ALTAM April 2025 Model Solutions

Question 1

The solution to this question is in the spreadsheet. It should be noted that as stated in the instructions for the exam, only work in the spreadsheet will be graded. Any work on paper is NOT graded for Excel problems.

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Question 2

(a) Examples of acceptable solutions:

- This model satisfies the Markov property because, given the state at time *t*, the probability of moving to any other state by time *t*+*s*, say, for any $s > 0$, does not depend on any history of the process before time *t*.
- ${}_s p_{x+t}^{ij}$ does not depend on how long the process has been in the current state, or on any other information about the process prior to being in state *i* at time *t*.
- Let $Y(t)$ denote the state that (x) is in at age $x+t$. Then

$$\Pr\left[Y(t+k) = j \middle| Y(t)\right] = \Pr\left[Y(t+k) = j \middle| Y(s)\, \forall s \le t\right]$$

Notes: Any equivalent answer should be given full credit. Answer must be coherent, and <u>must specify, in words or symbols, that the future probability does not depend on the past history.</u>

- *Examiners' Comments: Successful candidates were able to determine that the model satisfied the Markov property. The most successful candidates were also able to clearly explain how the model satisfied the Markov property. Candidates that contradicted themselves or gave an explanation but did not identify whether it satisfied the Markov property received little credit for their work.*

(b) (i)

$$\frac{d}{dt}\,{}_t p_x^{00} = -{}_t p_x^{00}(\mu_{x+t}^{01} + \mu_{x+t}^{03})$$

$$\frac{d}{dt}\,{}_t p_x^{01} = {}_t p_x^{00}\mu_{x+t}^{01} - {}_t p_x^{01}(\mu_{x+t}^{12} + \mu_{x+t}^{13})$$

Boundary Conditions: ${}_0 p_x^{00} = 1$ and ${}_0 p_x^{01} = 0$

(ii)

$$
\begin{aligned}
{}_tp_x^{01} &= \int_0^t {}_rp_x^{00}\ \mu_{x+r}^{01}\ {}_{t-r}p_{x+r}^{11}\ dr \\
&= \int_0^t e^{-0.07r}(0.02)\quad e^{-0.14(t-r)}\ dr \\
&= 0.02(e^{-0.14t})\int_0^t e^{0.07r}\ dr \\
&= 0.02(e^{-0.14t})\left(\frac{e^{0.07t}-1}{0.07}\right) \\
&= \frac{2}{7}(e^{-0.07t}-e^{-0.14t})\ \text{ as required.}
\end{aligned}
$$

Examiners' Comments:

- *The most successful candidates remembered to identify the boundary conditions and used the appropriate subscripts throughout the Kolmogorov forward differential equations for the first part of the question.*
- *For the second part of the question, the most successful candidates showed enough work through the derivation of the integral for ${}_tp_x^{01}$ to confirm their understanding of the proof being requested.*

(c)

(i) $$\bar{a}_x^{01} = \int_0^\infty {}_tp_x^{01}\, e^{-\delta t}\, dt$$

$$= \int_0^\infty \frac{2}{7}\left(e^{-0.07t} - e^{-0.14t} \right)\, e^{-0.05t}\, dt$$

$$= \frac{2}{7}\left(\frac{1}{0.12} - \frac{1}{0.19} \right) = 0.87719$$

(ii) $$\bar{A}_x^{02} = \int_0^\infty {}_t\, p_x^{01}\, \mu_{x+t}^{12}\, e^{-\delta t}\, dt$$

$$= \int_0^\infty 0.04\ {}_tp_x^{01}\, e^{-\delta t}\, dt = 0.04\ \bar{a}_x^{01}$$

Examiners' Comments:

- *Most candidates did well on this part of this question. The most successful candidates remembered to follow the rounding rules requested within the question.*
- *Most candidates also did well on the second part of this question. The most successful candidates remembered to utilize the force of interest notation within the integral and identified that the transition intensity was constant and could be moved outside the integral.*

