Reported Claims from Dec. 31, 2021 through Mar. 31, 2022		
AY	Actual	Expected	Difference
2015	39,000	41,205	-2,205
2016	44,600	43,297	1,303
2017	83,609	80,907	2,702
2018	652,780	454,956	197,824
2019	1,231,600	1,126,481	105,119
2020	1,301,000	1,034,523	266,477
2021	-446,545	-320,630	-125,915
Total	2,906,044	2,460,739	445,305

e.g., 2017:

(6) = (3) – (2): 83,609 = 36,055,609 – 35,972,000

(7) = [(1) – (2)]×[(5) – (4)]/[1 – (4)]:

80,907 = (36,660,362 – 35,972,000)×(98.34% - 98.12%)/(1 – 98.12%)

(8) = (6) – (7): 2,702 = 83,609 – 80,907

(h) Provide an interpretation of the results for the actual versus expected analysis derived in part (g).

- Actual values are mostly significantly higher than expected, suggesting development factors are too low.
- 2021 actual value is much lower than expected, suggesting the development factor for 2021 is too high.

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GIRR, Fall 2020, Q3

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

Commentary on Question:

This question tests the candidate's understanding of claims-made ratemaking.

Solution:

(a) Describe why the risk of reserve inadequacy is greatly reduced for claims-made policies compared to occurrence policies.

Claims-made policies incur no liability for pure IBNR claims.

(b) Calculate the total reported claims for each of the following:

(i) A first-year claims-made policy effective January 1, 2013

(ii) A third-year claims-made policy effective January 1, 2015

(iii) A tail policy purchased after the third-year claims-made policy from part (b)(ii)

Accident Year Lag by Report Year Matrix of Ultimate Claims								
Accident Year Lag	**Report Year**							
	2011	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
0	160	168	176	185	194	204	214	225
1	240	252	265	278	292	306	322	338
2	240	252	265	278	292	306	322	338
3	160	168	176	185	194	204	214	225

(i) $C_{0,3}$	176
(ii) $C_{0,5} + C_{1,5} + C_{2,5}$	778
(iii) $C_{1,6} + C_{2,6} + C_{3,6} + C_{2,7} + C_{3,7} + C_{3,8}$	1,577

(c) Calculate each of the following factors for this coverage:

(i) A second-year claims-made step factor

(ii) A mature claims-made tail factor

Accident Year Lag	Reported Years							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	200.00	220.00	242.00	266.20	292.82			
1	181.82	200.00	220.00	242.00	266.20	292.82		
2	165.29	181.82	200.00	220.00	242.00	266.20	292.82	
3		165.29	181.82	200.00	220.00	242.00	266.20	292.82

(i) second-year claims-made step factor = $\dfrac{266.20+242.00}{266.20+242.00+220.00+200.00}=0.54751$

(ii) mature claims-made tail factor = $\dfrac{266.20\times 3+242.00\times 2+220.00}{266.20+242.00+220.00+200.00}=1.61883$

GIRR, Fall 2020, Q10

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 36.

Commentary on Question:

This question tests the candidate's understanding of the NCCI approach to experience rating.

Solution:

(a) Describe how the NCCI split rating experience rating plan differentiates between the frequency and severity of an insured's experience.

The NCCI experience rating plan uses split rating to explicitly reflect the frequency and severity of an insured's experience.

The split rating segregates actual claims into primary claims and excess claims.

(b) Provide another way that an experience rating formula can differentiate between frequency and severity, other than the approach identified in part (a).

Cap individual claims.

(c) Calculate the following:

(i) Total actual excess claims

(ii) Total expected primary claims

(iii) Expected excess claims for Classification Code C

	(1)	(2)	(3)
		Actual Claims	
Claims ID	Reported	Primary	Excess
# 2	15,000	10,000	5,000
# 4	40,000	10,000	30,000
# 7	5,000	5,000	0
Claims less than 1,000	20,000	20,000	0
Total	80,000	45,000	35,000

	(4)	(5)	(6)	(7)	(8)
				Expected Claims	
Code	Payroll	ELR (per 100)	D-ratio	Primary	Excess
A	1,400,000	0.10	0.5	700	700
B	1,600,000	2.00	0.4	12,800	19,200
C	1,000,000	1.50	0.3	4,500	10,500
Total				18,000	30,400

(i)	45,000
(ii)	18,000
(iii)	10,500

Notes: (2) = (1) capped at 10,000
(3) = (1) – (2)
(7) = (4)(5)(6) / 100
(8) = (4)(5)[1 – (6)] / 100

(d) Calculate the NCCI experience rating modification factor using W = 0.5 and B = 50,000.

$$M = \frac{A_P + (1-W)\times E_{XS} + B + W \times A_{XS}}{E_P + (1-W)\times E_{XS} + B + W \times E_{XS}}$$

$$= \frac{45,000 + (1-0.5)\times 30,400 + 50,000 + 0.5\times 35,000}{18,000 + (1-0.5)\times 30,400 + 50,000 + 0.5\times 30,400} = \frac{127,700}{98,400} = 1.298$$

(e) Recommend two ways to increase responsiveness of this experience rating plan.

- Increase the limit (or cap) applied to the claims included in the experience rating formula
- Decrease the number of years in the experience period

GIRR, Fall 2020, Q11

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of deductible factors used in ratemaking.

Solution:

(a) Describe a potential issue related to the absence of complete data when using reported claim data from recent years.

You may only have access to the claim detail for the portion of the loss that the insurer covers.

(b) Describe a potential issue related to claim development when using individual reported claim data from recent years.

Claim development factors are selected based on aggregated claim experience by accident year and represent case development as well as pure IBNR. As such, claim development factors are not intended to be used on an individual claim file basis.

(c) Calculate the indicated deductible factors for deductibles of 500 and 1,000 relative to a base deductible of zero.

Average accident date in future rating period is March 1, 2022.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
						Trended Ultimate Indemnity		
		Ground Up	Trending Period		Trend			
Claim #	Date of Claim	Ultimate Claims	Months	Years	Factor @5%	No Deductible	Deductible of 500	Deductible of 1000
1	Jan. 1, 2017	7,500	62	5.17	1.2867	9,650.27	9,150.27	8,650.27
2	Jul. 1, 2017	800	56	4.67	1.2557	1,004.55	504.55	4.55
3	Jul. 1, 2017	1,600	56	4.67	1.2557	2,009.11	1,509.11	1,009.11
4	Jan. 1, 2018	2,400	50	4.17	1.2254	2,941.03	2,441.03	1,941.03
5	Jan. 1, 2018	6,700	50	4.17	1.2254	8,210.39	7,710.39	7,210.39
6	Jul. 1, 2018	2,300	44	3.67	1.1959	2,750.57	2,250.57	1,750.57
7	Jan. 1, 2019	700	38	3.17	1.1671	816.95	316.95	0.00
8	Jul. 1, 2019	300	32	2.67	1.1390	341.69	0.00	0.00
9	Jul. 1, 2019	1,100	32	2.67	1.1390	1,252.85	752.85	252.85
10	Jul. 1, 2019	4,500	32	2.67	1.1390	5,125.28	4,625.28	4,125.28
					Total	34,102.67	29,260.99	24,944.04
					Deductible factor:		0.858	0.731

Notes: (4) = $1.05^{(3)}$

(5) = (1)(4)

(6) = Greater of 0 and (5) – 500

(7) = Greater of 0 and (5) – 1,000

Deductible factors:

0.85829,260.99 / 34,102.67; 0.731 = 24,944.04 / 34,102.67

(d) Explain why the deductible factors would be higher if an annual severity trend greater than 5% is used in part (c).

A trend greater than that used (5%) in part (c) would result in each loss being larger in size. But the deductible stays the same. So, this increase would result in a greater percentage of total losses above the deductible.

(e) Evaluate the reasonability of the deductible factors calculated in part (c) by performing a consistency test.

The marginal rate should decrease as the value of the deductible increases.

Deductible	Deductible Factor	Marginal Rate Per Thousand
0	1.000	
500	0.858	0.284
1000	0.731	0.253

Since there is a decrease, the deductible factors are considered to be consistent.

GIRR, Spring 2021, Q6

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6a) Understand and apply classification ratemaking methods.

(6b) Explain the issues and considerations regarding classification ratemaking.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:

This question tests the candidate's understanding of risk classification.

Solution:

(a) Describe why grouping risks into more homogeneous classes can improve the effectiveness of a risk classification system.

By grouping together risks into relatively homogeneous classes, the risk classification system reduces the adverse selection that occurs when high-risk and low-risk participants are offered identical coverage at the same price.

(b) Describe how an effective risk classification system can contribute to availability of coverage.

If Auto Insurer was aware of the different costs underlying its portfolio of risks but was not allowed to differentiate its price based on the expected costs, there would be no incentive to provide coverage to risks that have higher than average expected costs.

(c) Evaluate each of the following risk characteristics for use in a risk classification system for automobile insurance:

(i) Gender

(ii) Credit score

(iii) Age

(iv) Telematics data

(i) Gender is easy to measure and not subject to manipulation, so it satisfies the objectivity. However, use of gender for risk classification is prohibited in some jurisdictions.

(ii) Credit score has been known to have positive correlation with claim experience, but it is difficult to show the causality. Also, the possibility of using credit score as a rating factor depends on a jurisdiction.

(iii) Age is easy to measure and not subject to manipulation, so it satisfies the objectivity. It has been shown that age of the primary driver has strong relationship with the claim behavior.

(iv) Telematics data is objective, and it can be used to measure the exposure of an insurance contract more precisely. However, one should be careful since use of telematics data might require additional managerial support such as IT, human resources, and financial requirements.

(d) Describe two problems encountered with a one-way analysis of a risk classification system.

Inability to adjust for distributional bias between risk classes, which occurs when there are differences in the distribution of exposures by risk characteristic between risk classes.

Inability to adjust for dependence between risk classes, which occurs when knowing the risk class of an insured within one risk characteristic changes the true relativities for the risk classes in another risk characteristic from what they would be without that knowledge.

GIRR, Spring 2021, Q10

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of coinsurance in property policies, deductible factors, and increased limits factors.

Solution:

(a) Explain the effect of a straight per-event deductible on each of the following:

(i) An insurer's claim frequency

(ii) An insurer's claim severity

Commentary on Question:

Candidates need to explain how deductibles can either increase or decrease severity. Simply stating that deductibles can increase or decrease severity is insufficient.

(i) Deductibles reduce an insurer's claim frequency because claims below the deductible are no longer the insurer's responsibility which reduces claim counts.

(ii) Deductibles can increase or decrease claim severity. An increase can occur when small claims are eliminated leaving larger claims with higher average severity. A decrease can occur when claims exceed the deductible amount. A portion of these claims is eliminated making each claim smaller which lowers average severity.

(b) Describe the reason for a coinsurance clause in a property insurance policy.

Coinsurance is used to motivate insureds to purchase the appropriate amount of insurance and to penalize those that do not. If the insured chooses to insure the property for a lesser amount than that required by coinsurance, then any payments for claims arising from insured events would be reduced in direct relationship with the ratio of the insured value, selected by the insured, to the property's required insurable value, determined by the insurer's rating rules.

(c) Calculate the claims paid by the insurer under the following scenarios:

(i) Loss amount is 800,000 and the deductible is 10,000

(ii) Loss amount is 900,000 and the deductible is 0

Coinsurance penalty percentage = $1 - 500{,}000/(1{,}000{,}000 \times 0.8) = 0.375$

(i) Paid by the insurer = $\min[(1 - 0.375) \times 800{,}000,\ 500{,}000] - 10{,}000$
$= 490{,}000$

(ii) Paid by the insurer = $\min[(1 - 0.375) \times 900{,}000,\ 500{,}000] - 0$
$= 500{,}000$

(d) Calculate the elimination ratio to be used for pricing a deductible option of 1,000.

Claims eliminated by 1,000 deductible:

Indemnity Range	Claims Eliminated
0 – 1,000	1,049,000
Over 1,000	1,000×10,620 = 10,620,000
Total	11,669,000

Elimination ratio = 11,669,000 / 60,459,000 = 0.193.

(e) Calculate a rate for the 1,000 deductible option using results from part (d).

Commentary on Question:
Applying the elimination ratio directly to the rate did not get full credit since it does not account for expenses properly.

Reduce claims for claims eliminated by deductible:

$110 \times 0.7 \times (1 - 0.193) = 62.138 = P \times CR$

Using the premium equation, $P = P \times CR + P \times V + F$, solve for P:

$P = 62.138 + P \times 0.2 + 110 \times 0.1$

$P = 91.42.$

(f) Calculate the increased limits factors relative to a basic limit of 10,000 for:

(i) 20,000 limit, and

(ii) 100,000 limit.

Claims limited to 10,000 = 35,000,000 + 10,000×(1,500 + 500) = 55,000,000

(i) Claims limited to 20,000 = 35,000,000 + 25,000,000 + 20,000×500
= 70,000,000
Therefore, increased limits factor for 20,000 limit:
= 70,000,000 / 55,000,000 = 1.273

(ii) Claims limited to 100,000 = 75,000,000
Therefore, increased limits factor for 100,000 limit:
= 75,000,000 / 55,000,000 = 1.364.

GIRR, Spring 2021, Q13

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

Commentary on Question:

This question tests the candidate's understanding of claims-made ratemaking.

Solution:

(a) Describe why the risk of a reserve inadequacy is greatly reduced for claims-made policies compared to occurrence policies.

Claims-made policies incur no pure IBNR claims as only claims reported during the policy year are covered.

(b) Explain how a coverage gap can be created when the insured switches:

(i) From claims-made to occurrence coverage

(ii) From occurrence to claims-made coverage

(i) Claims-made to occurrence: tail of claims-made is not covered by occurrence unless purchased separately.

(ii) Occurrence to claims-made: usually no issue, unless there is a timing issue between expiration date of the old policy and effective date of new policy.

(c) Construct a numerical example demonstrating this principle.

Commentary on Question:

Any example that properly demonstrates the principle is acceptable.

Assume 3 year reported, 100 reported each year, annual trend of 10%:

	AY Lag by Report Year Matrix			
AY Lag	1	2	3	4
0	*100*	110	121	133.1
1	*100*	*110*	121	133.1
2	*100*	110	*121*	133.1

Report year 1 claims-made policy = 100 + 100 + 100 =	300
Report year 1 occurrence policy = 100 + 110 + 121 =	331

(d) Construct a numerical example demonstrating this principle.

Commentary on Question:
Any example that properly demonstrates the principle is acceptable.

The solution below uses the same example as part (c), except with a trend after reported year 1 of 20%.

	AY Lag by Report Year Matrix			
AY Lag	1	2	3	4
0	100	120	144	172.8
1	100	120	144	172.8
2	100	120	144	172.8

	RY2 @10% Trend	RY2 @20% Trend
RY2 Claims-made policy:	330.00	360.00
RY2 Occurrence policy:	364.10	436.80

Change in claims-made = 360 / 330 – 1 =	9.1%
Change in occurrence = 436.80 / 364.10 – 1 =	20.0%

GIRR, Spring 2021, Q17

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 36.

Commentary on Question:

This question tests the candidate's understanding of individual risk rating.

Solution:

(a) Evaluate the suitability of each of the following individual risk rating programs for LMN:

(i) Schedule rating

(ii) Prospective experience rating

(iii) Retrospective experience rating

(i) Schedule rating
- Schedule rating should still be used as only 1 year of experience would reflect the new safety system.
- However, something less than 10% is recommended as only some of the past experience would reflect the new safety program.

(ii) Prospective experience rating
- This is a good option as LMNs future premiums can be based on its claim experience.
- Experience rating would help with fluctuations as it would hold LMN accountable for their claims.
- Experience rating should include the schedule rating adjustment.

(iii) Retrospective experience rating
- This is not a very large risk, so it is not ideal for retrospective rating.
- There are significant fluctuations, which suggests this is likely not an ideal candidate for retrospective rating.
- A company with strong financials is normally a good candidate for retrospective rating.

(b) Explain how this principle can be considered in the design of LMN’s prospective experience rating program.

- Actual claims should be capped and/or split into primary and excess components.
- Primary claims are expected to be more predictable because they are typically less volatile and have a shorter period of development than excess claims.

GIRR, Fall 2021, Q3

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of deductible factors and increased limit factors, including calculating elimination ratios and checking deductible factors for consistency.

Solution:

(a) Provide two reasons insurers use deductibles, other than to directly reduce the amount of claims paid.

Any two of the following is acceptable:

- Assist in reducing moral and morale hazard
- Encourage insureds to adhere to some measure of risk control
- Eliminate the processing costs associated with small claims
- Reduce exposure to catastrophic events

(b) Provide two reasons insurers use limits, other than to directly reduce the amount of claims paid.

Any two of the following is acceptable:

- To accommodate the financial needs and risk preferences of insureds
- To reflect the capacity of insurers
- To substitute for exclusions (in property policies)

(c) Explain why an analysis of increased limits factors is more likely to use a statistical distribution.

A limits analysis is working with the right tail of the distribution. Often times, there are not enough claims in the right tail of the distribution to credibly measure increased limits factors at higher limits. Therefore, an analysis of increased limits factors is more likely to use a statistical distribution.

(d) Determine the elimination ratios and deductible factors for each of the deductible options.

	Elimination Ratios by Accident Year		
Accident Year (AY)	250	500	1,000
2015	3.85%	8.04%	11.07%
2016	7.53%	12.92%	16.88%
2017	8.98%	14.40%	18.30%
2018	7.86%	13.32%	17.28%
2019	9.96%	15.75%	19.82%
2020	7.27%	12.37%	16.06%
All years average	7.58%	12.80%	16.57%
All years average excl. 2015	8.32%	13.75%	17.67%
Selected elimination ratio	8.32%	13.75%	17.67%
Deductible factor	0.9168	0.8625	0.8233

e.g.,

Elimination ratios for AY2015:

- 250: 3.85% = (1,128,906 – 1,085,419) / 1,128,906
- 500: 8.04% = (1,128,906 – 1,038,175) / 1,128,906
- 1,000: 11.07% = (1,128,906 – 1,003,976) / 1,128,906

Deductible factor for 250 deductible: 0.9168 = 1 – .0832

AY2015 seems to be an outlier, so the all years average excluding 2015 is selected.

(e) Evaluate the reasonability of the deductible factors calculated in part (d) using a consistency test.

