QFI IRM Model Solutions Spring 2023

1. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

Learning Outcomes:

(1b) Describe and apply various risk identification tools.

Sources:

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 19: Behavioral Risk Management – Page 581 / 19.5 Cumulative Prospect Theory

Commentary on Question:

This question is taken directly from the source reading. Candidates who have thoroughly read through the source material are expected to answer this question well.

Solution:

- (a) Explain the differences between expected utility theory and prospect theory in terms of
 - (i) reference points
 - (ii) value function
 - (iii) weighting function

Commentary on Question:

Candidates performed above average on this part. Candidates receive full credits as long as the main differences between the two theories are identified. Partial credits are given to incomplete answers.

Reference Points

Under expected utility theory, we consider the impact of the payoffs on the decision maker's total wealth.

Under prospect theory, we consider the impact of the payoffs relative to some reference point.

If the reference point is the decision-maker's current wealth, then this is similar to maximizing expected incremental utility.

Value Function

Under expected utility theory, we assume that individuals are risk-averse so that the same form of utility applies whether there is a gain or loss.

Individuals are in fact loss averse more than they are risk-averse.

Prospect theory incorporates a value function in place of the utility function, where the magnitude of the value function for a loss is larger than for an equivalent gain.

The function is also concave for gains and convex for losses to reflect risk seeking for losses and risk-averse for profits.

The Weighting Function

Under expected utility theory, individuals are assumed to know the objective probabilities associated with the payoff.

Under prospect theory, individuals may be biased on the probability distribution. The mental heuristic that people seem to employ is to divide events into three categories – 'will not happen', 'will certainly happen' and 'may happen' We tend to assign equal probability to all outcomes in the 'may happen' category.

Under prospect theory, we replace objective probabilities with a weighting function on the distribution function of x

(b) Determine which prospect gives the higher subjective value under the prospect theory.

Commentary on Question:

Candidates did as expected on this question. Those who read the source material generally were able to carry out the calculation of the subjective value. Partial credits given to the intermediary steps to arrive at the final solution.

Xi V piXj -4000 (3,490)0 -200 (235) 0.25 -5 (9) 0.25 100 63 0.25 600 316 0.25 12000 4,691 0 $SV_1 = (-3490)^*(0) + (-235)^*(0.25) + \dots + (4691)^*(0) = 33.89$ $SV_2 = 50^{0.9} = 33.81$

- (c)
- (i) Explain why the CRO thinks reframing the decision is more appropriate.
- (ii) Determine which prospect gives a higher subjective value under the new reference point.
- (iii) Explain the difference in decision, if any, between part b and part c(ii).

Commentary on Question:

Candidates did as expected on part (i). Candidates who were able to answer part (b) generally did well on part (ii). Candidates did poorly on (iii). The key is to identify the +50M causes a shift in value function and people are loss averse. Partial credit given to candidates who stated there's a shift in value function.

(i)

Prospect II has a probability of 1 that the company will receive \$50 million dollars. Because of this certainty, it makes more sense to evaluate the two prospects with a reference point of initial wealth + \$50 million.

xj-50	V	$p_i x_j$
-4050	(3,530)	0
-250	(288)	0.25
-55	(74)	0.25
50	34	0.25
550	293	0.25
11950	4,673	0

$$\begin{split} SV_1 &= (-3530)^*(0) + (-288)^*(0.25) + \ldots + (4673)(0) = -8.77259 \\ SV_2 &= 0^{\circ}0.9 = 0 \end{split}$$

(iii)

It is clear that the payouts under the first framing in the example are exactly the same as the payouts under the second framing, but the decision is different under prospect theory.

The difference is caused by the shift in value function as the perception has changed and people are loss averse. By subtracting the 50 million out, the value of having a loss is now larger than before and caused the change in decision.

2. Understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

- (2a) Explain the advantages and limitations of different risk metrics.
- (2b) Explain how different approaches and tests form a set of complementary investment risk metrics.

Sources:

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 3: Risk Measures

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)

- (i) Calculate the probability of a loss for the product.
- (ii) Calculate the 95% VaR of loss L.

Commentary on Question:

Most Candidates did well for this question. Some candidates use the normal distribution instead of the lognormal distribution to model the stock price and received partial credit.

 $Pr[L = 0] = Pr[S_{10} > 1] = 1 - Pr[S_{10} < 1] = 1 - 0.07656 = 0.92344$

As this is less than 0.95, the 95% quantile lies in the continuous part of the loss distribution.

$$\Pr\left[L \le Q_{0.95}\right] = 0.95$$

$$\begin{aligned} &\Pr[10,000(1-S_{10}) \leq Q_{0.95}] = 0.95 \\ &\Pr\left[S_{10} > 1 - \frac{Q_{0.95}}{10,000}\right] = 0.95 \end{aligned}$$

$$\Phi\left(\frac{\ln\left(1 - \frac{Q_{0.95}}{10,000}\right) - \mu}{\sigma}\right) = 0.05$$

$$\frac{\ln\left(1 - \frac{Q_{0.95}}{10,000}\right) - \mu}{\sigma} = -1.64485$$

$$Q_{0.95} = 1404.94$$

(b) Calculate the 90% and 95% Expected Shortfalls for the product.

Commentary on Question:

Most Candidates did not know the expected shortfall formula and did not do well for this question. Partial credits were given to candidates who completed parts of the calculation.

$$ES_{95} = 10,000 * \left\{ 1 - \frac{e^{\mu + \frac{1}{2}\sigma^2}}{1 - \alpha} * \Phi(z_{1 - \alpha} - \sigma) \right\}$$
$$ES_{95} = 3389.42$$

See intermediate calculations below:

1-lpha	0.05
$e^{\mu + \frac{1}{2}\sigma^2}$	3.472935
$\frac{e^{\mu+\frac{1}{2}\sigma^2}}{1-\alpha}$	69.4587
$z_{1-\alpha}$	-1.64485
$z_{1-\alpha}-\sigma$	-2.34485
$\Phi(z_{1-\alpha}-\sigma)$	0.009517
$\frac{e^{\mu+\frac{1}{2}\sigma^2}}{1-\alpha}\Phi(z_{1-\alpha}-\sigma)$	0.661058
$\left\{1-\frac{e^{\mu+\frac{1}{2}\sigma^2}}{1-\alpha}\Phi(z_{1-\alpha}-\sigma)\right\}$	0.338942

