SOCIETY OF ACTUARIES

Early Detection of Insurer Insolvency

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Executive Summary

Early detection and assessment of insurer's insolvency risk are critically important to regulators, insurers, policyholders, and investors for fiduciary assessment of insurer's financial stability and for protection of potential financial loss associated with insurer's insolvency. This proposed study aims to develop a market-based insolvency prediction model to detect financially distressed insurers at an early stage with the information content of higher moments from the market. This information can further complement the current regulatory capital requirement and credit risk assessment through ratings. More specifically, we significantly extend the traditional Merton's model and propose the use of the Gram-Charlier series expansion incorporating non-normal skewness and excess kurtosis. Since the market timely incorporates information relevant to the pricing of securities on each trading day, our proposed market-based model is able to extract useful information from financial prices for early detection of insurer's insolvency risk on a daily basis. The proposed model's results will provide new tools and additional insights for early detection of insurer insolvency, which can help to minimize the potential costs associated with the financial distress of insurance firms.

Section 1. Introduction and Project Motivation

The insurance industry is built on an important presumption of trust and that the insurance contracts will be fulfilled, and eligible claims paid when the insured event confidence trust occurs, which could be long in the future. An insurer experiences an insolvency risk when it has difficulty to cover its financial obligations. Insurer insolvency exposes the policyholders, beneficiaries, and investors to an unexpected and substantial financial loss and considerable personal and economic cost.

Given its critical role in the insurance industry, the detection and prediction of insurers' financial distress are critically important to regulators, insurers, policyholders, and other stakeholders. In addition, effective detection and prediction of insurer insolvency are critical for investor's asset pricing, credit risk assessment of loan portfolios, and the valuation of other financial products that are exposed to corporate default. An effective monitoring of insolvency risk should focus on the early detection of financially weak insurers, which can help regulators to intervene as early as possible and to minimize the potential costs associated with the financial distress of insurance firms.

As National Association of Insurance Commissioners (NAIC) (March 2020) recently states, "one of the primary objectives of insurance regulators is to identify, as early as possible, insurance companies that are showing signs of becoming financially troubled so corrective action can be taken to protect policyholders, claimants, and creditors from financial loss."[1] To answer this call, this study aims to develop a market-based insolvency prediction model to detect financially distressed insurers at an early stage. It is worth noting that, while we focus on US firms mainly North American insurers in this paper, some references to the Canadian framework are made and the proposed methodology could be naturally adopted globally wherever that a developed equity market for insurer exists.

[1] https://content.naic.org/cipr_topics/topic_troubled_companies.htm

Section 2. Literature Review

2.1 LITERATURE ON PREDICTING INSURER'S INSOLVENCY

Previous researchers have attempted to predict the solvency status of insurers using a variety of methodologies. Much of the earlier literature focused on estimating default probabilities using financial accounting data (e.g. Pinches and Trieschmann 1974; BarNiv and McDonald 1992; Ambrose and Carroll 1994; Rauch and Wende 2015).

Cummins, Harrington, and Klein (1995) analyze the accuracy of the NAIC risk-based capital measures and show that the risk-based capital alone has very low predictive accuracy and suggest that it has to be combined with other information to facilitate prompt corrective action and reduce insolvency costs. As an anecdotal evidence, when AIG's P&C subsidiaries were on the verge of bankruptcy at the end of the third quarter of 2008, they actually appeared reasonably well capitalized with a risk-based capital (RBC) ratio of 452 percent, well above the 200 percent regulatory criterion for company action to improve financial strength (Schimek, 2008).

The AIG crisis in 2008 was mainly due to its credit default swap (CDS) portfolio written by AIG Financial Products (AIGFP), a non-insurance entity which was not subject to insurance regulation. When the underlying securities of the CDS were downgraded in rating and their values declined, AIGFP had to post large amounts of collateral. AIG's life insurance subsidiaries also ran into major problem with its securities lending program when borrowers requested the return of a large amount of collateral. As a result, AIG's overall investment portfolio was exposed to a huge amount of loss. While AIG as a group was subject to consolidated regulation and oversight by the federal office of thrift supervision, the crisis reflected the weakness of its regulation system in detecting insolvency, especially for large composite groups.

In an earlier SOA finance research report, Klein et al. (2009) presents several important lessons that insurers should learn from the 2008 financial crisis. One of the key lessons identified in this research is that the "Information System (IRIS) and the Financial Analysis Solvency Tools (FAST) systems to monitor insurers...tend to lag behind actual events with calculated ratios that only crudely indicate insurers' exposures to losses…"

In the light of the AIG failure, NAIC voted to adopt the U.S. Own Risk and Solvency Assessment (ORSA) as a significant new addition to U.S. insurance regulation in 2011. In 2010, the NAIC Solvency Modernization Initiative task force suggested the "Windows and Walls" approach2 to incorporate certain prudential benefits of group supervision. Under ORSA, larger and medium size US insurer and insurance groups are required to regularly perform an annual exam and file report upon request from the regulator. The ORSA serves as an internal process to assess the adequacy of its risk management and current and prospective solvency positions under normal and severe stress scenarios by an insurer or insurance group. In 2012, NAIC created the Principle-Based Reserving that requires the net premium reserve, the deterministic reserve and the more sensitive stochastic reserve.3 Van Laere and Baesens (2010) also discussed about the internal and external credit rating under Solvency II, and they point out that a big challenge in setting up such a model is the inference of the probability of default. To address these challenges, our proposed methodology can serve as an integrated part of the ORSA and the credit rating criteria.

[2] https://www.naic.org/documents/index_smi_group_solvency_windows_and_walls.pdf
[3] https://content.naic.org/cipr_topics/topic_principlebased_reserving_pbr.htm

2.2 PROS AND CONS OF THE EXISTING METHODS

Insolvency of insurance firms has been a subject of study and a matter of concern for over 100 years. It can trace back to Elizur Wright who examined insurance companies in Massachusetts, established the principle of financial solvency to fulfil contractual promises to insured, and initiated regulation of the insurance industry in the US.

In modern society, Pinches and Trieschmann (1974) and Ambrose and Seward (1988) proposed the usage of financial ratios to detect insurers at risk of insolvency. Harrington and Nelson (1986) employed a regression-based methodology to detect firms in financial distress. Based on the early studies, the U.S. insurance regulators developed an early warning system for insurance insolvency. Originally, the regulators simply used the fixed capital standards for monitoring the financial solvency of insurance companies, regardless of the financial condition of the company. The NAIC later adopted the RBC standard in 1990s, which measures the minimum amount of capital a company must hold based on its level of risk, including asset risk, underwriting risk, and other risks.[4]

These accounting-based insolvency studies are reviewed in Browne and Hoyt (1995). Accounting-based models have inherent limitations because

1) Financial statements are constructed under the going-concern principle (i.e., under the assumption that the firm will not go bankrupt); which typically assumes a near-term, deterministic view of the entity's ability to remain in business.
2) Financial statements, accounting estimates in particular, are subject to management's judgement, reflecting any management bias, even if unintentional (e.g., usage of voluntary reserves which results in unrealistic ratios) 5.
3) Financial statements have a low frequency of reporting, generally quarterly or semi-annually, which becomes a growing concern during a financial crisis or a period of volatility, when the financial health status of firms may change very quickly over a relatively short period of time.
4) Notwithstanding the market-based nature of measurement of assets and liabilities in financial statements prepared under International Financial Reporting Standards, they may fail to incorporate important pricing information such as underlying asset volatility (see Hillegeist et al. 2004), and they reflect any speculative bias of market-makers, who may not give proper weight to adverse scenarios.

Study shows that there is a high risk that the current prediction models often can only detect the troubled insurance company too late to save it from insolvency and liquidation. For instance, Leadbetter and Dibra (2008) examined 35 involuntary exits of P&C insurers in Canada, providing evidence of the failure of existing insolvency prediction models. As a result, insolvencies of insurers have drawn widespread media attention and deep public concerns. For instance, a Wall Street Journal article "Collapse of Long-Term Care Insurer Reflects Deep Industry Woes" (Scism Dec 4, 2016) describes how "a pair of small Pennsylvania insurers focused on long-term care could soon become one of the nation's costliest insurance failures ever." The deep public concern is amplified given the fact that the solvency of insurance firms is already

[4] The RBC calculations are maintained by the NAIC Capital Adequacy (E) Task Force and the most recent version is available through https://content.naic.org/cipr_topics/topic_risk_based_capital.htm

[5] Note that the upcoming IFRS 17 Standard aims to standardize insurance accounting globally to improve comparability and increase transparency, and to provide users of accounts with the information they need to meaningfully understand the insurer's financial position, performance and risk exposure. Therefore, this "improved" accounting standard will help reduce these biases.

tightly regulated in order to protect the policyholders by ensuring that the insurer will be able to meet its financial obligations in the future.

Additionally, insurer ratings have traditionally been used as measures of insolvency risk and financial quality. The insurer financial strength ratings assess the overall claims-paying ability to meet its ongoing insurance policy and contract obligations. However, there are no regulatory requirements to obtain insurance ratings. The market for insurance ratings was largely dominated by A.M. Best until the late 1980s when other agencies with a long history of rating corporate and government debt entered the insurance ratings market including S&P and Moody's. One limitation of these ratings is that they are often subject to low frequency revision and some firms may not be covered by the rating agencies. In section 4.4, we will use S&P financial strength rating and long-term local issuer credit rating for insurers for model evaluation.

2.3 INTRODUCTION TO THE MERTON'S MODEL

In contrast, more recently, motivated by Merton's (1974) structural model of default risk, there has been renewed interest in the application of market-based insolvency prediction models that are based on the contingent claims valuation approach. Market-based measures, at least in theory, are believed to overcome some of the limitations associated with accounting-based models since they are forward-looking by design and incorporate all information relevant to the pricing of securities.

Merton (1974) shows how the probability of company default can be inferred from the market valuation of companies[6] under specific assumptions on how assets and liabilities evolve. There is growing empirical evidence that the Merton model provides significantly superior information than purely accounting-based measures of default probabilities (see for example, Hillegeist et al., 2004; Bharath and Shumway, 2008). Sharara (2012) provides a comparative overview of the U.S. Risk-Based Capital (RBC), the Canadian Minimum Continuing Capital and Surplus Requirements (MCCSR) that was replaced by Life Insurance Capital Adequacy Test (LICAT) effective on Jan 1, 2018 in Canada, and the Solvency II standard capital formulas on the life side, and advocates the market-based framework for solvency assessment given the market valuation paradigm for insurers' assets and liabilities.

2.3.1 DESCRIPTION OF THE MERTON'S MODEL

The Merton (1974) model proposed an approach to use option pricing theory to measure the probability of default of a firm. Latterly, after slight modification and relaxation of the assumptions, the Merton (1974) model has been broadly used to predict default. Essentially, the equity in a firm is a residual claim, i.e., equity holders lay claim to all cashflows left over other financial claim holders (debt, preferred stock etc.) have been satisfied. The principle of limited liability protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm, analogous to a call option.

The Merton (1974) model assumes the market value of a firm's asset follows a Geometric Brownian motion:

[6] According to the NAIC, there were 5,965 insurance companies in the U.S. (including territories) in 2019. According to Fintel, there are 349 publicly traded U.S. insurance companies, identified by Standard Industrial Classification code 6311, 6632, 6633, 6635, 6636, and 6639. These public traded insurance companies consist primarily of large multiline firms. Although we are limited by the public insurance firms, these public traded firm represents economically significant portion of the industry based on premiums written and market capitalization. (https://www.iii.org/fact-statistic/facts-statistics-insurance-company-rankings)

$$dV_t = \mu V_t dt + \sigma_V V_t dW_t, \qquad (1)$$

where, V is the market value of the firm's assets with an instantaneous drift μ, and an instantaneous volatility σ_V, and W is a standard Wiener process.