Deductible	Deductible Factor	Marginal Rate Per 1,000
100	1.0000	
250	0.9168	0.5548
500	0.8625	0.2173
1000	0.8233	0.0782

e.g., $0.2667 = 1{,}000 \times \dfrac{(1.000 - .9618)}{(250 - 100)}$

Since the marginal rates are strictly decreasing, the deductible factors are reasonable.

GIRR, Fall 2021, Q10

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

Commentary on Question:

This question tests the understanding of claims-made ratemaking.

Solution:

(a) State one advantage and one disadvantage of claims-made coverage from an insurer's perspective.

Advantage: more predictable loss cost
Disadvantage: less opportunity for investment income (or have to offer tail policy)

(b) Demonstrate, with a numerical example, a situation in which the claims-made loss cost is greater than the occurrence loss cost.

Commentary on Question:

Other solutions are possible.

Example:
Consider the case where the reporting period is two years with a reporting pattern of 50% in year 1 and 50% in year 2. Assume claims cost trend is –20%. For an occurrence claims cost of 100, the claims-made claims cost would be $50\times\left(1+\frac{1}{1-0.20}\right)=112.50$. Thus, the claims-made claims cost is greater.

(c) Calculate tail factors for a claims-made policy for the following maturities:

(i) Second-year

(ii) Mature

Report Year

AY Lag	1	2	3	4
0	**0.4**	0.4	0.4	0.4
1	**0.2**	***0.2***	0.2	0.2
2	0.2	***0.2***	***0.2***	0.2
3	0.2	0.2	***0.2***	***0.2***

(i) Second-year tail factor = (***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2***) / (**0.4** + **0.2**) = 1.667

	Report Year			
AY Lag	1	2	3	4
0	**0.4**	0.4	0.4	0.4
1	**0.2**	***0.2***	0.2	0.2
2	**0.2**	***0.2***	***0.2***	0.2
3	**0.2**	***0.2***	***0.2***	***0.2***

(ii) Mature tail factor = (***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2***) / (**0.4** + **0.2** + **0.2** + **0.2**) = 1.2

(d) Calculate CM's earned premium for 2021, 2022 and 2023 for a mature tail policy effective January 1, 2021 with a premium of 25,000.

With a 25,000 tail premium split into six units, the earning would be as follows:
2015: (3/6) of 25,000 = 12,500
2016: (2/6) of 25,000 = 8,333.33
2017: (1/6) of 25,000 = 4,116.67

GIRR, Spring 2022, Q3

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

Commentary on Question:

This question tests the candidate's understanding of claims-made and occurrence policies.

Solution:

(a) Provide two reasons why AV might decide to purchase coverage.

- It offers protection against the possibility of a claim, especially considering the possibility of a claim from a past incident.
- Purchasing coverage also provides the opportunity to obtain coverage for the incident that could be a claim, provided the retroactive date of the policy is on or before the date of the incident.

(b) Recommend two exposure base options for XYZ to consider in providing insurance coverage. Justify your recommendations.

Commentary on Question:

Other answers are possible.

- Number of full-time equivalent professionals is a typical measure.
- Revenue could be used also because of increasing revenue.

(c) Provide one advantage and one disadvantage to AV in purchasing a *claims-made* policy.

- An advantage is that it is lower cost than an occurrence policy.
- A disadvantage is that nose or tail coverage may be required.

(d) Provide one advantage and one disadvantage to AV in purchasing an *occurrence* policy.

- Advantage: it covers claims if occurrence coincides with policy period.
- Disadvantage: it is more expensive than a claims-made policy, unless there is a charge for an old retroactive date.

GIRR, Spring 2022, Q6

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 36.

Commentary on Question:

This question tests the candidate's understanding of individual risk rating.

Solution:

(a) Define the following terms in the context of individual risk rating:

(i) Prospective experience rating plan

(ii) Retrospective experience rating plan

(iii) Expense modification plan

(i) In a prospective experience rating program, the insurer adjusts the insured's future premiums, through discounts or surcharges, based on its claim experience in prior years.

(ii) In a retrospective experience rating program, the insured pays an initial deposit premium at the start of the policy term, and then, after the policy term is completed, retrospective refunds or surcharges are determined based on the actual claims during the policy term.

(iii) An expense modification plan is a form of rating plan (or rating procedure) where the variation of the premium for a particular insured is based on the variation in the expenses of the insurer with regard to this insured from those contemplated in the development of the manual rate.

(b) Provide one benefit of insurance company reliance on an insured's historical claims to project future claims for a prospective experience rating plan.

In relying on an insured's historical claims to project future claims, and in doing so to influence the determination of its premiums, the insurer provides incentives for the insured to manage its losses that result in claims to the insurer.

(c) Critique the use of a prospective experience rating plan for personal property coverage from an insurance company's perspective.

Commentary on Question:
Other answers are possible.

- It is difficult to hold insureds responsible as the cost would be significant from just one significant claim (volatility a concern).
- It would encourage risk control activities.
- With such low credibility, it is questionable that this would improve the predictive accuracy of premiums.

(d) Critique each characteristic in the new plan.

- Including only the most recent 3 years should improve responsiveness, but it might reduce credibility.
- Using a split rating formula will allow the plan to explicitly reflect the frequency and severity of an insured's experience.

(e) Explain why retrospective experience rating is typically not appropriate for each of the following:

(i) Insureds with low premium volume

(ii) Insureds with poor claims experience

(i) Insureds with small premium size are likely to have variable claims experience and one large claim may result in a maximum premium.

(ii) Insureds with poor claims experience will pay greater than the average premium and could have claims resulting in maximum premium.

GIRR, Spring 2022, Q7

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6a) Understand and apply classification ratemaking methods.

(6b) Explain the issues and considerations regarding classification ratemaking.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:

This question tests the candidate's understanding of classification ratemaking.

Solution:

(a) Determine if there is distributional bias in the exposure data. Support your conclusion.

Commentary on Question:

Only one inconsistency needs to be found to demonstrate distributional bias.

	Exposures			Ratios to territory 1 for each class:		
	Territory			Territory		
Class	1	2	3	1	2	3
A	2,700	2,700	2,025	1.00	1.00	0.75
B	1,350	2,025	2,700	1.00	1.50	2.00
C	1,350	675	4,050	1.00	0.50	3.00

e.g., Class A: 1.00 = 2,700/2,700; 1.00 = 2,700/2,700; 0.75 = 2,025/2,700

Since the ratios are not consistent for each class, there is distributional bias.

(b) Calculate the rebalanced pure premiums using the one-way analysis relativities for each rating variable combination.

			One-Way Relativities			
Class	Territory	Exposures	Class	Territory	Pure Premium	Rebalanced Pure Premium
A	1	2,700	0.8150	0.7567	213.94	209.93
A	2	2,700	0.8150	0.6522	184.40	180.94
A	3	2,025	0.8150	1.3637	385.57	378.34
B	1	1,350	0.9897	0.7567	259.80	254.92
B	2	2,025	0.9897	0.6522	223.92	219.72
B	3	2,700	0.9897	1.3637	468.21	459.43
C	1	1,350	1.2364	0.7567	324.54	318.45
C	2	675	1.2364	0.6522	279.72	274.48
C	3	4,050	1.2364	1.3637	584.89	573.92
Overall					353.53	346.90

e.g., Class A factor: 0.8150 = 282.73 / 346.90
Territory 3 factor: 1.3637 = 473.08 / 346.90
Class A, Territory 1 pure premium: 385.57 = 346.90×0.8150×1.3637
Overall pure premium: 353.53 = Sumproduct(exposures,pure premiums)
Class A, Territory 1 rebalanced pure premium:
378.34 = 385.57×353.53/346.90

(c) Calculate the revised relativities by class that result from a single iteration of the minimum bias method.

Commentary on Question:
Candidates need to start with one-way territory relativities to solve for class relativities.

One-way territory relativities :
- Territory 1: 262.50/346.90 = 0.7567
- Territory 2: 226.25/346.90 = 0.6522
- Territory 3: 473.08/346.90 = 1.3637

Total expected claims for each class:
- Class A: 240×2,700 + 200×2,700 + 450×2,025 = 2,099,250
- Class B: 270×1,350 + 250×2,025 + 450×2,700 = 2,085,750
- Class C: 300×1,350 + 260×675 + 500×4,050 = 2,605,500

First iteration for new class relativities using one-way territory relativities as starting point:

- Class A: 2,099,250/[(0.7567×2,700 + 0.6522×2,700 + 1.3637×2,025)×346.90] = 0.9217
- Class B: 2,085,750/[(0.7567×1,350 + 0.6522×2,025 + 1.3637×2,700)×346.90] = 0.9980
- Class C: 2,605,500/[(0.7567×1,350 + 0.6522×675 + 1.3637×4,050)×346.90] = 1.0753

(d) Describe the condition under which the converged results of the minimum bias method will be factors that reproduce all nine observed trended ultimate pure premiums.

The observed pure premiums must be independent for the minimum bias method to reproduce them.

GIRR, Spring 2022, Q19

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of deductible factors.

Solution:

(a) Calculate the indicated deductible factor for a deductible of 1,000.

Indemnity eliminated at 500 deductible:	
886,650 + 7,070 × 500	4,421,650
Total Indemnity at 500 deductible:	
12,605,205 – 4,421,650	8,183,555
Indemnity eliminated at 1,000 deductible:	
886,650 + 1,976,260 + 4,210 × 1,000	7,072,910
Total Indemnity at 1,000 deductible:	
12,605,205 – 7,072,910	5,532,295
Deductible relativity: 5,532,295 / 8,183,555	0.676

(b) Recommend a factor for a deductible of 1,500. Justify your recommendation.

First need to know the 2,000 deductible factor:
Indemnity eliminated at 2,000 deductible:

886,650 + 1,976,260 + 3,256,395 + 1,975 × 2,000 = 10,069,305

Total Indemnity at 2,000 deductible: 12,605,205 – 10,069,305 = 2,535,900

Deductible relativity = 2,535,900 / 8,183,555 = 0.310

Therefore, relativity needs to be between 0.676 and 0.310 → can use consistency test to find the appropriate range for a factor.

Relativity for 1,500 deductible = x
Based on consistency test,
Difference between 500 & 1000, and 1000 & 1500: $1 - 0.676 > 0.676 - x$
(note: can ignore denominators since all are 500)
solves for $x > 0.352$

Difference between 1000 & 1500, and 1500 & 2000: $0.676 - x > x - 0.310$
solves for $x < 0.493$

Therefore, recommend any factor higher than 0.352 and lower than 0.493.

(c) Describe why you would not be able to use data from policies with a 2,000 deductible to determine the deductible factor for a 1,000 deductible if the data was censored.

There may have been claims for amounts between 1000 and 2000 that we don't know about, and we would need to include those claims in the calculation.

(d) Provide a reason why you would choose to determine deductible factors using a classification ratemaking approach instead of using the elimination ratio approach.

Claimants' behavior and claim experience may differ between different deductibles.

GIRR, Fall 2022, Q1

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of deductibles and coinsurance used in property insurance.

Solution:

(a) Calculate the losses retained by the garage owner under each of the following deductible scenarios:

(i) Straight deductible of 500 per vehicle

(ii) Deductible of 20% of the garage owner's liability

(iii) Diminishing deductible per event where:

- The garage owner would fully retain any losses less than 50,000,
- The insurer would pay the total value of any covered loss greater than 100,000, and
- Losses with a total value between 50,000 and 100,000 would be proportionately shared between the garage owner and the insurer.

Total amount of loss = 80×1,000 = 80,000

(i) Amount retained = 80×500 = 40,000

(ii) Amount retained = 80×1,000×0.2 = 16,000

(iii) Multiplier = 100,000 / (100,000 – 50,000) = 2
Amount paid by insurer = (80,000 – 50,000)×2 = 60,000
Therefore, retained amount = 80,000 – 60,000 = 20,000

(b) State one advantage of a deductible from an insurer's perspective.

Any one of the following is acceptable:

- Moral and morale hazard
- Risk control
- Processing costs associated with small claims
- Exposure to catastrophic events

(c) Calculate the claims paid by the insurer under each of the following scenarios:

(i) The insured purchased coverage of 200,000 with a 50% coinsurance requirement.

(ii) The insured purchased coverage of 500,000 with an 80% coinsurance requirement.

(iii) The insured purchased coverage of 750,000 with a 90% coinsurance requirement.

Scenario	(1) Amount Purchased	(2) Coinsurance Percentage	(3) Amount of Insurance Required	(4) Coinsurance Penalty Percentage	(5) Amount Paid by Insured
(i)	200,000	50%	400,000	50.000%	200,000
(ii)	500,000	80%	640,000	21.875%	351,563
(iii)	750,000	90%	720,000	0.000%	450,000

Notes: $(3) = 800{,}000 \times (2)$

$(4) = \max\{[1 - (1) / (3)], 0\}$

$(5) = \min[(1 - (2)) \times 450{,}000, (1)]$

(d) State one reason why insurers favor including a coinsurance requirement in property policies.

Any one of the following is acceptable:

- Coinsurance is a technique used by insurers to limit their liability and assist insureds in managing their costs of coverage (or sharing the risk with the insureds).
- Essentially, coinsurance is used to motivate insureds to purchase the appropriate amount of insurance (close to full coverage) and to penalize those that do not.

GIRR, Fall 2022, Q8

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6a) Understand and apply classification ratemaking methods.

(6b) Explain the issues and considerations regarding classification ratemaking.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:

This question tests the candidate's understanding of classification ratemaking.

Solution:

(a) Critique your colleague's recommendation.

If the company raises the base rate from 100 to 110, it might be able to achieve breakeven provided the distribution of the portfolio does not change.

However, it is very likely that the lower risks (25 and over and/or female) leave the company while the higher risks (under age 25 and/or male) migrate to the portfolio more, which would decrease the profitability again.

(b) Calculate A_2, S_2, and μ with the single variable risk classification analysis, by setting the base class as "25 and over", "male.".

Age Group (i)	Number of the Insureds	Total Claims	Pure Premium	Relativity (A_i)
25 and over (1)	640	60,000	93.75	1.000
Under 25 (2)	360	50,000	138.89	1.481
Total	1000	110,000	110.00	

Sex (j)	Number of the Insureds	Total Claims	Pure Premium	Relativity (S_j)
Male (1)	720	90,000	125.00	1.000
Female (2)	280	20,000	71.43	0.571
	1000	110,000	110.00	

e.g., A_2: *1.481 = 138.89 / 93.75*

Need to solve for: 110,000 = $\mu \sum_i \sum_j X_{ij} A_i S_j$,

where X_{ij} = number of insureds for rating combination i,j

110,000 = μ×(480×1.000×1.000 + 160×1.000×0.571 + 240×1.481×1.000 + 120×1.481×0.571)

Solves for: μ = 106.94

(c) Describe two possible issues, in general, with the use of a single variable risk classification analysis.

Distributional bias:
It occurs when there are differences in the distribution of exposures by risk characteristic between risk classes.

Dependence:
It occurs when knowing the risk class of an insured within one risk characteristic changes the true relativities for the risk classes in another risk characteristic from what they would be without that knowledge.

(d) Describe two approaches that address the issues identified in part (c).

Minimum bias procedure/method:
Solve the multiple non-linear equations with the unknown multiplicative factors iteratively.

Generalized Linear Models (GLMs):
Set a relationship between a response variable and several predictor variables using a linear predictor and an appropriate link function.

(e) Describe this conflict.

Increasing homogeneity of a class typically means a smaller class which may have lower credibility due to smaller size.

or

There is an inverse relationship been credibility and homogeneity.

GIRR, Fall 2022, Q19

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 36.

Commentary on Question:

This question tests the calculation of an experience rating modification.

Solution:

Calculate the experience rating modification.

Policy Year	Claim ID	Claims at July 1, 2022 – Indemnity – Total Limits	Claims at July 1, 2022 – Indemnity – Basic Limits	Claims at July 1, 2022 – ALAE	MSL	Reported Indemnity & ALAE at July 1, 2022 Limited by Basic Limits and MSL
July 1, 2019 – June 30, 2020	1	14,000	14,000	35,000	45,000	45,000
	2	32,000	20,000	20,000	45,000	40,000
July 1, 2020 – June 30, 2021	3	22,000	20,000	16,000	45,000	36,000
	4	10,000	10,000	3,000	45,000	13,000
Total						134,000

Expected unreported claims at July 1, 2022	
= 88,600×0.16×0.67 + 92,200×0.38×0.67 =	32,972
Projected ultimate losses & ALAE Limited by Basic Limits & MSL:	
= Reported Losses & ALAE (134,000) + Expected unreported (32,972) =	166,972
Basic Limits Premiums Subject to Experience Rating	180,800
AER =	
Projected ultimate losses & ALAE Limited by basic limits & MSL / CSLC =	0.9235
AELR	0.67
$Z = \sqrt{180{,}800 / 2{,}000{,}000} =$	30%
Experience (credit)/debit = Z × (AER – AELR)/AELR =	11.38%

GIFREU, Fall 2020, Q15

Part (b) is not in GI 301

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Teng, M. and Perkins, M., "Estimating the Premium Asset on Retrospectively Rated Policies"

Commentary on Question:

This question tests a candidate's ability to calculate the premium asset on retrospectively rated policies and the statutory accounting treatment of this amount. This question required the candidate to use Excel. The model solution for this question included in this document does not represent the actual model solution. It is for explanatory purposes only. Refer to the Excel solution file for an example of a full credit solution in Excel.

Solution:

(a) Calculate the premium asset on WFH's retrospectively rated policies as of Dec. 31, 2019.

Commentary on Question:

There are many different ways that this calculation can be displayed in Excel. The model solution in the Excel file is an example of a full credit solution. The solution shown in this file outlines the calculation from the Excel solution. Amounts are shown in millions of dollars.

Note that as at 12/31/19, Policy Year 2018 is at 24 months of development so a development factor of 1.325 applies to calculate the ultimate value of losses. Applicable development factors for other policy years follows from this (e.g., Policy Year 2017 at 12/31/19 is at 36 months of development).