 $ES_{90\%}$

$$ES_{\alpha} = \frac{(\beta - \alpha)Q_{\alpha} + (1 - \beta)E[L|L > Q_{\alpha}]}{1 - \alpha}$$

Pr[L=0]=1-.07656 = .92344 from (a)

$$E[L|L > Q_{\alpha}] = ES_{0.92344} = 10,000 * 0.244988 = 2449.88$$

See intermediate calculations below:

$1-\alpha$	0.076564
$e^{\mu+\frac{1}{2}\sigma^2}$	3.472935
$\frac{e^{\mu + \frac{1}{2}\sigma^2}}{1 - \alpha}$	45.36005
$z_{1-\alpha}$	-1.42857
$z_{1-\alpha} - \sigma$	-2.12857
$\Phi(z_{1-\alpha}-\sigma)$	0.016645
$\frac{e^{\mu+\frac{1}{2}\sigma^2}}{1-\alpha}\Phi(z_{1-\alpha}-\sigma)$	0.755012
$\left\{1-\frac{e^{\mu+\frac{1}{2}\sigma^2}}{1-\alpha}\Phi(z_{1-\alpha}-\sigma)\right\}$	0.244988

 $ES_{0.90} = [(.92344 - .90)*0 + (1 - .92344)(2449.88)]/(1-.9)$

$$ES_{90\%} = 1875.72$$

(c) Calculate the expected loss for ABC net of reinsurance $E[\min(L, Q_{\alpha})]$.

Commentary on Question:

Most Candidates did not know formula for calculating $E[\min(L, Q_{\alpha})]$ from the text and did not attempt this question.

$$ES_{0.95} = Q_{.95} + \frac{1}{1 - \alpha} (E[L] - E[L \land Q_{.95}])$$

Where $E[L \land Q_{.95}] = E[\min(L, Q_{\alpha})]$

$$E[\min(L,Q_{\alpha})] = 88.3477$$

- (d)
- (i) Calculate the smoothed empirical estimate for VaR 95% from the Monte Carlo simulation presented in the Excel Spreadsheet.
- (ii) Calculate the 90% confidence interval for VaR 95% from the Monte Carlo simulation presented in the Excel Spreadsheet using the order statistics method. Interpolate between the integer value of the order statistics losses to get the results.
- (iii) Describe the advantages and disadvantages of the order statistics method vs. repeat simulation method for calculating confidence intervals.

Commentary on Question:

Candidates did reasonably well on this question. Most candidates received partial credits for completing parts of the calculation or for providing some of the advantages/disadvantages for the order statistics or the repeat simulation method.

(d) (i)

 $Q_{.95} \approx L_{.95*1001} = L_{950.95} = 0.05L_{950} + 0.95L_{951} = 1280.27$

(d) (ii)

 $L_{938.66} = 863.58, L_{961.34} = 1758.64$

(d) (iii)

The repeat simulation method is reliable, but it requires substantially more computational resources. If the sample is not generated by simulation, it may not be possible to use the repeat method.

The order statistics method is non-parametric; it usually generates a wider confidence interval (although in this case is not true due to the small number of repeat simulations – 100), but it does not require any additional simulations nor any additional assumptions or approximations.

(e)

- (i) Calculate the Expected Shortfall $\widehat{ES_{95\%}}$ from the Monte Carlo simulation presented in the Excel Spreadsheet.
- (ii) Critique the use of sample standard deviation $\frac{s_u}{\sqrt{N(1-\alpha)}}$, where s_u is the standard deviation of the upper $100(1-\alpha)\%$ of the simulated losses.
- (iii) Calculate the standard deviation of the Expected Shortfall $\widehat{ES_{95\%}}$ using the Manistre-Hancock formula.

Commentary on Question:

Most Candidates did well on part (i) of this question but did poorly on part (ii) & (iii). For part (ii), partial credits were received by candidates who identified that the sample standard deviation will underestimate the uncertainty. For part (iii), candidates who knew how to apply the Manistre-Hancock formula to calculate expected shortfall mostly received full credits.

(e) (i)

3161.87

(e).(ii)

This will underestimate the uncertainty on average because it does not consider the uncertainty conditioning on the quantile estimator $\widehat{Q_{\alpha}}$. In general

$$Var[\widehat{ES}_{\alpha}] = E\left[Var[\widehat{ES}_{\alpha}|\widehat{Q}_{\alpha}]\right] + Var[E[\widehat{ES}_{\alpha}|\widehat{Q}_{\alpha}]]$$

 $\frac{s_u^2}{N(1-\alpha)}$ estimates the first term; we need to make allowance also for the second term which considers the effect of the uncertainty in the quantile.

(e) (iii)

Expected Shortfall estimate is

$$S_{ES_{\alpha}} = \sqrt{\frac{s_{u}^{2} + \alpha (\widehat{ES_{\alpha}} - \widehat{Q_{\alpha}})^{2}}{N(1 - \alpha)}}$$

S_u^2	2511555
ES	3161.868
Q	1280.272
$s_u^2 + \alpha (\widehat{ES}_{\alpha} - \widehat{Q_{\alpha}})^2$	5874938
$S_{ES_{\alpha}}$	342.7809

 $S_{ES_{\alpha}} = 342.78$

2. Understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

- (2e) Understand, evaluate, and apply credit risk models.
- (2f) Apply different techniques of assessing rare event risks, including stress testing and scenario analysis.

Sources:

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 7 (pages 239-240, 241, 249-250)

Commentary on Question:

Most candidates performed well on this question particularly in parts (a) and (c).

Solution:

(a) Identify four uses of stress testing on investment portfolio.

Commentary on Question:

Most candidates successfully identified at least four uses of stress testing.