Assume the firm has outstanding debt with a face value of F[7] and maturity of T. Since shareholders are the residual claimants of the firm's asset, the market value of equity, E, can be thought as a call option on V with time to maturity of T and strike price of F. Therefore, E can be valued by the standard Black-Scholes formula for a European call option (for simplification of notation, we do not include the subscript of t):

$$E = V\mathcal{N}(d_1) - Fe^{-rT}\mathcal{N}(d_2), \qquad (2)$$

where $d_1 = \frac{\ln(V/F)+(r+0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}$, and $d_2 = d_1 - \sigma_V\sqrt{T}$.

In the above equation (2), only the market value of equity, E_t, is observable. The one-year Federal treasury bill rate can be used as proxy[8] of the risk free rate, r. Vassalou and Xing (2004) and Bharath and Shamway (2008) and others, use one year for maturity, T. They also approximate the face value of debt, F, by the sum of the current debt (COMPUSTAT item DLC or DLCQ) and 50% of long-term debt (COMPUSTAT item DLTT or DLTTQ). The remaining two variables, V and σ_V, are not observable.

2.3.2 VASSALOU AND XING (2004) IMPLEMENTATION

Vassalou and Xing (2004) propose an iterative procedure to estimate the two unknown variables simultaneously. They use daily data from the past 12 months to obtain an estimate of the volatility of equity σ_E, which is then used as an initial value for the estimation of σ_V.

Using the Black–Scholes formula, equation (2), and for each trading day of the past 12 months, they compute V using the market value of equity of that day as E. This results daily values for V. Then the standard deviation of the daily logarithm returns of those V's. is used as the value of σ_V for the next iteration. This procedure is repeated until the values of σ_V from two consecutive iterations converge. The tolerance level for convergence is set to 0.0001. The estimated σ_V is then used to back out the asset value V for each day using equation (2).

The above process is repeated at the end of every month, resulting in the estimation of monthly values of σ_V. The estimation window is always kept equal to 12 months. The risk-free rate used for each monthly iterative process is the 1-year T-bill rate observed at the end of the month. Once daily values of V are estimated, the drift, μ, is then the mean of the change in lnV, the logarithm daily return of assets. The advantage of this approach is that daily values of the asset and the volatility of the asset returns can be computed simultaneously.

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[7] Merton (1974), examined the valuation of corporate debt in three possible manifestations: zero-coupon debt, coupon-bearing debt, and callable debt. Following most other literature, we focus on the zero-coupon debt version in this research.

[8] London Inter-bank Offered Rates (LIBOR) rate may also be considered as proxy for the risk-free rate. However, LIBOR's credibility has been undermined after the LIBOR scandal in 2012. After that, a great deal of efforts have been done to try to replace LIBOR with Secured Overnight Funding Rate (SOFR)—which is a median of rates that market participants pay to borrow cash on an overnight basis, using Treasuries as collateral. As SOFR are structurally different to LIBORs, its transition is a long and complex process till the end of 2022, which may be further delayed due to the 2020 financial crisis.

Once the estimations for both V, σ_V and μ are obtained, the distance to default can be calculated as:

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}.$$

Based on the standard normal distribution assumption, the implied probability of default is just $\mathcal{N}(-DD)$ which is the probability of $\ln(V/F) < 0$ in the physical probability space, or more intuitively, the probability of the asset growth rate being less than debt rate.

2.4 CONS OF THE MERTON'S MODEL AND CONTRIBUTION OF THE PROPOSED METHOD

2.4.1 CONS OF THE MERTON'S MODEL

Other than the obvious limitation of the Merton's model that requires the firm to be publicly traded, the basic Merton's model and its various implementations (e.g., Vassalou and Xing 2004; Bharath and Shumway 2008; Miao, Ramchander, Ryan and Wang 2018) depend on the assumption that the asset log returns are normally distributed and can be fully characterized by the first two moments of mean and variance, and assumption on constant risk free rate and volatility. However, such assumptions fail to fit the real-world data. The previous finance and insurance literature has shown extensive evidence that the asset returns distributions cannot be adequately characterized by mean and variance alone (see, for example, Corrado and Su 1996; Conrad, Dittmar and Ghysels 2013).

The existence of higher moments clearly violates the underlying assumption of the basic Merton's model and may have a significant impact on the accuracy of insolvency detection. For example, if insurers increase their leverage and equity value falls, the wealth is transferred from bondholder to shareholder by increasing the moneyness of the option. Since skewness captures asymmetric risk, it is especially important for the downside risk and may have a substantial impact on the probability of default. Similarly, Kurtosis risk is commonly referred to as "fat tail" risk. Ignoring kurtosis risk will cause the Merton's model to understate the risk of an asset with high kurtosis and underestimate the possibility of default. More recently, Adcock, Eling, and Loperfido (2015) provide an overview of the literature on the presence of skewness and kurtosis in the insurance industry. They present that insurance risks have highly skewed distributions and may exhibit heavy tails with exposures to catastrophic risks, and document significant levels of non-normality in both life and P&C insurance stock returns.

2.4.2 CONTRIBUTION OF THE PROPOSED METHOD

Insolvency risk is a critical risk factor for insurers and all stakeholders. The impact and magnitude of a list of notable insurer insolvencies such as AIG during the recent financial crisis and the current volatile market environment suggest that a close reexamination of the prediction model for the insolvency risk profiles of insurers is warranted in order to minimize any loss to policyholders and creditors. This proposed work will contribute to a growing literature on predicting insurer's insolvency.

The contribution of our paper lies in proposing a new market-based insolvency prediction model to measure insolvency probabilities, which significantly expands current boundary and provides valuable information to insurers, regulators, policyholders, investors, and researchers. We propose the use of the Gram-Charlier series expansion and derive the probability of default (PD) with the adjustment of non-normal skewness and kurtosis. We suggest that the combination of both skewness and kurtosis into the Merton distance-to-default model should provide a more accurate prediction of insurer insolvency risk for early detection than the current models. In addition, it would aid in their choice of means of regulatory intervention to minimize harm of the insolvency risks.

Section 3. Methodology

3.1 BLACK-SCHOLES CORRECTIONS

To correct the well-documented Black and Scholes option pricing model biases, several authors have proposed series expansions of a given probability density in order to approximate the "true" underlying risk-neutral implied return distribution. Under this approach, skewness and kurtosis may have significant impact on option prices and correction terms in the Black-Scholes formula might lead to a plausible explanation of strike price and time-to-maturity biases.

In an expanded Black-Sholes option pricing framework incorporating skewness and kurtosis, Jarrow and Rudd (1982) model the distribution of stock prices with an Edgeworth series expansion. Corrado and Su (1996) model the distribution of stock log prices with a Gram-Charlier series expansion. Under risk-neutral probability, they apply the Gram-Charlier density function to derive European call price formula. More specifically, Corrado and Su (1996) expand the Black-Scholes formula with two adjustment terms accounting for non-normal skewness and kurtosis by truncating the expansion after the fourth moment. The first two moments of the approximating distribution remain the same as that of the normal distribution, but third and fourth moments are introduced as higher order terms of the density expansion.

However, the current literature mainly focused on Black-Sholes option pricing correction written on equity not on the Merton's default model written on asset or total value of the firm. To our best knowledge, this is the first research on higher moments adjusted Merton's model.

3.2 THE GRAM-CHARLIER EXPANSIONS

In an expanded Merton's framework incorporating skewness and kurtosis, we propose to model the distribution of stock log prices with a Gram-Charlier series expansion to early detect insolvency of insurers. A Gram-Charlier series expansion of a density function $\phi_{GC}(x)$ is analogous to the Taylor series expansion for analytic functions. It is calculated as the infinite sum of terms from the product of Hermite polynomials and normal density function as opposed to the values of the function's derivatives at a single point in the Taylor series expansion. The underlying intuition of Hermite polynomial approach is to transform the underlying process closer to the density of distribution of the non-normal distribution into a standard normal distribution (see Corrado and Su 1996) in order to integrate higher moments while keeping the computational tractability.

More formally, we can define Gram-Charlier series expansion of $\phi_{GC}(x)$ as

$$\phi_{GC}(x) = \sum_{n=0}^{\infty} c_n H_n(x)\, \phi_N(x,\mu,\sigma) \qquad (3)$$

where, $\phi_N(x,\mu,\sigma)$ is the normal density function with mean μ and standard deviation σ, $H_n(x)$ are Hermite polynomial defined by the relation $H_n(x)\phi(x,0,1) = (-1)^n d^n \phi(x,0,1)/dx^n$. For example, $H_0(x) = 1$, $H_1(x) = x$, $H_2(x) = x^2 - 1$, $H_3(x) = x^3 - 3x$, $H_4(x) = x^4 - 6x^2 + 3$. The coefficient c_n are determined by moments of the distribution function $N_{GC}(x)$.

Note that the m_{th} moment for a standard normal variable $X \sim N(0,1)$ is

$$E(X^m) = \begin{cases} 0 & m \text{ is odd} \\ 2^{-m/2}\dfrac{m!}{m/2!} & m \text{ is even} \end{cases}$$

Thus, $E(X^0) = 1$, $E(X^1) = 0$, $E(X^2) = 1$, $E(X^3) = 0$, $E(X^4) = 3$.

Because the "probability density function (PDF)" obtained by truncating an infinite series is not a true PDF and can sometimes assume negative values, Chernozhukov et al. (2010) present a method called the increasing rearrangement for improvement as detailed in the Appendix.

3.3 SKEWNESS AND KURTOSIS ADJUSTED MERTON'S MODEL

Under this proposed framework, we will apply the Gram-Charlier density function to expand the traditional Merton's model with two adjustment terms accounting for non-normal skewness and kurtosis by truncating the expansion after the fourth moment.

Specifically, after standardizing to a zero mean and unit variance, the Gram-Charlier series accounts for skewness μ_3 and kurtosis μ_4 and yields the following density function,

$$\phi_{GC}(z) = \phi_N(z)[1 - \frac{\mu_3}{3!}H_3(z) + \frac{\mu_4 - 3}{4!}H_4(z)], \quad (4)$$

where, $\phi_N(z) = \frac{1}{\sqrt{2\pi}}exp(-\frac{z^2}{2})$ is the standard normal density function. While expansion in Equation (3) is an infinite series, we will focus on the first four moments in Equation (4) in this paper for our analysis. We can naturally extend our framework to include more higher moments in a straightforward manner.

Notice that E(z) = 0, E(z2) = 1, E(z3) = μ3, E(z4) = μ4. Thus, the normal distribution will be a special case, with skewness μ3 = 0 and kurtosis μ4 = 3, $\phi_{GC}(z) = \phi_N(z)$. Therefore, the Gram-Charlier series allow us to adjust the higher order non-normal risk measures when the log return is deviated from the normal distribution.