$CPDLD_1 = (76\% \times 1.755 + 12\% \times 0.625 + 6\% \times 0.475 + 4\% \times 0.325 + 2\% \times 0.0) / (76\% + 12\% + 6\% + 4\% + 2\%) = 1.4053$

$CPDLD_2 = (12\% \times 0.625 + 6\% \times 0.475 + 4\% \times 0.325 + 2\% \times 0.0) / (12\% + 6\% + 4\% + 2\%) = 0.4854$

$CPDLD_3 = (6\% \times 0.475 + 4\% \times 0.325 + 2\% \times 0.0) / (6\% + 4\% + 2\%) = 0.3458$

$CPDLD_4 = (4\% \times 0.325 + 2\% \times 0.0) / (4\% + 2\%) = 0.2167$

$CPDLD_5 = (2\% \times 0.0) / (2\%) = 0$

		2014	2015	2016	2017	2018	Total
Reported Losses as of 12/31/19 ($M)	A	180	169	108	102	78	
Development factor	B	1	1.008	1.03	1.133	1.325	
Percent earned	C	100%	100%	100%	100%	100%	
Ultimate losses	D = A×B×C	180.00	170.35	111.24	115.57	103.35	
Losses Reported at Prior Retrospective Adjustment ($M)	E	179	166	104	90	0	
Expected loss emergence	F = D – E	1.00	4.35	7.24	25.57	103.35	
CPDLD	G as calculated	0	0.2167	0.3458	0.4854	1.4503	
Premium Booked at Prior Retrospective Adjustment ($M)	H	230	228	170	162	0	
Premium Booked as of 12/31/2019 ($M)	I	230	226	170	165	155	
Estimated Total Premium	J = H+(F×G)	230.00	228.94	172.50	174.41	149.89	
Estimated Premium Asset	K = J – I	0	2.94	2.50	9.41	(5.11)	9.75

(b) ~~Calculate the admitted portion of the premium asset from part (a) under U.S. statutory accounting.~~

Part (b) is not in GI 301

~~**Commentary on Question:**~~

~~*Note that 10% of unsecured receivables not yet due are nonadmitted assets. Therefore, the admitted portion of the premium asset is 90% of the calculated amount.*~~ ~~Amounts are shown in millions of dollars.~~

~~90% of 9.75 = 8.77~~

GIFREU, Fall 2021, Q5

Part (a) is not in GI 301

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Teng, M. and Perkins, M., "Estimating the Premium Asset on Retrospectively Rated Policies"

Commentary on Question:

This question tests a candidate's understanding of the accounting for retrospectively rated policies and the ability to calculate the premium asset on them. This question required the candidate to respond in Excel for parts (c), (d) and (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (c), (d) and (e) are for explanatory purposes only.

Solution:

(a) ~~Describe the recording of the amounts for *accrued additional retrospective premiums* in the statutory financial statement as per SSAP No. 66 with respect to the following:~~

Part (a) is not in GI 301

~~(i) the accounting transaction including where amounts are recorded~~

~~(ii) the timing of when amounts are recorded~~

~~(i) A receivable asset with a corresponding entry made either to written premiums or as an adjustment to earned premiums.~~

(ii) ~~Premiums not recorded through written premium when accrued shall be recorded through written premium when billed.~~

(b) Describe a benefit of using the formula approach over an approach using historical data for calculating PDLD ratios.

Commentary on Question:

There are several benefits to using the formula approach. Only one benefit was required for full credit. The model solution is an example of a full credit solution.

Policy terms may have changed since the policies included in the historical data were written. The formula approach ensures that the PDLD ratios reflect current terms.

(c) Calculate the Premium Development to Loss Development (PDLD) ratios under the formula approach for the first and second retrospective premium adjustments.

Commentary on Question:
BPF = basic premium factor, ELR = expected loss ratio, TM = tax multiplier, $\%Loss_x$ = expected percentage of loss emerged for the x retro adjustment, (CL1/PL1) = Loss capping ratio at the first retrospective adjustment, LCF = loss conversion factor, ICAP2 = Incremental loss capping ratio for the second retrospective adjustment period

$PDLD_1 = [BPF \times TM / (ELR \times \%Loss_1)] + [(CL1/L1) \times LCF \times TM]$
$[0.225 \times 1.035 / (0.7 \times 85\%)] + [0.875 \times 1.20 \times 1.035]$
$= 1.478$

$PDLD_2 = ICAP2 \times LCF \times TM = 0.59 \times 1.20 \times 1.035$
$= 0.733$

(d) Calculate the cumulative PDLD (i.e., CPDLD) ratios for the first and second retrospective premium adjustments.

$CPDLD_1 = (PDLD_1 \times \%Loss_1 + PDLD_2 \times \%Loss_2 + PDLD_3 \times \%Loss_3) /$
$(\%Loss_1 + \%Loss_2 + \%Loss_3)$
$= (1.478 \times 85\% + 0.733 \times 13\% + 0 \times 2\%) / 1$
$= 1.352$

$CPDLD_2 = (PDLD_2 \times \%Loss_2 + PDLD_3 \times \%Loss_3) / (\%Loss_2 + \%Loss_3)$
$= (0.733 \times 13\% + 0 \times 2\%) / 0.15$
$= 0.635$

(e) Calculate the premium asset on retrospectively rated policies for policy years 2019 and 2020 combined as of December 31, 2020.

Premium Asset = Estimated Total Premium − Premium Booked as of 12/31/20

Estimated Total Premium = Premium Booked from Prior + Expected Future Premium
= [0 + Expected Future Loss Emergence × CPDLD Ratio]

Premium Asset = [0 + 479,250 × 1.352] – 573,750 = 74,041

GIADV, Spring 2023, Q2

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating and experience rating.

Solution:

(a) Describe how an insured's risk control activities affect each of the following individual risk rating plans:

(i) Schedule rating

(ii) Prospective experience rating

(iii) Retrospective experience rating

(i) The insured receives a credit for the presence of a risk control measure at the time the policy is written.
(ii) Risk control measures must have actually lowered claims from the expected amounts in the historical experience period used for rating to be reflected in the rate.
(iii) Risk control measures must actually lower claims from the expected amounts during the policy period to be reflected in the rate.

(b) Explain why insurers use schedule rating.

To incorporate judgment about specific risk characteristics of the insured that are either not considered at all or are not adequately reflected in the manual rating process.

(c) Describe how the NCCI formula differs from the basic formula.

In the numerator, actual claims are split into primary and excess claims.

Actual primary claims are given full credibility while actual excess claims are credibility weighted with expected excess claims. Also, a ballast amount is added to both the numerator and the denominator to limit year-to-year variability.

(d) Identify two other characteristics of insureds that would make retrospective experience rating inappropriate.

Commentary on Question:
There are more than two other characteristics. The model solution is an example of a full credit solution.

- Significant fluctuations in premium volume from year to year
- Poor claims experience

GIADV, Spring 2023, Q7

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Commentary on Question:

This question required the candidate to respond in Excel for part (d). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (d) is for explanatory purposes only.

Solution:

(a) Describe what each of A, B, and x represent.

A is (Premium with no incurred losses / Standard premium) minus one.
B is the slope factor relating premium changes with loss changes.
x is the standard loss ratio.

(b) Describe how the PDLD method differs from Fitzgibbon's method with respect to the function relating retrospective premium to losses incurred.

The PDLD method assumes the function is a set of line segments of decreasing slope for increasing incurred losses.

(c) Describe the two methods for calculating PDLD ratios.

- Use the retrospective rating parameters to derive them.
- Use historical booked premium and reported loss development to estimate them.

(d) Calculate the premium asset on retrospectively rated policies as of December 31, 2022 arising from policy years 2020 and 2021 using the PDLD method.

Commentary on Question:

CPDLD = Cumulative PDLD, PLE = Percentage of Loss Emerged Since Prior Evaluation, ELE = Expected Loss Emergence after Last Completed Retrospective Adjustment, PY = Policy Year

For each PY calculate:

- Cumulative PDLD (CPDLD) ratios as follows: sum (PDLD ratio times PLE) from the next adjustment for the PY to the last adjustment divided by sum of PLE from the next adjustment for the PY to the last
    - For example, the PY 2020 CPDLD ratio = sum (PDLD ratio times PLE) from 2nd to 4th adjustment divided by sum of PLE from 2nd to 4th adjustment.
- Expected future premium = CPDLD ratio times ELE
- Estimated total premium = Premium booked from prior retrospective adjustment plus expected future premium

Premium asset = sum over PYs estimated total premium minus sum over PYs premium booked as of year-end 2022.

GIADV, Spring 2023, Q11

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question required the candidate to respond in Excel for part (b). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (b) is for explanatory purposes only.

Solution:

(a) Describe the following terms with respect to claims-made insurance:

(i) Step factor

(ii) Tail policy

(iii) Tail factor

Commentary on Question:

The model solution is an example of a full credit solution.

(i) Factor used to determine claims-made rate for non-mature claims-made policy and is expressed as a relativity to the mature claims-made rate.

(ii) Policy that covers claims that are reported after a claims-made policy has expired provided the claim arises from an incident that occurred during the period for which claims-made coverage was in effect.

(iii) Factor used to determine a tail policy rate dependent on number of years for which claims-made coverage was purchased and is expressed relative to the rate at a given claims-made maturity.

(b) Calculate tail factors for a claims-made policy for the following maturities:

(i) First year

(ii) Third year

(iii) Mature

Commentary on Question:
AY = accident year, RY = report year,
$C_{i,j}$ = claims incurred (%) for AY lag *i* that are reported in RY *j*

(i) First year
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values on the diagonal with the reporting pattern provided.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by RY1 $C_{i,j}$.*

(ii) Third year
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values for the diagonal and 1 and 2 years below the diagonal. Each year is detrended by 5%.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by the sum of the $C_{i,j}$ for RY1.*

(iii) Mature
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values for the diagonal and AY lags below the diagonal. Each year is detrended by 5%.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by the sum of the $C_{i,j}$ for RY1.*

GIADV, Spring 2023, Q12

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) to (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (b) to (d) is for explanatory purposes only.

Solution:

(a) Describe two issues that should be investigated with respect to the industry data used in this analysis.

Commentary on Question:

There are more than two issues. The model solution is an example of a full credit solution.

- Is the industry distribution by territory, policy limit or type of business comparable to the business the company will be writing?
- Has any trend been applied to the claims data? If it was applied, what were the trending parameters and how were they selected?

(b) Calculate the observed increased limits factors (ILFs) for the following indemnity limits, relative to a basic indemnity limit of 1,000,000:

(i) 1,500,000

(ii) 2,500,000

(iii) 3,500,000

(iv) 5,000,000

- Combine first two rows of data table (so it represents range 0 to 1,000,000)
- For each row, calculate:

A. Claims in interval = count × indemnity severity in interval × (1 + ALAE % of indemnity)
B. Cumulative amount of claims from A
C. Claims by limit = cumulative amount from B + limit × sum of counts for all limits greater than the limit for the row + sum of ALAE for all limits greater than the limit for the row
D. Severity by limit = claims by limit from C ÷ total count over all limits
E. ILF = severity by limit from D ÷ severity for 1,000,000 limit from D

(c) Test the consistency of the ILFs calculated in part (b).

Using the ILFs from part (b), calculate the marginal rates as the difference of successive ILFs divided by the corresponding difference in successive limits. ILFs are consistent if the marginal rates are decreasing for increasing limits.

(d) Recommend an ILF for a 2,000,000 indemnity limit. Justify your recommendation.

Using trial and error, a value between 1.068 (ILF at 1.5 million) and 1.135 (ILF at 2.5 million) was selected such that the marginal rates are decreasing. The graph should show a smooth curve increasing at a decreasing rate.

GIADV, Fall 2023, Q2

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating and experience rating.

Solution:

(a) Explain how certain features included in prospective experience rating plans promote equity among insureds regarding the determination of premiums.

Commentary on Question:

There are several different features included in prospective experience rating plans that promote equity among insureds regarding the determination of premiums. Full credit was given for providing an explanation that included at least two features. The model solution is an example of a full credit solution with two features.

- The use of credibility in the experience rating formula ensures that the insured's experience is included only to the extent that it is a reliable predictor. Additionally, the use of at least several years of experience in the experience rating formula ensures that one bad year does not have an overly large influence on the premium.

(b) Describe split rating as it pertains to the NCCI experience rating plan.

Split rating separates actual claims into primary and excess amounts. Primary claims represent an insured's frequency and excess claims represent an insured's severity.

(c) Explain why the use of prospective experience rating for an insured does not eliminate the need for schedule rating of that insured.

Commentary on Question:

There are several reasons for this. Full credit was given for providing an explanation that included at least two reasons. The model solution is an example of a full credit solution with two features.

Schedule rating differs in that it modifies the rates based on a subjective assessment of the insured's risk characteristics. Furthermore, risk characteristics from schedule rating adjustments may not have been in place during the experience period used for experience rating.

(d) Identify two examples of risk characteristics used in schedule rating plans.

Commentary on Question:
There are many examples to choose from. Only two were required for full credit. The model solution is an example of a full credit solution.

- Quality of police and fire protection
- Condition and upkeep of the premises and equipment

GIADV, Fall 2023, Q7

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

EPLE = Cumulative Expected Percentage of Loss Emerged, IEPLE = Incremental EPLE, ILCR = Incremental Loss Capping Ratio

Solution:

(a) Calculate the implied Cumulative Premium Development to Loss Development (CPDLD) ratios for the first to fourth retrospective rating adjustments using the formula approach.

$PDLD_1 = (BPF \times TM / (ELR \times EPLE_1)) + (TM \times LCF \times ILCR_1)$
For $i = 2$ to 4, $PDLD_i = TM \times LCF \times ILCR_i$

$IEPLE_i = EPLE_i - EPLE_{i-1}$

$$CPDLD_j = \sum_{i=j}^{4} PDLD_i \times IEPLE_i \div \sum_{i=j}^{4} IEPLE_i$$

(b) Provide two situations in which one would favor the formula approach to estimating PDLD ratios over the empirical approach assuming there is sufficient data to use the empirical approach.

1. Retro rating parameters are changing significantly over time.
2. Historical EPLEs are not indicative of current EPLEs.

(c) Calculate the premium asset on retrospectively rated policies as of December 31, 2022.

For each policy year:

Expected Future Loss Emergence
= Ultimate Losses – Losses Reported at Prior Retro Adjustment

Estimated Future Premium
= Expected Future Loss Emergence × CPDLD

Estimated Total Premium
= Estimated Future Premium + Premium Booked from Prior Adjustment

Premium Asset
= Estimated Total Premium − Premium Booked as of Dec. 31, 2022

GIADV, Fall 2023, Q11

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Identify two advantages of claims-made coverage.

- Less uncertainty in pricing as ultimate amounts do not require pure IBNR.
- Less effect on annual claim amounts due to sudden changes in either the trend or the reporting pattern.

(b) Identify two advantages of occurrence coverage.

- Greater opportunity for investment income.
- Less risk of coverage gap.

(c) Compare the size of expected ultimate claims for report year 2024 to expected ultimate claims for accident year 2024.

Commentary on Question:

One may use percentages or assume a base value for claims. The model solution uses a base value assuming that the total claims for report year 2021 equals 100.

- Create a matrix of report year (RY) claims by accident year (AY) lag.
- The first column is RY 2021 with values by lag equal to 40, 25, 20 and 15.
- The second column is RY 2022 with values by lag equal to RY 2021 values multiplied by the trend factor of 1.085.
- Each additional column is the prior column increased by the trend factor and represents the next RY.

- Since we are interested in AY 2024, we need only show lags 1, 2 and 3 for RY 2025, lags 2 and 3 for RY 2026 and lag 3 for RY 2027.
- RY 2024 claims is the sum over lags 0 to 3 for RY 2024 = 127.73.
- AY 2024 claims are calculated as RY 2024 lag 0 + RY 2025 lag 1+ RY 2026 lag 2 + RY 2027 lag 3 = 140.28.

RY 2024 expected ultimate claims are 91.1% of the AY 2024 expected ultimate claims.

(d) Explain why members with claims-made policies for prior years will have a coverage gap if they decide to get coverage with the association on January 1, 2024.

This is because they will lack coverage for claims incurred prior to January 1, 2024, but not yet reported. This would entail claims incurred in 2023 with accident year lags 1 to 3, claims incurred in 2022 with accident year lags 2 and 3, and claims incurred in 2021 with accident year lag 3.

GIADV, Fall 2023, Q12

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain how insurance policy deductibles assist in reducing both moral and morale hazard.

Moral hazard involves the character of the insured regarding the potential to gain from insurance. Morale hazard involves an insured's indifference to loss because they have insurance protection. A deductible shifts some of the burden from a loss to the insured. This will assist in reducing acts of both moral and morale hazard as there will not be full compensation for the loss.

(b) Define the following terms:

(i) Franchise deductible

(ii) Disappearing deductible

(i) Franchise deductible – the insurer pays the full amount of the loss if the covered loss exceeds the deductible amount; otherwise the insurer pays nothing.

(ii) Disappearing deductible – a combination of a straight deductible and a franchise deductible.

(c) Determine the total amount paid by the insurance company if the following loss amounts occurred on <u>each</u> of policies A to D:

(i) 3,500

(ii) 350,000

Commentary on Question:
Max = maximum of, Min = minimum of, limit = insured limit, L = loss amount

For policies with a coinsurance requirement, the coinsurance penalty is 1 minus the limit divided by (coinsurance requirement times property value).

Insurance company paid after coinsurance, limit and deductible for L
= Max(0 and (Min(limit and L × (1 – coinsurance penalty)) – deductible))

GIADV, Spring 2024, Q2

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating, experience rating and dividend plans.

Solution:

(a) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding individual risk rating.

(i) The schedule rating adjustment is typically applied _______ premium discounts. [*after, before*]

(ii) An experience modification factor of _______ is referred to as a credit modification. [*greater than 0, greater than 1, less than 0, less than 1*]

(iii) Increasing the cap applied to claims _______ <u>the responsiveness of an experience rating formula</u>. [*decreases, does not affect, increases*]

(iv) D-ratio curves relate to _______ in experience rating. [*application of premium discounts, determination of credibility, limiting of claims*]

(i) before
(ii) less than 1
(iii) increases
(iv) limiting of claims

(b) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding insurer dividend plans.