- Stress tests can explore extreme scenarios
- Stress tests can challenge the models and assumptions used in quantitative analysis
- New products may require it because of insufficient data for statistical modeling
- Can be used to focus effect of extreme scenarios on capital and liquidity
- Should serve to facilitate communication within an institution
- Can be used to establish and communicate a firm's risk appetite
- Integral component of deterministic micro-simulation modeling
- Often required by regulators in assessing capital adequacy
- (b) You begin stress testing for company XYZ and ask for input from employees who design insurance products.
 - Employee A recommends stressing the interest rate in isolation by 1-2 basis points, which is a typical movement in the interest rate.
 - Employee B recommends stressing the interest rate by 1000-2000 basis points while also stressing the tax rate on bonds.
 - Employee C recommends stress testing based on the market conditions during the 2020 pandemic.

Critique each of the employees' recommendations.

Commentary on Question:

Many candidates did not receive full credit on this part because the critique did not include enough information or both positive and negative aspects of the employee's recommendation.

Employee A

- The event is given in isolation, which is a scenario, not a stress test
- The event is not extreme

Employee B

- The event is so extreme, it may not be plausible
- Perhaps other factors should also be stressed

Employee C

- Historical scenarios are commonly used
- Constrained by the severity of the 2020 pandemic. Are other extreme but plausible events worthy of testing?
- (c)
- (i) Critique the firm's current use of stress testing.
- (ii) Recommend three changes the firm can implement to more effectively utilize stress testing.

Commentary on Question:

Most candidates performed well on this part, providing a valid critique of the firm's current practices and offering at least three recommendations.

(i)

- Positive that objectives were shared with the Board.
- The stress tests should inform business decisions as well as be communicated to the Board
- Regular review of the models and results is positive
- Are the events plausible and extreme or implausible? Perhaps the employees' bias is preventing them from seeing the value in the stress testing

(ii)

- Better promote the formally adopted objectives and share results widely.
- Use the stress testing to inform business decisions as well as satisfy regulatory requirements
- Educate employees on value of stress testing and seek input on plausible, yet extreme scenarios to be included
- Be sure that data and IT systems are sufficient for the stress testing
- Continue to regularly review and challenge the results

2. Understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

- (2a) Explain the advantages and limitations of different risk metrics.
- (2b) Explain how different approaches and tests form a set of complementary investment risk metrics.
- (2c) Analyze and evaluate the use and misuse of correlation, integrated risk distributions and copulas.
- (2f) Apply different techniques of assessing rare event risks, including stress testing and scenario analysis.

Sources:

QFII-123-21: IAA Note on Stress Testing and Scenario Analysis (pp. 1-6, 14-17, 19-26& 32-39)

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 3: Risk Measures

Investment Risk Management, Baker & Filbeck, Ch. 8: Liquidity Risk

Commentary on Question:

Generally, candidates did not perform as expected on the exam and did not understand nor attempt the questions. Most candidates struggled in giving justification to support their insights.

Solution:

(a) Calculate the current Liquidity Risk for each company on Friday using:

- (i) Liquidity Ratio (LR)
- (ii) Hui and Heubel Liquidity Index (HH)

Commentary on Question:

Generally, candidates did not perform as expected on part (i) of the question. Candidates were asked to calculate the liquidity ratio as of Friday and most calculated the entire week

(a).(i)

 $\begin{aligned} & \mathsf{LR}_{it} = \mathsf{P}_{it} * \mathsf{V}_{it} / | \mathsf{P}_{it} - \mathsf{P}_{i(t-1)} | \\ & \mathsf{LR}_{X5} = 95 * 30 \text{MM} / | 95 - 115 | = 142.5 \text{MM} \\ & \mathsf{LR}_{Y5} = 42 * 5.6 \text{MM} / | 42 - 42.8 | = 294 \text{MM} \\ & \mathsf{LR}_{Z5} = 5 * 5.88 \text{MM} / | 5 - 4.9 | = 294 \text{MM} \end{aligned}$

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(a).(ii)

\begin{split} HH_i &= (P_{max} - P_{min}) / (P_{min} * P_{avg} * V_{total} / N) \\ HH_X &= (115 - 95) / (95 * 105 * 95.5 MM / 1000 MM) = 2.1\% \\ HH_Y &= (45 - 41) / (41 * 42.96 * 32.56 MM / 200 MM) = 1.4\% \\ HH_Z &= (5.1 - 4.6) / (4.6 * 4.92 * 37.24 MM / 100 MM) = 5.93\% \end{split}
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(b)

- (i) (0.5 points) Determine the ranking of each fund, from the highest liquidity to the lowest liquidity, using the values calculated in part (a).
- (ii) (*1 point*) Justify your ranking.

Commentary on Question:

Most candidates did as expected on the question in ranking and comparing the liquidity of each fund. The candidates that performed above average ranked and justified the funds based on both metrics

This question is meant to use both LR and HH measures to draw the conclusion. Fund Y and Z have the same LR, so HH should be used to rank between Y and Z.

Full Solution: Ranking from the highest liquidity to the lowest liquidity Fund Y > Fund Z > Fund X Justify

- 1. A higher liquidity ratio means that more of the fund value can be sold without affecting the price and thus more liquid at the current price. Fund X has a drastically lower LR than that of Fund Y and Z, which are same in value, leads to X having the lowest liquidity of the three funds.
- 2. A lower HH Index implies a higher liquidity of the fund. This would imply that Fund Y is more liquid than fund Z.

Alternative Solution: get only 1 point with appropriate justification using only 1 measure.

Fund Y > Fund X > Fund Z. Justify: based on HH only

- (c) Propose a rebalancing portfolio trade to have an expected weighted average liquidity ratio of 250 million based on the Friday close price, subject to the following constraints:
 - (i) No shorted shares are allowed
 - (ii) No remaining cash on hand is allowed
 - (iii) Fractional shares are allowed

(Goal Seek functionality is available within Excel under the Data tab, What-If Analysis)

Commentary on Question:

Candidates did below average on this question with most not attempting the question

Multiple designs are possible to answer the question. The target was to allocate shares from Y or Z to fund X to reach the target Liquidity ratio

Design 1 (move from Y to X): Sell 192,912 shares of Fund Y and buy 85,287 shares of Fund X

Design 2 (move from Z to X): Sell 1,620,462 shares of Fund Y and buy 85,287 shares of Fund X

Design 3 (move from Y and Z to X) Sell shared from Fund Y and X so that 85,287 shares of Fund X can be purchased. As long as the shares are calculated correctly.

