# Optimality of General Reinsurance Contracts under CTE Risk Measure

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**Abstract**

By formulating as a constrained optimization model, we address the problem of optimal reinsurance design using the criterion of minimizing the Conditional Tail Expectation (CTE) risk measure of the insurer's total risk. The constraints in our model reflect a profitability guarantee or a reinsurance purchase budget for the insurance company. Under the assumption of the expectation principle for the reinsurance premium, we derive analytically optimal solutions among all general ceded loss functions.

Our proposed optimal reinsurance model is not very tractable partly due to the fact that the objective function cannot be expressed explicitly. To derive the solutions, we first transform it into a certain equivalent model, and then analyze the latter for the optimal reinsurance treaties. The transformation is achieved by exploiting a key result in Rockafellar and Uryasev (2002) while the solutions are obtained using the Lagrangian approach based on the concept of directional derivative.

**Keywords:** Optimal reinsurance, ceded loss function, conditional tail expectation (CTE), expectation premium principle, convex analysis, Lagrangian method, directional derivative, subdifferential.