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Highlights on SOA Report “Reinvestment Strategies for Life Insurance Products in a Changing Economic Environment”

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Life insurance annuities are in force for many years and contain embedded options of very long duration, during which periods of rapid economic change and sustained extreme economic conditions can occur. In March 2017, the Society of Actuaries proposed research investigating possible reinvestment strategies for life insurance companies experiencing periods of such change. The result of that research[1] suggests a rule for the investment/reinvestment strategies in a changing economic environment. We propose a dynamic approach where year by year the reinvestment strategy is defined to maximize the expected result of the following year, taking into account its conditional tail expectation.

The problem with the long duration of life insurance products during periods of extreme financial scenarios has been exacerbated by the increasing presence of guarantees. The sustained low interest rate environment that began in 2009 endangered the sustainability of life insurance products with embedded options because the primary account values of policyholders was performing poorly over the period, while their shadow accounts were remaining positive over a longer part of that time.

To handle this situation, it was necessary to rethink the investment and reinvestment strategies for these products on a more frequent basis. The SOA sought an in-depth analysis based on asset liability management modeling for the original investment strategy, repeated with stochastic scenarios to evaluate new reinvestment strategies and compare the results. To accomplish this, they suggested considering two sets of stochastic scenarios: the most current corporate assumptions and where the mean reversion target of the interest rates should be identical to the starting yield assumption. In the instance of a sustained low interest rate environment, the first set of scenarios could represent the company outlook and the second set could consider the impact of interest rates continuing to remain low far into the future. A side benefit of using two sets of scenarios for this analysis is the comparative value of understanding status quo environment results against the corporate philosophy embedded in the company assumption results.

According to this suggestion, we evaluate, at the beginning of each year, the investment strategy in terms of the optimal portfolio weights, looking at the stochastic evolution of the cash flows during the prior year. We do not evaluate the cash flows during the whole maturity of the contract, to avoid the problem of choosing a single discounted rate. In a contest of stochastic evolution of interest rates, the choice of a constant discounted rate deeply shows its weakness because valuing liability cash flows with any single interest rate loses the interest rate sensitivity of the cash flows and the tail distribution of the results. Instead, this research explicitly evaluates this sensitivity and tail distribution to inform the reinvestment strategy. In particular, we design a dynamic strategy where, at the beginning of each year, we evaluate how to modify the investment strategy on the basis of the results realized during the prior year. The aim of this model is to select the investment/reinvestment strategy dynamically, with a continuous fit to a given set of criteria, like the optimization of an opportune objective function. The function of the maximization problem could be defined according to the goals of the strategic asset allocation. In our model, we decided to deal with the maximization of the expected value of the distributable earnings, taking into account also the dispersion of the simulated distribution. Another way to take into account the dispersion could be the maximization of the expected value minus the expected loss in the CTE. From a practical point of view, we have evaluated the statutory reserve in terms of working reserve as required by the National Association of Insurance Commissioners’ Actuarial Guideline 43, the projected cash flows achieved on the asset side, and the difference in terms of accumulated deficiency, its mean and dispersion under different strategies.

THE MODEL

Let $\pi_t = \left(\pi_t^1,\ \pi_t^2, \ldots, \pi_t^j\right)$ be the weights of the portfolio composed by j asset classes with statutory bounds $\pi_t^1 \le b^i$. Let W_0 be the premium paid by the policyholder and invested in the portfolio after the initial expenses. Let A_t be the value of the assets at the end of the year t, with $A_0 = W_0$, and R_t be the statutory reserve at the end of the year t influenced by $g(t)$, the amount guaranteed at t. Starting from $t = 0$, we simulate B path of the evolution of the assets and the contractual obligations, considering the interaction of financial, demographic and behavioral factors. For each path at the end of each year, we calculate the statutory reserve able to achieve the future contractual obligations according to the statutory prescriptions and evaluate the investment portfolio. Based on the investment returns achieved on the asset, the accumulated deficiency is calculated.