(d) Describe the key considerations of using historical and synthetic scenarios to understand the liquidity risk of the portfolio.

Commentary on Question:

Candidates performed below average on the question by only providing 1 or 2 considerations for each scenario

Historical Scenario

- 1. External market forces are different based on the timing.
 - a. 1980s energy shortage applied to the rate changes during the period
 - b. Current environment is due to different forces: war and pandemic
- 2. Blocks of business for most companies was focused on Whole life and term instead of IUL.

- 3. Type of funds invested in during that time and now could have large deviations
- 4. Transaction price, price transparency, and speed of transactions have increased and can allow for faster response to changes in interest rates, inflation, and price changes
- 5. Interest rates were at near zero values for an extended period before the current increase while during the historical time frame they were stable at drastically higher. The overall percentage increase in interest rates is less for the historical period than the current period
- 6. Financial markets and regulatory frameworks have evolved, and the impact can be larger or smaller than previously experienced
- 7. The historical scenario can be used as a guideline for developing scenarios with a modification for credibility
- 8. Historical scenarios can be easily understood and communicated to stakeholders

Synthetic Scenario

- 1. Requires more assumptions than a historical scenario. They are subject to greater challenges and are more difficult to communicate to stakeholders
- 2. Using a singular set of deterministically modelled rates can lead a good luck scenario and the risk metrics be inadequately calculated This will allow the company to test new product designs or changes to the current product to test mitigation techniques for the increase surrender rates
- (e) Recommend a modification to improve each scenario.

Commentary on Question:

Candidates performed below average on the question by not providing justification for their recommended modification to the scenario

Example 1 Solution

The company should use a Historical Scenarios. The period of rates to use should be reduced and focused on years 1975 to 1980 years. The model should then use a mean reversion process to align to future expectations of rates during the current unforeseen actuals.

Example 2 Solution

The company should use the Synthetic Scenarios. A change to make to the process would be to stochastically model the interest and inflation rates to better calibrate the model results to the dynamic nature of the financial markets.

3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

(3a) Identify and describe various approaches for managing portfolio risks including VaR/ ES methods, risk budgeting, position limits, etc.

Sources:

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 3: Risk Measures

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 9: Short Term Portfolio Risk

Commentary on Question:

This question tested candidates' knowledge of common risk measures and their ability to apply their knowledge in an investment portfolio scenario.

Solution:

(a)

- (i) Define Value at Risk (VaR) and Expected Shortfall (ES).
- (ii) Identify two advantages of ES over VaR.

Commentary on Question:

For this section, candidates performed better than expected, with most receiving at least partial credit. While most were able to define value at risk appropriately, a smaller number of candidates were also able to define expected shortfall sufficiently for full credit.

(i) VaR: For a given confidence level a, the a-VaR represents the loss that with probability a will not be exceeded.

Expected shortfall (ES), ES takes the average loss, given the loss lies in the right tail of the distribution.

ES accounts not just for the point estimate indicated by VaR, but also the tail.
 ES is coherent while VaR is not.

(b) Describe two advantages and two disadvantages of using this assumption.

Commentary on Question:

Candidates performed as expected for this section. Generally, candidates received more credit for stating advantages, but tended to earn less than full credit for disadvantages, due to focusing on normal distribution disadvantages rather than the disadvantages of applying this distribution to calculate VaR and ES.

Advantages: closed form; tractable Disadvantages: Not realistic over long periods; may not match historical data

- (c)
- (i) Calculate the value of the portfolio at time 0.
- (ii) Calculate the 1-day 99% ES of the portfolio using the Delta-Normal method.

Commentary on Question:

For this section, candidates performed as expected. Many were able to correctly calculate the portfolio value but fewer accurately calculated the 99% expected shortfall and so only received partial credit for ii).

(i) Value of stocks at time 0 = 5*150+8*125 = 1750Value of each call at time 1 = 20.74 (see below for details) Sum = portfolio value = 1750 + 20.74 = 1957.43

Value of each call at time 1 details:

Ka	=	130	\mathbf{d}_1	=	2.0122
r	=	0.02	d_2	=	1.9372
t	=	0.25	N(d ₁)	=	0.9779
S	=	150	N(d ₂)	=	0.9736
volatility	=	0.15			

(ii) 1-day 99% ES of the portfolio = 61.49 (see below for details)

Delta of calls on stock A = Number of calls x $N(d_1) = 9.779$ Delta of portf for stock A = = Number of units of A + (Delta of calls on stock A) = 14.779 Delta of calls on stock B = Number of units of B = 8

Note it's given that stocks A and B are uncorrelated, so the formula on page 309 in Source 2 reduces to the sum of the two terms below (i.e., the term with the correlation becomes 0).

(Delta of portf for stock A x S_A x vol)^2 = (14.779*150*0.15)^2 = 110574.45

(Delta of portf for stock B x S_B x vol) $^2 = (8*0.15*125)^2 = 22500$ Variance of loss of portfolio = 133074.45, and s.d. of loss of portfolio = 364.794

Applying Excel functions NORM.DIST and NORM.INV, NORM.DIST (NORM.INV (0.99,0,1),0,1,FALSE) / 0.01*(1/250)^0.5*364.794 = 61.49

(d) Your manager wants to calculate the 10-day ES for a different portfolio with derivatives and 10 underlying assets. Your colleague suggests that the Delta-Gamma-Normal method would be appropriate in this case.

Critique your colleague's suggestion.

Commentary on Question:

Candidates performed poorly on this section relative to the rest of the question. Although several candidates understood the computational strain with the number of assets involved, only a small number were able to sufficiently critique the Delta-Gamma-Normal method in this case.

Gamma adjustment may somewhat improve accuracy but the method is still insufficient given the number of assets and horizon Normal distribution of returns is not accurate in the tails Estimating correlations with more than one asset may be problematic given historical returns are not consistent with a multivariate normal assumption 2nd derivative or similar related to 2nd order of Taylor series not appropriately captured.

- 2. Understand and be able to apply different approaches to measuring and assessing risk exposures.
- 3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

- (2c) Analyze and evaluate the use and misuse of correlation, integrated risk distributions and copulas.
- (3a) Identify and describe various approaches for managing portfolio risks including VaR/ ES methods, risk budgeting, position limits, etc.

