53RD ARC REPORT

EVALUATION OF THE RUIN PROBABILITY IN ORDERED RISK MODELS

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Contents

1 ABSTRACT

A study on numerical methods for computing non-ruin probabilities under a classical risk process is conducted. A Monte Carlo simulation-based method to compute ruin probabilities in the ordered risk model is proposed. A numerical comparison, in terms of accuracy and computing time, between a Monte Carlo based estimator relying on Appell polynomials and a standard Monte Carlo evaluation is made. After selecting a numerical method, the sensitivity of the ruin probability with respect to the claim sizes distribution and the claim arrival process is studied.

2 INTRODUCTION TO RISK THEORY

A non-life insurance company holds an initial capital of amounts u, receives premiums and pays claims as time goes by. The premiums are collected linearly in time at a rate c. The number of claims up to time T is modeled by a counting process $\{N(t)\ ,\ t \geq 0\}$. Each claim filed is associated to a loss, a compensation paid to the policyholder. The claim sizes $W_1, \dots W_{N(T)}$ form a sequence of iid (independent, identically distributed) non-negative random variables, independent of $N(T)$. We define the surplus of the insurance company at time $t \geq 0$ as

$$R(t) = u + ct - \sum_{i=1}^{N(t)} W_i \ . \tag{1}$$

The claim arrival process $N(t)$ is governed by an order statistic point process (OSPP) which means that conditioned upon $N(t) = n$, the successive jump times $(\tau_1, \tau_2, ...\tau_n)$ are distributed as the order statistics of n iid random variables. We are interested in the distribution of the ruin time at which $R(t)$ becomes negative:

$$\tau_u = \inf\{t \geq 0 : R(t) < 0\}\ . \tag{2}$$

Ruin occurs while $R(t)$ is making a downward jump and crosses the horizontal axis, meaning that the company went bankrupt. Denote by

$$\Phi(u, T) = \mathbb{P}(\tau_u > T) \tag{3}$$

the survival probability up to time T and $\Psi(u,T) = 1 - \Phi(u,T)$ as the ruin probability by time T.

In the Cramer-Lundberg risk model, the claim arrivals are governed by a Poisson process. We assume that the claim arrival process enjoys the order statistic property. By doing so, we consider a broad class of risk processes which encompasses the standard Cramer-Lundberg risk model. The ruin probabilities admit a tractable expression only in a few particular cases, which motivates the development of numerical methods.

The ruin probability is a risk measure, useful for decision making. This allows, for instance, an actuary to determine how likely a company is to become insolvent given a certain time horizon T and initial capital u. Actuaries are in charge of two things: rate making and claim reserving. Using the model, an actuary can determine the premium he needs to charge in order to remain insolvent with a certain probability. Figure 1 is an example of a trajectory until some time T. The amount of claims accumulate through jumps by size, W_i, at times τ_i. If the aggregate claim amount, $S(t) = \sum_{i=1}^{N(t)} W_i$, ever crosses the boundary $u + ct$ (the amount of premium), then ruin has occurred.

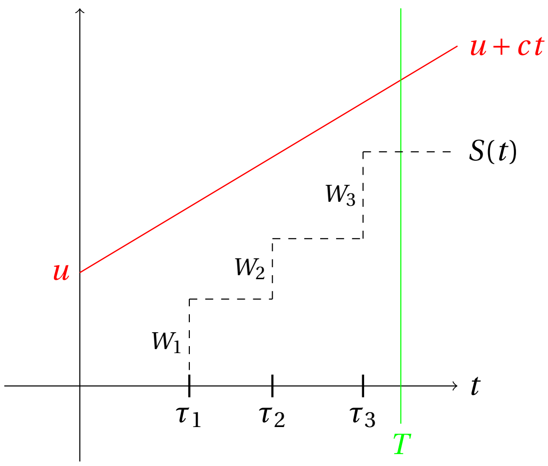


Figure 1: Risk Process Visualization

3 Preliminaries and Notation

3.1 Order Statistic Point Process

Recall that a Poisson process is a stochastic process in which the inter-arrival times are distributed as exponential random variables. The Poisson process is actually a special case of a more general family of point processes called Order Statistic Point Process (OSPP).

Definition 1. Let $\{N(t), t \geq 0\}$ be a point process generated by arrival times $\tau_1, \tau_2, \ldots$. Then $N(t)$ is an OSPP if for every $k \geq 1$, given $N(t) = k$, the successive jump times $\tau_1, \tau_2, ..., \tau_k$ are distributed as the order statistics of k iid random variables with the distribution function $F_t(x)$ for $x \leq t$.