Following Corrado and Su (1996), if we define $\mu = ln(V_0) + (r - 0.5\sigma^2)t$, then

$$z = \frac{ln(V_t) - \mu}{\sigma\sqrt{t}} = \frac{ln(V_t/V_0) - (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

Based on the Skewness and Kurtosis adjusted Black-Scholes equation for option prices (Corrado and Su 1996; Brown and Robinson 2002), the Skewness and Kurtosis adjusted Black-Scholes equation for valuing the firm's equity under these conditions is given by:

$$E = VN(d_1) - Fe^{-rT}N(d_2) + \mu_{3V}Q_3 + (\mu_{4V} - 3)Q_4 \quad (5)$$

where

$$Q_3 = \frac{1}{3!}V\sigma_V\sqrt{t}[(2\sigma_V\sqrt{t} - d_1)\phi(d_1) + \sigma_V^2 tN(d_1)]\ Q_4 = \frac{1}{4!}V\sigma_V\sqrt{t}[(d_1^2 - 1 - 3\sigma_V\sqrt{t}(d_1 - \sigma_V\sqrt{t}))\phi(d_1) + \sigma_V^3 t^{\frac{3}{2}}N(d_1)]$$

3.4 SKEWNESS AND KURTOSIS ADJUSTED PROBABILITY OF DEFAULT

Assuming risk neutrality, we can now apply the density function $\phi_{GC}(z)$ to derive a theoretical adjusted probability of default ($PD_{adjusted}$) as

$$PD_{adjusted} = \int_{-\infty}^{F} \phi_{GC}(V_t) dV_t$$

Now we can apply the change of variable $z = \frac{ln(V_t)-\mu}{\sigma\sqrt{t}}$ to the $PD_{adjusted}$ equation

$$\begin{aligned} PD_{adjusted} &= N_{GC}(-d_2) = \int_{-\infty}^{-d_2} \phi_{GC}(z) dz \\ &= \int_{-\infty}^{-d_2} \phi_N(z)[1 - \tfrac{\mu_3}{3!} H_3(z) + \tfrac{\mu_4 - 3}{4!} H_4(z)] dz \\ &= N(-d_2) - \tfrac{\mu_3}{3!} \int_{-\infty}^{-d_2} H_3(z)\phi_N(z) dz + \tfrac{\mu_4 - 3}{4!} \int_{-\infty}^{-d_2} H_4(z)\phi_N(z) dz \end{aligned}$$

In the formula, $d_2 = (\mu - ln(F))/\sigma\sqrt{t}$ contains the drift and volatility terms.

According to Kendall et al. (1987), we have the following lemma with constant number C.

$\int H_{n+1}(z)\phi(z) dz = -H_n(z)\phi(z) + C$ Thus,

$$PD_{adjusted} = N_{GC}(-d_2) = N(-d_2) + \tfrac{\mu_3}{3!}(d_2^2 - 1)\phi_N(-d_2) + \tfrac{\mu_4 - 3}{4!}(d_2^3 - 3d_2)\phi_N(-d_2) \quad (6)$$

Notice that the first term $N(-d_2)$ is the risk-neutral probability of default of the firm in Merton's model. The second and third term are corresponding adjustments for the skewness and kurtosis. When the underlying distribution is indeed normal, $PD_{adjusted} = N(-d_2)$ with Merton's model as a special case.

3.5 SKEWNESS AND KURTOSIS ADJUSTED IMPLEMENTATION

Let V denote asset value, and μ_V the mean, σ_V the volatility, μ_{3V} the skewness, and μ_{4V} the kurtosis, all of which are not observable, and therefore must be estimated. The other three variables, F, T, and r can be approximated. In the same spirit of Vassalou and Xing (2004) to estimate probability of default, we use the one-year Treasury rate as a proxy of the risk free interest rate r, and use daily frequency since the annualized higher moments will be normalized. To approximate the strike price, F, the model uses the sum of the current debt and 50% of the long-term debt following the literature (e.g. Vassalou and Xing 2004). The remaining five variables, namely, the market value of the firm's assets V, the instantaneous drift μ_V and instantaneous volatility σ_V, skewness μ_{3V}, kurtosis μ_{4V} are estimated by following an iterative procedure using daily equity data from the previous 12 months.

The first two moments of the approximating distribution remain the same as that of the normal distribution. Using daily data from the past 12 months, σ_E, the volatility of equity returns is estimated and used as an initial value for the estimation of σ_V, the volatility of asset returns. Third and fourth moments are introduced as higher order terms of the density expansion. Similarly, we use equity skewness and kurtosis as an initial value for the corresponding estimation of the firm value. Then using the skewness and kurtosis adjusted Merton's model in Equation (5), asset value V is computed for each trading day using the corresponding market value of equity for that day, E. The volatility, skewness, and kurtosis of the daily logarithmic returns of those V's are then used as the value of σ_V, μ_{3V}, μ_{4V} for the next iteration. This procedure is repeated until the values of σ_V, μ_{3V}, μ_{4V} from two consecutive iterations converge. The estimated σ_V, μ_{3V}, μ_{4V} are then used to back out the asset value V for each day. The drift, μ_V, is the mean of the change in lnV, the logarithm daily "return" of assets. Finally, the estimated drift μ_V is plugged into Equation (6) incorporating skewness and kurtosis to compute the probability of default.

Section 4. Empirical Analysis

4.1 DATA DESCRIPTION

In this project, we examine the default prediction ability of the higher moments adjusted Merton's model. Firm accounting information is obtained from COMPUSTAT, and daily equity markets data are from CRSP. The bankruptcy data is from Bloomberg that contains bankruptcy cases for all public companies. The previous accounting-based literature on insurance insolvency is built upon the low frequency of accounting-based characteristics and the natural lag behind actual events and market movements. Our proposed market-based model will significantly mitigate this limitation of accounting-based models with the use of considerably more frequent market price data. Since the market timely incorporates information relevant to the pricing of securities on each trading day, our proposed market-based model is able to extract useful information from financial prices for early detection of insurer's insolvency risk on a daily basis.

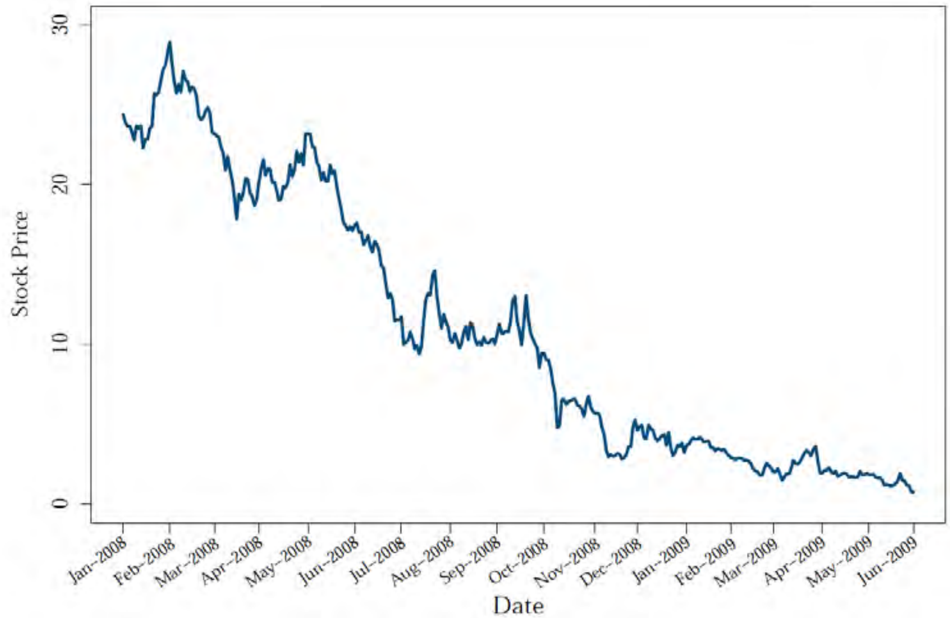
4.2 CASE EXAMPLES

4.2.1 GENERAL INSOLVENCY CASE OF NON-INSURER: GM

While the focus of this research is on insurer insolvency detection, the proposed method is general for all firms that are publicly traded. Therefore, we first conducted an empirical test of the probability of default for a non-insurance firm General Motors (GM) in 2008-2009 to illustrate the proposed higher moment adjusted Merton's model approach's flexibly for a general public firm.

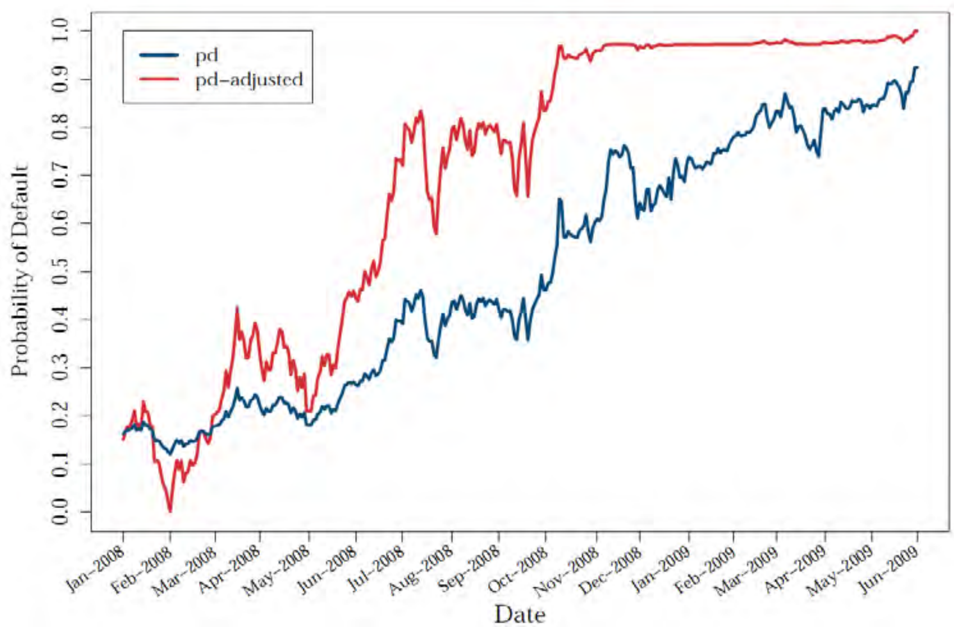
The Chapter 11 filing of GM was the fourth-largest bankruptcy in U.S. history. GM was already financially vulnerable prior to 2008 and was further severely affected by the 2008 financial crisis. Its stock price significantly declined throughout 2008 as shown in Figure 1 and GM eventually filed for Chapter 11 bankruptcy on June 1, 2009. It was then delisted and removed from the Dow Jones Industrial Average index.

Figure 1
DAILY STOCK PRICE FOR GM



Using the sample data of GM, we calculated two separate probability of default prediction estimates at the end of each day: the traditional Merton's model using the Vassalou and Xing (2004) approach, and our proposed higher moments adjusted Merton's model approach.

Figure 2
DAILY LEVELS OF PROBABILITY OF DEFAULT FOR GM



Note: hereafter, we use pd to denote the probability of default from the Merton's model, and use pd-adjusted to denote the probability default from the proposed higher moments adjusted Merton's model.

Figure 2 plots the daily probability of default by both approaches for GM. The daily probability to default of GM increase for both the traditional Merton's model and the proposed higher moments adjusted Merton's model approach since 2008, showing that both approaches capture the financial vulnerability of GM from the market implied information. More interestingly, the higher moment adjusted probability of default is much larger than the traditional Merton's model and jumps higher much earlier than the traditional Merton's model. This implies that with the additional information from the higher moments, the adjusted Merton's model predicts the bankruptcy of GM much earlier and with a much stronger signal than the traditional Merton's model does, which totally ignores such information. Note that, in the very late stage of the GM before it filed for bankruptcy, the traditional Merton's model still showed a less than 90% of probability of default, while the proposed approach assigned a close to 100% probability of default almost half year ahead of the Chapter 11 filing of GM.

4.2.2 INSOLVENCY INSURANCE CASE: AIG

We next move on to the insolvency case of probably the most well-known insurance company AIG to illustrate the proposed approach for insurers. As we discussed in the early part of the paper, AIG was seriously affected by the 2008 financial crisis when its overall investment portfolio was exposed to huge amount of loss mainly due to its credit default swap (CDS) portfolio written by AIG Financial Products (AIGFP). On the verge of bankruptcy at the end of the third quarter of 2008, AIG's risk-based capital (RBC) ratio was still at the level of 452 percent that was well above the 200 percent regulatory criterion for company action to improve financial strength. Such disparity reflects the potential flaws of the accounting-based measures. However, the risk of insolvency is clearly reflected in the significant decline of AIG's stock prices as shown in Figure 3.