Sources:

Introduction to Risk Parity and Risk Budgeting, T. Roncalli. Ch. 2: Risk Budgeting Approach

Commentary on Question:

This question was designed to test a candidate's understanding of risk budgeting from both a quantitative and qualitative perspective.

Solution:

(a) Calculate the volatility of the portfolio.

Commentary on Question:

Most candidates did well on this part. The most common mistakes were either due to minor typos in the Excel formula or misunderstandings on how matrix calculations are handled in Excel.

This is a special case of formula 2.1, with c=1, and drift = 0.

 $SD_c(x) = -x^{\top} \mu + c \cdot \sqrt{x^{\top} \Sigma x}$ (2.1)

The covariance matrix can generally be defined as $\sum_{i,j} \rho_{i,j} \sigma_i \sigma_j$

i.e.

/ 0.09	0.03	-0.018	
0.03	0.04	-0.012	
\-0.018	-0.012	0.0144/	

It follows that the variance of the portfolio is

$$\sigma^{2} = 0.2^{2} \times 0.09 + 0.3^{2} \times 0.04 + 0.5^{2} \times 0.0144 + 2 \times 0.2 \times 0.3 \times 0.03 + 2 \times 0.2 \times 0.5 \times -0.018 + 2 \times 0.3 \times 0.5 \times -0.012 = 0.72\%$$

The standard deviation is then $\sqrt{0.72\%} = 8.5\%$

(b) Calculate the marginal volatility and risk contribution of each asset in the portfolio.

Commentary on Question:

Candidates generally either did very well or very poorly on this question. Candidates who made minor calculation errors in part a) were not penalized for incorrect answers in part b) so long as the proper formulas were applied.

For risk contributions, apply formula 2.10

$$\mathcal{RC}_i = x_i \cdot \left(-\mu_i + c \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \right)$$
(2.10)

Note that the textbook describes the portion in brackets as the vector of marginal volatilities, e.g. bottom of page 79:

The previous formulas can be extended to the case n > 2. Because $\sigma(x) = \sqrt{x^{\top} \Sigma x}$, it follows that the vector of marginal volatilities is:

$$\frac{\partial \sigma (x)}{\partial x} = \frac{1}{2} (x^{\top} \Sigma x)^{-1} (2\Sigma x)$$
$$= \frac{\Sigma x}{\sqrt{x^{\top} \Sigma x}}$$

Marginal risk is calculated as follows:

$$\frac{1}{8.5\%} \begin{pmatrix} 0.09 & 0.03 & -0.018 \\ 0.03 & 0.04 & -0.012 \\ -0.018 & -0.012 & 0.0144 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.3 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0.212 \\ 0.141 \\ 0.000 \end{pmatrix}$$

Risk contribution is obtained by multiplying the weights by the marginal volatilities:

$$\begin{pmatrix} 0.20\\ 0.3\\ 0.5 \end{pmatrix} * \begin{pmatrix} 0.212\\ 0.141\\ 0.000 \end{pmatrix} = \begin{pmatrix} 0.042\\ 0.042\\ 0.000 \end{pmatrix}$$

(c) Describe two ways of interpreting risk contributions.

Commentary on Question:

Many candidates did not perform well on this part and only provided the risk contribution interpretation from a marginal risk perspective. Full marks were only awarded for including quantitative and qualitative aspects in each description.

(i) The marginal analysis of risk. If we increase the weight on an asset by a small amount h, then the risk measure increases by h x the marginal risk (also can use formula 2.13, which is the same thing; however the formula alone isn't sufficient for full credit.)

(ii) The performance analysis. Define a specific utility function

$$U(x) = \mathbb{E}(R(x)) - \frac{1}{2}\phi R(x)$$

Then at the optimum portfolio, performance contributions (weighted performance over total performance) for each asset class are equal to the risk contributions (weighted risk contribution over total risk). (also see Equation 2.14; again the formula alone is not sufficient for full credit)

(d)

- (i) Assess whether your colleague's suggested portfolio meets the risk budget constraints.
- (ii) Assess whether your colleague's suggested portfolio is the optimal portfolio subject to these constraints.

Commentary on Question:

Many candidates did not perform well on this part, perhaps not understanding how to relate risk contribution with the risk budget framework. There were multiple ways to demonstrate that the colleague's suggested portfolio was not optimal, and marks were awarded for correct reasoning.

Part (i): Meets Constraint (Calculations are analogous to those in part a, using the new portfolio weights.)

Part (ii): Not optimal.

The negative marginal volatility for asset three indicates this cannot be the optimal portfolio, because the risk budget portfolio is the unique portfolio that the following relationship:

$$\begin{cases} \mathcal{RC}_{i} = x_{i} \cdot \partial_{x_{i}} \mathcal{R}(x) = b_{i} & \text{if } i \notin \mathcal{N} \\ \begin{cases} x_{i} = 0 \text{ and } \partial_{x_{i}} \mathcal{R}(x) > 0 & (i) \\ \text{or} & \text{if } i \in \mathcal{N} \end{cases} \\ x_{i} > 0 \text{ and } \partial_{x_{i}} \mathcal{R}(x) = 0 & (ii) \end{cases}$$
(2.32)

That is, for assets with 0 budget, either the weights are 0 are the risk contribution is positive; or, the risk contribution is 0 and the weights are positive. The colleague's suggested portfolio does not meet these requirements.

3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

(3a) Identify and describe various approaches for managing portfolio risks including VaR/ ES methods, risk budgeting, position limits, etc.

Sources:

QFIP-128-23: Liquidity Risk Management: Best Risk Management Practices CRO Forum, October 2008

Commentary on Question:

The question is intended to test the candidates' knowledge and understanding of liquidity risk management and how to apply the information to practical applications. Most candidates are doing great on the first section as they are more generic questions related to liquidity risk. Very few candidates could get full marks on the last section as it is related to practical applications.

Solution:

(a)

- (i) Define liquidity risk.
- (ii) Identify key types of cash sources and cash needs for life insurance companies.

Commentary on Question:

Most candidates are doing great on the first section as they are more generic questions related to liquidity risk. Some candidates are confused with cash and asset when we deal with liquidity risk.