(d)

$$P\bar{a}_x^{00} = 50{,}438.60 + 10{,}000\bar{a}_x^{01} + 120{,}000\bar{A}_x^{02}$$

$$\bar{a}_x^{00} = \int_0^\infty {}_tp_x^{00}\, e^{-0.05t}dt = \int_0^\infty e^{-0.12t}dt = \frac{1}{0.12} = 8.333$$

$$P = \frac{50{,}438.6 + 8{,}771.9 + 4{,}210.5}{8.333} = 7610.5$$

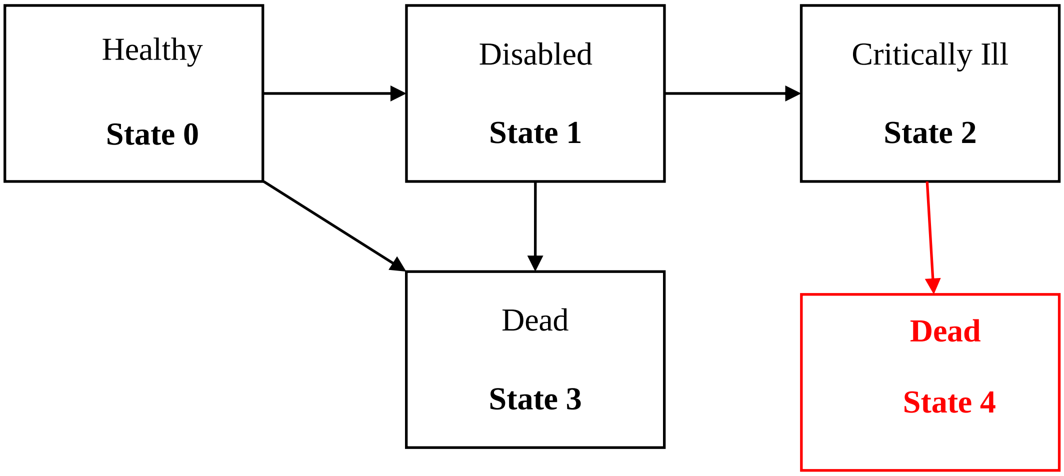
- *Examiners' Comments: Most candidates did well on this part of the question. The most successful candidates realized that there were three components to the benefit – the death benefit of 100,000 paid immediately, the disability income annuity of 10,000 per year paid continuously, and the critical illness benefit of 120,000 paid immediately.*

(e)

$$\frac{d}{dt}\,{}_tV^{(1)} = \delta\,{}_tV^{(1)} - 10{,}000 - \mu^{12}\left(120{,}000 - {}_tV^{(1)}\right) - \mu^{13}\left(100{,}000 - {}_tV^{(1)}\right)$$

Examiners' Notes: Most candidates did well on this part of the question. The most successful candidates avoided using the generic formula for the Thiele differential equation and realized that ${}_tV^{(2)}$ and ${}_tV^{(3)}$ were zero.

(f)



Examiners' Notes: Most candidates did well on this part of the question. The most successful candidates realized that a separate Dead state was needed to identify the unique benefit being offered to individuals moving from the Critically Ill state to the Dead state.

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Question 3

(a)

The probability is:

$$1-{}_{20}q_{60}\ {}_{20}q_{70} = 1-\left(1-\frac{75{,}657.2}{96{,}634.1}\right)\left(1-\frac{41841.1}{91{,}082.4}\right)$$

$$= 1 - 0.21708 \times 0.54062 = 0.88264$$

Examiners' Comments: Most candidates earned full credit on this part. A few students calculated the present value of the endowment instead of the probability.

(b) (i) The EPV is:

$$10{,}000 A_{\overline{60:70}:\overline{20|}} = 10{,}000\left(A_{60:\overline{20|}} + A_{70:\overline{20|}} - A_{60:70:\overline{20|}}\right)$$

$$A_{60:\overline{20|}} = 0.41040; \quad A_{70:\overline{20|}} = 0.47091 \quad \text{from tables}$$

$$A_{60:70:\overline{20|}} = A_{60:70} - {}_{20}p_{60}\,{}_{20}p_{70}\,v^{20} A_{80:90} + {}_{20}p_{60}\,{}_{20}p_{70}\,v^{20} =$$

$$0.46562 - (1-0.21708)(1-0.54062)(1.05)^{-20}0.78538 + (1-0.21708)(1-0.54062)(1.05)^{-20}$$

$$= 0.49471$$

$$\Rightarrow A_{\overline{60:70}:\overline{20|}} = 0.41040 + 0.47091 - 0.49471 = 0.386596$$