(i) Insurers offer dividend plans to U.S. insureds for_______coverage. [*commercial automobile, professional liability, workers compensation*]

(ii) Dividend plans closely resemble ________rating plans. [*prospective, retrospective, schedule*]

(iii) Dividend plans are also referred to as ________. [*participating policies, predictive rating plans, risk-control plans*]

(iv) Dividend payments may require approval by the ________. [*insurer's board of directors, insurer's shareholders, regulatory authority*]

(v) In a sliding-scale dividend plan, the insured's claims experience ________ dividend payments. [*affects, does not affect*]

(vi) An insurer's board of directors may ________dividend payments for all dividend plan policies. [*not reject, reject*]

(vii) Dividend payments occur after ________. [*the end of the policy period, the filing of the financial statements, settlement of the claims on the policy*]

(viii) A combined dividend plan is a combination of the sliding-scale and ________dividend plans. [*flat, schedule-rated, split-rated*]

(i) workers' compensation
(ii) retrospective
(iii) participating
(iv) regulatory authority
(v) affects
(vi) reject
(vii) the end of the policy period
(viii) flat

(c) Describe the use of safety groups for U.S. workers compensation dividend plans.

Commentary on Question:
There are several ways to describe these plans. The model solution is an example of a full credit solution.

Safety groups are used to pool insureds' premiums and claims for similar employers. The dividends of a safety group are determined based on the aggregated experience of the group and are not based on an individual member's experience.

GIADV, Spring 2024, Q7

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Commentary on Question:

This question tested a candidate's ability to calculate the premium asset on retrospectively rated policies. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

BP = Basic Premium, SP = Standard Premium, LCF = Loss Conversion Factor,
TM = Tax Multiplier, EPLE = Cumulative Expected Percentage of Loss Emerged,
IEPLE = Incremental Expected Percentage of Loss Emerged,
ELR = Expected Loss Ratio, EMLR = Emerged Loss Ratio, LCR = Loss Capping Ratio,
ILCR = Incremental LCR, CLCR = Cumulative LCR,
LEPA = Loss elimination ratio from per accident limit,
LEMM = Loss elimination ratio from retro formula maximum and minimum

Solution:

(a) Calculate the cumulative premium development to loss development (CPDLD) ratio for each retrospective adjustment period using the formula approach.

$IEPLE_i = EPLE_i - EPLE_{i-1}$
$EMLR_i = ELR \times EPLE_i$
$CLCR_i = 100\% - LEPA_i - LEMM_i$
$ILCR_1 = CLCR_1$
For $i = 2$ to 4,
$ILCR_i = (EMLR_i \times CLCR_i - EMLR_{i-1} \times CLCR_{i-1}) / (EMLR_i - EMLR_{i-1})$

$PDLD_1 = (BPF \times TM / (EMLR_1)) + (TM \times LCF \times ILCR_1)$
For $i = 2$ to 4, $PDLD_i = TM \times LCF \times ILCR_i$

For $j = 1$ to 4,

$$\text{CPDLD}_j = \sum_{i=j}^{4} \text{PDLD}_i \times \text{IEPLE}_i \div \sum_{i=j}^{4} \text{IEPLE}_i$$

(b) State the formula to estimate the premium asset that includes the CPLD ratio as one of the elements in the formula.

As of the valuation date, for each policy year,
Premium Asset =
Expected Future Loss Emergence × CPDLD + Premium Booked from Prior Adjustment – Premium Booked as of the valuation date.

(c) Identify two situations where an empirical approach to estimating PDLD ratios would be preferred to the formula approach.

Commentary on Question:
There are several ways that this could be answered correctly. The model solution is an example of a full credit solution.

- Different retrospective parameters apply to many different regions.
- Historical patterns of PDLD show stability.

(d) Provide a reason the PDLD method might be preferred to Fitzgibbon's method.

Commentary on Question:
There are several ways that this could be answered correctly. The model solution is an example of a full credit solution.

It follows the actual retrospective premium adjustment formula, so it's easier to explain and justify to underwriters.

GIADV, Spring 2024, Q11

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question tested a candidate's understanding of the reasons for claims-made policies and the principles of claims-made ratemaking.

Solution:

(a) Describe these circumstances.

Commentary on Question:

This question tested a candidate's understanding of the fundamentals of claims-made ratemaking. There are a number of circumstances that may be referenced for this. Full credit was given for providing a description with at least two appropriate circumstances. The model solution is an example of a full credit solution.

The industry was faced with a basic inability to accurately set the price for the occurrence policy form, because most of the claims arising out of any given year's professional services would not be reported until well after the insurer had accepted a fixed price for an open-ended promise to indemnify. Many insurers ceased writing this kind of insurance, and others decided to charge prices they deemed high enough creating an availability and affordability problem.

(b) Identify two reasons that this shift to claims-made coverage was not as prevalent outside of the United States.

Commentary on Question:

There are more than two reasons. The model solution is an example of a full credit solution.

- Greater role of socialized health care in other countries.
- Greater use of mechanisms for dispute resolution in other countries.

(c) Define the claims-made coverage retroactive date.

The earliest accident date for which coverage is provided under a claims-made policy.

(d) Marker and Mohl identified five principles of claims-made ratemaking.

State four of these principles.

Commentary on Question:
The model solution includes all five. Only four were required for full credit.

- A claims-made policy should always cost less than an occurrence policy, as long as claim costs are increasing.
- Whenever there is a sudden, unpredictable change in the underlying trend, claims-made policies priced on the basis of the prior trend will be closer to the correct price than occurrence policies priced in the same way. [
- Whenever there is a sudden unexpected shift in the reporting pattern, the cost of mature claims-made coverage will be affected very little if at all relative to occurrence coverage.
- Claims-made policies incur no liability for pure IBNR claims so the risk of reserve inadequacy is greatly reduced.
- The investment income earned from claims-made policies is substantially less than under occurrence policies

GIADV, Spring 2024, Q12

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2^{nd} Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question tested a candidate's understanding of deductibles and self-insured retentions. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe three ways that a self-insured retention (SIR) differs from the use of a deductible.

Commentary on Question:

There are more than three ways. The model solution is an example of a full credit solution.

- In most traditional GI policies with relatively small deductibles, defense costs are outside of the deductible and outside of the policy limits, and insurers are responsible to pay for all defense costs. In contrast, with an SIR program the insured would typically take the lead on the defense of all claims until the SIR is breached.
- In most traditional GI policies with a deductible, the policy limit is eroded by the deductible. However, under an SIR, there is no erosion of the limit.
- In most traditional GI policies with a large deductible, the insurer is typically required to hold collateral to ensure that the insurer can pay the entire claim regardless of whether or not the insured will reimburse the insurer for the deductible. Collateral is not generally required for an SIR program.

(b) Provide the following with respect to an insurer's application of this approach.

(i) Definition of elimination ratio

(ii) Formula for elimination ratio

Commentary on Question:
The model solution is an example of a full credit solution.

(i) Definition of elimination ratio
The proportion of claims eliminated by the deductible relative to the claims underlying the estimate of the base rate, which is associated with an insurer's base deductible.

(ii) Formula for elimination ratio
(claims eliminated by the deductible − claims eliminated by the base deductible)
divided by
(total ground up claims − claims eliminated by the base deductible)

(c) The reduction in premium is not proportional to the size of the deductible for many lines of general insurance, particularly automobile physical damage coverages and personal property insurance.

Explain why this should be expected.

Commentary on Question:
The model solution is an example of a full credit solution.

This is because lines like automobile physical damage have a claims distribution with many smaller claims. As such, a change in the deductible will tend to have a disproportionate effect on premium as it will have a much larger effect on smaller claims.

(d) Determine the amount the insurer would pay to the insured for this loss under the following scenarios. State any assumptions required.

(i) The policy has no deductible.

(ii) The policy has a deductible of 2,500.

Commentary on Question:
The model solution is an example of a full credit solution.

(i) The policy has no deductible.
Coinsurance penalty % = 1-100,000/(200,000*60%) = 16.7%
Claim payment = 40,000 × (1-0.167) = 33,333

(ii) The policy has a deductible of 2,500.
The policy deductible may come before or after the coinsurance penalty, depending on the policy wording.

- If before the coinsurance penalty, the claim payment is:
  (40,000 – 2,500) × (1 – 0.167) = 31,250

- If before the coinsurance penalty, the claim payment is:
  40,000 × (1 – 0.167) – 2,500 = 30,833

GIADV, Fall 2024, Q2

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6e) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of individual risk rating methods.

Solution:

(a) Compare schedule rating with judgement rating.

Commentary on Question:
The model solution is an example of a full credit solution.

Judgment rating differs from schedule rating as the insurance rate is determined entirely by the underwriter based on their subjective evaluation of the risk. In contrast, for schedule rating, the underwriter uses judgement to determine credits and surcharges to a schedule of risk characteristics that are applied to the manual rate.

(b) Insurance companies typically only use schedule rating for certain types of general insurance policies.

Describe these types of policies.

Commentary on Question:
There are several ways that these types of policies can be described. The model solution is an example of a full credit solution.

Commercial general insurance including lines of business such as commercial multi-peril and general liability.

(c) Identify three primary objectives typically used by insurers in this determination.

Commentary on Question:
There are more than three objectives that can be considered as primary. The model solution is an example of a full credit solution.

- Holding insureds responsible for claims
- Encouraging insureds to participate in risk control activities
- Enhancing market competitiveness

GIADV, Fall 2024, Q7

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6f) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

LR = Loss Ratio, EMLR = Emerged LR, ELR = Expected LR, TM = Tax Multiplier,
BPF = Basic Premium Factor, LCF = Loss Conversion Factor,
EPLE = Expected Percentage of Loss Emerged,
PLEMM = Expected Percentage of Losses Eliminated by Retro Min/Max,
CLCR = Cumulative Loss Capping Ratio, ILCR = Incremental Loss Capping Ratio

Solution:

(a) Calculate the incremental loss capping ratio by retro adjustment period using the Teng and Perkins methodology.

For each retro adjustment *t* period calculate:

- EMLR = Standard LR × Cumulative EPLE
- PLEMM = Ins. Charge at Retro Max – Ins. Savings at Retro Min
- CLCR = 1 – PLEMM – % of Losses Eliminated by a Per Accident Limit

For $t = 1$

- $ILCR_1 = CLCR_1$

For $t > 1$

$ILCR_t = (EMLR_t \times CLCR_t - EMLR_{t-1} \times CLCR_{t-1}) / (EMLR_t - EMLR_{t-1})$

(b) Calculate the implied PDLD ratios at each retro adjustment period based upon the retrospective rating parameters and the selected incremental loss capping ratios.

For retro adjustment period 1

- $PDLD_1 = (BPF \times TM / (ELR \times EPLE_1)) + (ILCR_1 \times LCF \times TM)$

For retro adjustment period $t > 1$

- $PDLD_t = ILCR_t \times LCF \times TM$

(c) Calculate the premium asset as of December 31, 2023, for the policy period subject to the second retrospective adjustment using the PDLD ratios from part (b).

Commentary on Question:
Amounts in millions

Premium Asset
= Estimated total premium – Premium booked
= [Expected future loss emergence × $CPDLD_2$ + Premium booked from prior adjustment] – Premium booked

For retro adjustment period 1

- $EPLE_1$ = Cumulative $EPLE_1$

For retro adjustment period $t > 1$

- $EPLE_t = EPLE_t - EPLE_{t-1}$

$$CPDLD_2 = \frac{\sum_{t=2}^{5} EPLE_t \times PDLD_t}{\sum_{t=2}^{5} EPLE_t} = 0.741$$

Premium Asset
= [72.65 × 0.741 + 302.38] – 298.62
= 57.56

GIADV, Fall 2024, Q11

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6d) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Determine the following:

(i) (*0.5 points*) Average annual accident year trend rate

(ii) (*1 point*) Accident year reporting pattern as a percent of total

(iii) (*1.5 points*) Step factor at each year of claims-made maturity

(iv) (*0.5 points*) Tail factor applicable to coverage following a first-year claims-made maturity policy

(v) (*0.5 points*) Tail factor applicable to coverage following a third-year claims-made maturity policy

Commentary on Question:

Comments are included within the solution as italicized text.

(i) Average annual accident year trend rate

One should look at the trend in claims over several AYs. The model solution uses AY 6 to AY10 as they are all the completed AYs in the information provided. Claims ($C_{AY\text{-}Lag,\ RY}$) for AYs 6 to 10 are as follows:

$C(AY6) = C_{0,6}+C_{1,7}+C_{2,8}+C_{3,9}+C_{4,10}$

$= 541.93+451.63+451.63+180.65+180.65 = 1{,}806.49$

$C(AY7) = C_{0,7}+C_{1,8}+C_{2,9}+C_{3,10}+C_{4,11}$

$= 585.30+487.75+487.75+195.09+195.09 = 1{,}950.98$

and similarly

C(AY8) = 2,107.06, C(AY9) = 2,275.59 and C(AY10) = 2,457.67

AY claims year-over-year changes = C(AYx) / C(AYx−1) − 1

providing the following: 7.998%, 8%, 7.998%, 8%

Note that one could also look at the changes to AY lag claims such as $C_{0,7}/C_{0,6} - 1 = 8\%$, $C_{1,8}/C_{1,7} - 1 = 7.998\%$, *etc. to see the 8% trend.*

Average annual accident year trend rate = 8%

(ii) Accident year reporting pattern as a percent of total

AY reporting pattern for AYx is given by $C_{0,x}/C(AYx)$, $C_{1,x+1}/CY(AYx)$, *etc.*

Using AY6 as an example to calculate the pattern:

AY Lag 0 report % = 541.93 / 1,806.49 = 30%,

AY Lag 1 report % = 451.63 / 1,806.49 = 25%, etc. so we have

AY Lag	0	1	2	3	4
Pattern	**30.0%**	**25.0%**	**25.0%**	**10.0%**	**10.0%**

(iii) Step factor at each year of claims-made maturity

SF(1) = 1st year step factor, SF(2) = 2nd year step factor, etc.

Using RY 10 as an example.

$SF(1) = C_{0,10}/(C_{0,10}+C_{1,10}+ C_{2,10}+C_{3,10}+ C_{4,10}) = 0.33$

$SF(2) = SF(1) + C_{1,10}/(C_{0,10}+C_{1,10}+ C_{2,10}+C_{3,10}+ C_{4,10}) = 0.59$

$SF(3) = SF(2) + C_{2,10}/(C_{0,10}+C_{1,10}+ C_{2,10}+C_{3,10}+ C_{4,10}) = 0.83$

$SF(4) = SF(3) + C_{3,10}/(C_{0,10}+C_{1,10}+ C_{2,10}+C_{3,10}+ C_{4,10}) = 0.92$

$SF(mature) = SF(4) + C_{4,10}/(C_{0,10}+C_{1,10}+ C_{2,10}+C_{3,10}+ C_{4,10}) = 1.00$

(iv) Tail factor applicable to coverage following a first-year claims-made maturity policy

Using RY 10 as an example.

Tail factor 1st year = $(C_{1,11}+C_{2,12}+ C_{3,13}+C_{4,14}) / C_{0,10}$

= (614.42+614.42+245.76+245.76) / 737.71 = 2.33

(v) Tail factor applicable to coverage following a third-year claims-made maturity policy

Using RY 10 as an example.

Tail factor 3rd year

$= (C_{1,11}+C_{2,11}+ C_{3,11}+C_{2,12}+C_{3,12}+C_{4,12}+C_{3,13}+C_{4,13}+C_{4,14}) / (C_{0,10}+C_{1,10}+C_{2,10}) = 1.73$

GIADV, Fall 2024, Q12

Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

Learning Outcomes:

(6c) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question tested a candidate's knowledge of policy limits and deductibles.

Solution:

(a) Identify two other reasons insurers use deductibles in their policies.

Commentary on Question:

There are more than two other reasons. The model solution is an example of a full credit solution.

- Encourage insureds to adhere to some measure of risk control
- Eliminate the processing costs associated with small claims

(b) Provide an example of an action taken by an insured that would be considered:

(i) Moral hazard

(ii) Morale hazard

Commentary on Question:

There are many possible examples for this. The model solution is an example of a full credit solution.

(i) Moral Risk:
An insured fraudulently puts forth a claim for a stolen item when the item was not stolen because they sold it.

(ii) Morale Risk:
An insured does not properly safeguard their property against theft because they know they are insured.

(c) Describe a problem with the use of a percentage deductible for property insurance.

Commentary on Question:
The model solution is an example of a full credit solution.

It can provide an incentive for insureds to purchase insurance with a lower sum insured than the full value of the property. This is because those who purchase insurance policies with a smaller total insured value will automatically have a lower deductible and a lower premium. But they will be underinsured.

(d) Describe how a coinsurance clause in a property policy limits claims.

Commentary on Question:
The model solution is an example of a full credit solution.

It creates a penalty for underinsuring a property. The penalty is based on the percentage that the policy limit is below the property value.

GI 301 Learning Objective 7 Curated Past Exam Solutions

GIADV, Fall 2020, Q1 2

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GIADV, Fall 2020, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

Commentary on Question:

The solution for this question can be displayed in several different ways. The model solution in the Excel file is an example of one approach to display the solution.

Step 1: Select the proper trend period for each loss, in years, as the difference between the treaty year being priced (2021) and the year of the loss because all losses occurred at the midyear point.

Accident Date	Trend Period
7/1/2017	4
7/1/2017	4
7/1/2018	3
7/1/2018	3
7/1/2019	2
7/1/2019	2

Step 2: Calculate the trended loss in layer as the loss increased by the trend rate (5%) over the trend period adjusted by the layer.

Accident Date	Loss	Trended Loss	Trended Loss in Layer
7/1/2017	200,000	243,101	0
7/1/2017	350,000	425,427	175,427
7/1/2018	225,000	260,466	10,466
7/1/2018	900,000	1,041,863	750,000
7/1/2019	250,000	275,625	25,625
7/1/2019	800,000	882,000	632,000

Step 3: Calculate the covered ALAE as the ALAE increased by the trend rate (5%) over the trend period and allocated to the layer by using the ratio of the trended loss in layer to the trended loss.

Accident Date	ALAE	Trended ALAE	Covered ALAE
7/1/2017	150,000	182,326	-
7/1/2017	400,000	486,203	200,488
7/1/2018	-	-	-
7/1/2018	450,000	520,931	390,698
7/1/2019	50,000	55,125	5,125
7/1/2019	275,000	303,188	217,250

Step 4: Calculate the developed trended loss and ALAE for the layer by applying the appropriate development factor based on the age of the loss to the combined trended claim and ALAE for the layer.

