# Formula Sheet Used on Exam ALTAM

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## **Interest Functions**

$$\alpha(m) = \frac{id}{i^{(m)}d^{(m)}}$$
 and  $\beta(m) = \frac{i-i^{(m)}}{i^{(m)}d^{(m)}}$ 

### Makeham's Law

$$\mu_x = A + Bc^x$$
 and  $_t p_x = \exp\left(-At - \frac{B}{\log c}c^x(c^t - 1)\right)$ 

#### Three-term Woolhouse's Formula in a single decrement context

$$\ddot{a}_{x}^{(m)} \approx \ddot{a}_{x} - \frac{m-1}{2m} - \frac{m^{2}-1}{12m^{2}} (\delta + \mu_{x})$$

# Three-term Woolhouse's Formula in a multiple state context

$$\ddot{a}_x^{(m)ii} \approx \overline{a}_x^{ii} + \frac{1}{2m} + \frac{\mu_x^{i\bullet} + \delta}{12m^2} \text{ where } \mu_x^{i\bullet} = \sum_{j \neq i} \mu_x^{ij}$$

$$\ddot{a}_x^{(m)ij} = \overline{a}_x^{ij} - \frac{\mu_x^y}{12m^2} \quad i \neq j$$

Note that in the "three-term" equation for  $\ddot{a}_x^{(m)ij}$ , where  $i \neq j$ , the second term is equal to zero.

#### **Black Scholes Put Option Value**

$$p(t) = Ke^{-r(n-t)}\Phi\left(-d_2(t)\right) - S_t\Phi\left(-d_1(t)\right)$$

where 
$$d_1(t) = \frac{\log(S_t/K) + (r + \sigma^2/2)(n-t)}{\sigma\sqrt{n-t}}$$
 and  $d_2(t) = d_1(t) - \sigma\sqrt{n-t}$ 

#### **GMMB Embedded Option Value**

Assume management charges of m per year payable continuously and a GMMB of kP. Let  $F_t$  denote the fund value immediately after any expense deduction at  $t \ge 0$ .

$$p(t) = kPe^{-r(n-t)}\Phi(-d_2(t)) - F_t e^{-m(n-t)}\Phi(-d_1(t)) \text{ and } \pi(t) = \int_{n-t}^{(\tau)} p(t) dt$$

where 
$$d_1(t) = \frac{\log(F_t e^{-m(n-t)}/kP) + (r+\sigma^2/2)(n-t)}{\sigma\sqrt{n-t}}$$
 and  $d_2(t) = d_1(t) - \sigma\sqrt{n-t}$ 

Candidates are expected to use Excel to calculate Normal distribution probabilities.