$\Rightarrow$ EPV of benefits is $10{,}000(0.386596) = 3{,}865.96$

Or

$$A_{60:70:\overline{20|}} = 1 - d\ddot{a}_{60:70:\overline{20|}} = 1 - \left(\frac{0.05}{1.05}\right)(10.61105) = 0.49471$$

where:

$$\ddot{a}_{60:70:\overline{20|}} = \ddot{a}_{60:70} - {}_{20}p_{60}\,{}_{20}p_{70}\,v^{20}\,\ddot{a}_{80:90}$$

$$= 11.2220 - (1-0.21708)(1-0.54062)(1.05)^{-20}4.5071 = 10.61105$$

(ii) The premium is P

$$P\ddot{a}_{\overline{60:70}:\overline{20|}} = 3865.96$$

$$\ddot{a}_{\overline{60:70}:\overline{20|}} = \frac{1 - A_{\overline{60:70}:\overline{20|}}}{d} = 12.8815$$

$$\Rightarrow P = 300.12$$

Examiners' Comments:

1. *This part was generally done pretty well, not as well as part (a) but still quite well.*
2. *An answer not supported by the candidate's work was not given any credit since the answer was given.*
3. *The most common error was to double count the endowment by adding the pure endowment to the value of the endowment insurance which already include the pure endowment. The candidate was equating the value of the endowment insurance to be the value of the term insurance.*

(c) (i) Let 0 denote the initial state.

$$ {}_{10}V^{(0)} = 10,000 A_{\overline{70:80}:\overline{10|}} - P\ddot{a}_{\overline{70:80}:\overline{10|}} $$

$$ \ddot{a}_{\overline{70:80}:\overline{10|}} = 7.9879 \quad \text{(given)} $$

$$ A_{\overline{70:80}:\overline{10|}} = 1 - d\ddot{a}_{\overline{70:80}:\overline{10|}} = 0.61962 $$

$$ \Rightarrow \ {}_{10}V^{(0)} = 10,000(0.61962) - 300.12(7.9879) = 3,798.93 $$

Or

$$ A_{\overline{70:80}:\overline{10|}} = A_{70:\overline{10|}} + A_{80:\overline{10|}} - A_{70:80:\overline{10|}} $$

$$ = 0.63576 + 0.67674 - [1 - d(6.4497)] = 0.61963 $$

(ii) Let 1 denote the state in which only Kim is alive:

$$ {}_{10}V^{(1)} = 10,000 A_{70:\overline{10|}} - P\ddot{a}_{70:\overline{10|}} = (10,000)(0.63576) - (300.12)(7.6491) = 4061.95 $$

Examiners' Comments:

1. *Most candidates did not get part (i) correct. Once again, many students double counted the pure endowment benefit.*
2. *Candidates did better on part (ii). Most candidates did understand that the reserve was just a single life reserve since only Kim was alive.*

(d)

(i) The extra premium is R such that

$$(P+R)\,\ddot{a}_{60:70:\overline{20}|}=3865.96$$

$$\ddot{a}_{60:70:\overline{20}|}=\frac{1-A_{60:70:\overline{20}|}}{d}=\frac{1-0.49471}{0.047619}=10.6110$$

or

$$\ddot{a}_{60:70:\overline{20}|}=\ddot{a}_{60:70}-{}_{20}E_{60}\cdot{}_{20}E_{70}(1.05)^{20}\ddot{a}_{70:80}=11.2220-0.13555(4.5071)=10.6110$$

$$\Rightarrow R=\frac{3865.96}{10.6110}-P=64.21$$

(ii) Relevant factors include:

- Whether it might be a burden for the survivor to continue paying premiums after the first life dies
- Are there other options for investing/utilizing the extra 64.21 per year up to the first death that could compensate for paying an extra 300.12 per year after the first death.
- What cash value (or viatical market value) would be paid if the policy is surrendered (or transferred) before maturity – ie how much risk is there if they do not select the contract in which the premiums stop on first death?

Examiners' Comments:

1. *For part (i), many candidates got most of the points. Quite a few candidates did not understand that question asked for the additional amount instead of the total premium.*
2. *For part (ii), candidates did very well although some candidates did not explain why a consideration should be taken into account.*

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Question 4

(a)

EPV Premiums – Expenses: $\left(0.95\ddot{a}_{60}-0.45\right)P=13.7089P$

EPV of Benefits:

$$100,000A_{60}+100,000\ {}_{20}E_{60}\,A_{80}=46,524.23$$

Or:

$$200,000\left(A_{60}-0.5A^{\,1}_{60:\overline{20}|}\right)=46,524.23$$

Or:

$$100,000A^{\,1}_{60:\overline{20}|}+200,000\,{}_{20}E_{60}\,A_{80}=46,524.23$$

$$\text{So Premium}=\frac{46,524}{13.7089}=3393.73$$

Examiners' Comments: Most candidates did pretty well on this. Common issues were candidates treating this as an endowment.