Accident Date	Losses + ALAE	Development Factor	Developed Trended Layer Loss and ALAE
7/1/2017	-	1.10	-
7/1/2017	375,915	1.10	413,507
7/1/2018	10,466	1.40	14,652
7/1/2018	1,140,698	1.40	1,596,978
7/1/2019	30,750	2.40	73,800
7/1/2019	849,250	2.40	2,038,200
			4,137,137

Step 5: Calculate the rate as the total developed trended layer loss and ALAE divided by the total on level subject premium for the period 2017 to 2019.

$4,137,137 / (3 \times 10,000,000) = 13.8\%$

GIADV, Fall 2020, Q8

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7f) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) and (b). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) and (b) are for explanatory purposes only.

Solution:

(a) Complete the following aggregate loss probability table:

Aggregate Losses (millions)	Probability
0	
1	0.1673
2	0.1966
3	0.1496
4	
5	
6	0.0411
7	
8	
9	
10	
11	
12	
13	
14	0.0001
15	0.0000

Commentary on Question:

Note that the calculation of the probability of 0 for aggregate losses is from the Poisson distribution and is equal to $e^{-\lambda}$. The probability of 0 for aggregate losses could have been estimated as 1 minus the sum of the probabilities for aggregate *losses from 1 to 15 million. This is not exact because aggregate losses could exceed 15 million. There was a minor deduction for using this approximation.*

Let p_y = probability of loss size of y million.

The annual number of losses is Poisson with mean 1.5. That is, $\lambda = 1.5$.

For aggregate losses of 0, the probability is $e^{-\lambda} = 0.2231$.

For aggregate losses of x, in millions, the probability is given by the formula:

$$(\lambda/x) \times [(x-1) \times p_1 + 2 \times (x-2) \times p_2 + 3 \times (x-3) \times p_3]$$

The table of values is then given by:

Aggregate Losses (million)	**Probability**
0	0.2231
1	0.1673
2	0.1966
3	0.1496
4	0.1059
5	0.0695
6	0.0411
7	0.0231
8	0.0122
9	0.0062
10	0.0030
11	0.0014
12	0.0006
13	0.0003
14	0.0001
15	0.0000

(b) Verify the following underwriting results for Specialist:

(i) A profit of 0.3 million if aggregate losses are 2 million.

(ii) A loss of 1.125 million if aggregate losses are 5 million.

Profit is premium less the losses and margin.

- For (i) this is 0.25
- For (ii) this is −2.75

When profit is greater than zero, there is a profit commission of 80% of the profit.

- For (i) this is 0.2
- For (ii) this is 0

When profit is less than zero (i.e., a loss), there is additional premium of 50% of (losses plus the margin minus the annual premium).

- For (i) this is 0
- For (ii) this is 1.375

The underwriting result is premium plus additional premium less losses less profit commission.

- For (i) this is $2.5 + 0 - 2 - 0.2 = 0.3$
- For (ii) this is $2.5 + 1.375 - 5 - 0 = -1.125$

(c) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

- The reinsurer must assume significant insurance risk.
- It must be reasonably possible that the reinsurer will realize a significant loss.

(d) Explain whether the finite reinsurance can be considered insurance by Ceding Insurance Company.

Commentary on Question:
There is no single correct answer to this question. However, an explanation for this should consider loss sizes relative to the premium and their probabilities. The model solution is an example of a full credit solution.

We may consider that a significant loss is one that is at least 25% of the premium. This level of loss has a probability of over 25% which can be considered reasonably possible. As such, it can be considered as insurance.

GIADV, Spring 2021, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) The underlying losses have the following characteristics:

- Mean 300
- Standard Deviation 1,200

The estimated parameters of the lognormal distribution based on the method of moments are:

- mu (μ) 4.287
- sigma (σ) 1.683

Demonstrate that this is true.

Commentary on Question:

The appropriate formulas for the lognormal distribution are
sigma = (ln[(sd/mean)^2+1])^0.5 and mu = ln(mean)-(sigma^2)/2.
Using these formulas one can demonstrate that mu and sigma are as shown.
An alternative solution is to use the estimated parameters to calculate the mean and standard deviation.

sigma = ln[(1,200/300)^2+1]^0.5 = 1.683
mu = ln(300) − (1.683^2)/2 = 4.287

(b) Demonstrate that the ILF at policy limit 1,500 is 1.44.

Commentary on Question:
The appropriate formula for the ILF[L, U] is E[x; U] / E[x; L] where E[x; A] in Excel is EXP(mu + (sigma^2)/2) × NORM.S.DIST((LN(A) – mu − sigma^2)/sigma, TRUE) + A × (1 − NORM.S.DIST((LN(A) − mu)/sigma, TRUE)).
The model solution in the Excel solutions spreadsheet uses the values of sigma and mu as presented in the question (i.e., rounded to 3 decimal places) to calculate the ILF. It was equally acceptable to use the mu and sigma values calculated in part (a) for this calculation.

E[x; 500] = 151.59 and E[x; 1,500] = 217.78
ILF[500; 1,500] = 217.78 / 151.59 = 1.44

(c) Calculate the expected losses in the layer using an exposure rating approach.

Expected losses in reinsured layer calculation

- Calculate reinsurance exposure factors for each of the four UL/PL combinations
    - (E[x; 1,000] – E[x; 500]) / (E[x; 1,000] – E[x; 0]) = (1.28 – 1.00) / (1.28 – 0) = 0.219
    - (E[x; 1,500] – E[x; 500]) / (E[x; 1,500] – E[x; 0]) = (1.44 – 1.00) / (1.44 – 0) = 0.306
    - (E[x; 1,500] – E[x; 1,000]) / (E[x; 1,500] – E[x; 500]) = (1.44 – 1.28) / (1.44 – 1.00) = 0.364
    - (E[x; 2,000] – E[x; 1,000]) / (E[x; 2,000] – E[x; 500]) = (1.53 – 1.28) / (1.53 – 1.00) = 0.472
- Calculate reinsurance exposure factor times premium for each of the four UL/PL combinations
    - 0.219 × 2,000 = 438
    - 0.306 × 2,500 = 764
    - 0.364 × 4,000 = 1,455
    - 0.472 × 4,500 = 2,123
- Total exposed premium is 4,779 (= 438 + 764 + 1,455 + 2,123)
- Calculate expected loss as expected loss ratio times the total exposed premium: 4,779 × 55% = 2,628

GIADV, Spring 2021, Q8

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) through (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) through (d) are for explanatory purposes only.

Solution:

(a) Show that with the expected loss ratio of 54.0%, the 2021 treaty profit is 7.0% of ceded premium.

Commission at a 54% loss ratio is 25% + (60% − 54%) = 31%
Profit = 100% − 54% − 31% − 8% = 7%

(b) Show that using the loss distribution above, the expected 2021 treaty profit is 3.6% of ceded premium.

First compute the commission at each loss amount.

Treaty Loss	Probability	Loss Ratio	Commission
0	0.0916	0.0%	45.0%
40,000	0.1465	12.1%	45.0%
110,000	0.1954	33.2%	43.4%
180,000	0.1954	54.4%	30.6%
250,000	0.1563	75.5%	25.0%
320,000	0.1042	96.7%	25.0%
390,000	0.0595	117.8%	25.0%
400,000	0.0511	120.8%	25.0%

The expected loss is 53.95%.
The expected commission is 34.45%.
Expected profit = 100% − 53.95% − 34.45% − 8% = 3.59%

(c) State whether or not the sliding scale commission structure is "balanced." Justify your answer.

It is not balanced because the expected loss ratio is close to one end of the commission slide.

(d) Recalculate the expected 2021 treaty profit from (b) as a percentage of ceded premium, allowing for the loss ratio in 2020.

The carryforward loss ratio is that in excess of 60%, the top loss ratio in the commission scale slide. Therefore, the carryforward loss ratio adjustment to the commission slide with a 2020 loss ratio of 75.5% is 15.5%.

Adjusted Loss Ratio	Adjusted Commission
14.5% or below	45%
14.5%-34.5%	Sliding 0.5:1
34.5%-44.5%	Sliding 1:1
44.5% or above	25%

Treaty Loss	Probability	Loss Ratio	Commission
0	0.0916	0.0%	45.0%
40,000	0.1465	12.1%	45.0%
110,000	0.1954	33.2%	35.6%
180,000	0.1954	54.4%	25.0%
250,000	0.1563	75.5%	25.0%
320,000	0.1042	96.7%	25.0%
390,000	0.0595	117.8%	25.0%
400,000	0.0511	120.8%	25.0%

The expected adjusted commission is 31.84%.
Expected profit = 100% − 53.95% − 31.84% − 8% = 6.21%

(e) Another approach to assessing the effect of a carryforward provision is to look at the "long run" of the contract.

State two problems with this approach.

One problem is that the contract may not be renewed in which case there is no "long run" of the contract.

Another problem is that an assessment of the variance over a multiyear period may be complex.

GIADV, Fall 2021, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7f) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the probability that aggregate losses will be 5 billion.

Commentary on Question:

This question could be answered using recursion or first principles. Both approaches were acceptable for full credit. Both solutions are presented.

Recursion:

For negative binomial with α=1 and p=0.5, $A(k) = \sum [1/2 \times S(i) \times A(k-i)]$.
Let L be the loss size and A the aggregate losses. The probability (Pr) that A will be 5 billion (B) is equal to:

$1/2 \times [\Pr(L = 1B) \times \Pr(A = 4B) + \Pr(L = 2B) \times \Pr(A = 3B) + \Pr(L = 3B) \times \Pr(A = 2B)]$
$= 0.5 \times [0.6 \times 0.05055 + 0.3 \times 0.08350 + 0.1 \times 0.12000]$
$= 0.03369$

First principles:

The probability of aggregate losses equal to 5 billion is the sum of the probabilities for the following events:

- 2 losses with 1 loss of 2B and 1 loss of 3B
- 3 losses with 2 losses of 1B and 1 loss of 3B
- 3 losses with 1 loss of 1B and 2 losses of 2B
- 4 losses with 3 losses of 1B and 1 loss of 2B
- 5 losses with all losses equal to 1B

$= (0.5^3 \times 2 \times 0.3 \times 0.1)$
$+ (0.5^4 \times 3 \times 0.6^2 \times 0.1)$
$+ (0.5^4 \times 3 \times 0.6 \times 0.3^2)$
$+ (0.5^5 \times 4 \times 0.6^3 \times 0.3)$
$+ (0.5^6 \times 0.6^5)$
$= 0.03369$

(b) Calculate the mean and coefficient of variation of aggregate catastrophe losses.

$E(n) = 1$
$E(L) = 0.6 \times 1 + 0.3 \times 2 + 0.1 \times 3 = 1.5$
$Var(L) = E(L^2) - E(L)^2$
$E(L)^2 = 0.6 \times 1^2 + 0.3 \times 2^2 + 0.1 \times 3^2 = 2.7$
$Var(L) = 2.7 - 1.5^2 = 0.45$

$E(A) = E(n) \times E(L) = 1.5$
$Var(A) = E(n) \times Var(L) + Var(n) \times E(L)^2 = 4.95$
Coefficient of variation of A = Std Dev(A) / E(A) = $4.95^{0.5} / 1.5 = 1.48324$

(c) Identify one disadvantage of using a recursive formula to calculate aggregate distribution probabilities.

Commentary on Question:
Only one disadvantage was required for full credit.

The calculation is inconvenient when E(n) is large.

Only a single severity distribution can be used.

GIADV, Fall 2021, Q8

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Estimate the experience rating loss cost as a percentage of the subject premium.

For each loss, calculate the developed trended layer loss as follows:

- The untrended loss amount times the trend factor gives the trended loss. The trend factor is 1.05^(2022 – AY for loss).
- Take the layer portion of the trended loss to get the trended layer loss.
- Apply the LDF to the trended layer loss to get the developed trended layer loss. Note that the LDF for AY 2018 layer losses is 1.00, the LDF for AY 2019 layer losses is 1.25 and the LDF for AY 2020 layer losses is 1.50.

The rate is the sum of the developed trended layer losses, 4,194,542, divided by the total subject premium for the three years, 3 × 5,000,000. This equals 28.0%.

(b) Define free cover.

This refers to an experience rating in which no losses trend into the highest portion of the layer being priced.

(c) Calculate a revised loss cost as a percentage of the subject premium using these exposure factors to estimate the cost of free cover.

Select 2,000,000 excess of 1,000,000 for experience rating and 1,000,000 excess of 3,000,000 as free cover for exposure rating. Experience rating gives 28.0% for 2,000,000 excess of 1,000,000.

For each layer, calculate the percentage of insured value for the top and bottom of the layer and obtain the exposure factors for these percentages using the table provided.

Then calculate the difference of these exposure factors for each layer. The rate for the free cover is the experience rate times the ratio of the exposure factor difference for the free cover layer to the exposure factor difference for the experience rating layer. This equals 7.4%.

The revised loss cost is then 28.0% + 7.4% = 35.3%.

(d) Assess whether the loss cost percentage you calculated in part (c) would be appropriate for pricing coverage on properties with insured values of 12 million.

This would not be appropriate as the factors used are based on properties with insured values of 6 million.

GIADV, Spring 2022, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the annual treaty loss ratios including ALAE for each accident year, 2019-2021 (at the 2023 level).

- The trend period is 2 years for accident year (AY) 2021, 3 years for AY 2020 and 4 years for AY 2019.
- The loss development factors are 2.30 for AY 2021, 1.35 for AY 2020 and 1.10 for AY 2019.
- For each AY, the treaty premium is 15% of the subject premium base (i.e., 600,000).

For each AY:

- trended loss is the untrended loss trended by 5% per year for the trend period
- trended loss in the layer is determined from the trended loss
- trended ALAE is the untrended ALAE trended by 5% per year for the trend period
- ALAE covered for the layer is the trended ALAE times the ratio of trended loss in the layer to the trended loss
- developed loss and ALAE for the layer is the trended loss in the layer plus the ALAE covered for the layer times the loss development factor
- the treaty loss ratio is the developed loss and ALAE for the layer divided by the treaty premium

AY	Trended Loss	Loss in Layer	Trended ALAE	Developed Loss + ALAE	Treaty Premium	Treaty Loss Ratio
2019	486,203	236,203	303,877	422,212	600,000	70.4%
2020	1,041,863	500,000	694,575	1,125,000	600,000	187.5%
2021	275,625	25,625	330,750	129,663	600,000	21.6%

(b) Calculate the annual treaty loss ratios including ALAE with the proposed swing plan for each accident year, 2019-2021 (at the 2023 level).

For each AY:

- the loaded amount of loss and ALAE is the developed loss and ALAE for the layer from part (a) loaded by the retro premium factor of 100/80
- the layer swing-rated premium is the loaded amount of loss and ALAE for the layer subject to a minimum of 10% of subject premium (400,000) and a maximum of 20% of subject premium (800,000)
- the revised treaty loss ratio is the developed loss and ALAE for the layer divided by the layer premium

AY	Developed Loss + ALAE	Loaded Loss + ALAE	Swing-rated Premium	Treaty Loss Ratio
2019	422,212	527,765	527,765	80.0%
2020	1,125,000	1,406,250	800,000	140.6%
2021	129,663	162,078	400,000	32.4%

(c) Provide one argument for and one argument against introducing the swing plan.

Commentary on Question:

There are several arguments that could be made for and against the swing plan. Only one of each was required for full credit. The model solution is an example of a full credit solution.

For: It can reduce the volatility in the annual treaty loss ratios.
Against: The treaty loss ratio for the 3-year period combined with swing rating is higher than the treaty loss ratio without swing rating.

GIADV, Spring 2022, Q8

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7g) Describe considerations involved in pricing property catastrophe covers.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only. "M" represents million in the solution that follows.

Solution:

(a) Calculate the nominal rate on line.

Nominal rate on line = Annual premium / Occurrence limit
= 15M / 100M = 15.0%

(b) Calculate the underwriting loss (excluding expenses) to ABC Reinsurance if a loss fully exhausts the limit.

Annual premium – Occurrence limit + Additional premium
= 15M – 100M + 50% of (100M + 10% of 15M – 15M)
= – 41.75M

(c) Calculate the premium for an equivalent traditional risk cover.

Annual premium – Profit commission
= 15M – (100% – 10%) × 15M × 80%
= 4.2M

(d) Calculate the rate on line for an equivalent traditional risk cover.

Premium / Cover
= 4.2M / (41.75M + 4.2M)
= 9.14%

(e) Calculate the minimum value of *N* that would allow ABC Reinsurance Company to avoid an expected underwriting loss with the finite risk cover.

Check reciprocal of the rate on line for the equivalent traditional risk cover:
1 / 0.0914 = 10.94. Therefore, the minimum value of *N* would be 11.

(f) A further consideration when comparing a traditional risk cover to a finite risk cover is credit risk.

Explain how credit risk affects the comparison.

The reinsurer will need to consider the credit risk of the ceding company because the reinsurer will need to rely upon the ceding company's ability to pay the additional premium in the event of a full loss. At this point, the ceding company may be financially weakened.

GIADV, Fall 2022, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the loss cost rate for the treaty.

1. Calculate the ELF at the bottom of the layer (250,000) and the top of the layer (750,000) for each state/hazard group using the NCCI ELF parameters for the appropriate hazard group.
2. Calculate the layer loss cost for each state/hazard group as the ELF difference (bottom of the layer minus top of the layer) times the standard premium for the state/hazard group times the expected loss ratio for the state.
3. Calculate the loss cost rate for the treaty as the total loss cost divided by the total standard premium. This equals 2.52%.

(b) Describe the characteristics of the following categories:

(i) Working Layer

(ii) Exposed Excess

(iii) Clash Cover

Commentary on Question:

The model solution is an example of a full credit solution.

(i) Working Layer: Layer that regularly has claims over the experience period.

(ii) Exposed Excess: Layer likely to have a small number of claims over the experience period.

(iii) Clash Cover: Layer may have only a few claims, or no claims over the experience period.

(c) Compare the use of experience rating in pricing treaties in the three categories.

Commentary on Question:
The model solution is an example of a full credit solution.

(i) Working Layer: Experience rating is generally used and is likely to give a rate representative of the risk.

(ii) Exposed Excess: Experience rating is of limited usefulness.

(iii) Clash Cover: Experience rating is generally not useful.

GIADV, Fall 2022, Q8

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7f) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only. This solution uses "B" to denote billion.

Solution:

(a) Describe what is meant by "collective risk model."

Separate modeling of the claim frequency distribution and the severity distribution and then they are combined to produce the aggregate loss distribution.