The homogeneous Poisson process is the most famous process enjoying this property, in which case $F_t(x)$ is the Cumulative Distribution Function (CDF) of the uniform distribution on [0,t]. The Poisson process is the only process with independent increments which possesses the order statistic property.

Other OSPPs also include the Polya-Lundberg process, the linear birth process with immigration, the linear death process, and the mixed Poisson process. Our study emphasizes on the mixed Poisson process.

3.2 Mixed Poisson Process

Definition 2 (Mixed Poisson Process)**.** A Mixed Poisson Process is a generalization of a Poisson process where the intensity Λ is itself a random variable.

A key property of the Mixed Poisson Process is conditioning on $N(T) = n$, the successive jump times $(\tau_1, \tau_2, \ldots, \tau_n)$ are distributed as the order statistics of n iid uniform random variables on [0,T].

It is worth mentioning that the Poisson process is a special case of the Mixed Poisson Process where Λ is a constant λ.

A comprehensive overview on mixed Poisson process is provided in Grandell's monograph [Gra97].

Let $N(T)$ be the number of claims that an insurance company experiences during a year $T = 1$.

It is common in actuarial science to model the claim frequency, $N(T)$, with the Poisson distribution with intensity λ. However, It may be more practical to use a Mixed Poisson process if we have a non-homogeneous population.

For example, assume we have two groups of policy holders: risky and non-risky. Then we can model the claim frequency as a Mixed Poisson process $N(\Lambda t)$, where

$$\Lambda = \begin{cases} \lambda_1, & p \\ \lambda_2, & (1-p) \end{cases}$$

In this model, we take into account the different frequencies at which each group experiences claims; p in this context may be proportion of risky policy holders.

3.3 Appell Polynomials

Let $U = \{u_i, i \geq 1\}$ be a sequence of real, non-decreasing numbers. Then define the unique family of Appell polynomials of degree n, $A_n(x|U)$, as follows.

$$A_n(x|U) = n! \int_{u_n}^{x} \left[\int_{u_{n-1}}^{y_n} dy_{n-1} \cdots \int_{u_1}^{y_2} dy_1 \right] dy_n, \ n \geq 1,$$

$$A_0(x|U) = 1,$$

$$A_n(u_n|U) = 0, n \geq 1.$$

Appell polynomials satisfy recursive relationships, useful for numerical calculations:

$$A_n(x|U) = \sum_{k=0}^{n} \binom{n}{k} A_{n-k}(0|U) x^k, n \geq 1 \text{ , and}$$

$$A_n(0|U) = -\sum_{k=1}^{n} \binom{n}{k} A_{n-k}(0|U) u_n^k, n \geq 1$$

3.3.1 Probabilistic Interpretation

Let $U_{1:n},...,U_{n:n}$ be the order statistics associated to the sample of n iid random variables uniformly distributed on [0,1]. Then

$$f_{U_{1:n},\dots,U_{n:n}}(u_1,...,u_n) = n!\mathbb{1}_{0<u_1\leq\ldots\leq u_n\leq 1}$$

is the joint probability density function, and

$$\mathbb{P}(U_{1:n} > u_1,...,U_{n:n} > u_n) = n!\int_{u_n}^{1}\int_{u_{n-1}}^{U_{n:n}} dU_{n-1:n}\ldots\int_{u_1}^{U_{2:n}} dU_{1:n}dU_{n:n} = A_n(1|U)$$

is the probability that each order statistic is larger than its corresponding index in the set U.

3.4 Constant Claim Sizes

Let $N(t)$ from model (1) be a Mixed Poisson process. Then the jump times of the process given $N(t) = n$ are distributed as n uniform order statistics $U_{1:n},\ldots,U_{n:n}$. Also, let β_n denote the time at which $h(t) = u + ct$ reaches n. Then $A_n(1|\beta)$ is the probability that each jump occurs after the accumulated premium income has reached the level n:

$$\mathbb{P}(U_{1:n} > \beta_1,\ldots,U_{n:n} > \beta_n) = A_n(1|\beta).$$

The following result gives an explicit formula for the ruin probabilities.