Figure 3

DAILY STOCK PRICE FOR AIG

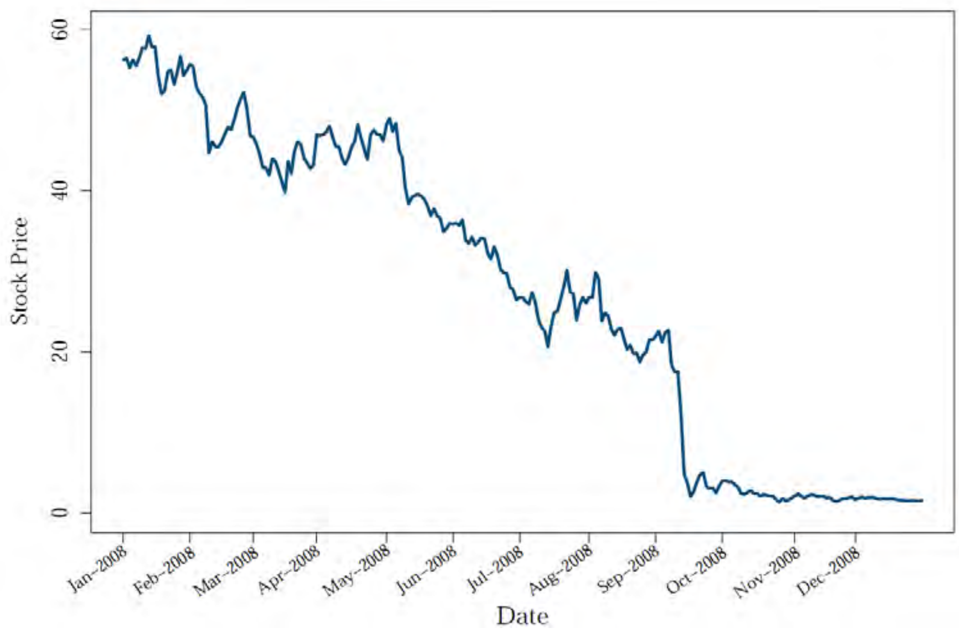
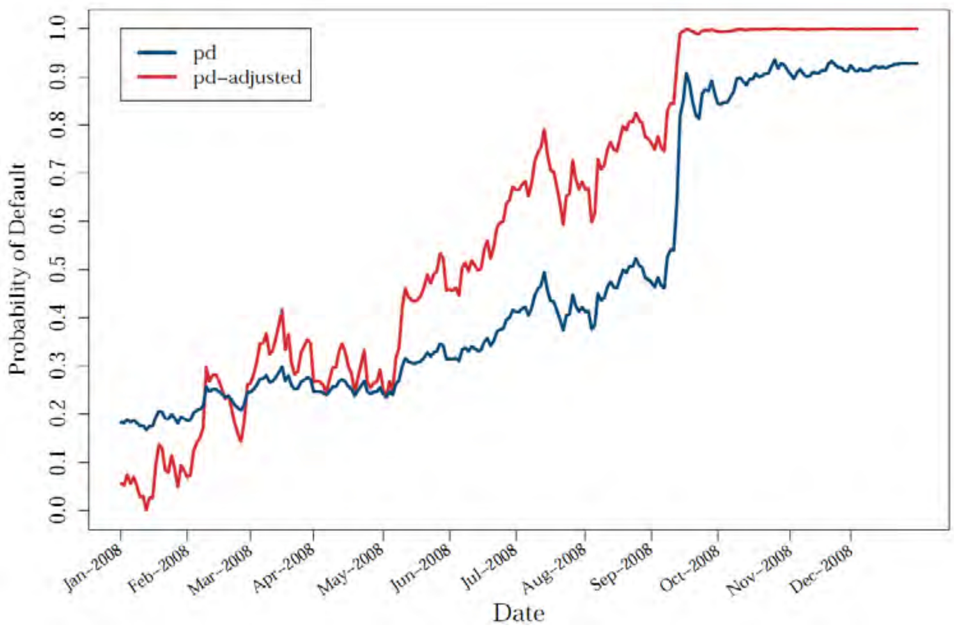


Figure 4 plots the daily probability of default by both the traditional Merton's model and the proposed higher moments adjusted Merton's model approaches for AIG. Note that loss reserves for an insurance company represent a liability of the insurer. However, such liability (obligations to policyholders) is fundamentally different from claims by other creditors. Following the previous literature in insurance and actuarial science (see, for example, Pottier, and Sommer, 1999; Gaver and Pottier 2005), the definition of debt in our proposed research will include both short-term and long-term obligations to creditors and exclude the insurer's loss reserve liability. Consistent with our findings in the case of GM, the higher moment adjusted probability of default jumps up much higher and much earlier than the traditional Merton's model, sending a clear and strong signal from the skewness and kurtosis of market information.

Figure 4

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR AIG



While severely damaged, AIG was bailed out by American Federal Reserve (FED) with the total amount of 205 billion USD. AIG was imposed a drastic restructuring plan to cede a large number of non-strategic assets and was taken into receivership, which is the bankruptcy process for an insurance company.

4.2.3 FINANCIALLY DISTRESSED INSURANCE CASE: AMTRUST

As not all financially distressed insurers filed for bankruptcy, next we illustrate the proposed approach to help identify the insolvency problem in early stage with the case of AmTrust Financial Services, Inc.

Although AmTrust's accounting-based reserves were solid for regulators, according to a Wall Street Journal article in April 11, 2017, a whistleblower who was an auditor from BDO USA LLP, an accounting firm, cooperated with the Securities and Exchange Commission (SEC) to record casual conversations for the Federal Bureau of Investigation (FBI). Since 2014, there has been an ongoing investigation into the company's accounting practices. The whistleblower indicated that AmTrust may have overstated its operating income by over $250 million USD. The company's stock price suffered from a sequence of decline in 2016 and 2017 as showed in Figure 5.

Figure 5
DAILY STOCK PRICE FOR AMTRUST

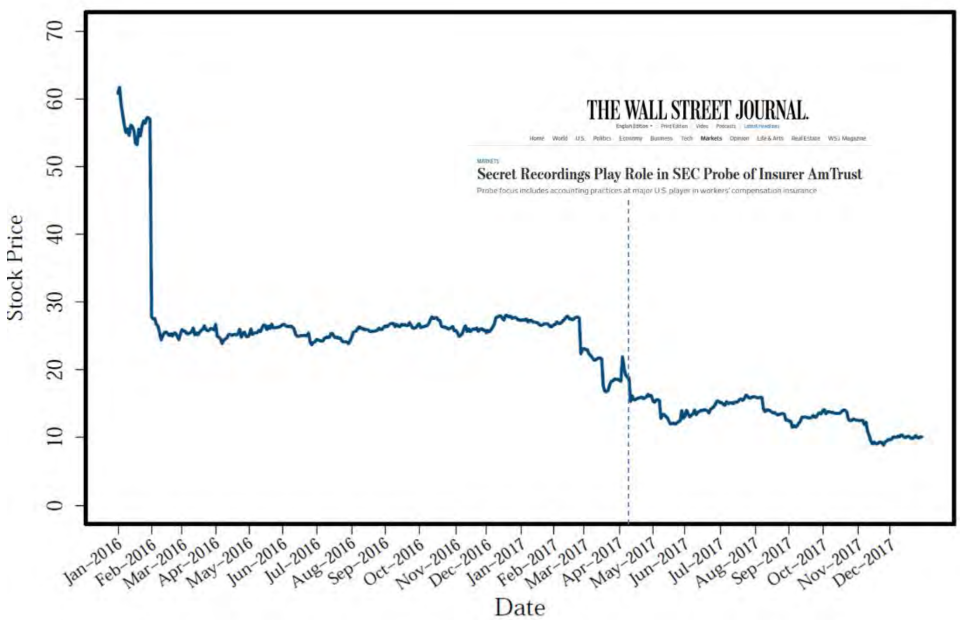
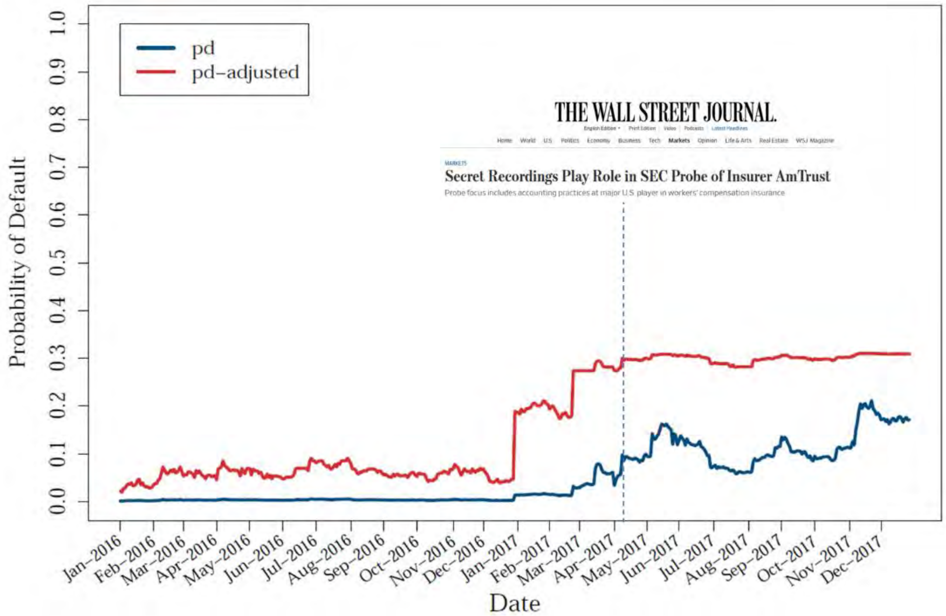


Figure 6 plots the daily probability of default by both the traditional Merton's model and the proposed higher moments adjusted Merton's model approaches for AmTrust. We can see that while the Merton's model reported a close to 0 probability of default all the way before the Wall Street Journal's article, the higher moments adjusted Merton's model already showed some warning signs before that. And since the end of 2016, the proposed method already detected a jump of the probability of default almost half year before the report of whistleblower on AmTrust's potential underwriting and accounting malpractice in the WSJ article. In 2018, AmTrust insurer's founding family took the company private.[9] AmTrust serves as an example that the insurer didn't go bankruptcy, but the proposed method helps identify the problem from the market implied information.