(b) Calculate the mean and coefficient of variation for the aggregate losses.

Mean

= count mean × severity mean

= 1.5 × [1B × 0.6 + 2B × 0.3 + 3B × 0.1] = 2.25B

Coefficient of variation

= $[1B^2 \times 0.6 + 2B^2 \times 0.3 + 3B^2 \times 0.1]^{0.5}$ / Mean

= 0.8944

(c) Complete the following aggregate loss probability table:

Aggregate Losses (billions)	Probability
0	
1	
2	
3	
4	
5	
6	
7	

8	
9	
10	
11	
12	
13	
14	
15	

At 0 aggregate losses, the probability is given by the Poisson formula with mean 1.5 for 0 losses. $P(0) = 1.5^0 \times e^{-1.5} / 0! = 0.2231$

For amounts of aggregate losses (where values are integer amounts in B), we use the recursion formula for the remaining values of X in the table. The recursion formula is as follows:

$$P(X) = 1.5 \times (P(X-1) \times 0.6 + 2 \times P(X-2) \times 0.3 + 3 \times P(X-3) \times 0.1) / X$$

So, we have:

$P(1) = 1.5(0.2231 \times 0.6)/1 = 0.2008$
$P(2) = 1.5(0.2008 \times 0.6 + 2 \times 0.2231 \times 0.3) / 2 = 0.1908$
$P(3) = 1.5(0.1908 \times 0.6 + 2 \times 0.2008 \times 0.3 + 3 \times 0.2231 \times 0.1) / 3 = 0.1509$
$P(4) = 1.5(0.1509 \times 0.6 + 2 \times 0.1908 \times 0.3 + 3 \times 0.2008 \times 0.1) / 4 = 0.0995$
And so on, to complete the table.

Aggregate Losses (billions)	**Probability**
0	0.2231
1	0.2008
2	0.1908
3	0.1509
4	0.0995
5	0.0622
6	0.0356
7	0.0190
8	0.0096
9	0.0046
10	0.0021
11	0.0009
12	0.0004
13	0.0002
14	0.0001
15	0.0000

GIADV, Spring 2023, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the expected losses in the layer using an exposure rating approach.

Commentary on Question:

This part required understanding how an underlying limit affects the pricing for an excess layer. ILF = increased limits factor.

For each of the five rows in the table:

Step 1: The coverage starts at the underlying limit and ends at the underlying limit plus the policy limit.

Step 2: The reinsured layer starts at 1 million plus the underlying limit and ends at the minimum of 4 million plus the underlying limit and the policy limit plus the underlying limit.

Step 3: The reinsurance exposure factor is [ILF at reinsurance end point – ILF at reinsurance start point] divided by [ILF at policy end point – ILF at policy start point].

Step 4: Expected loss in layer is the expected loss ratio times the subject premium times the reinsurance exposure factor.

The sum of the expected loss in layer over the five rows is the total expected loss in layer.

(b) Calculate the expected technical ratio (loss ratio plus commission ratio) for the treaty.

The commission at each loss ratio (LR) is determined by the sliding scale. The commission "slides" from 30% at a 50% LR to 10% at a 90% LR in which the commission reduces by 0.5% for every 1% incremental increase in LR. The expected technical ratio is the sum over all possibilities of the probability of a LR times the technical ratio (LR plus commission ratio) at that LR.

(c) Assess whether the sliding scale commission is balanced.

Commentary on Question:
This may be done by comparing the commission at the expected LR (ELR) and the expected commission. In this scenario, these results are reasonably close so one may conclude that the scale is balanced. Because it is not exactly balanced, an answer stating that it is not balanced was also acceptable.

ELR = 57.2% [= 40% × 15% + 50% × 35% + … + 90% × 2%]
Commission at ELR = 26.4% [using the scale for a 57.2% LR]
Expected commission = 25.7% [= 30% × 15% + 30% × 35% + … + 10% × 2%]

Yes, the scale is balanced because the expected commission is close to the commission at the ELR.

GIADV, Spring 2023, Q9

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7a) Understand the types of reinsurance and key reinsurance terms.

(7i) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) and (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (b) and (c) is for explanatory purposes only.

Solution:

(a) Explain why the risk transfer in this reinsurance contract would not be categorized as "reasonably self-evident" to permit reinsurance accounting.

Commentary on Question:

The model solution is an example of a full credit solution.

Aggregate excess of loss reinsurance is not one of the types of insurance usually categorized as reasonably self-evident. Furthermore, this contract has characteristics that limit risk transfer such as dictating the timing of payments at a specific date in a lump sum.

(b) Determine whether or not this reinsurance contract transfers sufficient risk to permit reinsurance accounting using the Expected Reinsurer Deficit (ERD) test with a threshold of 1%.

Commentary on Question:

M = Million

For each claim amount:

- Reinsurance before loss participation = minimum of [100M and maximum of (0 and (claim amount – 150M))]
- Cedant's loss participation = 65% × maximum of (0 and (reinsurance before loss participation – 48M))
- Reinsurance after loss participation = reinsurance before loss participation − cedant's loss participation
- Reinsurer's net economic loss = maximum of [0 and −1× (48M − reinsurance after loss participation × ($1.04^{-3.5}$))]

p = probability of reinsurer having a net economic loss = 6%
T = average reinsurer net economic loss when one occurs = sum of probability × reinsurer's net economic loss ÷ p = 7.83M
P = 48M

ERD = p × T ÷ P = 6% × 7.83M ÷ 48M = 0.98%
The ERD is less than the threshold of 1% so it fails the risk transfer test. It does not transfer sufficient risk to permit reinsurance accounting.

(c) Reinsurance accounting may be applicable even if the risk transfer in this reinsurance contract is <u>not</u> categorized as "reasonably self-evident" and the contract does <u>not</u> meet the conditions for risk transfer from a quantitative test.

Describe when this may apply.

Commentary on Question:
The model solution is an example of a full credit solution.

This can apply when the reinsurer has assumed substantially all risk from the cedant.

GIADV, Spring 2023, Q13

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7c) Analyze and describe the various types of reinsurance.

(7h) Understand the application of a reinstatement premium.

Sources:

Basics of Reinsurance Pricing, Clark

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 10: A Reinsurance Primer

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total losses recoverable under each treaty.

Surplus Share (4 lines with 1,000 retained line)

For each property & loss combination:

- Surplus cover = minimum of [4 × 1,000 and maximum of (0 and insured value – 1,000)]
- Recoverable from surplus = loss × (surplus cover / insured value)

Total recoverable from surplus = 6,620

For each loss, retention after surplus = loss – recoverable from surplus share.

Per Risk Excess of Loss (2,000 excess 1,000)

For each loss:

- Recoverable = minimum of [2,000 and maximum of (0 and retention after surplus – 1,000)]

Total recoverable from per risk excess of loss = 2,500

For each loss, retention after surplus and excess = retention after surplus – recoverable from per risk excess of loss.

Catastrophe (6,000 in excess of 4,000)

- Recoverable = minimum of [6,000 and maximum of (0 and sum of retention after surplus and excess for all losses – 4,000)] = 1,080

(b) Calculate the reinstatement premium for the catastrophe treaty.

Reinstatement premium = annual premium × pro-rata provision × recoverable from catastrophe treaty / catastrophe cover = 600 × 125% × 1,080 / 6,000 = 135

(c) Calculate the amount retained by ABC for each claim.

Commentary on Question:
This question included an AAD for a per risk excess of loss reinsurance treaty. There are two readings in the syllabus resources that discuss an AAD: the Clark reinsurance pricing paper and Chapter 10 from the Friedland text. The Clark reading is used for this item.

- Retention below attachment = minimum of (2,000 and the claim amount)
- Retention above limit = maximum of (0 and the claim amount – (6,000 + 2,000))
- Reinsurance before AAD = minimum of [6,000 and maximum of (0 and the claim amount – attachment)]
- Cumulative reinsurance before AAD = accumulation of reinsurance before AAD starting with claim 1 going sequentially to claim 5
- Cumulative reinsurance AAD = minimum of (10,000 and cumulative reinsurance before AAD)
- AAD from claim = incremental amounts by claim using cumulative reinsurance AAD
- Retained by ABC = retention below attachment + retention above limit + AAD from claim

GIADV, Fall 2023, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the annual experience rating loss cost for each year in 2019-2022.

Commentary on Question:

This part required understanding how to apply trend and development to individual layer losses. The following outlines the steps involved in the calculation that is included in the Excel solutions spreadsheet.

Step 1: Apply trend to the individual ground-up losses to get the individual trended ground-up losses.

Step 2: Apply the reinsurance terms to the individual trended ground-up losses to get the individual trended layer losses.

Step 3: Multiply the individual trended layer losses by the appropriate development factors to get the individual trended ultimate layer losses.

Step 4: Aggregate the individual trended ultimate layer losses by accident year.

Step 5: Divide the trended ultimate layer losses by the subject premium to get the loss cost for each year from 2019 to 2022.

(b) Calculate the revised expected loss cost for each year in 2019-2022.

Commentary on Question:

This part involved understanding that the trended ultimate layer losses need to be calculated for each scenario and then weighted by the scenario probabilities. The following outlines the steps involved in the calculation that is included in the Excel solutions spreadsheet.

For each scenario (0% increase, 10% increase, 20% increase):

Step 1: Multiply the individual trended ground-up losses (from part (a) Step 1) by the factor from each scenario (1, 1.1, 1.2) to get the individual trended ground-up losses by scenario.

Step 2: Apply the reinsurance terms to the individual trended ground-up losses to get the individual trended layer losses by scenario.

Step 3: Multiply the individual trended layer losses by the appropriate development factors to get the individual trended ultimate layer losses by scenario.

Step 4: Weight the individual trended ultimate layer losses by the scenario probabilities to get the revised individual trended ultimate layer losses.

Step 5: Aggregate the revised individual trended ultimate layer losses by accident year.

Step 6: Divide the revised trended ultimate layer losses by the subject premium to get the loss cost for each year from 2019 to 2022.

(c) Explain why using the average of all years may not be appropriate for pricing the 2024 treaty.

The loss cost is increasing by accident year, so using the average would be inappropriate. There is a need to reevaluate the model.

GIADV, Fall 2023, Q9

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7i) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why the "10% - 10% rule" is often not considered appropriate for determining the existence of sufficient risk transfer in a reinsurance agreement.

It does not capture risk transfer in many valid types of reinsurance, such as those with a high loss amount offsetting a low probability of loss.

(b) Define the expected reinsurer deficit (ERD) metric as used for determining the existence of sufficient risk transfer in a reinsurance agreement.

The ERD measure is derived from the probability distribution of net economic outcomes. ERD = pT/P where p = probability of net income loss, T = average severity of net economic loss when it occurs, and P = expected premium.

(c) Determine the reinsurance premium. [Using Excel's Goal Seek function is an acceptable method for determining this amount.]

Commentary on Question:

The model solution was set up to be solved using Excel's Goal Seek function. However, it was also acceptable to solve this problem using trial and error by changing P such that it provides an ERD of 5%.

Amounts in millions.

Min = minimum of, Max = maximum of, RD = reinsurer deficit

For each row in the table, we need to calculate the following:

- Layer Loss = Min[800 and Max[0 and (UVW Direct Loss minus 200)]]
- Reinsured Amount = Layer Loss times 0.85
- PV of RD = Max[0 and − (P − Reinsured Amount/(1.035^3))]
- Probability of RD = Probability if PV of RD > 0, else 0%

Set up cells with P, p, T, ERD and (ERD – 5%).

- P is the value that is changed by goal seek so that the target cell of (ERD – 5%) is equal to 0. Any reasonable starting value should work. The model solution used a starting value of 250.
- p is the sum of the column for Probability of RD.
- T is the sum of the product of PV of RD times Probability of RD divided by p.
- ERD is pT/P.
- The target cell is ERD – 5% with a goal of 0 by changing P.

(d) Explain why UVW would likely not need to test for risk transfer with respect to this reinsurance agreement.

Testing for risk transfer is not required when risk transfer is self-evident. Risk transfer in most excess of loss reinsurance contracts is self-evident. This aggregate excess of loss reinsurance agreement is priced such that it is exposed to significant risk.

GIADV, Fall 2023, Q13

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7f) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Demonstrate that the mean and coefficient of variation of aggregate losses are 2 billion and 1.118, respectively.

Commentary on Question:

CoV = coefficient of variation, x = loss size in billions, p = probability,
λ = Poisson mean for annual number of losses

Mean = $\sum px = 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) = 2$
$\sum px^2 = 0.4(1^2) + 0.3(2^2) + 0.2(3^2) + 0.1(4^2) = 5$
CoV = $(\lambda \sum px^2)^{0.5}$ / Mean = 2.2361 / 2 = 1.118

(b) Complete the following aggregate loss probability table:

Aggregate Losses (billion)	Probability
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Commentary on Question:
L = aggregate loss in billions, Prob(L) = probability of L,
p(x) = probability of loss size x

- For L = 0, Prob(0) = $\lambda^0 \times e^{-\lambda} / L! = 0.3679$.
- For L = 1, Prob(1) = $(\lambda / 1) \times (Prob(0) \times p(1))$
- For L = 2, Prob(2) = $(\lambda / 2) \times (Prob(1) \times p(1) + 2 \times Prob(0) \times p(2))$
- For L = 3, Prob(3) = $(\lambda / 3) \times (Prob(2) \times p(1) + 2 \times Prob(1) \times p(2) + 3 \times Prob(0) \times p(3))$
- For L > 3, Prob(L) = $(\lambda / L) \times (Prob(L-1) \times p(1) + 2 \times Prob(L-2) \times p(2) + 3 \times Prob(L-3) \times p(3) + 4 \times Prob(L-4) \times p(4))$

(c) Calculate the method of moments estimates for μ and σ^2.

Commentary on Question:
ln = natural logarithm

σ^2 = ln(aggregate CoV^2 + 1) = ln(1.118^2 + 1) = 0.8109
μ = ln(aggregate mean) − σ^2 / 2 = ln(2) – 0.8109 / 2 = 0.2877

GIADV, Spring 2024, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to analyze aspects of a pricing analysis for a proportional treaty. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the probability of a combined ratio of more than 100%.

Commentary on Question:

Note that the question did not indicate whether the loss distribution provided was discreet or continuous. Assuming either form for the distribution was acceptable and would earn full credit if answered correctly with that assumption. The model solution assumes that the loss distribution is continuous. Because of this assumption, x is set to the midpoint of loss amounts. Note that if the assumption was a discrete loss distribution, x would be set to the loss amount.

1. Set up columns for Loss, x, F(x) and p(x). F(x) is the cumulative probability. Then p(x), the probability of x, is the successive differences of the cumulative probability.

2. Set up columns for the reinsurance loss ratio (LR) and the reinsurance combined ratio (CR). The LR(x) is x times one million divided by the reinsurance premium of eight million. The CR(x) is the LR(x) plus the ceding commission (30%) plus brokerage fees (5%) plus other expenses (2%).

3. Find the CR just above and just below 100%, and its associated cumulative probability for the x that produces it.

Reins. CR	F(x)
93.3%	0.7191
105.8%	0.8182

4. Using interpolation between the two points in step 3 above:

Reins. CR	F(x)
100.0%	0.7726

Therefore, the probability is 22.7% (= 1 – .7726 in % form).

(b) Calculate the expected loss ratio after the loss corridor.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

1. Set up columns for the losses in the loss corridor at x and the associated revised reinsured LR(x).

   Losses in the corridor at x equal the maximum of 0% and (LR(x) from part (a) minus 60%) capped at 40% (i.e., the full amount in the corridor from 60% to 100% for LR(x) greater than 100%).

   The revised reinsured LR(x) is LR(x) from part (a) minus 75% of the losses in the corridor at x.

2. The expected LR is the sum of the revised reinsured LR(x) times the p(x) from part (a) divided by the sum of p(x) for all x.

The expected LR is 45.0%.

(c) Calculate the expected combined ratio.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

1. Set up columns for the commission at x, (from the sliding scale based upon the revised LR(x) from part (b)) and the revised combined ratio CR(x).

   - If the revised LR(x) is less than, or equal to 40%, the commission is 35%

- If the revised LR(x) is greater than, or equal to 70%, the commission is 15%
- If the revised LR(x) is between 60% and 70%, the commission is 15% plus (70% minus LR(x))
- If the revised LR(x) is between 40% and 60%, the commission is 25% plus one half of (LR(x) minus 40%).

The revised reinsured CR(x) is the sum of the revised LR(x) from part (b), the commission at x as determined by the scale (as noted in the bullet points above), brokerage fees (5%) and other expenses (2%).

2. The expected CR is the sum of the revised reinsured CR(x) from step 1 times the p(x) from part (a) divided by the sum of p(x) for all x.

The expected CR is 81.1%.

(d) Assess whether the sliding scale commission is balanced.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

The expected commission is the sum of the commission at x from part (c) times the p(x) from part (a) divided by the sum of p(x) for all x.

If the expected commission is approximately equal to the provisional commission, it is balanced.

Expected commision	29.1%
Provisional commission	30.0%
Difference (%)	-3.02%

These appear different enough, so the scale may be considered as imbalanced.

GIADV, Spring 2024, Q9

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7i) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question tested a candidate's understanding of methods to measure the existence of risk transfer. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe two examples of contracts where the risk transfer is "reasonably self-evident."

Commentary on Question:

The model solution is an example of a full credit solution.

- Straight quota share with fixed terms
- Fixed premium per-risk excess of loss

(b) Describe two advantages of using ERD and RCR versus using VaR and TVaR.

Commentary on Question:

The model solution is an example of a full credit solution.

- ERD and RCR do not rely on a fixed (arbitrary) selection of a percentile. TVaR and VaR rely on a selection of a percentile.
- ERD and RCR capture all capital-destroying loss events, while TVaR and VaR generally do not.

(c) Determine whether risk transfer exists in this contract using the Max QP test with α equal to 4.

Commentary on Question:
P(x) is the probability of loss amount x.

Calculate F(x) as the cumulative probability at loss amount, x.
$F^*(x) = 1 - [1 - F(x)]^{1/2}$
$P^*(x)$ values are the incremental values of $F^*(x)$
$E(x) = \sum xP(x)$
$E^*(x) = \sum xP^*(x)$
$RTD = E^*(x) - E(x)$

For this contract, 4 times the RTD of 30.7 is 122.7. This is greater than the premium of 48 so it passes the risk transfer test based on the Max QP test methodology.