Proposition 1. Assume the claim size is constant equal to d, then we have

$$\mathbb{P}(\tau_\beta > t) = \sum_{n=0}^{\frac{u+ct}{d}} \frac{(\lambda t)^n e^{-\lambda t}}{n!} A_n(1|\beta_1,\ldots,\beta_n),$$

where $\beta_n = max\left((\frac{dn-u}{ct}),0\right), n = 0,1,\ldots$

Proof. The number of claims $N(t)$ cannot exceed $\frac{u+ct}{d}$ since $u+ct > N(t)d$. We condition on $N(t) = n$, then add the probabilities that each jump time occurs after the surplus has reached the level n, thus ensuring no ruin has occurred. This is a direct application of proposition 4.1 of [GL17]. □

4 Monte Carlo Method to Evaluate Ruin Probabilities

Monte Carlo methods rely on the simulation of a process many times, which is useful in the case where there exists no closed-form solution. A Crude Monte Carlo (CMC) method is a brute force approach. While this technique can be used for almost any process, it has a few pitfalls. Primarily, the CMC method is computationally expensive and takes a lot of time to execute, and for events with low probability, it is inefficient in capturing the true probability. Tailor-made Monte Carlo procedures may be put together to handle specific risk models. We present in this work a Monte Carlo estimator, named the Appell Polynomial Monte Carlo (APMC), and we show its superiority over the standard Crude Monte Carlo approach.

4.1 Crude Monte Carlo

Monte Carlo simulations are advantageous in that we can run them for any claim size distribution $W_i > 0$. The CMC method replicates the entire insurance model $R(t)$ by simulating the claims arrival process $N(t)$ and claim amounts W_i and then checking if the total losses $\sum_{i=1}^{N(\tau_k)} W_i$ have surpassed the premium income $u + c\tau_k$ for each jump time τ_k, $0 < \tau_k < T$. If claims surpass the premium income, the simulation stops and counts one ruin. However, if the premium earnings absorb the losses due to claims, the simulation has to go until time T, and then counts no ruin. This process is repeated many times and the final fraction: $\frac{\text{number of non-ruins}}{\text{number of simulations}}$ is the estimated non-ruin probability. This corresponds to the empirical evaluation of the expectation of a Bernoulli random variable that can be observed as:

$$\mathbb{1}_{\tau_u > T} = \begin{cases} 1 & \mathbb{P}(\tau_u > T) \\ 0 & \mathbb{P}(\tau_u \leq T) \end{cases},$$

an indicator function equal to one if ruin occurred and zero otherwise. As a result, the non-ruin probability can be defined as an expected value. This means that we can take advantage of the law of large numbers and approximate the expected value by an empirical mean of a sample of replications of the random variable inside the expectation: τ_u will be greater than T:

$$\Phi(u, t) = \mathbb{P}(\tau_u > T) = E[\mathbb{1}_{\tau_u > T}].$$

4.2 Appell Polynomial Monte Carlo

Another method is the APMC simulation, which uses the recursive Appell structure described in Section 2, and only applies to processes that have claim arrivals following an OSPP. This method is based on the formula:

$$\mathbb{P}(\tau_u > T) = E\Big\{A_{N(T)}\left[1\middle|F_T\left(\frac{S_1 - u}{c}\right)_+, \ldots, F_T\left(\frac{S_{N(T)} - u}{c}l\right)_+\right]\mathbb{1}_{S_{N(T)}\leq u+cT}\Big\},$$

from Picard and Lefèvre [PL97].

We can calculate this finite-time non-ruin probability as the expectation of the APMC random variable, X, that can be observed as:

$$X = \begin{cases} 1, & S_{N(T)} < u \\ A_{N(T)}\left[1\middle|F_T\left(\frac{S_1-u}{c}\right)_+, \ldots, F_T\left(\frac{S_{N(T)}-u}{c}\right)_+\right], & u \leq S_{N(T)} < u + cT \\ 0, & S_{N(T)} \leq u + cT \end{cases}, \text{ so that}$$

the Monte Carlo evaluation of the expectation above defines the APMC estimator. We compare the error of the CMC and APMC procedure so as to keep the most accurate estimator; the variance is the common way to measure accuracy in this context.

The simulation process generates n observations x_i where $x_1, \ldots, x_n \overset{iid}{\sim} X$, and for

- CMC: $x_i = 0, 1$
- APMC: $0 \leq x_i \leq 1$

The non-ruin probability is approximated by $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$, and we want our observations to be as close to $\overline{X}$ as possible. For a large enough n, we then can construct a (1-α)% confidence interval around $\overline{X}$:

$$\overline{X} \pm z^*_{1-\alpha}\sqrt{\frac{Var[X]}{n}}$$

Our goal is to minimize this interval without sacrificing computational time, so we hold n constant. Thus, to minimize the range of the confidence interval, we have to minimize $Var[X]$. Therefore, the most accurate estimator is the estimator with the smallest variance.

Proposition 2. The variance associated to the APMC estimator will never be larger than the variance associated with the CMC estimator. Namely, we have

$$\sigma^2_{APMC} \leq \sigma^2_{CMC},$$

where σ^2_{CMC} is the variance of the CMC estimator and σ^2_{APMC} is the variance of the APMC estimator.