Liquidity risk can be defined as the risk that cash sources are insufficient to meet cash needs under either current conditions or possible future environments.

Cash sources reflect cash inflows from insurance products (premiums), asset cash flows, sales of assets. Cash needs include product cash outflows (claims, withdrawals, commissions), operating cash outflows (including overhead), dividend payments, and contingent cash needs arising from environment driven factors.

(b) Assess whether the MRR is met.

Commentary on Question:

The majority of candidates who tried this section got this section right. Some candidates are confused with cash and asset.

Cash sources from bond = 100000*5%=5000 Dividend from equity = 20000*10%=2000

Current Cash sources = 1000+5000+2000=8000 Current cash needs = 3500+1500=5000

Current liquidity coverage ratio = 8000/5000 = 1.6Current liquidity coverage ratio is slightly higher than the company current minimum required ratio

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(c)
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- (i) (0.5 points) Evaluate your manager's assertion.
- (ii) (0.5 points) Explain your conclusion.

Commentary on Question:

This section is mainly intended to test the capital and liquidity risk.

Current cash needs = 3500+1500+500=5500 Current liquidity coverage ratio = 8000/5500 =1.46 Current liquidity coverage ratio is slightly lower than the company current minimum required ratio The new product should not be launched right away

Disagree.

Liquidity risk is a risk to be managed before during and after a liquidity stress event.

It is inappropriate to expect any amount of required capital to protect against insolvency arising from the liquidity risk

(d) Propose four improvements to the liquidity standard.

Commentary on Question:

No candidates got full marks on this section. This section requires candidates some knowledge on liquidity risk management.

- Besides products & asset needs, Include additional cash needs including those from shareholders (dividends, buybacks, etc.) and operations (overhead)
- Additional time horizon such as 3 month, 6 month, 1 year, 2 year and so on should be included / Project the cash source estimates in both amount and timing.
- Consider the amount of cash that can be obtained through different funding channels such as secured funding
- Consider proceeds from sales of assets at various horizons
- Should examine cash flow behavior under a variety of different scenarios.

For example:

- Increasing interest rates as well as surrender options grand to the policyholder that cause additional surrender/withdrawal
- The new priced products attract the existing clients which the company will experience increasing withdrawal rates.
- The stress tests that combine product, asset, and market scenarios should be considered as well.

- 1. The candidate will understand and be able to identify and describe types of risk present in investment management.
- 4. Understand and be able to apply different approaches to mitigate investment risks using derivatives.

Learning Outcomes:

- (1c) Identify behavioral risks and explain how they factor into investment management.
- (4c) Understand credit derivatives and use them to mitigate credit risk.
- (4d) Understand CDS valuations and marking-to-market counterparty risk in credit derivatives.

Sources:

Handbook of Fixed Income Securities, Fabozzi, Frank J., 9th Edition, 2021. Ch. 68 and Ch. 69.

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)

- (i) Explain what "buying the index" means.
- (ii) Calculate the net cash flow of ABC Life Insurance on September 20, 2021 with respect to the CDS index, assuming no default since the inception of the index.

Commentary on Question:

Many candidates demonstrated strong performance in question a(i); however, some candidates mistakenly wrote about the definition of a CDS index, which was not awarded any credit.

Many candidates were unable to correctly perform the calculations required in *question a(ii)*.

- (i) "Buying the index" means assuming the credit risk by selling protection and receiving the coupon.
- (ii) Fixed coupon is 1% and it is quarterly payment.
 Coupon payment = 1% / 4 * 1million = \$2,500
 Company is receiving the cash flow (net cash flow is positive)

- (b) Explain whether the following values change due to the credit deterioration.
 - (i) Coupon set on the issue date of the CDS index
 - (ii) Upfront value of this CDS index series if you were to enter a position today

Commentary on Question:

Candidates did well in parts b(i) and (ii); however, in b(ii), some candidates only mentioned that the upfront value would change without specifying the direction of the change. In such cases, the candidates did not receive full credit.

- (i) CDS index coupon payment is fixed. It does not change with credit quality of the index.
- (ii) Changes in the market perceived credit quality of the index causes the upfront value to change. The protection buyer will have to pay more money upfront in order to offset the fact that the 100bps coupon is not high enough to compensate the protection seller for the market perceived credit risk.

(c)

- (i) List the possible "credit events" that will trigger the payment of the CDS protection.
- (ii) List the steps that the existing index series and the reference credit will need to go through after a credit event.

Commentary on Question:

Candidates performed well in c(i). For c(ii), candidates would receive full credits if they provided two of the steps outlined in the solution.

- (i) Examples of credit events include bankruptcy, failure to pay, obligation acceleration, restructuring.
- (ii) The credit is immediately removed from the index series and the new index without the credit is thereafter assigned a new version number to distinguish it from earlier versions of the index series.
 The reference credit that was removed then becomes a standalone CDS position which then take part in the standard auction procedure.
 The surviving notional of the index is reduced in proportion to the number of credits remaining. The premium leg of a CDS index is the payment of the coupon to the buyer of the index and the actual payment amounts are determined by multiplying the index fixed coupon by the day count fraction that has elapsed since the previous coupon date and then by the surviving notional of the index reference portfolio.

- (d) Calculate the net cash flow of ABC Life Insurance with respect to the CDS index on:
 - (i) November 1, 2021
 - (ii) September 20, 2022

Commentary on Question:

Most candidates were unable to correctly perform the calculations required in this part. Many candidates did not attempt this part.

- (i) Notional of the stand alone CDS contract = 1 million / 125 = \$8000\$8000 * (1 - 40%) = \$4,800The company (protection seller) pays the protection buyer \$4800
- (ii) Company receives coupon = 1% / 4 * 1 million * 124 / 125 = \$2,480

4. Understand and be able to apply different approaches to mitigate investment risks using derivatives.

Learning Outcomes:

- (4a) Explain and implement techniques used to mitigate market risks.
- (4b) Understand interest rate derivatives and use them to mitigate interest rate risk.

Sources:

Fixed Income Securities: Valuation, Risk, and Risk Management, Pietro Veronesi, Chapter 5 and 6

Commentary on Question:

This question tests candidates' knowledge on using various financial instruments such as forward rate agreement (FRA) to mitigate market risks. It also tests candidates' ability to compute spot rates and the value of the FRA at various durations, as well as their ability to correctly apply the pull call parity in a particular scenario. To receive full credits, candidates are expected to provide support (whether in words or in mathematical terms) for the final answer.