GIADV, Spring 2024, Q13

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7c) Analyze and describe the various types of reinsurance.

(7e) Calculate the price of a reinsurance contract.

(7g) Describe considerations involved in pricing property catastrophe covers.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to analyze per risk excess treaties. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the nominal rate on line.

This is the annual premium divided by the occurrence limit
= 20 million / 150 million
= 13.3%

(b) Calculate the underwriting loss (excluding expenses) to ABC Re if a loss fully exhausts the limit.

This is the underwriting result (times −1 because we want the loss) produced with a claim amount of 150 million.

Underwriting loss
= −1×[annual premium – claim amount + additional premium]

Additional premium
= 50% × (150 million + 10%×150 million – 150 million)
= 66 million

Underwriting loss
= −1×[20 million −150 million + 66 million]
= 64 million

(c) Calculate the premium for an equivalent traditional risk cover.

Premium
= 20 million – (1 – 10%) × 80% × 20 million
= 5.6 million

(d) Calculate the rate on line for an equivalent traditional risk cover.

Rate on line
= 5.6 million / (5.6 million + 64 million)
= 8.0%

(e) Construct a counterproposal that should be acceptable to both ABC Re and JKL. Justify your answer.

Commentary on Question:
There are many different counterproposals that should be acceptable to both parties. These counterproposals should do one or more of the following: increase premium, decrease profit commission, increase margin, increase additional premium. This should be done to ensure that the rate on line for the equivalent traditional risk cover is greater than 10% (because there is a full loss once every 10 years) . The model solution is an example of a full credit solution increasing premium and the margin so that the rate on line for the equivalent traditional risk cover is just above 10%. A full credit solution should calculate this amount to show that their counterproposal upholds the target of being just above 10%.

Increasing the premium to 23 million, and the margin to 15%, should be acceptable to both parties with a rate on line of just over 10%.

The rate on line for the equivalent traditional risk cover is 10.6%

GIADV, Fall 2024, Q1

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to analyze aspects of a pricing analysis for workers compensation excess of loss reinsurance. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the values of *a* and *b* for each hazard group.

Step 1: Calculate ELF[200,000] / ELF [1,000,000] for J and K using the provided ELF tables. We get 5.000 for J and 3.6316 for K.

Step 2: Calculate the natural logarithm of the step 1 amounts for J and K. We get 1.6094 for J and 1.2897 for K.

Step 3: Calculate the natural logarithm of 200,000 / 1,000,000. We get −1.6094

Step 4: Calculate *b* as (−step 2 amounts) divided by step 3 amount. We get 1.0000 for J and 0.8013 for K.

Step 5: Calculate *a* as ELF[200,000] divided [$200{,}000^{(-\text{step 4 amounts})}$]. We get 6,000 for J and 1,220.8056 for K.

(b) Calculate the loss cost rate for the treaty.

Step 1: Calculate the ELFs for amounts 100,000 and 400,000 for each of J and K as $a \times \text{amount}^{-b}$. We get

Loss Size	J	K
100,000	0.060	0.120
400,000	0.015	0.040

Step 2: Calculate the layer ELF each of J and K using amounts from step 1 as ELF[100,000] – ELF[400,000]

Step 3: Calculate the Treaty Loss for each State and Hazard combination as Standard Premium (SP) × Expected Loss Ratio (ELR) × layer ELF

State	Hazard	SP	Treaty Loss
X	J	70,000	1,575
X	K	120,000	4,839
Y	J	110,000	3,465
Y	K	100,000	5,646

Step 4: Treaty Loss Cost is total Treaty Loss divided total SP.
We get 15,525 / 400,000 = 3.9%

(c) Explain how excluding state X will affect the loss cost rate for the treaty.

Commentary on Question:
The model solution is an example of a full credit solution.

For state Y only, we get a loss cost of 9,111 / 210,000 = 4.3%.
Therefore, excluding state X would increase the loss cost from 3.9% to 4.3% which is an 11.8% increase in the rate.

GIADV, Fall 2024, Q9

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7i) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question tested a candidate's understanding of testing for risk transfer in reinsurance contracts.

Solution:

(a) Complete the following table with the attributes listed above. *Note that an attribute may be included in more than one cell and a cell may include more than one attribute.*

Quantitative Test	Advantage(s)	Disadvantage(s)
Value-at-Risk (VaR)		
Tail Value-at-Risk (TVaR)		
Expected Reinsurer Deficit (ERD)		

Commentary on Question:

The model solution is an example of a full credit solution. Note that some attributes are subjective and could be included in different cells from what is included in the model solution. For example, it was also acceptable to include II as an advantage for TVaR. Also, while VI applies to ERD, arguments may be made for it either being an advantage or a disadvantage. However, some of the attributes could only be correctly placed in one cell. For example, I is an advantage for ERD only and III is a disadvantage for VaR only.

Quantitative Test	Advantage(s)	Disadvantage(s)
Value-at-Risk (VaR)	II	III, V, VII, VIII
Tail Value-at-Risk (TVaR)	IX	V, VII
Expected Reinsurer Deficit (ERD)	I, VI	V

(b) State the following:

(i) The accounting treatment for a reinsurance contract that is categorized as not transferring sufficient insurance risk.

(ii) A type of reinsurance coverage deemed to transfer sufficient risk transfer despite being not "reasonably self-evident" and not fulfilling quantitative risk transfer tests.

(i) Deposit accounting (*or accounted for as a financial instrument*)

(ii) A reinsurance coverage that assumes substantially all the risks from the primary contract (*e.g., straight quota share*).

(c) Compare the risk measurement in the ERD test with that in the Risk Coverage Ratio (RCR) test.

Commentary on Question:
The model solution is an example of a full credit solution.

Both measure tail risk. However, ERD is a risk/premium measure, while RCR is the corresponding risk/return measure.

(d) Show the formula for RCR (in percent form) that includes ERD as a term in the formula. Define all terms in the formula, excluding ERD.

RCR (% form) = ERD / ($E[G]/P$)

- $E[G]$ = expected economic gain across all possibilities
- P = reinsurance premium

GIADV, Fall 2024, Q13

Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(7e) Calculate the price of a reinsurance contract.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

Step 1: Determine the trend period (in years) for each claim as the difference between the accident date of the claim and the average accident date of the policy period being priced (accident year 2025, so July 1, 2025)

Step 2: Calculate trended losses and trended ALAE separately for each claim. The trended loss uses the loss trend (5%) over the trend period for the claim multiplied by the untrended loss. The trended ALAE uses the ALAE trend (10%) over the trend period for the claim multiplied by the untrended ALAE.

Step 3: Calculate the layer trended loss and ALAE combined for each claim. For each claim, sum the trended loss and the trended ALAE. The amount in the layer is that combined trended loss and ALAE amount above the excess attachment point (200,000) limited by the size of the layer (800,000) for each claim.

Step 4: Calculate the total developed trended loss and ALAE in the layer. For each claim, apply the applicable development factor (based on the accident year of the claim) to the trended loss and ALAE in the layer. Sum this amount for all six of the claims to get the total developed trended loss and ALAE in the layer (4,082,185).

Step 5: Calculate the experience rating loss cost. This is the total developed trended loss and ALAE in the layer (step 4 amount) divided by the total premium over the experience period (30,000,000 because it is 10,000,000 for each of the three years in the experience period). This is equal to 6.80%

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GI 301 Learning Objective 8 Curated Past Exam Solutions

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GIRR, Fall 2020, Q12

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8a) Understand the purpose and development of catastrophe models.
(8b) Understand the type of output produced by catastrophe models.
(8c) Understand how catastrophe model output can be used in actuarial tasks..

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) State four applications of catastrophe modeling for insurance.

Any four of the following are acceptable:

- Ratemaking
- Underwriting and Risk Selection
- Loss Mitigation
- Catastrophe Reinsurance
- State and federal public policymakers use catastrophe models to address public policy issues.
- Capital adequacy (sensitivity) testing
- For reserving purposes

(b) Recommend which portfolio you would add to the book. Justify your recommendation.

Account Y is recommended because it has a relatively high AAL, but it could be in an area with low concentration in the current book, since it doesn't impact the total book's PML too much.

(c) Calculate the premium for this other portfolio assuming hurricane shutters are installed on all properties in the portfolio.

Adjusted AAL = AAL × (1 – Discount) = 5,000 × (1 – 0.137) = 4,315
Premium = (Adjusted AAL + Risk Load)/(1 – expense load factor)
= (4,315 + 440) / (1 – 0.27) = 6,514

(d) Provide a consideration in the selection of a risk load in this situation.

The variability (i.e., standard deviation or CV) or uncertainty in the loss estimates.

(e) Recommend a way this risk could be managed.

The company could manage this exposure by transferred the risks to other parties (e.g., investors or reinsurers with worldwide portfolios).

GIRR, Spring 2021, Q8

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8a) Understand the purpose and development of catastrophe models.
(8b) Understand the type of output produced by catastrophe models.
(8c) Understand how catastrophe model output can be used in actuarial tasks..

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Describe four limitations of relying on historical data to analyze catastrophe events.

Any four of the following are acceptable:

- Traditional actuarial methods rely on incurred historical data to derive indications.
- Frequency and severity of catastrophe activity has not been constant over time.
- The attributes of historical events may be quite different from future events.
- Geographical patterns and physical characteristics of the historical record do not reflect the full range of possible catastrophe events.
- Property distributions and characteristics have changed.
- Many important property characteristics are not available in historical records.
- Claim payment records may be limited or inaccurate and claim practice may have changed over time.
- Information related to older events is not always reliable.

(b) Explain how catastrophe model output can be used to evaluate alternative loss mitigation strategies.

Any of the following is acceptable:

- The impact of the loss mitigation features can be evaluated by seeing how AALs and other measures react to the presence or absence of these features.

- Cost/Benefit tradeoffs can be evaluated.
- Strategies to encourage desired choices can be tied to potential loss dollar changes.

(c) Calculate the hurricane wind premium by county for a 207,500 Coverage A limit.

Commentary on Question:
The risk load needs to be included in the Hurricane Wind Premium Per $1,000 Coverage A before multiplying by the average Coverage A limit.

County	(1) Modeled Gross Hurricane Wind Loss Per 1,000 Coverage A	(2) Selected Risk Load (Standard Deviation)	(3) = [(1)+(2)]/[1 – 0.27] – [(1)+(2)] Expense Load	(4) = (1)+(2)+(3) Hurricane Wind Premium Per 1,000 Coverage A	(5) = (4)×207,500/1,000 Hurricane Wind Premium for 207.5K Coverage A Limit
Monroe	13.82	27.65	15.34	56.81	11,788
Broward	5.54	11.08	6.15	22.77	4,724
Palm Beach	5.26	10.51	5.83	21.60	4,483
Miami-Dade	7.60	15.21	8.44	31.25	6,484
Hillsborough	0.75	1.51	0.84	3.10	642
Orange	0.36	0.72	0.40	1.48	307
Okeechobee	1.91	3.81	2.12	7.84	1,626
Duval	0.25	0.49	0.27	1.01	210
Sarasota	1.74	3.48	1.93	7.15	1,484

GIRR, Fall 2021, Q17

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8a) Understand the purpose and development of catastrophe models.
(8b) Understand the type of output produced by catastrophe models.
(8c) Understand how catastrophe model output can be used in actuarial tasks.

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Calculate the probability of reaching an amount of loss that activates reinsurance coverage for each of the reinsurance layers.

1/100 = 1.0%, for layer 1 (insured losses between the 100-year and 250-year PMLs from these two perils)

1/250 = 0.4%, for layer 2 (insured losses between the 250-year and 250-year PMLs from these two perils)

(b) Calculate Primary's reinsurance recoverables from this catastrophic event for each of the two layers.

Amount in 000s:
Layer 1 losses from 664,515 to 1,089,697
Layer 2 losses from 1,089,697 to 1,605,179

Insured loss total = 1,098,085 + 132,325, = 1,230,410

Layer 1 losses = min(1,089,697,1,230,410) – 664,515 = 425,182
Layer 2 losses = min(1,605,179,1,230,410) – 1,089,697 = 140,713

(c) Estimate Primary's reinsurance premium for each layer of coverage.

Amount in 000s:

	Layer 1	Layer 2
AAL	661	233
Risk load	5,838	3,718
Expense load	2,052	1,248
Premium	8,551	5,199

e.g., Risk load for layer 1: 5,838 = 0.85×6,868

Expense load for layer 1:

$$\frac{(661+5838)}{1-0.24}-(661+5838)$$

Premium for layer 1: 8,551 = 661 + 5,838 + 2,052 + 8,551

(d) Provide two reasons why Primary should not calculate the total reinsurance premium using the underwriter's recommendation.

If ABC wants to cover insured losses in the layer between the 100-year and 500-year PMLs, it must get the combined perils PMLs because PMLs are not additive.

Premiums cannot be added because this will overstate the risk load due to the fact that the SDs are not additive. The combined perils SD is less than the sum of the SDs for all the perils covered.

GIRR, Spring 2022, Q13

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8a) Understand the purpose and development of catastrophe models.
(8b) Understand the type of output produced by catastrophe models.
(8c) Understand how catastrophe model output can be used in actuarial tasks..

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Explain why the 100-year PML for hurricane wind losses and the 100-year PML for tornado wind losses should not be added together to determine the 100-year PML for hurricane and tornado wind losses.

PMLs are not additive. This is because the probability that all causes have a one in 100-year event in the same year is much less than 1 percent. As such, the sum of the one in 100-year PMLs is associated with a much longer return period.

(b) Describe how an insurer could use each of the following loss metrics to understand the risk of an individual insured.

(i) AAL to TIV ratio

(ii) PML to TIV ratio

(i) AAL to TIV ratio: This ratio shows long-term risk at a location. It can be used to compare the long-term risk for properties that are close geographically.

(ii) PML to TIV ratio: This ratio gives an indication of possible loss severity at a location. It can be used to compare properties that have similar AALs to determine which one has a higher loss potential from extreme events.

(c) Explain how catastrophe models can be used by an insurer for portfolio optimization with respect to risk.

An insurer chooses a modeled metric that it considers important. It then builds a portfolio that optimizes that metric relative to a level of premium or exposure using outputs from catastrophe model runs.

(d) Provide two other examples of requirements that have been established to govern the use of catastrophe models.

Any two of the following are acceptable:

- The American Academy of Actuaries and insurance regulatory bodies have developed requirements and guidance for Actuaries in their development, use, and reliance on catastrophe models.
- Actuaries in the U.S. must follow ASOPs of which two are specifically focused on the use of catastrophe models.
- The State of Florida has a uses a legislated methodology for evaluating hurricane models that can be used.
- The NAIC in the U.S. requires model use for completion of RBC and ORSA.
- Enterprise Risk Management (ERM), rating agencies, and state insurance regulators mandate certain model output to be provided for use in evaluation of risk-bearing entities.

GIRR, Fall 2022, Q20

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8a) Understand the purpose and development of catastrophe models.
(8b) Understand the type of output produced by catastrophe models.
(8c) Understand how catastrophe model output can be used in actuarial tasks..

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Calculate the expected Average Annual Loss (AAL) per $1,000 of building coverage.

AAL = Sumproduct of the Annual Probability of Hurricane (p) and the Expected Loss (L) Per $1,000 of Building Coverage

$= (1.00\% \times 50 + \ldots + 1.20\% \times 100) = 8.785$

(b) Calculate Hurricane Wind Premium for the average building in the zip code using the method described in the American Academy of Actuaries monograph, Uses of Catastrophe Model Output.

Event #	$p \times L^2$	$p \times L^2 - AAL^2$
1	25.00	-52.18
2	20.00	-57.18
3	62.50	-14.68
4	135.00	57.82
5	0.56	-76.61
6	200.00	122.82
7	227.81	150.64
8	312.50	235.32
9	440.00	362.82
10	120.00	42.82
Total		771.61

Risk load = $771.61^{0.5} = 27.78$

$$\text{Expense Load} = \frac{(8.785+27.78)}{(1-0.25)} - (8.785+27.78) = 12.19$$

Hurricane Wind Premium per Coverage A = 8.785 + 27.78 + 12.19 = 48.75
Hurricane Wind Cover in Zip-code per $1,000 of building coverage
= 48.75×200,000/1,000 = 9,750.11

(c) Describe why hurricane deductibles tend to be larger in inland areas compared to coastal regions.

Coastal regions experience higher wind speeds and losses are more likely to be severe, so deductibles tend to be a smaller portion of the overall loss. Because inland counties' hurricane wind losses are likely to be lower, deductibles tend to be a higher percentage of overall loss.

(d) Identify which zip code has the highest potential for loss from hurricane events. Justify your selection.

Zip Code	PML/AAL
A	106.13
B	77.43

106.13 / 77.43 = 1.37

While zip code B has the higher AAL, the ratio of PML/AAL is 37% higher for zip code A indicating there is higher loss potential in zip code A.

GIADV, Fall 2020, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the renewal risk load for each account using the Marginal Variance method.

- The risk load for each account is the account marginal variance times λ.
- Account marginal variance is the total variance less the variance of the other account.
  - $\text{Var}(X+Y) = \Sigma_i \text{Var}(X_i+Y_i) = \Sigma_i (X_i+Y_i)^2 \times p(i) \times (1-p(i)) = 14{,}419{,}500$
  - $\text{Var}(X) = \Sigma_i \text{Var}(X_i) = \Sigma_i (X_i)^2 \times p(i) \times (1-p(i)) = 6{,}880{,}000$
  - $\text{Var}(Y) = \Sigma_i \text{Var}(Y_i) = \Sigma_i (Y_i)^2 \times p(i) \times (1-p(i)) = 1{,}655{,}500$
  - X marginal variance = 14,419,500 – 1,655,500 = 12,764,000
  - Y marginal variance = 14,419,500 – 6,880,000 = 7,539,500

- X renewal risk load = 12,764,000 × λ = 306.34
- Y renewal risk load = 7,539,500 × λ = 180.95

(b) Demonstrate that the Marginal Variance method is not renewal additive.

- Risk load for both accounts combined = 14,419,500 × λ = 346.06
- The risk load from account X plus the risk load from account Y = 487.28
- The risk load for both accounts combined does not equal the sum of the risk loads for each account.

(c) Calculate the risk load for each account using the Covariance Share method.

