

# Advanced Long-Term Actuarial Mathematics Exam

SPRING 2023

The ALTAM Exam is a three-hour exam consisting of 60 points of written-answer questions. The exam will be taken at Prometric testing centers. The written-answer questions will be displayed on the computer and answered in a paper answer booklet to be provided and collected by Prometric. Tables will be provided in an Excel workbook, which may also be used for calculations. However only the paper answer booklet will be submitted for grading. Paper versions of the exam and tables will not be provided. Candidates will be provided with a scratchpad.

Knowledge of the FAM Exam material is assumed for the ALTAM Exam.

[Exam Registration](#)

Candidates may register online or with an application.

[Introductory Study Note](#)

The Introductory Study Note has a complete listing of all study notes as well as errata and other important information.

[Past Exams](#)

Past LTAM Exams from October 2018 through present are available on the SOA website. There are no past ALTAM Exams.

[Updates](#)

Candidates should be sure to check the Updates page on the exam home page periodically for additional corrections or notices.

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**1. Topic: Survival Models for Multiple State Contingent Cashflows (10-20%)**

**Learning Objectives**

The Candidate will understand key concepts concerning multiple state mortality/morbidity and joint life mortality models for insurance and annuity contracts.

**Learning Outcomes**

The Candidate will be able to:

- a) Apply Markov multiple state models to state-contingent life and long-term health insurance benefits, and Continuing Care Retirement Communities (CCRCs).
- b) Understand and critique the assumptions underlying Markov multiple state models for long-term insurance.
- c) Derive and apply Kolmogorov's forward equations for continuous time Markov multiple state models.
- d) Calculate state-dependent probabilities for continuous time Markov models.
- e) Apply the Chapman-Kolmogorov equations to calculate discrete time transition probabilities in the Markov model.
- f) Construct and deconstruct multiple decrement tables using the associated single decrement models and appropriate fractional age assumptions.
- g) Calculate maximum likelihood estimates of transition intensities and probabilities for multiple state and multiple decrement models, assuming piecewise constant transition intensities.
- h) Calculate approximate confidence intervals for the estimators in Topic 1(g), using asymptotic properties of the maximum likelihood estimators.

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**2. Topic: Premium and Policy Valuation for Long-Term State-Dependent Coverages (12-20%)**

**Learning Objectives**

The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for single life, joint life, CCRCs, or other state-dependent insurance and annuity contracts.

**Learning Outcomes**

The Candidate will be able to:

- a) Define and interpret state-dependent insurance and annuity present value random variables and identify and calculate their expected values.
- b) Derive and apply two-term and three-term Woolhouse approximations for calculating expected present values of state-dependent cash flows.
- c) Calculate premiums for state-dependent life insurance, long-term health insurance, and CCRCs using the equivalence principle.
- d) Calculate policy values for state-dependent life insurance, long-term health insurance, and CCRCs.
- e) Identify and apply Thiele's differential equation in a single life or multiple state setting.

**3. Topic: Joint Life Insurance and Annuities (8-16%)**

**Learning Objectives**

The Candidate will understand key concepts concerning estimation and construction of multiple state and joint life models for insurance and annuity contracts.

**Learning Outcomes**

The Candidate will be able to:

- a) Understand how joint-life mortality can be modelled using (i) a time-to-status-failure random variable, and (ii) a multiple state model.
- b) Understand the implications of independence or dependence of future lifetimes in both versions of the joint life model from 3(a). Identify sources of dependence and understand how they are accommodated in the models.
- c) Calculate premiums for insurance and annuities on joint lives using the equivalence principle.
- d) Calculate policy values for insurance and annuities on joint lives.

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**4. Topic: Profit Analysis (10-20%)**

**Learning Objectives**

The Candidate will be able to analyze emerging surplus, and to set premiums and reserves, using profit testing.