(b)

(i) Lapse supported products are priced taking into account that some policies will lapse without payment of any benefit (or with a very low cash value relative to asset share). The unused funds from policies that lapse are used to subsidize costs for those who stay, leading to lower premiums.

(ii) The problem arises if policyholders lapse in lower numbers than expected.

Suppose the insurer charges a lower premium based on an assumption that a significant proportion of policyholders will lapse. If the insurer overestimates the proportion lapsing then it is making an expected loss on each contract. With the increase in viatical/third party market, policyholders will often do much better by selling their policy to third parties than by lapsing, so rates based on the historical lapse experience may significantly underestimate future lapse experience.

Examiners' Comments: Candidates struggled with the pricing implications of lapse supported products, the subsidization that applies and the impacts that they have to the insurer. The candidates that did well provided justification tying to the context of the problem.

(c)

(i)

$$\text{EPV Benefits} = 100,000\left(A_{60}^{01} + {}_{20}p_{60}^{00}\, v^{20} A_{80}^{01}\right)$$

$$= 100,000(0.10330 + 0.04282) = 14,612$$

Or

$$100{,}000 A_{60:\overline{20}|}^{01} + 200,000\left({}_{20}p_{60}^{00}\, v^{20} A_{80}^{01}\right)$$

$$A_{60:\overline{20}|}^{01} = A_{60}^{01} - {}_{20}p_{60}^{00}\, v^{20} A_{80}^{01} = 0.06048$$

$$\Rightarrow \text{EPVBenefits} = 100,000(0.06048) + 200,000(0.04282) = 14,612$$

$$\Rightarrow P = \frac{14,612}{0.95\, \ddot{a}_{60}^{00} - 0.45} = 1846.77$$

(ii) At time 10:

$$ {}_{10}V^{(0)} = 100,000\left(A^{01}_{70} + {}_{10}p^{00}_{70}\, v^{10} A^{01}_{80} \right) - P\left(0.95\, \ddot{a}^{00}_{70} \right) $$

$$ = 100,000(0.380964) - 1846.76(0.95 \times 8.7967) = 22,663.3 $$

Examiners' Comments:

1. *Most candidates did pretty well.*
2. *A few candidates incorrectly included first time expenses in the reserve calculations.*
3. *Some candidates calculated values that were provided in the questions.*

(d)

(i)

$$ {}_{t+h}p^{00}_{70} \approx {}_{t}p^{00}_{70} - h\ {}_{t}p^{00}_{70}\left(\mu^{01}_{70+t} + \mu^{02}_{70+t} \right) $$

Let $h = 0.5,\ t = 0$:

$$ {}_{0.5}p^{00}_{70} \approx 1 - 0.5(1)(0.009881 + 0.04) = 0.97506 $$

Let $h = 0.5,\ t = 0.5$:

$$ {}_{1}p^{00}_{70} \approx 0.97506 - 0.5(0.97506)(0.010462 + 0.045) = 0.94802 $$

(ii)

$$ {}_{t+h}p_{70}^{01} \approx {}_{t}p_{70}^{01} + h\ {}_{t}p_{70}^{00}\left(\mu_{70+t}^{01}\right) $$

Let $h = 0.5,\ t = 0$:

$$ {}_{0.5}p_{70}^{01} \approx 0 + 0.5(1)(0.009881) = 0.00494 $$

Let $h = 0.5,\ t = 0.5$:

$$ {}_{1}p_{70}^{01} \approx 0.00494 + 0.5(0.97506)(0.010462) = 0.01004 $$

$$ \left({}_{10}V + 0.95P\right)(1+i) = p_{70}^{01}(100{,}000) + p_{70}^{00}\ {}_{11}V $$

$$ \Rightarrow\ {}_{11}V = 25{,}985.21 $$

Examiners' Comments: Results were generally mixed. Candidates generally understood the formulas but struggled to apply to the question being asked. Specially most struggled on the first two parts but most did well in part 3 with many receiving full credit on part (iii). Some candidates miss the premium based expenses.