Proof. Since CMC estimator $E[\mathbb{1}_{\tau_u>t}]$ takes the expected value of a Bernoulli random variable $\mathbb{1}_{\tau_u>t}$, where $t>0$ and τ_u is the time of ruin, the variance is $\Phi(1-\Phi)$, where $\Phi = \mathbb{P}(\tau_u > t)$ is the non-ruin probability.

Also, since the APMC estimates the non-ruin probability as

$$\Phi(u,t) = \sum_{n=0}^{\lfloor u+ct \rfloor} A_{N(t)}[1|\{F_t(\beta_1),...,F_t(\beta_n)\}]P[N(t)=n],$$

we can use the law of total variance to find σ^2_{APMC} as

$$\sigma^2_{APMC} = E[Var(Y|N(t)] + Var[E(Y|N(t)],$$

where $Y = A_{N(t)}[1|\{F_t(\beta_1),...,F_t(\beta_n)\}]$. Let $M = E[A_{N(t)}[1|\{F_t(\beta_1),...,F_t(\beta_n)\}]]$ and $P_N = P[N(t)=N]$. Also, let $\sigma^2_N = Var(Y|N(t))$. Then the variance becomes

$$\sigma^2_{APMC} = \sum_{n=0}^{\infty} \sigma^2_N P_N + \sum_{n=0}^{\infty} M^2 P_N - \left(\sum_{n=0}^{\infty} MP_N \right)^2.$$

Since $0 \leq y \leq 1$, $E[y^2] \leq E[y]$ and $E[y^2] - E[y]^2 + E[y]^2 \leq E[y]$. Then $\sigma^2_N + M^2 \leq M$.

Thus

$$\sum_{n=0}^{\infty} \sigma^2_N P_N + \sum_{n=0}^{\infty} M^2 P_N - (\sum_{n=0}^{\infty} MP_N)^2 \leq \phi(1-\phi),$$

and

$$\sigma^2_{APMC} \leq \sigma^2_{CMC}.$$

□

To actually calculate the non-ruin probability, we follow these steps:

1. Simulate $N(T)$, set $n = N(T)$
2. Generate n claims, with aggregated claim sizes: $S_k, k = 0, 1, 2, \cdots, n$
3. Calculate $v_k = (\frac{S_k - u}{c})_+, k = 0, 1, 2, \cdots, n$
4. Check:
   (a) If $S_n < u$, then not ruined, return a non-ruin probability of one
   (b) If $S_n \geq u + cT$, then ruined, return a non-ruin probability of zero
   (c) If $u < S_n \leq u + cT$, then continue
5. Substitute the vector V that contains all of the v_k's into an Appell Polynomial of the form: $A_{N(T)}(1|F_T(V))$, producing a non-ruin probability

This process is repeated many times and the estimated non-ruin probability is the empirical mean of the Appell polynomials computed over each Monte Carlo run.

One should note that the APMC method only require the simulation of the number of claims and their size while CMC requires also the arrival times of the claims.

Figure 2 to the right is a visual representation of understanding the Appell Polynomial method. We take our total claims: $S_k, k = 1, 2, \cdots, N(T)$ and compare them to our premium: $u + ct, t \in (0, T)$, generating our v_k's, $k = 1, 2, \cdots, N(T)$. We then observe the distribution of v_k's vs the distribution of τ_k's $(P[(\tau_1 > v_1) \cap (\tau_2 > v_2) \cap \cdots \cap (\tau_{N(T)} > v_{N(T)})])$, which can be evaluated using Appell polynomials for OSPPs.

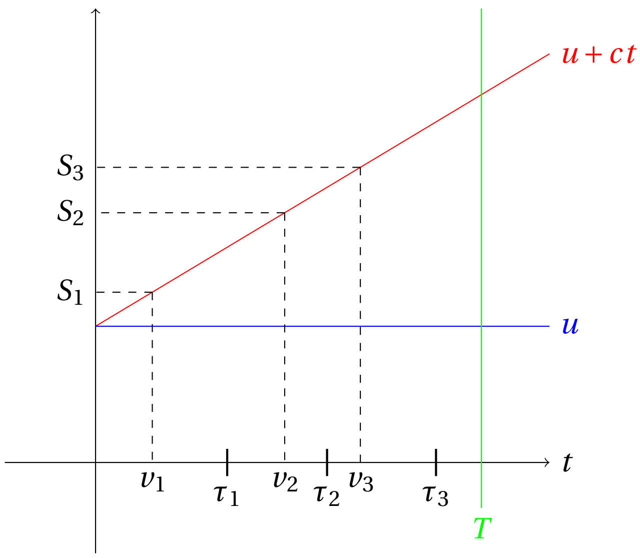


Figure 2: APMC Method

5 Numerical Illustrations

While the CMC works for any situation, it suffers from a lack of accuracy in the rare event simulation situation. The computation of small ruin probabilities associated to large initial reserve is interesting in practice. Variance reduction techniques are designed to cope with this flaw. We aim at quantifying the difference between the variances associated to the APMC and the CMC. We also demonstrate that it goes along with a reduction in process time.