Solution:

(a) Explain whether the price of a futures contract may be different from the price of an otherwise equivalent forward contract.

Commentary on Question:

Candidates generally did poorly on this question. Very few candidates recognized that the price of a futures contract is greater than the price of an otherwise equivalent forward contract, when the interest rates are positively correlated with futures prices. Most candidates received partial credits for stating that the daily settlement of futures could result in differences in the price, if all else equal.

Price of a futures contract is greater than the price of an otherwise equivalent forward contract.

If interest rates are positively correlated with futures prices, interest earned on cash from daily settlement gains on futures contract will be greater than the opportunity cost of interest on daily settlement losses, and a futures contract will have a higher price than an otherwise equivalent forward contract that does not feature daily settlement.

- (b)
- (i) Determine the semi-annually compounded forward rate of the contract.
- (ii) Calculate the value of the FRA at inception.

Commentary on Question:

Candidates overall did well on this question. For part (ii), most candidates were able to receive full credits by explaining that the value of the forward at inception is zero as no money changes hands.

- (i) F(0,0.5,1) = Z(0,1)/Z(0,0.5) = 93.51/96.79 = 0.96611f= 2*(1/F(0,0.5,1)-1) = 7.02%
- (ii) The value of the forward at inception is zero as no money changes hands.
- (c)
- (i) Calculate the value of the FRA on t_1 , July 1, 2000.
- (ii) Calculate the current semi-annually compounded spot interest rate.
- (iii) Determine the net amount to be paid at the settlement of the FRA on t_2 , January 1, 2001 and which party will be responsible for it.

Commentary on Question:

Candidates generally did well in part (ii), but poorly in part (i) and part (iii) of this question. In part (ii), most candidates were able to calculate the spot rate correctly, but some candidates calculated the continuously compounded rate instead of the semi-annually compounded rate and therefore, receive partial credits.

(i) At inception, M = Z(0,0.5)/Z(0,1) = 96.79/93.51 = 1.0351 T-bills maturing in 1 year.

FRA(0.5) = 100(M*Z(0.5,1) - Z(0.5,0.5)) = 100*(1.0351*0.9692 - 1) = \$0.3219

- (ii) r(0, 0.5) = 2*(1/Z(0, 0.5) 1) = 6.36%
- (iii) f2(0, 0.5, 1) is from b,i) r2(0.5,1) is from c,ii)

Net payment of the firm at T2 = N/2 * [f2(0, 0.5, 1) - r2(0.5, 1)] = \$100 / 2 * (7.02% - 6.36%) = \$0.3298million (paid by the bank to the firm)

- (d)
- (i) Demonstrate that the securities are priced incorrectly.
- (ii) Recommend a strategy to take advantage of the arbitrage opportunity.
- (iii) Determine the net cashflow of d (ii).

Commentary on Question:

Candidates generally performed poorly on this question. Many candidates were able to receive full credits or partial credits in part (i) by demonstrating the correct formula of the pull-call parity. Few candidates correctly stated the strategy and the net cashflow from the arbitrage opportunity.

(i) The securities are not correctly priced, as the put-call parity is violated.

Pfwd = 100*(0.9545/0.9692) = 98.4833Call (K) = Put (K) + Z(0, 0.05)* (Pfwd - K), where Z(0, 0.05)* (Pfwd - K) = 0.9692*(98.4833 - 99.12) = -0.6171 Call from Put-Call Parity = 0.1044 - 0.6171 = \$-0.5127

- (ii) Strategy should be to long call, short put, and short forward. This will give a positive cashflow of \$0.8061 at no risk.
- (iii) \$0.2934 (-\$0.5127) = \$0.8061

5. The candidate will understand the importance of risk culture and governance.

Learning Outcomes:

- (5a) Explain the importance of risk culture and ethics in an investment firm.
- (5b) Identify sources of unethical conduct and explain the role of a fiduciary.
- (5d) Explain how governance may be structured to gain competitive advantages and efficiencies.

Sources:

Investment Ethics, Peck, Sarah, 2011 Ch. 1-2

Commentary on Question:

This question is to test candidate's knowledge and understanding of Risk Culture and Governance and identification of risk types. The cognitive skill levels are retrieval and comprehension.

Solution:

(a) Describe each of the fundamental principles of investment ethics for your audience.

Commentary on Question:

Part A: It appeared that candidates understood the question and had studies this part of the reading and memorized the list, and the list was retrieved successfully for almost all candidates.

- 1. Ethical understanding Not knowingly engage in investment transaction that either you or other do not sufficiently understand.
- Ethical use of information To ensure you and others have access to relevant information and do not misuse or distort information in investment transactions.
- 3. Responsible investing Not knowingly make recommendation that support activities that harm others.
- 4. Trust and Fairness Not abuse the trust in you and treat others fairly
- (b)
- (i) List four considerations when assessing ABC's risk tolerance and thus the type of investments that would be suitable.
- (ii) Determine any unethical behaviors in this scenario.
- (iii) Recommend any corrective actions.

Commentary on Question:

Part B(i): Although the way the question was written is similar to Part A, candidates appeared to misinterpret the question or did not pay attention to the reading, many candidates answered it using common sense rather than providing listing from textbook, therefore only partial credits or no credit were assigned to those candidates. Only small number of candidates received full credit.

Part B(ii): It is a straight-forward question; more candidates answered the question correctly. Majority candidates captured the IPS limit part, some missed the commission part.

Part B(iii): It is a better question in term of differentiating candidate's knowledge and understanding, only about half of candidates answered it correctly.

- (i) Considerations:
 - o Holding horizon
 - o Liquidity
 - Income needs
 - Tax considerations
- (ii) Investment policy statement needs to be followed to make right investment decisions, deviation from IPS (exceeding 5% limit for alternatives) can be legal grounds for contract violation, even if temporary. Allocating to funds that have higher commissions could violate Trust and Fairness if done for selfish purposes.
- (iii) If higher yield is indeed needed, then the IPS should be amended, in consultation with ABC so that they fully understand it, to reflect the new goals and thus loosen certain constraints, such as the limit on alternatives.
- (c)
- (i) Identify the relevant regulations or laws that management needs to consider in its action plans.
- (ii) Assess the appropriateness of each action plan.