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Question 5

(a) The projected final average salary is

$$FAS = \frac{S_{62} + S_{63} + S_{64}}{3}$$

$$= S_{50} \frac{1.02^{12} + 1.02^{13} + 1.02^{14}}{3} = 100,914.51$$

Then $AL = \alpha\, n\,(FAS)\ {}_{15}E_{50}\, \ddot{a}_{65}^{(12)}$

$${}_{15}E_{50} = \frac{l_{65}}{l_{50}} v^{15} = 0.46151$$

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} = 13.0915$$

$$\Rightarrow AL = (0.02)(25)(100,914.5)(0.46151)(13.0915) = 304,858$$

Examiners' Comments: Candidates did very well on this part.

(b) (i) The projected monthly pension is

$$\frac{(0.02)(40)(FAS)}{12} = \frac{80,731.6}{12} = 6,727.63$$

(ii) The projected replacement ratio is $\frac{80,731.6}{S_{50}(1.02)^{14}} = 78.44\%$

Examiners' Comments: This part was also done quite well, but some candidates missed the extra 15 years of service in the pension valuation.

(c)

(i) The monthly income from the DB part is $(0.02)(25)(FAS)/12 = 4204.77$

The accumulated DC assets under option A are:

$$(0.12)S_{50}(1.06^{14} + (1.02)(1.06^{13}) + (1.02^2)(1.06^{12}) + \ldots + 1.02^{14})$$

$$= (0.12)S_{50}(1.06)^{14}\left(1 + \left(\frac{1.02}{1.06}\right) + \left(\frac{1.02}{1.06}\right)^2 + \cdots + \left(\frac{1.02}{1.06}\right)^{14}\right)$$

$$= (0.12)S_{50}(1.06)^{14}\left(\frac{1-\left(\frac{1.02}{1.06}\right)^{15}}{1-\left(\frac{1.02}{1.06}\right)}\right)$$

$$= (0.12)S_{50}\left(\frac{1.06^{15}-1.02^{15}}{1.06-1.02}\right) = (0.12)(78{,}000)(26.2672) = 245{,}861$$

Converting these assets to a pension gives a monthly income from the DC account of

$$\frac{245{,}861}{12\ddot{a}_{65}^{(12)}} = \frac{245{,}861}{157.1} = 1565.02$$

==> total projected monthly income under Option A is
$4204.77 + 1565.02 = 5769.80$

(ii) Under Option B the projected DC assets at retirement are

$$304{,}858(1.06^{15}) + 245{,}861 = 976{,}471$$

which converts to a monthly pension of

$$\frac{976{,}471}{12\,\ddot{a}_{65}^{(12)}} = \frac{976{,}471}{157.1} = 6215.7$$

Examiners' Comments: Quite a few candidates omitted this part. Good candidates understood the correct point in time at which benefits should be calculated. Many candidates were off by a single year.

(d)

Advantages

- The DB part of the pension carries no investment risk during the accumulation (pre-retirement) period; it is guaranteed (given FAS) as long as the employer survives. It is also increasing with salary. Under Option B all assets are vulnerable to investment risk.
- Annuitization of DC assets depends on mortality and interest rates at retirement. Under Option A at least the DB part of the income is safe from interest rate / annuity pricing risk.

Disadvantages:

- The projected income is lower than Option B;
- The DB part is vulnerable to employer default
- There is less opportunity to benefit from stronger market returns.
- No death benefit from DB part.

Examiners' Comments: High performing candidates were able to recognize that Option B produced higher benefits, and recognized that as a disadvantage of Option A. Candidates need to have clear labeling of what is an advantage and which is a disadvantage, as the graders can not reward points if it is not clear.

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Question 6

Examiners' General Comments:

There were some candidates who noted that they were unable to find values from a normal distribution. Per the introductory study note, candidates are expected to know how to use Excel to generate normal distribution values in addition to other distributions.

Some candidates noted they used Excel, provided an answer, but showed no work. Candidates are welcome to use Excel to aid in their calculations but are unlikely to receive full credit for a correct answer without evidence of their work. In cases where an answer is incorrect, we need to see evidence of work to award any points. For non-Excel questions, only the work shown on paper is considered for grading.