5.1 Checking Consistency

We first begin by comparing the CMC and APMC non-ruin probabilities to ensure that they produce the same results. To generate fair results, we used the same simulation process for both methods. The parameters we used were as follows:

- $u = 10$
- $T = 1$
- $\lambda \in \{5, 10, 15, 20, 25\}$
- $N(T) \sim Poi(\lambda T)$
- $W_i \sim Exp(1)$
- $\eta = 0.1$
- $c = (1+\eta)E[N(T)]E[W_i]$
- $N \in \{10^2, 10^3, 10^4, 10^5, 10^6\}$

The table in Figure 3 below depicts the ratios of CMC non ruin to APMC non ruin. Most of the values are the same, but they differ in low values of λ where the CMC model suffers from lack of accuracy in rare event simulations. The CMC cannot accurately capture the few ruins that do happen when the probability of ruin is so small. This shows that our codes coincide.

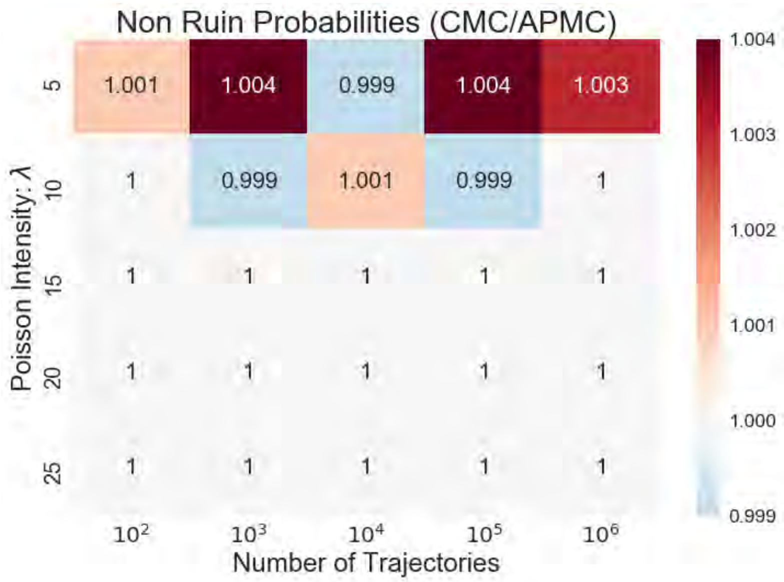


Figure 3: Non Ruin Comparison

5.2 Time Comparison

Since both the CMC and APMC methods produce similar non-ruin probabilities, we want the method that is more efficient in running time. To generate fair times, we use the same claim sizes and counts for each simulation for both methods. The parameters we use are as follows:

- $u = 10$
- $T = 1$
- $N(T) \sim Poi(\lambda T)$, $\lambda \in \{5, 10, 15, 20, 25\}$
- $W_i \sim Exp(1)$
- $\eta = 0.1$
- $c = (1+\eta)E[N(T)]E[W_i]$
- $N \in \{10^2, 10^3, 10^4, 10^5\}$

We iterated through various processes having a different number of trajectories ($N = 10^2, 10^3, 10^4, 10^5$) and different claim arrival rates ($Poi(\lambda), \lambda = 5, 10, 15, 20, 25$) and simulated each process 30 times. We calculated $\frac{\text{CMC time}}{\text{APMC time}}$ so if the ratio is greater than 1, then the CMC time is larger and if the ratio is less than 1, then the APMC time is larger. Figure 4 below compares the CMC to APMC with respect to total time. We note that for overall time, the CMC is always worse than the APMC, with the difference increasing as we increase the claim arrival process intensity and number of trajectories.