Commentary on Question:

Part C(i): only a few candidates answered the question correctly. Either they are not familiar with the reading or have no experience in practice, there is a possibility that the way the question was written won't be able to lead them to the intended answer.

Part C(ii): This is a better question as it asks candidates to judge and provide reasons to support the conclusion. This helps graders to know the thoughts behind the judgement and the candidates' understanding of the situation and solution. Majority received partial credits.

Part (i) SOX.

Part (ii)

SOX Act of 2002 applies to these situations.

1) Primary auditors are required to rotate every five years. There is nothing wrong for the existing auditing firm to stay.

2) An accounting firm cannot offer non-audit service to a firm they are also auditing.

3) SOX requires the auditing firm to identify any accounting discrepancies that are used to alter the financial statement within GAAP.

4) Top executives cannot have been on the auditing committee for at least one year before employment

5. The candidate will understand the importance of risk culture and governance.

Learning Outcomes:

- (5c) Compare the interests of key stakeholders and describe governance mechanisms that attempt to address conflicts.
- (5d) Explain how governance may be structured to gain competitive advantages and efficiencies.

Sources:

QFII-101-14: Chapter 11, pp. 378-384 only, up to "Agency Theory" (including example 11.1 "Strategy in Action")

Investment Ethics, Peck, Sarah, 2011, Ch 7

Commentary on Question:

The candidates had to analyse and critique some governance issues resulting from a strategic change in the future operations of a company. It includes its impacts on different stakeholders, the evaluation of the compensation structure of executives, and the assessment of the effectiveness of the board in the management of the company.

The general results are satisfactory, in particular in part c).

However, for each item of part a) and b), many candidates limited themselves to a single argument while they could have explained more.

Also, in part b), they had more difficulties to recommend and justify the use of stocks as an alternative type of compensation, in particular in using stock appreciation right (phantom stock), and restricted stock.

Solution:

- (a) Describe the interests of each of the following stakeholders and explain how this strategy will or will not serve their interests:
 - (i) stockholders
 - (ii) employees
 - (iii) customers

(i) **Stockholders** perspective

Stockholders provide the company with risk capital and in exchange expect management to maximize the return on their investment through higher dividends and capital appreciation in the MV of a share.

If ROIC downfall is expected to be short-term for a reasonable investment for the long-term profit and profit growth, the stockholders will react positively.

If the strategy is viewed as too much emphasis on the future profit and profit growth over short-term profit, it can make the company less attractive to shareholders.

(ii) **Employee** perspective

Employees provide labor and skills and in exchange expect commensurate income, job satisfaction, job security, and good working conditions.

The strategy will negatively affect the employees who will be laid off from the factories. Other employees could view the strategy as a threat to their job security in the future.

On the other hand, some employees could see this opportunity for higher salaries and benefits if the strategy succeeds and the company's profit grows.

(iii) **Customers** perspective

Customers provide with the company's revenues and in exchange want high-quality, reliable products that represent value for money.

The customers sensitive to price will like the lower price of the product resulting from the company's cost-cutting measures.

The environmentally conscious customers will like that the product is made in an environment-friendly way.

- (b)
- (i) Critique the compensation stucture and its governance.
- (ii) Recommend two other types of compensation that could be appropriate.

(i)

Executive should be compensated to drive performance, sensitive to changes in stock prices so their incentives are aligned with shareholders' incentives. Because the cash bonus and salary is not sensitive to the stock prices, it disincentivizes the executives to make decisions benefiting the shareholders.

While the stock price can be influenced by many factors outside of management's control, managers have greater control over accounting performance. Because managers have control over accounting methods used to report the performance, they can also manipulate them.

The compensation committee should oversee the effectiveness of compensation for the interest of the shareholders. Because the committee keeps approving a compensation package that does not provide the right incentives, it could be a sign of dysfunction (governance failure).

(ii)

Make the bonuses part of long-term incentive plans (LTIPs): In LTIP, accounting performance is taken into consideration not only for the current year but for a longer time period, say, five years. This disincentivizes managers to manipulate accruals to improve the current year's performance at the expense of later years.

Restricted stock: The transfer of ownership is resticted for one to five years. To make money on these shares, managers better be sure the decisions they make will improve long-term share value.

Stock appreciation right (phantom stock): the firm agrees to pay out the difference in share value that has accumulated over a set period of time. After they pay out, managers no longer have an ownership interest in the firm. Managers have incentive to increase the longer-term stock price.

Stock options: it is the right to buy a stock at a prespecified price, the exercise price, at a prespecified date, the exercise or maturity date, in the future. The grant usually is made annually, and the exercise price is set based on the stock price on the grant date. Managers have incentive to work hard to increase the stock price beyond the grant price.

- (c) You have gathered the following observations regarding the PQR board:
 - 1) There are nine directors. All are nonmanagement directors, except for the chairman, who is the CEO of the company.
 - 2) There are three committees: Nominating, Compensation, and Audit. Each has three gray directors. The chairman does not serve on any committee.
 - 3) The term of each director is three years, and only a third of them are up for election in any given year.
 - 4) Directors in the Nominating and Compensation committees have served at least two terms.
 - 5) Five of the directors are currently serving on several other companies' boards.
 - 6) The compensation committee met 30 times before the cash bonus approval, while other board committees met 6 times on average.

Assess the effectiveness of the board.

- 1) The CEO is also the chairman of the board and thus can exert undue influence over board decisions.
- 2) The board has key committees but including gray directors who have relationships with the company on these committees could compromise their independence.
- 3) Board is staggered therefore it's hard to take over the board. This could increase the risk of board and CEO entrenchment.
- 4) Many nonmanagement directors have lengthy tenures, thus compromising their independence from the CEO.
- 5) Directors are too busy, serving too many directorships. They may be overcommitted and stretched too thin, making poor monitors.
- 6) The compensation committee met too many times, more than any other board committee, which could be warning sign of the board misaligned with the interests of the stockholders, especially when the compensation of the executives is unreasonable