[9] https://www.barrons.com/articles/in-buyout-amtrust-takes-its-problems-private-1515623142

Figure 6

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR AMTRUST

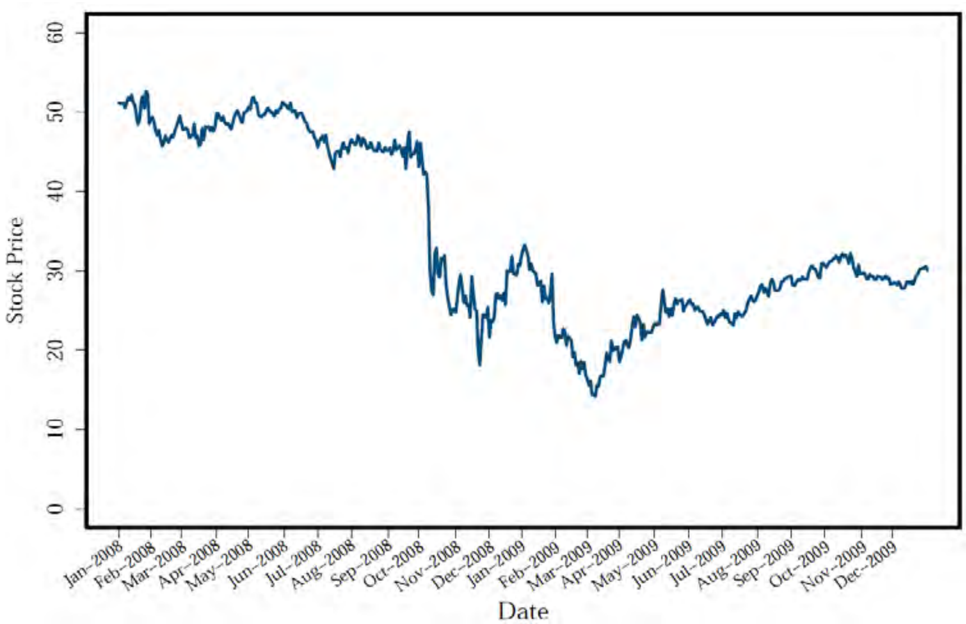


4.2.4 FINANCIALLY SOUND INSURANCE CASE: ALLSTATE

We also want to illustrate the proposed approach for a case of financially sound insurer Allstate. Founded on April 17, 1931, Allstate has been a pioneer of the insurance industry. Although the stock price of Allstate also suffered from the 2008-2009 financial crisis as shown in Figure 7, Allstate remains solid balance sheet strength, which AM Best categorizes as strongest.

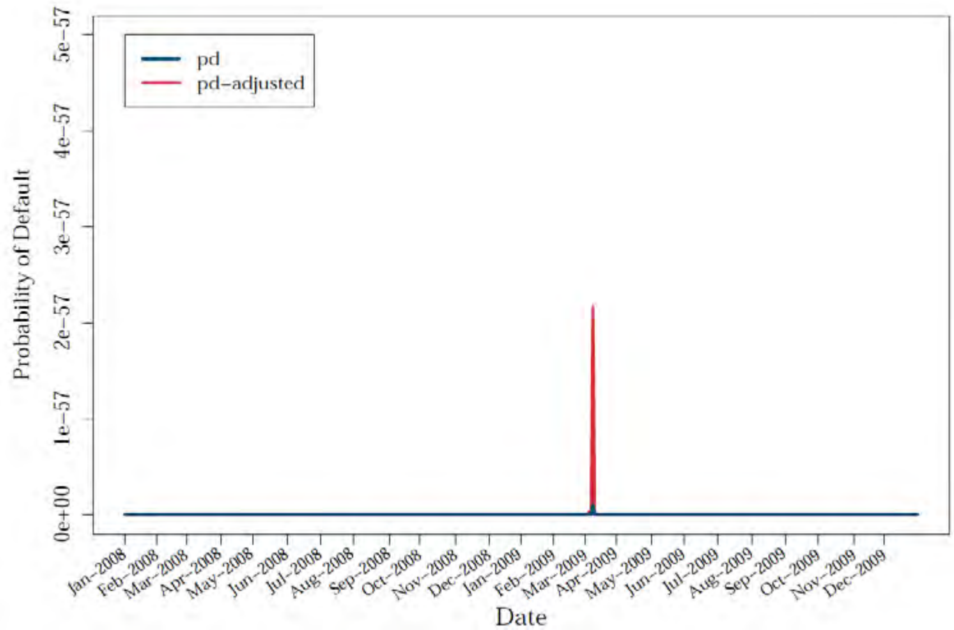
Figure 7

DAILY STOCK PRICE FOR ALLSTATE



Such strong financial rooting is also evident from the proposed approach as shown in Figure 8. In comparison to the case of AIG during the same sample period, Allstate is much healthier and the patterns in default probability are very different between these two firms for both traditional Merton's model and the higher moments adjusted Merton's model. All of the daily probability of default for Allstate are around zero which implies that it is almost impossible for Allstate to default in 2008-2009.

Figure 8
DAILY LEVELS OF PROBABILITY OF DEFAULT FOR ALLSTATE



Note: the scale of the axis for the probability of default is different in this chart due to the extreme small size of the probability of default.

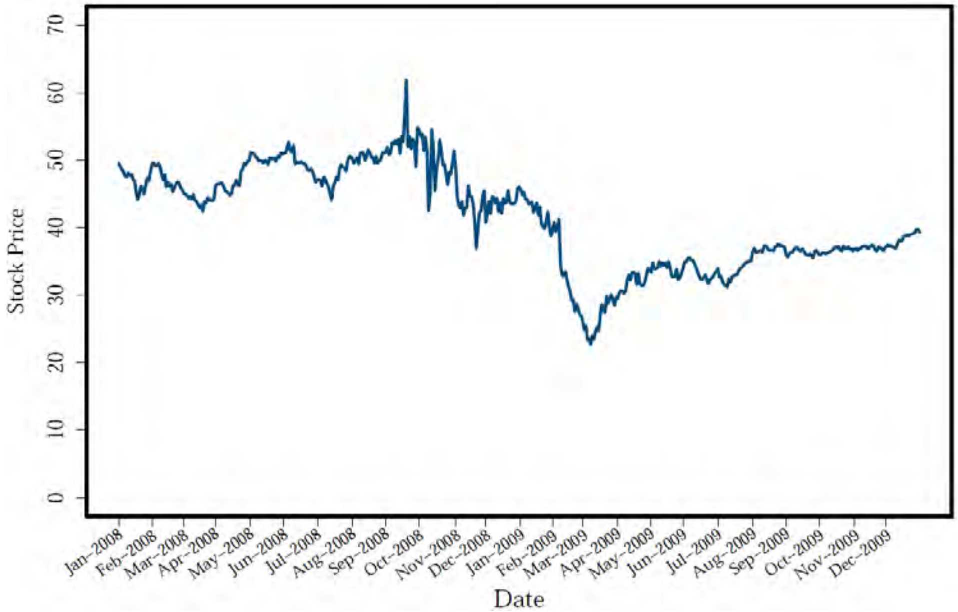
For financially healthier firms like Allstate and Mercury General Corporation (MCY) which will be explained below, the impact of higher moments is not significant, and the higher moment adjusted Merton's model is reduced to the traditional Merton's model. However, for financially distressed firms like AIG and AmTrust, the equity and asset returns are highly skewed and have excess kurtosis, the higher moment adjusted Merton's model incorporates the important information from skewness and kurtosis, therefore provides more accurate and earlier prediction on the probability of default.

4.2.5 FINANCIALLY SOUND INSURANCE CASE: MERCURY GENERAL CORPORATION

Lastly, while Allstate is a mature company with a long history, we also test a relatively younger and more regional company Mercury General Corporation (MCY) comparing to Allstate for the same period. Mercury General Corporation is a multiple-line insurance organization offering personal automobile, homeowners, renters and business insurance.

Figure 9

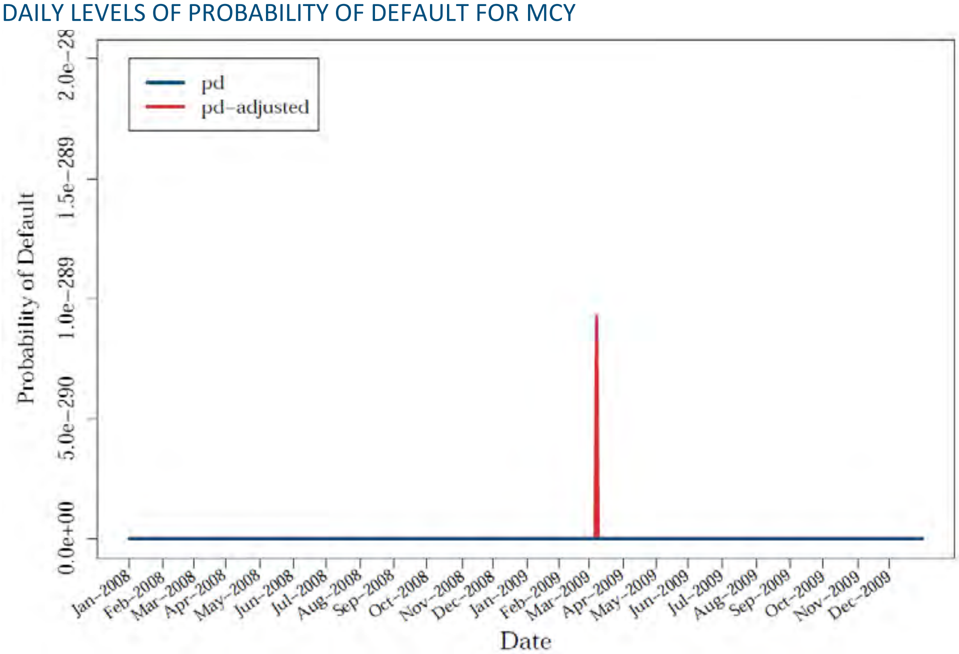
DAILY STOCK PRICE FOR MCY



AM Best has the Financial Strength Rating (FSR) of A (Excellent) and the Long-Term Issuer Credit Ratings (Long-Term ICR) of "a+" of the members of Mercury Casualty Group. From the stock price of MCY in 2008-2009 in Figure 9 and daily probability of default in Figure 10, we can see that similar to Allstate, although there is some price volatility due to the financial crisis, MCY's daily probability of default is around zero which implies that it is almost impossible for MCY to default in 2008-2009.

Figure 10

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR MCY



Note: the scale of the axis for the probability of default is different in this chart due to the extreme small size of the probability of default.

4.3 EMPIRICAL TEST OF BANKRUPTED INSURERS

In the previous literature on insurance insolvency, there exists a major data limitation due to the very sparse nature of insolvency data of insurance companies. The insolvency frequency rate in existing literature is estimated to be around 1% using the population of insurers (Pottier 1998). In this research, we are subject to the same sparse data of insolvent insurers just as all other research on this topic.

We start the sample selection process with all common stocks traded on NYSE, AMEX and NASDAQ. According to Bloomberg, we identify 12 major publicly traded insurance companies that filed for bankruptcy in the past 20 years. There are 4 major insurance bankruptcy cases that have complete information for COMPUSTAT quarterly database, and we will examine them one by one.

Table 1

LIST OF MAJOR PUBLIC INSURANCE FIRMS FILED FOR BANKRUPTCY

Name	Date	Industry	Assets	Liabilities
Life Partners Inc	5/19/2015	Life Insurance	NA	01.00B
Universal Health Care Group	2/6/2013	Life Insurance	28.96M	61.72M
Ambac Financial Group Inc	11/8/2010	P&C Insurance	-394.50M	01.68B
Answer Financial Inc	1/21/2008	P&C Insurance	2.41M	53.08M
Vesta Insurance Group Inc	7/18/2006	P&C Insurance	14.92M	214.28M
Frontier Insurance Group Inc	7/5/2005	P&C Insurance	79.10M	271.81M
Acceptance Insurance Cos Inc	1/7/2005	P&C Insurance	234.15M	338.06M
Metropolitan Mtg & Sec-Cl A	2/4/2004	Life Insurance	420.82M	415.25M
Trenwick Group Ltd	8/20/2003	Reinsurance	5.02B	04.79B
Conseco Inc	12/17/2002	Life Insurance	52.29B	51.18B
Highlands Insurance Group	10/31/2002	P&C Insurance	1.64B	01.82B
Inspire Insurance Solutions	2/15/2002	P&C Insurance	22.68M	15.87M

With the probabilities of default calculated, we further examine the accuracy of the proposed model using the S&P widely recognized financial strength rating and long-term local issuer credit rating for insurers from Bloomberg. The financial strength rating and long-term local issuer credit rating are S&P's assessment of the insurer's financial soundness and ability to meet ongoing obligations to its policyholders. Maintaining excellent financial strength ratings is important for insurers to attract and retain customers and to meet scrutiny from regulators and investors. Because insurers' liabilities are primary obligations to policyholders rather than interest-bearing debt, the financial strength rating is considered to be as important as the general credit rating for these firms.