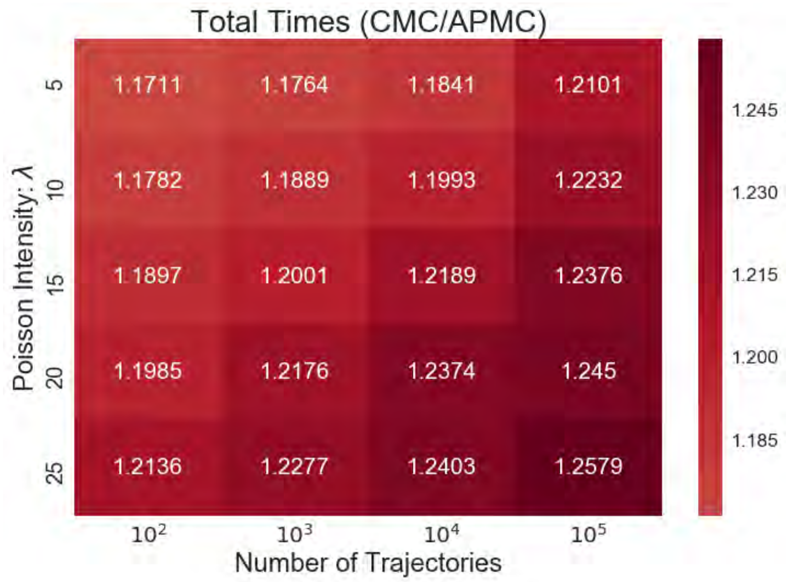


Figure 4: Total Time Comparison

5.3 Accuracy Comparison

As well as being faster, the APMC method is also more accurate. Proposition 2 proved that theoretically, the variance of the APMC is less than the variance of the CMC. We want to determine by how much APMC outperforms CMC. The parameters we use are as follows:

- $u \in \{1, 2, \ldots, 10\}$
- $T \in \{1, 2, 3, 4\}$
- $\lambda = 15$
- $N(T) \sim Poi(\lambda T)$
- $W_i \sim Exp(1)$

- $\eta = 0.1$
- $c = (1+\eta)E[N(T)]E[W_i]$
- $N = 10^5$

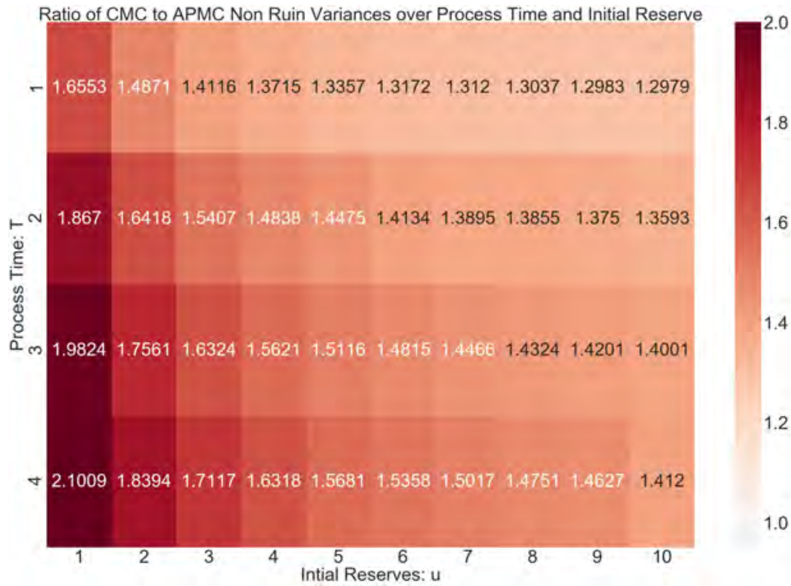


Figure 5: Total Time Comparison

Above is Figure 5, a table portraying the ratios of CMC variance to APMC variance. The CMC variance is always greater than APMC, with the difference increasing as we increase T and decrease u. This relates back to the rare event situation where the CMC cannot as accurately capture the small chances of ruin.

Overall, we have shown that the CMC and APMC methods produce similar non ruin probabilities. However, the CMC process takes longer to simulate. Additionally, the CMC process has a greater variance than the APMC process, meaning the CMC is less accurate. The APMC method is the superior estimator and we shall utilize it to perform analysis for the rest of the project.

6 Sensitivity Analysis

Equipped with our faster and more accurate estimator, the APMC method, we assess the effect of changing the parameters of the risk models on ruin probabilities. More specifically, we want to know what happens to ruin probability when we alter claim sizes and claim arrivals.

6.1 Claim Size Distributions

We adjust our original risk model by assuming the claim sizes to be Weibull distributed. The pdf is given by

$$f_X(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(\frac{x}{\beta})^{\alpha}}.$$

The shape parameter, α, allows us to tune the tail behavior, making it either heavier ($\alpha < 1$) or lighter ($\alpha > 1$) than the that of the exponential distribution ($\alpha = 1$).