We define the optimization problem as follows:

$$\begin{cases} max_{\pi_t} \left(E\left[A_{t^+} - R_{t^+}\right] | SD\left[A_{t^+} - R_{t^+}\right] \right) \; for \; each \; t \\ \pi_t^i \le b_i \\ \pi_t^1 + \pi_t^2 + \ldots + \pi_t^j = 1 \end{cases}$$

where the decision of a target allocation is done at the inception of each year t based on the simulated value of assets and reserve at the end of the same period t^+. In this formulation, the model produces different results depending on how the standard deviation is constrained—for example, fixing a given level of risk the insurer is willing to assume. The problem can be standardized considering the following formula:

$$\begin{cases} max_{\pi_t} \; E\left[A_{t^+} - R_{t^+}\right] | SD\left[A_{t^+} - R_{t^+}\right] \; for \; each \; t \\ \pi_t^i \le b_i \\ \pi_t^1 + \pi_t^2 + \ldots + \pi_t^j = 1. \end{cases}$$

An alternative optimization formula follows:

$$\begin{cases} max_{\pi_t} \left(EA_{t^+} - R_{t^+} - CTE(70)_{t^+} \right) \; \forall t \\ \pi_t^i \le b_i \\ \pi_t^1 + \pi_t^2 + \ldots + \pi_t^j = 1 \end{cases}$$

where $CTE_{(70)}$ is the conditional tail expectation of the simulated distribution of the accumulated deficiency.

The model is flexible and can be modified to meet specific needs. The central idea is that the strategy is dynamic based on the results obtained during the year for whatever formula you decide to maximize.

Following the ALM models, we project asset and liability cash flows based on financial and demographic assumption at the valuation date. The model takes into account the interaction of the following variables:

- **Financial variables.** Interest rate and return of other investments
- **Demographic variables.** Lapse and death
- **Investment choices by management**

NUMERICAL SIMULATIONS

The financial variables are simulated through the Financial Scenario Generator Version 7.1.201805 developed by the American Academy of Actuaries. The generator produces scenarios for the future paths of interest rates for U.S. treasury securities and several kinds of investment portfolios, including both equity and fixed-income portfolios. The U.S. Treasury yields are generated using the C-3 Phase I interest rate model designed by the American Academy of Actuaries. The model simulates Treasury bond yields according to a stochastic variance process with mean reversion under real-world probability measures. The equity return scenarios are generated from a monthly stochastic local volatility model wherein the natural logarithm of the annualized volatility follows a strong mean-reverting stochastic process and the annualized drift is a deterministic quadratic function of volatility. This model is able to capture many of the dynamics observed in the equity market data: the negative skewness and positive kurtosis ("fat tails") over short holding periods, the time-varying volatility and volatility clustering, and the increased volatility in bear markets.

The model is implemented through two financial sets of stochastic scenarios. In the first, the mean reversion target of the interest rates is derived by the yield curve of December 2016, while in the second set, the reference point for the interest rate curve is December 2000 before the financial crisis. In the instance of a sustained low interest rate environment, the first set of scenarios could represent the company's prudent outlook while the second set could consider the impact of interest rates not remaining low far into the future. Table 1 shows the parameters of the Financial Scenario Generator for the two sets of scenarios.

Table 1
Scenario Parameters

Starting date	December 2016	December 2000
Yield curve on starting date		
3 months	0.51%	5.89%
6 months	0.62%	5.70%
1 year	0.85%	5.32%
2 years	1.20%	5.11%
3 years	1.47%	5.06%
5 years	1.93%	4.99%
7 years	2.25%	5.16%
10 years	2.45%	5.12%
20 years	2.79%	5.59%
30 years	3.06%	5.46%
Mean reversion to		
	3.75%	6.50%