(a) From the formula sheet we have

$$p(0) = kPe^{-10r}\Phi\big(-d_2(0)\big) - P\xi\Phi\big(-d_1(0)\big)$$

where $k = 1$ and $\xi = 0.97e^{-10m}$

$$\Rightarrow p(0) = Pe^{-10r}\Phi\big(-d_2(0)\big) - P(0.97)e^{-10m}\Phi\big(-d_1(0)\big)$$

$$d_1(0) = \frac{log\left(\frac{P(0.97)e^{-10m}}{P}\right) + \left(r + \frac{\sigma^2}{2}\right)(10)}{\sigma\sqrt{10}} = 0.58430$$

$$d_2(0) = d_1(0) - \sigma\sqrt{10} = -0.04816$$

$$\Rightarrow \Phi\big(-d_1(0)\big) = 0.279511 \qquad \Phi\big(-d_2(0)\big) = 0.51921$$

$$\Rightarrow p(0) = 12{,}605.5$$

$${}_{10}p_{50} = \frac{l_{60}}{l_{50}} = \frac{96{,}634.1}{98{,}576.4} = 0.9803$$

$$\pi(0) = {}_{10}p_{50}\ p(0) = 12{,}357.2$$

Examiners' Comments: We expected this to be a straightforward question. The Black-Scholes formula variant was given in the formula sheet with the same notation as the problem. Many candidates attempted to re-derive the variant of the formula, sometimes with mistakes, while others made mistakes with the appropriate expense levels. Some candidates used discrete discounting instead of continuous.

(b) At $t = 6$:

$$p(6) = Pe^{-4r}\Phi\big(-d_2(6)\big) - F_6\ e^{-4m}\Phi\big(-d_1(6)\big)$$

$$\text{where}\quad d_1(6) = \frac{log(F_6\ e^{-4m}/P) + (r + \sigma^2/2)(4)}{\sigma\sqrt{4}} = 1.24118$$

$$d_2(6) = d_1(6) - \sigma\sqrt{4} = 0.84118$$

$$\Rightarrow \Phi\big(-d_1(6)\big) = 0.10727 \qquad \Phi\big(-d_2(6)\big) = 0.20012$$

$$\Rightarrow p(6) = 3190.27$$

$$_4p_{56} = \frac{l_{60}}{l_{56}} = \frac{96{,}634.1}{97{,}651.2} = 0.98958$$

$$\pi(6) = \ _4p_{56}p(6) = 3157.05$$

Examiners' Comments: Performance on part B was similar to part A. An additional common mistake here was candidates attempting to re-solve the benefit amount given in the problem or deducting additional expenses/discounting, not recognizing that we were looking for the value at time 6.

(c)

$$BB_{10} \geq max(P(1.03^{10}),\ F_3, F_6) = max(134{,}392,\ 103{,}000,\ 140{,}000)$$

$$\Rightarrow BB_{10} \geq 140{,}000$$

$\Rightarrow$ guaranteed minimum income annual benefit is

$$\gamma(140{,}000) = 0.08(140{,}000) = 11{,}200$$

Examiners' Comments: The primary goal with this question was to assess different benefit amounts as the benefit base. Many candidates missed different components to consider in calculating the benefit base. The most common mistake was recognizing that the fund value at time 6, 140,000, should have been considered (and is what we were looking for). Some candidates overcomplicated this and calculated an annuity factor.

(d)

$$BB_4 \geq 1.05 \times 140{,}000 = 147{,}000$$
$$\Rightarrow \text{ guaranteed minimum WB} = 0.06 \times 147{,}000 = 8820$$

Examiners' Comments: Many candidates made a similar mistake here as in part C - not recognizing that the time 6 fund value given in the problem, 140,000, should be used here. Some candidates attempted to forecast future values of the fund which was incorrect since we were concerned with the amount payable 'based on the information at the time of the exchange offer,' i.e. time 6.

(e)

Advantages:

- More flexible – can withdraw more or less than GMWB, as long as funds are available. The GMIB is a fixed annuitization.
- Continue to participate in investment returns – as long as the fund is not fully depleted. Under the GMIB all funds are converted to a fixed annuity.
- Any funds not withdrawn as GMWB can still be withdrawn on surrendering the policy, or they will pass onto the policyholder's estate on their death. Under the GMIB, as with all life annuities, the policyholder (or their heirs) has no entitlement to any of the underlying assets on death or surrender (except possibly for any additional payment guarantees under the GMIB terms).

Disadvantage

- Typically, as in this case, the minimum payment is lower under the GMWB than under a GMIB, with the same fund value at the start of the annuity.

Examiners' Comments: Most commonly, candidates simply stated an advantage or disadvantage of a GMWB or GMIB, missing that we were looking for a comparison of the two. That said, candidates were typically able to determine an advantage of a GMWB over an GMIB, but many missed the primary disadvantage (GMWBs typically offer less income than GMIBs).