For each event (i) calculate:

- Covariance to share as $\text{Cov}(i) = \text{Var}(X_i+Y_i) - \text{Var}(X_i) - \text{Var}(Y_i)$
- X-share of $\text{Cov}(i) = \text{Cov}(i) \times X_i / (X_i + Y_i)$
- Y-share of $\text{Cov}(i) = \text{Cov}(i) \times Y_i / (X_i + Y_i)$

X risk load $= \lambda \times \Sigma_i$ [$\text{Var}(X_i)$ + X-share of $\text{Cov}(i)$] = 258.12
Y risk load $= \lambda \times \Sigma_i$ [$\text{Var}(Y_i)$ + Y-share of $\text{Cov}(i)$] = 87.95

GIADV, Spring 2021, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel for part (c). An example of a full credit solution for part (c) is in the Excel solutions spreadsheet. The model solution in this file for part (c) is for explanatory purposes only.

Solution:

(a) Identify which risk load is larger.

Under the Marginal Variance method, the renewal risk load for account X is greater than the risk load for account X during build-up.

(b) Explain why there is this difference.

During build-up, the Marginal Variance risk load is a factor times the variance of account X, i.e., Var(X). On renewal, the Marginal Variance risk load is a factor times the variance of the combined accounts X and Y, less the variance of account Y. That is, Var(X+Y) – Var(Y).

Var(X+Y) – Var(Y) = Var (X) + 2Cov(X,Y) > Var(X) since Cov(X,Y) is greater than 0.

(c) Calculate the renewal risk load for each account using the following methods:

(i) Marginal Variance

(ii) Shapley

(i)

	X	Y	X+Y
Variance	5,764,994	2,883,138	11,719,844
Change in variance	8,836,706	5,954,850	
Marginal Variance - Risk Load	397.65	267.97	

(ii)

Covariances	X	Y
X	5,764,994	1,535,856
Y	1,535,856	2,883,138

	X	Y
Shapley value	7,300,850	4,418,994
Shapley - Risk Load	328.54	198.85

GIADV, Fall 2021, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the following for each of the four portfolios:

(i) Expected loss

(ii) Variance

(iii) Coefficient of variation

(i) Expected loss: Let Pr(i) be the probability for event i and L(A, i) be the loss to portfolio A for event i. Then the expected loss to portfolio A is $\sum_i \Pr(i) \times L(A, i)$.

(ii) Variance: $\sum_i \Pr(i) \times (1-\Pr(i)) \times L(A, i)^2$

(iii) Coefficient of Variation (CoV): Variance^0.5 / Expected loss

		Portfolio Q	Portfolio U	Portfolio V	Portfolio W
(i)	Expected loss	27,000	11,450	10,000	9,100
(ii)	Variance	10,090,000,000	2,860,173,750	1,284,480,000	1,097,625,000
(iii)	CoV	3.72	4.67	3.58	3.64

(b) Recommend which portfolio the reinsurance company should add if it wants to minimize the size of the total risk load. Justify your answer.

Commentary on Question:

Risk load is directly proportional to variance, so one should add the portfolio that produces the minimum variance for the combined portfolio.

For each of the combined portfolios (Q+U, Q+V and Q+W) we calculate the variance as $\sum_i \Pr(i) \times (1-\Pr(i)) \times L(U+\text{<added portfolio>}, i)^2$.

	Variance		
Event	Q+U	Q+V	Q+W
1	3,490,644,375	3,369,240,000	4,129,897,500
2	2,574,609,375	4,719,000,000	2,817,750,000
3	4,610,410,000	566,440,000	784,000,000
4	6,023,160,000	9,900,000,000	9,219,127,500
Total	16,698,823,750	18,554,680,000	16,950,775,000

The minimum variance is with Q and U, so the company should add portfolio U.

(c) Calculate the renewal risk loads for portfolio Q and the portfolio you recommended be added in part (b).

Total risk load for Q+U = $\lambda \times \text{Var}(Q+U) = 0.00002 \times 16{,}698{,}823{,}750 = 333{,}976$.

For the Covariance Share method, the Covariance Share of Q for Q+U
$= \sum_i [L(Q, i) / (L(Q, i) + L(U, i))] \times (\text{Var}(Q+U, i) - \text{Var}(Q, i) - \text{Var}(U, i))$.

Event	Covariance Share of Q+U for Q
1	1,025,368,421
2	76,800,000
3	454,639,175
4	428,365,385
Total	1,985,172,981

The Cov. Share renewal risk load for Q = $\lambda \times [\text{Var}(Q) + \text{Cov. Share for Q}]$
$= 0.00002 \times (10{,}090{,}000{,}000 + 1{,}985{,}172{,}981) = 241{,}503$.

The Cov. Share renewal risk load for U = Total risk load for Q+U less the Cov. Share renewal risk load for Q = 333,976 – 241,503 = 92,473.

GIADV, Spring 2022, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the following for each portfolio and for the two portfolios combined:

(i) Expected losses

(ii) Variance of losses

(iii) Coefficient of variation

- Expected losses = the sum over i of $p(i)$ times the loss to the portfolio for i
- Variance (Var) of losses = the sum over i of $p(i)$ times $(1 - p(i))$ times the square of the loss to the portfolio for i
- Coefficient of variation = the square root of the variance of the losses divided by the expected losses

	Portfolio A	Portfolio B	Portfolio A+B
(i) Expected losses	53,270	65,110	118,380
(ii) Var of losses	107,160,721,750	211,290,600,250	476,840,776,600
(iii) CoV	615%	706%	583%

(b) Calculate the renewal risk load by portfolio using each of the following methods:

(i) Marginal Variance

(ii) Shapley

(iii) Covariance Share

For (i), Risk Load = $\lambda \times$ Change in Var

- Change in Var for A is Var for A+B minus Var for B
- Change in Var for B is Var for A+B minus Var for A

	A	B
Change in Var	265,550,176,350	369,680,054,850
Risk Load	63,732	88,723

For (ii), Risk Load = $\lambda \times$ Shapley Value

- Shapley Value for A is the Var for A plus the Covariance of A and B
- Shapley Value for B is the Var for B plus the Covariance of A and B
- Covariance of A and B is the sum over *i* of loss to A for *i* times loss to B for *i* times $p(i)$ times $(1 - p(i))$

Covariances	A	B
A	107,160,721,750	79,194,727,300
B	79,194,727,300	211,290,600,250
	A	B
Shapley Value	186,355,449,050	290,485,327,550
Risk Load	44,725	69,716

For (iii), Risk Load = $\lambda \times$ (Var + Covariance to Share)

- Covariance to Share for A is the sum over *i* of Covariance(*i*) times loss to A for *i* / loss to A+B for *i*
- Covariance to Share for B is the sum over *i* of Covariance(*i*) times loss to B for *i* / loss to A+B for *i*

	A	B
Var + Cov to Share	172,783,414,690	304,057,361,910
Risk Load	41,468	72,974

(c) Demonstrate for each method in part (b) whether or not the risk load is renewal additive.

The risk load for the portfolio of A and B combined is $\lambda \times$ Var for A and B combined = $\lambda \times$ 476,840,776,600 = 114,442.

For the Marginal Variance method, the total renewal risk load = 63,732 + 88,723 = 152,455. This does not equal the risk load for the portfolio of A and B combined, so the method is not renewal additive.

For the Shapley method, the total renewal risk load = 44,725 + 69,716 = 114,442. This equals the risk load for the portfolio of A and B combined, so the method is renewal additive.

For the Covariance Share method, the total renewal risk load = 41,468 + 72,974 = 114,442. This equals the risk load for the portfolio of A and B combined, so the method is renewal additive.

GIADV, Fall 2022, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load multiplier using Kreps' formula.

y = return on margin surplus, selected as 20%
z = standard normal multiplier, selected as 2.0
The risk load multiplier using Kreps' formula is $yz / (1 + y) = 0.2 \times 2 / 1.2 = 1/3$.

(b) Describe how Mango converted this multiplier for use in the Marginal Variance method to ensure that the two methods have the same total risk load for the portfolio.

This multiplier is divided by the standard deviation of the portfolio.

(c) Mango refers to portfolio variance as a *super-additive characteristic function*. Explain what is meant by this reference.

Because of the covariance component, the variance of a portfolio is greater than the sum of the individual account variances.

(d) Compare the Shapley Value under a variance-based method to the Marginal Variance for calculating a risk load when adding a new account to an existing portfolio.

Shapley Value = Variance(n) + Covariance(L,n)
Marginal Variance = Variance(n) + 2×Covariance(L,n)
Difference is Covariance(L,n)
Marginal Variance > Shapley Value if the covariance of L and n is > 0

(e) Explain why Mango did not pursue the use of a Shapley Value under a standard deviation-based method.

This is due to the complex nature of the mathematics involved. There is no simplifying reduction formula as there exists when using the variance.

GIADV, Spring 2023, Q5

Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

Learning Outcomes:

(8d) Allocate a property catastrophe risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Note that the scenarios, U to Z, are not independent.
L = Loss, p = Probability, Var = Variance, Cov = Covariance,
MV = Marginal Var, RL = Risk Load

Solution:

(a) Calculate the renewal risk load for each treaty using the Marginal Variance method.

For each treaty (P, Q, and R), and the total of all combined, calculate:

- E(L) = sum over scenarios: $p \times L$
- E(L^2) = sum over scenarios: $p \times L^2$
- Var(L) = E(L^2) − E(L)^2

For each treaty combination, (P+Q, P+R, Q+R), calculate:

- E(L), E(L^2) and Var(L)
- Covariance = (Var – Var for the two treaties)/2

For each treaty (P, Q, and R):

- MV = Var for the total – Var for the other two treaties combined
- Renewal RL = λ × MV

(b) Calculate the renewal risk load for each treaty using the Shapley method.

For each treaty combination, (P+Q, P+R, Q+R), calculate:

- E(L), E(L^2) and Var(L), Covariance

For treaty P, Shapley Value = Var for P + Cov for P+Q + Cov for P+R

For treaty Q, Shapley Value = Var for Q + Cov for P+Q + Cov for Q+R
For treaty R, Shapley Value = Var for R + Cov for P+R + Cov for Q+R
For each treaty (P, Q, and R): Renewal RL = λ × Shapley Value

(c) Explain how the risk loads calculated using the Covariance Share method would differ from those using the Shapley method.

The Shapley method allocates the covariance equally between the accounts while the Covariance Share method allocates the covariance in proportion to the loss size.

GIADV, Fall 2023, Q5

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Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

EL = Expected Losses, Var = Variance, SD = standard deviation,
CoV = coefficient of variation, Corr = Correlation, RL = Risk Load

Solution:

(a) Calculate the renewal risk loads for each of the three accounts (X, Y and Z) using the Marginal Surplus method.

The RL multiplier for the Marginal Surplus method is the return on marginal surplus times the z-score divided by (1 plus the return on marginal surplus).

$$\mathrm{SD}(XYZ) = [\mathrm{Var}(X) + \mathrm{Var}(Y) + \mathrm{Var}(Z) + 2\mathrm{Corr}(X,Y)\mathrm{SD}(X)\mathrm{SD}(Y) + 2\mathrm{Corr}(X,Z)\mathrm{SD}(X)\mathrm{SD}(Z) + 2\mathrm{Corr}(Y,Z)\mathrm{SD}(Y)\mathrm{SD}(Z)]^{0.5}$$

$\mathrm{SD}(X) = \mathrm{CoV}(X)\times\mathrm{EL}(X)$, $\mathrm{SD}(Y) = \mathrm{CoV}(Y)\times\mathrm{EL}(Y)$, $\mathrm{SD}(Z) = \mathrm{CoV}(Z)\times\mathrm{EL}(Z)$
Marginal $\mathrm{SD}(X) = \mathrm{SD}(XYZ) - [\mathrm{Var}(Y) + \mathrm{Var}(Z) + 2\mathrm{Corr}(Y,Z)\mathrm{SD}(Y)\mathrm{SD}(Z)]^{0.5}$
Marginal $\mathrm{SD}(Y) = \mathrm{SD}(XYZ) - [\mathrm{Var}(X) + \mathrm{Var}(Z) + 2\mathrm{Corr}(X,Z)\mathrm{SD}(X)\mathrm{SD}(Z)]^{0.5}$
Marginal $\mathrm{SD}(Z) = \mathrm{SD}(XYZ) - [\mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Corr}(X,Y)\mathrm{SD}(X)\mathrm{SD}(Y)]^{0.5}$

For each account, RL = Marginal SD × RL multiplier for Marginal Surplus method.

(b) Calculate the renewal risk loads for each of the three accounts using the Marginal Variance method.

The RL multiplier for the Marginal Variance method is the RL multiplier for Marginal Surplus method divided by SD(XYZ).

Marginal Var(X) = Var(XYZ) − [Var(Y) + Var(Z) +2Corr(Y,Z)SD(Y)SD(Z)]
Marginal Var(Y) = Var(XYZ) − [Var(X) + Var(Z) +2Corr(X,Z)SD(X)SD(Z)]
Marginal Var(Z) = Var(XYZ) − [Var(X) + Var(Y) +2Corr(X,Y)SD(X)SD(Y)]

For each account, RL = Marginal Var × RL multiplier for Marginal Variance method.

(c) Demonstrate that the renewal risk loads for accounts X, Y and Z, as calculated in both parts (a) and (b), are not renewal additive.

For the combined total of all accounts, the RL is 454 under both the Marginal Surplus and Marginal Variance methods.

The sum of the RLs equals 408 under the Marginal Surplus method and 637 under the Marginal Variance method. Neither equals the RL for the combined total of all accounts. As such, they are both not renewal additive.

GIADV, Spring 2024, Q5

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Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question tested a candidate's ability to calculate property catastrophe risk loads based upon Mango's approach. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the renewal risk load for each account using the following methods:

(i) Marginal Variance

(ii) Shapley

Commentary on Question:

Note that the events are independent when calculating variance for the accounts.

(i) Marginal Variance

For each account j, the variance is equal to the sum over the events of
$\text{Loss}_j(i)^2 \times \text{p}(i) \times [1 - \text{p}(i)]$.

For the three accounts combined, the variance is calculated in a similar manner.

For each account, the marginal variance is the variance for the three accounts combined minus the sum of the variance for the other two accounts.

For each account, the risk load is the marginal variance for the account times λ.

(ii) Shapley

Create a variance-covariance matrix for the three accounts as follows:

For each cell (j, k) in the 3×3 matrix representing account j and account k the value is the sum over events i of $\text{Loss}_j(i) \times \text{Loss}_k(i)^2 \times \text{p}(i) \times [1 - \text{p}(i)]$.

For each account, the Shapley value is the sum of its variance and the covariances with the other two accounts (i.e., sum of the account column)

For each account, the risk load is the Shapley value for the account times λ.

(b) Demonstrate that the Shapley method is renewal additive.

Calculate the variance for the three accounts combined as follows:
For each event i, the loss, $L(i)$ is the sum of the losses for each account.
The variance for the three accounts combined is equal to the sum over the events i of $\text{Loss}(i)^2 \times p(i) \times [1 - p(i)]$.

The risk load for the three accounts combined is the variance for the three accounts combined times λ.

Add the risk load for the three accounts from the Shapley method.

These two risk load amounts should be equal which demonstrates that the Shapley method is renewal additive.

GIADV, Fall 2024, Q5

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Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question tested a candidate's ability to calculate property catastrophe risk loads based upon Mango's approach. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why using a premium risk load based upon the Marginal Surplus method is problematic.

Commentary on Question:

The model solution is an example of a full credit solution.

The Marginal Surplus method uses standard deviation of the risk. The square root operator is sub-additive. This means that the sum of the premiums calculated for each of the three accounts will be less than a premium calculated for the three accounts combined.

(b) Calculate the total premium to be received by WXY.

$L(J \mid K)$ is the loss for outcome J, portfolio K

p_J is the probility of outcome J

$$\text{Mean}(all) = E(L(total \mid all)) = \sum_{all\ J} \sum_{all\ K} p_J L(J \mid K)$$

$$\text{Variance} = \sum_{all\ J} p_J \left(\sum_{all\ K} L(J \mid K) \right)^2 - \text{Mean}(all)^2$$

$$\text{Premium} = \text{Mean}(all) + \lambda \times \text{Variance}$$

(c) Calculate the premium for each account using the Shapley method.

For each account K we have mean and variance as follows:

$$\text{Mean}(K) = E(L(total \mid K)) = \sum_{all\ J} p_J L(J \mid K)$$

$$\text{Var}(K) = \sum_{all\ J} p_J L(J \mid K)^2 - \text{Mean}(K)^2$$

For each account pairing (K_1, K_2) combined we have:

$$\text{Mean}(K_1 + K_2) = E(L(total \mid K)) = \sum_{all\ J} p_J \left[L(J \mid K_1) + L(J \mid K_2)\right]$$

$$\text{Var}(K_1 + K_2) = \sum_{all\ J} p_J \left[L(J \mid K_1) + L(J \mid K_2)\right]^2 - \text{Mean}(K_1 + K_2)^2$$

Covariances for each account pairing are calulated

$$\text{Cov}(K_1, K_2) = \left[\text{Var}(K_1 + K_2) - \text{Var}(K_1) - \text{Var}(K_2)\right] \times \tfrac{1}{2}$$

The Shapley values, SV, for each portfolio are calulated as:

SV(AA) = Var(AA) + Cov(AA, BB) + Cov(AA, CC)

SV(BB) = Var(BB) + Cov(AA, BB) + Cov(BB, CC)

SV(CC) = Var(CC) + Cov(AA, CC) + Cov(BB, CC)

Renewal Risk Load (RL) and premium for each account K is:

$$\text{RL}(K) = \text{SV}(K) \times \lambda$$

$$\text{Premium}(K) = \text{Mean}(K) + \text{RL}(K)$$

$L(J \mid \text{AA+BB})$, $L(J \mid \text{AA+CC})$, and $L(J \mid \text{BB+CC})$

(d) Demonstrate that the Shapley method does not have the problem identified in part (a).

Commentary on Question:
The model solution is an example of a full credit solution.

From part (b), the premium for all accounts combined is 4,366.
From part (c), if we sum the premiums across the three accounts ,we get 4,366 (1,811 + 1,283 + 1,272 = 4,366).

The sum of premiums across the accounts is equal to the total portfolio premium. Therefore, it does not have the problem that the Marginal Surplus method has as explained in part (a).