Table 2
Product Features

Policyholder	U.S. male, 50 years old	
Projection period	30 years? 40 years?	
Premium	10,000	
Percentage of investment in fixed account	20%	
Fund chosen for the variable account	Balance fund	
Separate account fund expense	0.01	
Fixed account credited rate formula	Min (New Money Rate – Base Expense Margin + Inversion Adjustment; 0%)	
Base expense margin	0.024	
Inversion adjustment	0.25% when the 10-year Treasury is less than the 2-year Treasury and the 10-year Treasury is less than 5.00%; 0.00% otherwise	
Guaranteed rate	0.01	
Administrative fee (on VA)	0.0015	
Mortality and expense (on VA)	0.0125	
Fixed account credited rate	Set to the maximum of the rate determined by the fixed account credited rate formula and the guaranteed interest rate; the credited rate is reset each policy anniversary	
GMDB	0.04	
Surrender charges	1 year	7%
	2 years	6%
	3 years	5%
	4 years	4%
	5 years	3%
	6 years	2%
	7 years	1%
	8 years+	0
Annuitization	No annuitization assumed	
Partial withdrawal	No partial withdrawal assumed	

LAPSE RATE	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
	0.007	0.0142	0.0214	0.0286	0.0358	0.043	0.05	0.22	0.15	0.05

Regarding the demographic variables, a deterministic mortality model is assumed justified by the fact that mortality risk can be diversified in a large portfolio; a prudent approach would be to use the appropriate projected mortality tables. Instead, the risk of lapse is not fully diversifiable. Both academics and practitioners try to explain and model the policyholder behavior and the factors driving the choice to lapse or not. From a financial point of view, during the period of decreasing markets, the value of the underlying fund will decrease and consequently the economic value of the guarantee will rise, with a potential incentive to not exercise the surrender option. In reality, insurance companies usually do not assume this behavior for all policyholders because some of them may not be rational or aware agents or well versed with the economic value of the guarantee. There could also be exogenous factors driving policyholder actions, such as the need for liquidity. A survey conducted by the Society of Actuaries in 2011 shows that "'company experience studies' continue to be the most popular source of lapse assumptions."[2] Based on this

Table 3
Initial Investment Strategies

	STR I	STR II	STR III	STR IV
Cash	5%	5%	5%	5%
U.S. Treasury (2 year)	30%	30%	30%	30%
U.S. Treasury (10 year)	60%	50%	40%	30%
INT.R EQUITY	5%	15%	25%	35%

consideration, we consider the evolution of lapse rate in line with the SOA study.

Starting from these assumptions, we have implemented our model considering the following variable annuity with guaranteed minimum death benefit option as seen in Table 2 (Pg. 14). The projection period is 10 years.

RESULTS

At $t = 0$, we evaluate different strategies in terms of proportion of investment in bond and equity and chose to follow a more or less aggressive strategy. Once the strategy has been chosen, the assets and liabilities are simulated and the values of distributable earnings at the end of the first year are collected for each path. The simulations are repeated for both sets of financial scenario. At the end of the year, a reinvestment strategy has to be defined. We consider that when positive distributable earnings occurs, it is not actually distributed but invested in additional assets according to the following reinvestment strategies.

A. **Base strategy.** Assets are only bought/sold when there are net positive/negative distributable earnings at the end of the year, maintaining the same proportion of assets as in the initial strategy.

B. **Base strategy + shift to aggressive equity fund.** When positive distributable earnings occur, assets are bought and the equity investment profile becomes aggressive.

C. **Base strategy + shift to balanced equity.** When negative distributable earnings occur, assets are sold and the equity investment profile becomes balanced.

D. **Combination of strategies B and C.** When distributable earnings are positive, assets are bought and the profile of investment in equity becomes aggressive, while when distributable earnings are negative, assets are sold and the equity investment profile becomes balanced.

E. **Base strategy + interest rate swap.** The company hedges the U.S. 10-year Treasury assets, paying the U.S. 10-year swap rate (historical value at December 2016) and receives the variable U.S. 10-year interest rate.

The costs of rebalancing are ignored. In practice, other more sophisticated investment strategies could be implemented but we have limited this example to theses five strategies for illustrative purposes.