**Learning Outcomes**

The Candidate will be able to:

- a) Calculate and interpret common profit measures such as expected profit, actual profit, gain, gain by source and period, profit signature, profit vector, net present value, internal rate of return, profit margin, and discounted payback period for long-term life and health insurance, and annuity contracts.
- b) Calculate premiums for long-term life and health insurance and annuity contracts based on a specified profit objective.
- c) Calculate reserves for long-term life and health insurance and annuity contracts using profit testing.

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**5. Topic: Pension Plans and Retirement Benefits (10-18%)**

**Learning Objectives**

The Candidate will understand how retirement benefits are accrued, valued, and funded.

**Learning Outcomes**

The Candidate will be able to:

- a) Calculate replacement ratios for Defined Contribution (DC), and Defined Benefit (DB) plans, including final average salary (FAS), career average earnings (CAE), and career average revalued earnings (CARE) plans.
- b) Calculate the required contribution rate to meet a target replacement ratio for a DC plan participant, using a deterministic approach.
- c) Identify, interpret, and apply service table and salary scale functions for pension plan valuation.
- d) For a DB plan, calculate and interpret replacement ratios, accrued benefits, including benefits on early exit from the plan.
- e) For a DB plan, calculate and interpret the actuarial accrued liability and the normal cost for benefits payable on age retirement or early exit using the projected unit credit (PUC) and traditional unit credit (TUC) valuation methods.
- f) Identify and interpret the assumptions and funding methods used for retiree health care valuation.
- g) Calculate and interpret the expected present value of future benefits, accumulated postretirement benefit obligation (APBO), and the normal cost or service cost for retiree health care plans.

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**6. Topic: Universal Life Insurance (10-18%)**

**Learning Objectives**

The Candidate will understand the policy design and benefits payable under Type A and Type B Universal Life contracts and be able to assess and quantify account values, premiums, and reserves.

**Learning Outcomes**

The Candidate will be able to:

- a) Understand the cashflows and calculate account values and benefits under Type A and Type B Universal Life policies.
- b) Calculate reserves for no-lapse guarantees.
- c) Use deterministic profit testing to calculate premiums and to assess emerging surplus for Universal Life insurance.

**7. Topic: Embedded Options in Life Insurance and Annuity Products (10-18%)**

**Learning Objectives**

The Candidate will understand various types of equity-linked life insurance guarantees, options that are embedded in life insurance and annuity contracts and be able to price, reserve, and hedge the risk inherent in these options.

**Learning Outcomes**

The Candidate will be able to:

- a) Define and calculate payoffs under each of the following options embedded in insurance and annuity contracts:
  - Guaranteed minimum death benefit
  - Guaranteed minimum maturity benefit
  - Guaranteed minimum income benefit
  - Guaranteed minimum withdrawal benefit
- b) Value the following options embedded in insurance and annuity contracts, using the Black-Scholes model:
  - Guaranteed minimum death benefit
  - Guaranteed minimum accumulation/maturity benefit
- c) Construct a replicating portfolio for the options in 7(b) using delta-hedging
- d) Understand and evaluate the costs associated with discrete-time rebalancing.

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Resources

- *Actuarial Mathematics for Life Contingent Risks, Third Edition* Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.
  - Chapter 7 (Sections 2.4 and 4)
  - Chapter 8
  - Chapter 9
  - Chapter 10
  - Chapter 11
  - Chapter 13 (except section 8)
  - Chapter 14
  - Chapter 15 (sections 1-3)
  - Chapter 17
  - Chapter 18 (Section 6)
- [ALTAM-21-23: Variable Annuity Guarantees](#)
- [Notation and Terminology used on Exam ALTAM](#)
- Exam ALTAM Tables  
[Excel Workbook for Exam ALTAM](#)
- [Formula Sheet](#)
- Sample [questions](#) and [solutions](#)

Note: The text and study notes will not be available with the examination booklet. The Excel Workbook for Exam ALTAM and a pdf copy of the Formula Sheet will be available.