Table 2 listed the S&P Ratings Change for the Public Insurance Firms Filed for Bankruptcy. As we will illustrate in detail, for each firm, we marked the date with dotted vertical lines for S&P rating downgrades on the Figure 12, 14, 16, 18 for Conseco, Trenwick, Acceptance, and Ambac, respectively. These Figures show that the proposed method aligned very well with the change of S&P ratings, i.e. the S&P downgrade are aligned with the sharp increase and often line up at some major threshold of the probability of default in the proposed higher moment adjusted Merton’s model.

Table 2

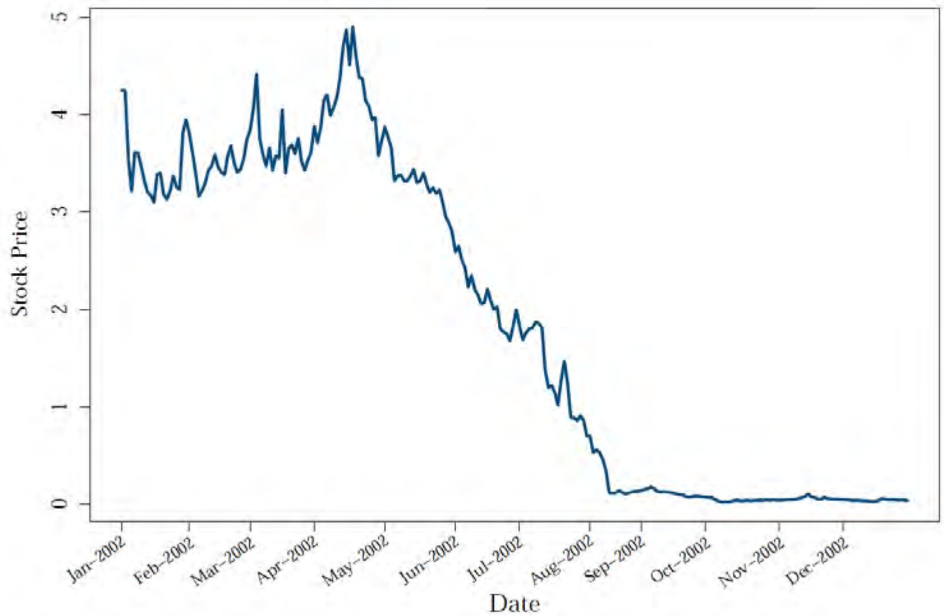
LIST OF S&P RATINGS CHANGE FOR PUBLIC INSURANCE FIRMS FILED FOR BANKRUPTCY

Company Name	Date	Rating Type	Agency	Curr Rtg	Last Rtg
Conseco Variable Insurance Co	12/18/2002	LT Local Issuer Credit	S&P	NR	B+ *-
Conseco Variable Insurance Co	08/09/2002	LT Local Issuer Credit	S&P	B+ *-	B+
Conseco Variable Insurance Co	08/02/2002	LT Local Issuer Credit	S&P	B+	BB+
Conseco Variable Insurance Co	01/16/2002	LT Local Issuer Credit	S&P	BB+	BBB-
Conseco Variable Insurance Co	12/18/2002	Financial Strength	S&P	NR	B+ *-
Conseco Variable Insurance Co	08/09/2002	Financial Strength	S&P	B+ *-	B+
Conseco Variable Insurance Co	08/02/2002	Financial Strength	S&P	B+	BB+
Conseco Variable Insurance Co	01/16/2002	Financial Strength	S&P	BB+	BBB-
Trenwick America Reinsurance Corp	06/24/2003	LT Local Issuer Credit	S&P	NR	CCC
Trenwick America Reinsurance Corp	06/23/2003	LT Local Issuer Credit	S&P	CCC	CCC *-
Trenwick America Reinsurance Corp	03/04/2003	LT Local Issuer Credit	S&P	CCC *-	CCC
Trenwick America Reinsurance Corp	01/31/2003	LT Local Issuer Credit	S&P	CCC	BB- *-
Trenwick America Reinsurance Corp	11/11/2002	LT Local Issuer Credit	S&P	BB- *-	BB+ *-
Trenwick America Reinsurance Corp	10/29/2002	LT Local Issuer Credit	S&P	BB+ *-	A-
Trenwick America Reinsurance Corp	06/24/2003	Financial Strength	S&P	NR	CCC
Trenwick America Reinsurance Corp	06/23/2003	Financial Strength	S&P	CCC	CCC *-
Trenwick America Reinsurance Corp	03/04/2003	Financial Strength	S&P	CCC *-	CCC
Trenwick America Reinsurance Corp	01/31/2003	Financial Strength	S&P	CCC	BB- *-
Trenwick America Reinsurance Corp	11/11/2002	Financial Strength	S&P	BB- *-	BB+ *-
Trenwick America Reinsurance Corp	10/29/2002	Financial Strength	S&P	BB+ *-	A-
Acceptance Insurance Cos Inc	11/26/2002	LT Local Issuer Credit	S&P	R	CC *-
Acceptance Insurance Cos Inc	11/19/2002	LT Local Issuer Credit	S&P	CC *-	CCC *-
Acceptance Insurance Cos Inc	11/18/2002	LT Local Issuer Credit	S&P	CCC *-	B+ *-
Acceptance Insurance Cos Inc	11/15/2002	LT Local Issuer Credit	S&P	B+ *-	B+
Acceptance Insurance Cos Inc	08/13/2001	LT Local Issuer Credit	S&P	B+	BB- *-
Acceptance Insurance Cos Inc	05/31/2001	LT Local Issuer Credit	S&P	BB- *-	BB-
Acceptance Insurance Cos Inc	12/07/1999	LT Local Issuer Credit	S&P	BB-	BB+ *-
Acceptance Insurance Cos Inc	11/16/1999	LT Local Issuer Credit	S&P	BB+ *-	BB+
Ambac Financial Group Inc/Old	11/30/2010	LT Local Issuer Credit	S&P	NR	D
Ambac Financial Group Inc/Old	11/02/2010	LT Local Issuer Credit	S&P	D	CC
Ambac Financial Group Inc/Old	07/28/2009	LT Local Issuer Credit	S&P	CC	BB *-
Ambac Financial Group Inc/Old	06/24/2009	LT Local Issuer Credit	S&P	BB *-	BBB

4.3.1 BANKRUPT INSURANCE CASE: CONSECO INC.

Conseco is an insurance company that was set up in 1979. Due to a variety of troubled investments including the acquisition of numerous companies in the 1990s, Conseco was pushed to bankruptcy in 2002 as the company sought protection from creditors while it sold its finance unit and tried to restructure. The stock price sharply declined in the second and third quarters of 2002 as shown in Figure 11.

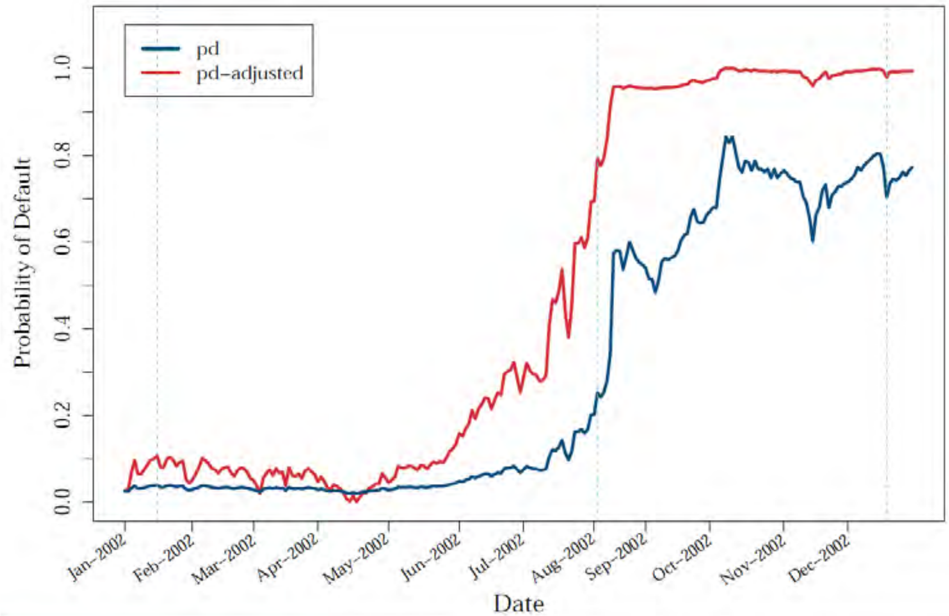
Figure 11
DAILY STOCK PRICE FOR CONSECO



In December 2002, Conseco became the third-largest U.S. bankruptcy in history of the time. Figure 12 shows that at the beginning of year 2002, the proposed model already started to jump higher and then quickly climbed up in comparison to the traditional Merton's model. The chance of default gradually increased to about 80% towards July of the year, while the Merton's model only assigned an about 20% probability of default at the time. In September 2002, the Merton's model only predicted about 60% chance of default, while the proposed model already predicted an almost 100% probability of default, three months ahead of the filing of bankruptcy of Conseco. More importantly, while the higher moments adjusted Merton's model predicted about 100% chance of default ever since September of year 2002, the Merton's model still only predicted of 70% chance of default when Conseco filed for bankruptcy in December of 2002.

Figure 12

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR CONSECO



Note: the dotted vertical lines marked the events when S&P lowered Conseco's rating.

4.3.2 BANKRUPT INSURANCE CASE: ACCEPTANCE INSURANCE COMPANIES INC.

According to SEC record, Acceptance Insurance Companies INC. filed a chapter 7 bankruptcy case on Jan 7, 2005.[10] While Acceptance finally filed for bankruptcy in 2005, its stock price already fell to almost zero in 2003. Therefore, we conducted our analysis from 1998 when the stock prices started to sharply decline to 2003 when the price became zero as shown in Figure 13.

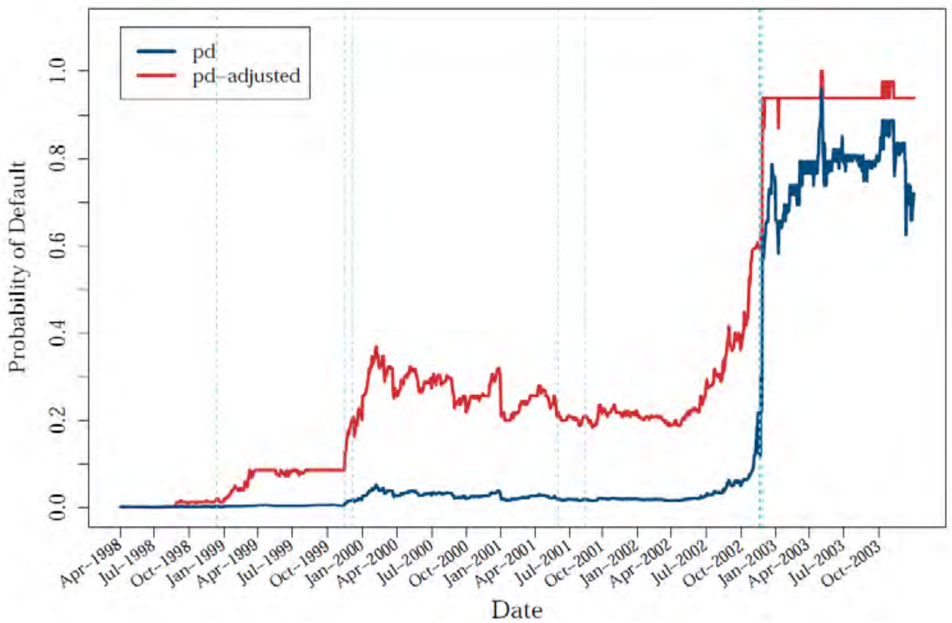
[10] https://www.sec.gov/Archives/edgar/data/74783/000119312510184703/d8k.htm

Figure 13
DAILY STOCK PRICE FOR ACCEPTANCE

Figure 14 shows the daily probability of default by both the traditional Merton's model and the proposed higher moments adjusted Merton's model approaches for Acceptance Insurance Companies. Consistent with our findings in other cases, the proposed model jumped higher much earlier than the traditional Merton's model. The chance of default gradually increased to about 60% towards October of 2002, while the Merton's model only assigned an about 20% probability of default at the time. In November 2002, the Merton's model only predicted about 60% chance of default, while the proposed model already predicted an almost 100% probability of default. Additionally, the higher moments adjusted Merton's model predicted about 100% chance of default ever since November of 2002, the Merton's model predicted of 60% chance of default when Acceptance stock price already fell to almost zero in 2003.