We generate 10,000 paths of the asset liability cash flows for both scenarios considered. At $t = 0$, the insurer chooses the first asset allocation and calculates for each of the selected scenarios the distributable earnings at the end of the first year. In the second step, at the end of the first year, given the results obtained for each path, the insurer uses the results to simulate the distributable earnings at the end of the following year and chooses the reinvestment strategy (A through E) that produces the best results. The policyholder can choose the risk/return investment profile of the separate account; we assume the policyholder opts for a diversified, balanced allocation portfolio. We simulate the liability cash flows of both fixed and variable accounts, taking into account the features of the product described in Table 2 and the guarantees, the investment choice of the policyholder, the occurrence of mortality and the lapse.

The aim of this model is to select the investment/reinvestment strategy dynamically, with a continuous fit to a given set of criteria, like the optimization of an opportune objective function.

On the asset side, we assume that at the inception of the contract, four explanatory investment strategies vary the proportion of investment between bonds and equity according to the parameters shown in Table 3. When compared to U.S. Treasury bonds with 2- and 10-year maturities, the equity asset class is the riskiest. At $t = 0$, the insurer chooses the first allocation between strategies I–IV.

Table 4
Simulated results at the end of the first year according to the investment strategies I-IV

	Scenario 2000				Scenario 2016			
	STR I	STR II	STR III	STR IV	STR I	STR II	STR III	STR IV
Mean	570.6482	613.2906	655.933	698.5754	323.294	392.6313	461.9686	531.3059
SD	812.9938	854.2247	932.6598	1039.915	784.647	827.3418	908.1488	1018.032
Mean/SD	0.70191	0.71795	0.703293	0.671762	0.412025	0.47457	0.508693	0.521895

Table 5
Simulated results at the end of the second year according to the reinvestment strategies A-E

	Scenario 2000					Scenario 2016				
	A	B	C	D	E	A	B	C	D	E
Mean/SD	0.678204	0.68023	0.67450	0.67657	0.78247	0.3691	0.37422	0.3608	0.36594	0.388479
E-|CTE|	446.9291	454.726	429.466	439.044	877.632	–752.541	–741.222	–802.006	–789.912	–683.806

The expected value and deviations of the simulated distributable earnings for each strategy are summarized in Table 4.

Under the hypothesis of scenario 2016, according to which the interest rates will remain low in the future, starting strategy IV has the greatest mean/SD, while under the hypothesis of scenario 2000, the second strategy dominates the others. Considering that the second strategy is more sensitive to the changes in interest rates that represent the focus of this research and also is more realistic given the real constraints to the asset allocation, we decided to implement the second strategy at the inception of the contract. Following this initial plan, we followed the path evolutions of assets and liabilities under both scenarios and evaluated the distributable earnings at the end of the first year. For each path of each scenario, starting with the results obtained from the first year, reinvestment strategies A–E were implemented to evaluate the distributable earnings at the end of the second year. Results are shown in Table 5.

Under both the scenarios, strategy E produces the best results in terms of expected mean of distributable earning per unit of risk measured by the standard deviation and in terms of expected mean of distributable earning minus the expected loss in the tail according to the requirements of AG43 (CTE).

The results given in this report are for explicative purposes only. More realistic strategies might be considered and different constraints introduced; other investment strategies may be optimal under different scenarios.

FINAL REMARKS

Taking into account the statutory requirements for products with variable accounts and guarantees, this paper has proposed a dynamic strategy for maximizing distributable earnings year by year. The risk measure introduced is inspired by the statutory requirements. We have analyzed the mean of distributable earnings and the CTE for two complementary metrics using the maximization formula; however, the model appears flexible, and other risk measures can easily be introduced. The stochastic simulation of the statutory reserve and investment portfolio permits us to consider the complex interaction of assets and liabilities, taking into account the relationship between financial, demographic and behavioral factors. ■

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ENDNOTES

1 Piscopo, Gabriella. 2018. "Reinvestment Strategies for Life Insurance Products in a Changing Economic Environment." Society of Actuaries. *www.soa.org/research-reports/2018/2018-reinvestment-strategies-life-insurance-prod/*.

2 Society of Actuaries. 2011. "Policyholder Behavior in the Tail: Variable Annuity Guaranteed Benefits." *www.soa.org/resources/research-reports/2012/research-policy-behavior-tail-results*.