We will look at three different Weibull distributions:

$$W_i \sim Weib(\alpha = 0.25, \beta = 0.042)$$

$$W_i \sim Weib(\alpha = 1, \beta = 1)$$

$$W_i \sim Weib(\alpha = 10, \beta = 1.051)$$

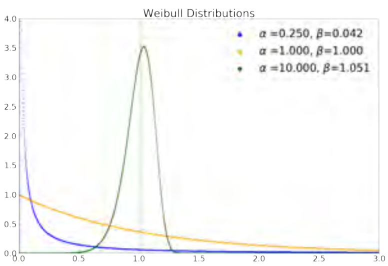


Figure 6: Left Tails

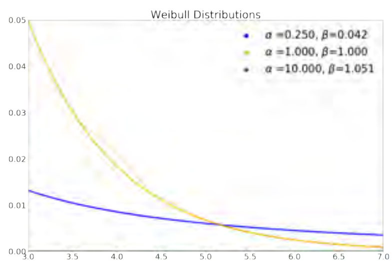


Figure 7: Center Tails

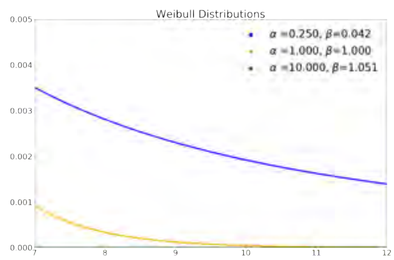


Figure 8: Right tails

The distributions are set such that the cost of one claim is 1 on average. However, the densities of the distributions significantly vary. In Figure 6, the $\alpha = 0.25$ distribution has most if its mass toward 0 while the $\alpha = 10$ distribution has a narrow range around 1, and the $\alpha = 1$ distribution is in-between. The vertical, dashed lines represent the medians of the distributions; as we increase α, we increase the median. In Figure 7, we shift our view further along the horizontal axis to inspect the tails of the distributions. The farther we go away from 0, the tail of the smallest α distribution gets heavier with respect to the other distributions. Finally, on Figure 8, the tail of the $\alpha = 0.25$ distribution is noticeably heavier than the other distributions. This is important because with small αs, most of our claims are small, but we could also have very large claims with significant probability.

To understand this effect, we analyze the processes for each claim size distribution. The parameters are set as follows:

- $u \in \{1, 2, \ldots, 20\}$
- $T = 1$
- $\lambda = 20$
- $N(T) \sim Poi(\lambda T)$
- $W_i \sim Weib(\alpha \in \{0.25, 1, 10\})$
- $\eta = 0.1$
- $c = (1 + \eta)E[N(T)]E[W_i]$
- $N = 10^5$

6.1.1 Non-Ruin Probabilities

The graph of the non-ruin probabilities for the different claim size distributions, Figure 9, shows that the process with the small α claim sizes does not change much when there is a decrease or increase in initial reserve. This is the same process with the greater variance, so a smaller alpha leads to a greater variance which results in more resistance to initial reserve. This is not good from an insurance company point of view, since no matter how much they increase their initial reserves, they are still highly prone to ruin.

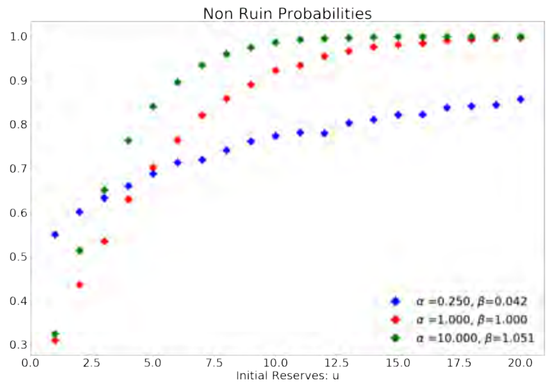


Figure 9: Non-ruin probabilities when changing claim sizes

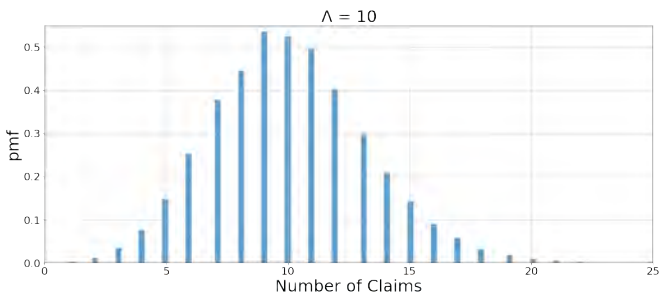
6.2 Claim Arrival Distributions

We will adjust our original risk model by letting our claim arrival process be a Mixed Poisson Process, $N(\Lambda T)$. We consider two distributions for Λ:

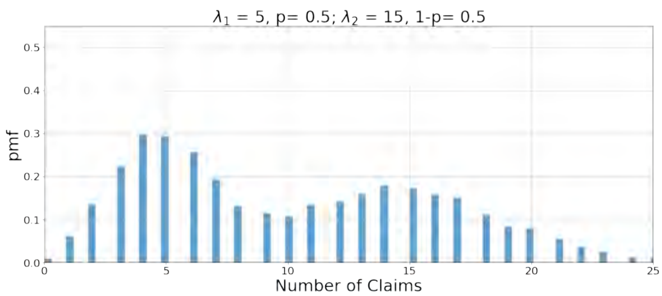
$$\Lambda = \text{Constant}$$

$$\Lambda = \begin{cases} \lambda_1, & p_1 \\ \lambda_2, & (1-p_1) \end{cases}$$

The second process follows the model in Section 3.2 that has two separate risk populations, and will be referred to as $Step(p_1, \lambda_1, \lambda_2)$. Below are two graphs depicting a simulation for number of claims when we have a constant Λ (Figure 10(a)) and when we have a Λ that can take on two different values (Figure 10(b)). Figure 10(a) illustrates that under a constant Λ,the number of claims follows a unimodal distribution with the peak at Λ. While Figure 10(b) shows that with a Λ that can take two values: λ_1, λ_2, the number of claims follows a bimodal distribution with peaks at λ_1, λ_2. This is important because with the Step model, we could either have a small amount of claims or a large amount of claims, greatly affecting our chances of ruin.



(a) Constant



(b) Step

Figure 10: Mixed Poisson Processes

To understand this effect, we analyze the processes for each claim arrival distribution. We simulate processes with the following parameters:

- $u \in \{1, 2, \dots, 20\}$
- $T = 1$
- $E[\Lambda] = 10$
- $N(T) \sim Poi(\Lambda T)$
  - $\Lambda = constant$
  - $\Lambda = \begin{cases} \lambda_1 = 5, & p_1 = 0.5 \\ \lambda_2 = 15, & (1 - p_1) = 0.5 \end{cases}$
- $W_i \sim Exp(4)$
- $\eta = 0.1$
- $c = (1+\eta)E[N(T)]E[W_i]$
- $N = 10^5$

6.2.1 Non Ruin Probabilities

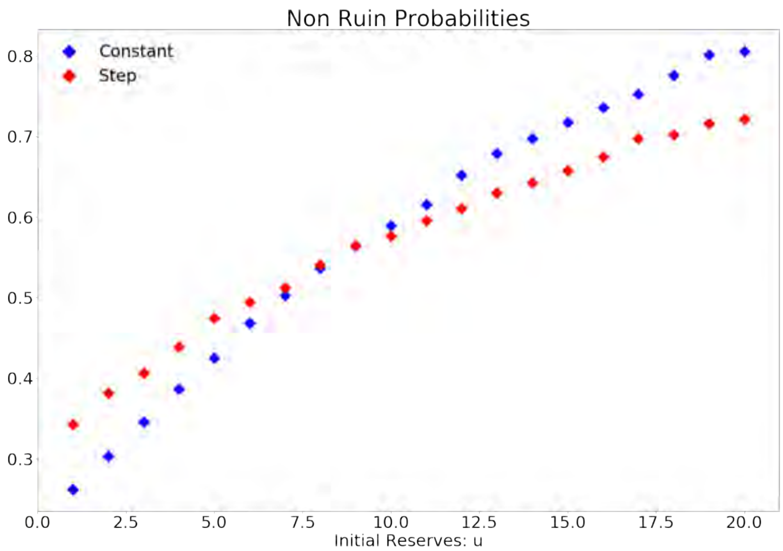


Figure 11: Non ruin probabilities when changing claim arrivals

The graph, Figure 11, of the non-ruin probabilities for the different claim arrival distributions shows that the non ruin probability for the step model does not change much when there is a decrease or increase in initial reserve. This process has a wider variety of risk, which leads to a greater variance, resulting in more resistance to initial reserve. This is not good from an insurance company point of view, since no matter how much they increase their initial reserves, they are still highly prone to ruin.

7 SUMMARY

A surplus process for an insurance company is introduced, where the company collects premium linearly over time but pays out claims when they occur, which follow an OSPP. The subject of interest is the ruin probability, which is the likelihood of the insurance company becoming insolvent. Two numerical methods are presented and compared to estimate the non-ruin probability: a standard Monte Carlo approach and a Monte Carlo method tailored to the case of OSPP claim arrivals referred to as APMC. While both estimators produce similar non-ruin probabilities, the APMC estimator is faster and more accurate. The APMC estimator is then used to assess the sensitivity of the ruin probabilities to the assumptions over the claim sizes and frequencies. Our main conclusion is that a larger variance of the risk process, for a given average cost of claim, requires a larger initial reserve to achieve targeted risk level.

References

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