Figure 14

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR ACCEPTANCE



Note: the dotted vertical lines marked the events when S&P lowered Acceptance's rating.

4.3.3 BANKRUPT INSURANCE CASE: TRENWICK AMERICA CORPORATION

On August 20, 2003, insolvency proceedings were initiated for Trenwick America Corporation. Trenwick filed for protection under chapter 11 bankruptcy. Similar to the previous cases, the stock price of Trenwick plummeted since 2002 as showed in Figure 15, and the proposed higher moments adjusted Merton's model approach detected a sharp rise of probability of default much earlier and on a more accurate magnitude than the traditional Merton's model suggested as shown in Figure 16.

Figure 15
DAILY STOCK PRICE FOR TRENWICK

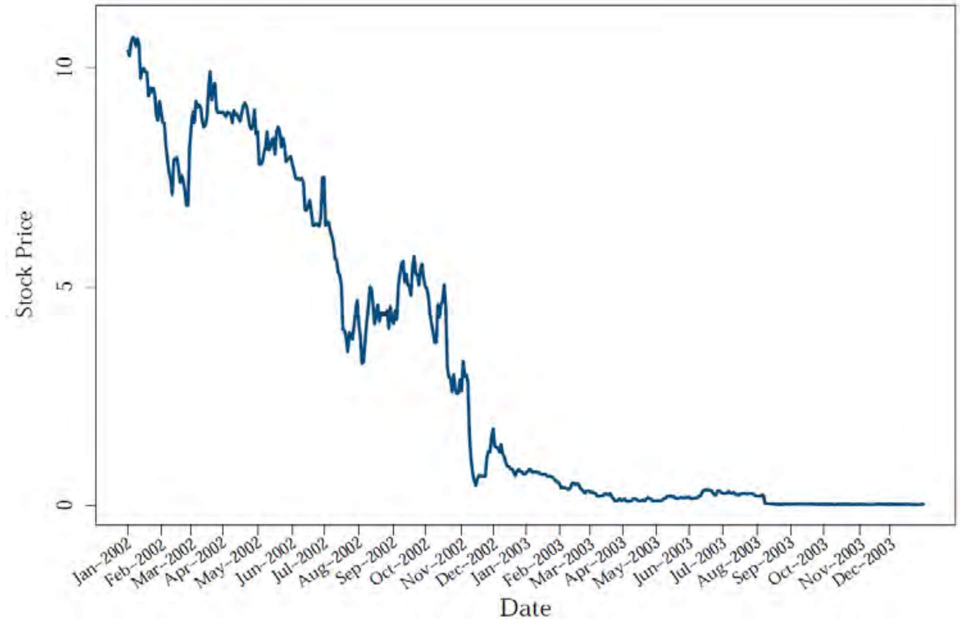
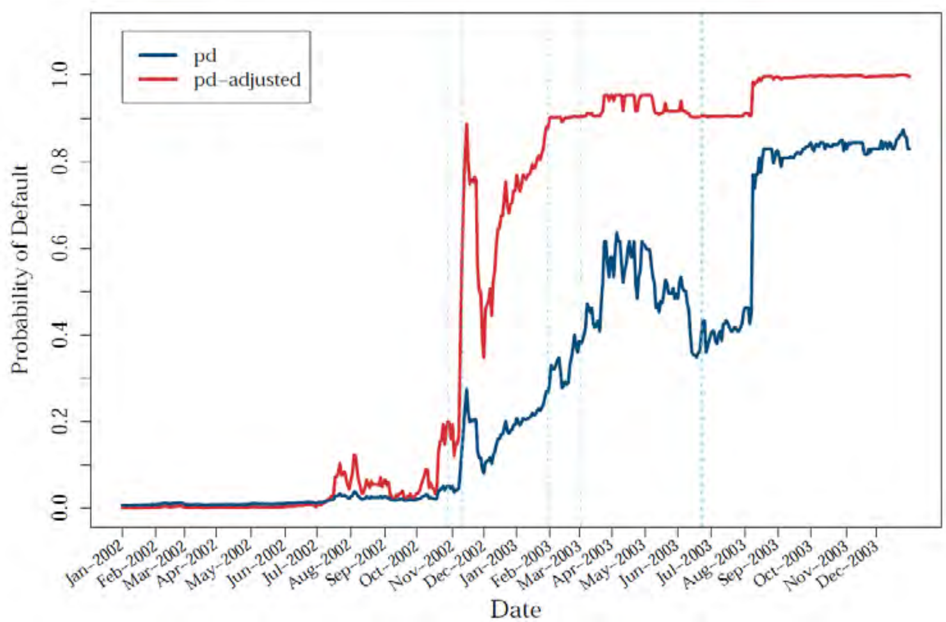


Figure 16
DAILY LEVELS OF PROBABILITY OF DEFAULT FOR TRENWICK



Note: the dotted vertical lines marked the events when S&P lowered Trenwick's rating.

4.3.4 BANKRUPT INSURANCE CASE: AMBAC FINANCIAL GROUP INC.

Ambac Financial Group Inc was once the second-largest U.S. bond insurer in the world. On June 8, 2010, Ambac announced that it would likely seek a pre-packaged bankruptcy as it was unable to pay dividends from its bond insurance unit to the holding company after suffering huge losses on risky mortgages and reported liabilities of $1.68 billion. On Nov 8, 2010, Ambac Financial Group filed for Chapter 11 bankruptcy. Under the bankruptcy protection, Ambac stock kept trading in 2011, and Ambac eventually exited Chapter 11 on May 1, 2013, issuing 45 million new common shares and approximately 5 million new warrants to holders of allowed claims.

According to Reuters, Ambac stock was a favorite target of short sellers although Ambac routinely dismissed the short sellers' concerns by stating "we have got more than sufficient capital against any claim." The stock price fluctuation reflected the very volatility nature of the company which plummeted to almost zero in 2011 as shown in Figure 17.

Figure 17

DAILY STOCK PRICE FOR AMBAC

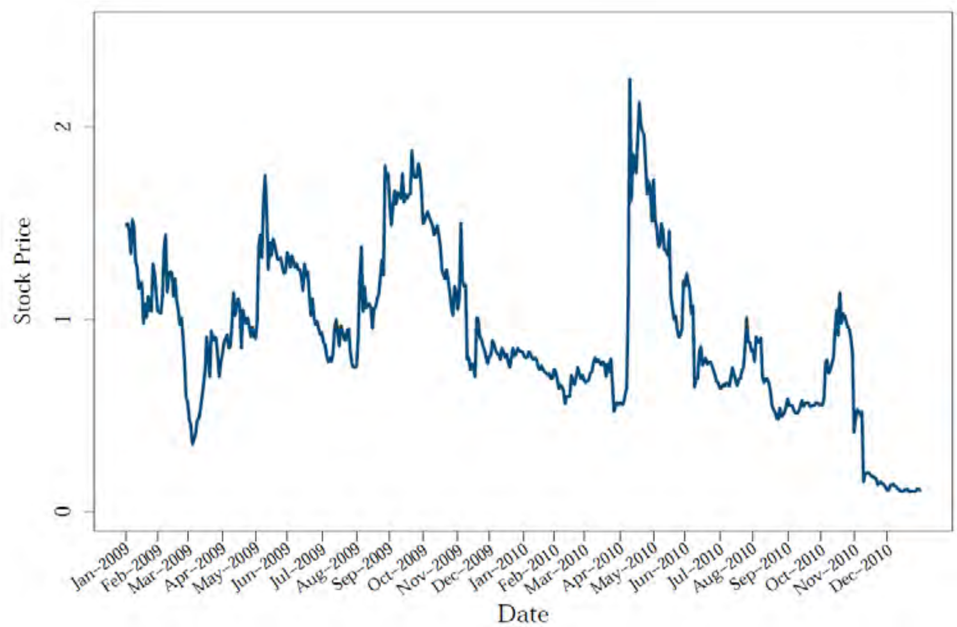
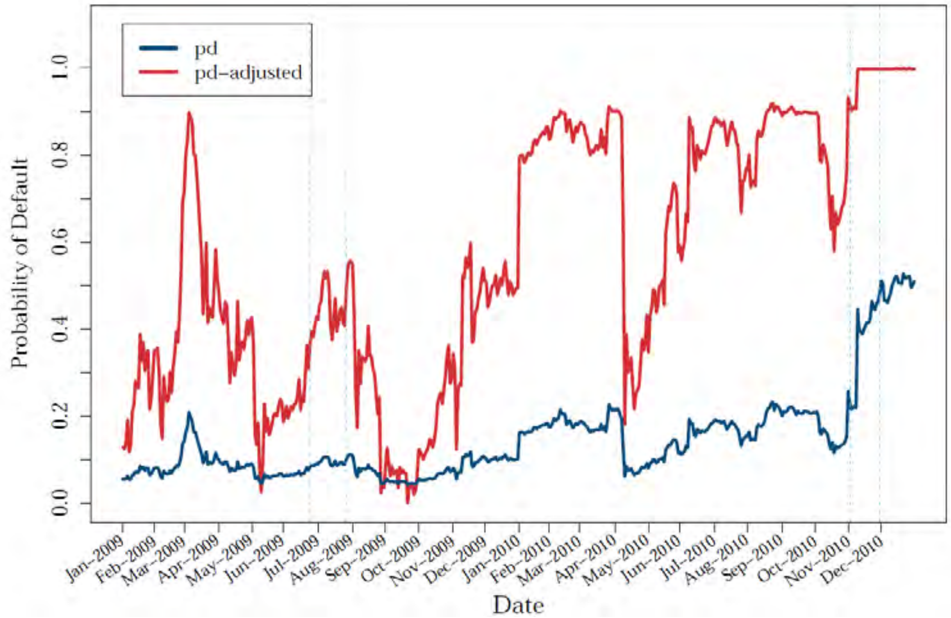


Figure 18 shows the daily probability of default by both the traditional Merton's model and the proposed higher moments adjusted Merton's model approaches for Ambac Financial Group. The proposed model was always well above the traditional Merton's model, and predicted an almost 100% probability of default in Nov 2010. Additionally, the higher moments adjusted Merton's model predicted about 90% chance of default ever since October of 2010, the Merton's model only predicted 50% chance of default at the peak.

Figure 18

DAILY LEVELS OF PROBABILITY OF DEFAULT FOR AMBAC



Note: the dotted vertical lines marked the events when S&P lowered Ambac's rating.

Section 5. Additional Discussion

5.1 MORE DISCUSSION ON THE RATING SYSTEM

According to Standard & Poor's Ratings Definitions[11], "An insurer rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the insurer to meet its financial commitments." And "Insurers rated 'BB', 'B', 'CCC', and 'CC' are regarded as having significant speculative characteristics. 'BB' indicates the least degree of speculation and 'CC' the highest. While such insurers will likely have some quality and protective characteristics, these may be outweighed by large uncertainties or major exposures to adverse conditions." These ratings may be modified by the addition of a plus (+), minus (-) and star (*) sign to show relative standing within the major rating categories.

We can observe that for a solvent firm, the normal range of higher moment adjusted probability of default during market disruptions usually should not exceed 10%. For instance, the higher moment adjusted probability of default of Allstate is way below 10% even during the most turbulent period of the 2008 financial crisis.

Once the higher moment adjusted probability of default exceeds the level of 10%, it is becoming the first warning sign for the investor and regulators. For instance, the higher moment adjusted probability of default of Conseco jumped above the 10% level at the beginning of 2002. At the same time, S&P lowered

[11] https://www.spratings.com/documents/20184/86966/Standard+%26+Poor%27s+Ratings+Definitions/fd2a2a96-be56-47b8-9ad2-390f3878d6c6

Conseco's financial strength rating from BBB- to BB+, and Conseco's Long-Term Local Issuer Credit rating from BBB- to BB+ on Jan 16, 2002. In another example, S&P lowered Acceptance's Long-Term Local Issuer Credit rating from BB+ to BB+ *- on November 16, 1999, consistent with the timing of the higher moment adjusted probability of default exceeded the level of 10%. Similarly, S&P lowered Trenwick's financial strength rating and Long-Term Local Issuer Credit rating from A- to BB+ *- on October 29, 2002, consistent with the timing of the higher moment adjusted probability of default exceeded the level of 10%. Although the traditional probability of default could still stay at almost zero at the time (for example in the case of Conseco and Acceptance), the move north of the 10% level should be concerning to the investors and regulators to consider taking some preliminary actions.

From the S&P Ratings changes, we can observe that the next key level of adjusted probability of default is at 80%. For instance, the higher moment adjusted probability of default of Conseco jumped above the 80% level at the beginning of 2002. At the same time, S&P lowered Conseco's financial strength rating and Long-Term Local Issuer Credit rating from BB+ to B+ on August 2, 2002. In another example, S&P lowered Acceptance's Long-Term Local Issuer Credit rating from B+ to B+ *- on November 15, 2002, consistent with the timing of the higher moment adjusted probability of default exceeded the level of 80%. Similarly, S&P lowered Trenwick's financial strength rating and Long-Term Local Issuer Credit rating from BB+ *- to BB- *- on November 11, 2002 and again from BB- *- to CCC on Jan 31, 2003, both consistent with the timing of the higher moment adjusted probability of default exceeded the level of 80%. Additionally, when Ambac's higher moment adjusted probability of default exceeded 80%, S&P lowered its ratings from CC to D on November 2, 2010.

These observations further confirm and justify the proposed approach in detecting insolvency. We also note that there are some obvious limitations of the S&P's rating. For instance, in the case of Conseco, S&P didn't adjust its ratings at all from Jan 16, 2002 to August 2, 2002 when its adjusted probability of default was increased from 10% to 80%. And when S&P finally adjusted its rating from BB+ to B+ on August 2, 2002, it was just 3 months away from its bankruptcy. Similarly, when the higher moment adjusted probability of default exceeded the level of 80%, S&P only lowered Acceptance's Long-Term Local Issuer Credit rating from B+ to B+ *- on November 15, 2002, and then S&P had to further lower its rating three times to R in just 11 days.

Also note that many of the firms do not have financial strength rating or long-term local issuer credit rating from S&P, but they can use the proposed method. In addition, while the frequency of the change of S&P rating is low, the proposed method provides a daily frequency on the change of probability of default, which could complement the S&P ratings system.

5.2 EARLY WARNING SYSTEM

An early warning system for companies at risk of insolvency is critical to regulators so that early remedial actions can be taken. A reliable early warning system with benchmarks will facilitate an accurate, fair and objective policy to guide regulators on the appropriate level of response. In the previous literature, insurers are often classified as solvent or insolvent using the probability cutoff scores that minimize the expected cost of misclassification in the estimation sample. Cummins and Derrig (2012) provide a good review of insurance insolvency literature. Following the previous literature, we will use the proposed framework to calculate the probability of default and determine appropriate cutoff scores as benchmarks for the early detection warning system.

Inspired by the existing warning systems and observations from the S&P rating system, we propose to develop either letter-coded benchmark as the credit rating systems or color-coded benchmarks as the disaster alert systems. For instance, Table 3 provides a possible example of a multi-color (multi-letter) early warning system.

Table 3
EXAMPLE OF COLOR-CODED AND LETTER-CODED EARLY WARNING SYSTEM

Color-coded system	Letter-coded system	Benchmarks for Probability of Default
Green	A	Probability of Default ≤ 10%
Light Yellow	B	10%<Probability of Default <20%
Bright Yellow	C	20%<Probability of Default <60%
Orange	D	60%<Probability of Default <80%
Red	E	Probability of Default ≥ 80%

Note: The benchmarks in the Table is only for example purpose and not recommendations by the authors or SOA.

The green level (or A rating grade under the letter-coded system) means "safe and low risk of insolvency" with Probability of Default ≤ 10%. Light Yellow (or B rating grade under the letter-coded system) means "caution and closely monitor for possible insolvency" with Probability of Default between 10% and 20%. Any probability of default above 20% would be very concerning for an insurance company. Bright Yellow (or C rating grade under the letter-coded system) means "escalating situation for possible insolvency" with Probability of Default between 20% and 60%. Orange (or D rating grade under the letter-coded system) means "deteriorating situation for possible insolvency" with Probability of Default between 60% and 80%. Red (or F rating grade under the letter-coded system), the highest level, means "severe risk of insolvency and immediate company action to improve financial strength" with Probability of Default ≥ 80%.

Note that the 10% and 80% cutoff points are given as a hypothetical example for illustration purpose that is consistent with the limited observations from the case examples and are not recommendations by the authors or SOA. The specific benchmarks of the probability of default for each grade can be determined by regulators based on the expected cost of misclassification and the fine-tune level of the system in assessing the insolvency risks for insurers. The regulators can further fine-tune the system with either more colors or variations of an alphabetical combination of lower-case and upper-case letters or plus/minus signs.

5.3 POLICY IMPLICATIONS

In the wake of the 2008 financial crisis when American International Group (AIG) faced severe financial insolvency problem, the U.S. state insurance regulators reassess the financial condition of insurer and the NAIC adopted the U.S. Own Risk and Solvency Assessment (ORSA) in November 2011, which requires insurance companies to issue their own assessment of their current and future risk through an internal risk self-assessment process. In Europe, Solvency II Directive was issued by the European Union in 2009 to determine the capital requirements reflecting the risk associated with an insurance company to limit the possibility of insurance company falling into financial ruin.

Both the internal process undertaken by an insurer and the external process undertaken by an insurance regulator require an adequate assessment of the current and prospective solvency positions under both normal and severe stress scenarios. Due to the lower frequency and backward-looking nature of the financial reporting data, it could be insufficient to analyze relevant insolvency risks that could have an impact on an insurer's ability to meet its policyholder obligations. One policy implication of the proposed default model is to gear insurers' capital requirements more closely to the actual and dynamic economic and market risk. It allows regulators to form an enhanced view of an insurer's ability to withstand financial stress and would help improve security and soundness of the financial system.

Section 6. Conclusions

Insolvency risk is a critical risk factor for insurers and all stakeholders. This work contributes to a growing literature on predicting corporate bankruptcy by examining the information content of higher moments in predicting defaults. The contribution of our paper lies in proposing a new market-based insolvency prediction model to measure insolvency probabilities, which significantly expands current boundary and provides valuable information to insurers, regulators, policyholders, investors, and researchers.

More specifically, we extend the Merton's model and propose the use of the Gram-Charlier series expansion and derive the adjusted probability of default with the adjustment of non-normal skewness and kurtosis. The proposed model's results are compared with predictions obtained from current popular models in the literature. We propose the use of the Gram-Charlier series expansion and derive the probability of default (PD) with the adjustment of non-normal skewness and kurtosis.

Our proposed framework shares all benefits of the basic Merton's model and significantly reduces its practical limitations of the normality assumption. The basic Merton's model is only a special case of the proposed framework when the asset log returns are indeed normally distributed. Given the violation of normality assumption from well-documented empirical evidence, our proposed framework will significantly expand the basic Merton's model by accounting for non-normal skewness and kurtosis with the use of Gram-Charlier approach. We suggest that the combination of both skewness and kurtosis into the Merton distance-to-default model should provide a more accurate prediction of insurer insolvency risk for early detection than the current models.

This research is subject to several limitations. First, as a market-based measure, it is only accessible to publicly traded firms and is not feasible to private firms. Second, due to the data limitation, the model validation is based on a small number of case example and there is some failure that couldn't be examined. For example, the 2017 Pen Treaty Network America liquidation can't be analyzed as the company stopped financial reporting years before liquidation, with SEC revoking the registration in 2013. Third, the Merton's model is sensitive to implied market volatility, so it is possible to generate false alarm when the market is very volatile. For example, during COVID, market volatility spiked but quickly mean reverted, during which a firm with high predicted probability of default is at no risk of bankruptcy. Therefore, the proposed method is only proposed to complement the existing internal models and regulatory requirements by the NAIC (ORSA and RBC ratios). In addition, it would aid in their choice of means of regulatory intervention to minimize harm of the insolvency risks.

Effective estimation and prediction of corporate defaults are critical for asset pricing, credit risk assessment of loan portfolios, and the valuation of other financial products that are exposed to corporate default. This proposed research will provide new tools and additional insights for early detection of financially weak insurance firms, which are valuable information to insurers, regulators, policyholders, investors, and researchers.

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Appendix A: The Increasing Rearrangement

It should be noted that the "probability density function" obtained by truncating an infinite series is not a true PDF and can assume negative values. Jondeau and Rockinger (2001) note that, since Gram-Charlier expansions are polynomial approximations, they have the important drawback of yielding negative values for a probability. Actually, it is not guaranteed to be positive, and therefore may violate the domain of validity of the probability distribution. This arises from the fact that the expansions are usually truncated after the fourth power, which may imply negative densities over some interval of their domain of variation (Leon, Mencia and Sentana 2007), therefore probabilities can be negative for such expansions.

Fortunately, Chernozhukov et al. (2010) present a method called the increasing rearrangement for any possible value of skewness and kurtosis. The increasing rearrangement is defined as follows: Let $f(x)$ be a measurable function mapping $[0,1] \to R$, and let $F_f(y) = \int 1\{f(u) \leq y\}du$ denote the distribution function of $f(X)$ when $X \sim U(0,1)$. The function

$$f^*(x) = inf\{y \in R: [\int 1\{f(u) \leq y\}du] \geq x\},$$

is called the increasing rearrangement of the function f. The rearrangement operator transforms the function f into its quantile function f^*, which is called the increasing rearrangement of the function f. The rearrangement is a tool extensively used in functional analysis and optimal transportation (e.g., Villani 2003).

In essence, given values of the function $f(x)$ evaluated at x in a fine enough mesh of equidistant points, sort the values in increasing order. Chernozhukov et al. (2010) prove that the rearranged function is at least as good as the originally estimated function in L_p norm, $p \in (1, \infty)$. If the originally estimated function is invalid, they show that the increasing rearrangement leads to a strictly better estimate in L_p norm.

We use this increasing rearrangement to compute the $PD_{adjusted}$ with Gram-Charlier expansion. Let $N_{GC}(x)$ be an approximation to a distribution, and $N_{GC}^*(x)$ be the rearrangement of $N_{GC}(x)$, then

$$PD_{adjusted} = N_{GC}^*(-d_2)$$

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