GI ADV Model Solutions
Spring 2025

1.

Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5b) Explain the principal functions of reinsurance.

(5c) Analyze and describe the various types of reinsurance.

(5f) Calculate the price for a property per risk excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 10: A Reinsurance Primer

Commentary on Question:

This question tested a candidate's understanding of the purpose of reinsurance and exposure rating excess of loss reinsurance after surplus share reinsurance.

Solution:

(a) Complete the following table (***in Excel***) based upon Friedland's six principal functions of reinsurance.

Principal Functions of Reinsurance	Suitably Addressed by Proportional Reinsurance	Suitably Addressed by Non-Proportional Reinsurance
1. Provision of technical service and expertise	Yes	Yes
2. Facilitation of withdrawal from a market segment	Yes	No
3.		
4.		
5.		
6.		

1. Continued

Principal Functions of Reinsurance	Suitably Addressed by Proportional Reinsurance	Suitably Addressed by Non-Proportional Reinsurance
1. Provision of technical service and expertise	Yes	Yes
2. Facilitation of withdrawal from a market segment	Yes	No
3. Increase capacity	Yes	Yes
4. Provide catastrophe protection	No	Yes
5. Stabilize claims experience	No	Yes
6. Strengthen financial position / capital relief	Yes	Yes

(b) Calculate XS-Re's expected losses for each property (Y and Z).

Commentary on Question:
With the information provided, one could calculate the expected loss on a direct and ceded to surplus share reinsurance (SS) basis. Then calculate the TIV net of SS. The model solution follows from this starting point to calculate the expected loss for the excess of loss reinsurance (XOL). The model solution is an example of a full credit solution. It is not the only acceptable approach.

For each property, Y and Z:

1. TIV (4,000 for Y and 15,000 for Z) and the expected loss as a percentage of TIV (25% of TIV for both X and Y) is provided so that we can calculate the direct expected loss (TIV×25%).
2. SS share is proportional reinsurance in which the percent ceded to SS is determined for each amount of TIV and the SS terms. This SS is 5 lines with a retained line of 1,000. This means that for each TIV, the percent ceded to SS is MIN(MAX(0, TIV – 1,000), 5,000) / TIV (e.g., for Y, MIN(MAX(0, 3000))/4000) = 75%)
3. ABC's expected loss after the SS is (1 – percent ceded to SS) × direct expected loss (from 1.)
4. ABC's retained TIV after the SS is (1 – percent ceded to SS) × direct TIV
5. For exposure rating, first calculate the % of TIV after SS at the amount (XS limit + XS attachment) and XS attachment. (e.g., for Z, (2,000 + 1,000) / 10,000 = 30% and 1,000 / 10,000 = 10%).
6. For each of the percentages calculated (from 5.), look up the exposure factor in the table provided. (e.g., for Z, % of TIV after SS at XS attachment is 10% so the exposure factor is 37%)
7. The exposure factor for the XS layer is exposure factor at (XS limit + XS attachment) minus the exposure factor at the XS attachment.

1. **Continued**

8. The expected loss in the XS layer is calculated as the XS layer exposure factor (from 7.) by the expected loss after SS (from 3.).

2. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6c) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the experience rating methodology.

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Calculate the experience modification for this plan. Assume that no trending procedures are used.

Commentary on Question:

Many candidates organized the losses by accident year (e.g., AY 2023 is Jan. 1, 2023 to Dec. 31, 2023). They should have been organized by policy year (e.g., PY 2023 is July 1, 2023 to June 30, 2024). Note that the first two bullet points in the question refer to PY. Additionally, the last table was by PY development month.

1. For each claim, determine the PY, calculate the limited indemnity (i.e., loss indemnity s subject to the basic limit for indemnity, 20,000), and claim total i.e., (limited indemnity + ALAE) limited to the MSL, 26,000.
2. Accumulate the claims by PY. PY 2023 is at 20 development months, PY 2022 is at 32 development months, and PY 2021 is at 44 development months.
3. For each PY, the adjustment to reflect ultimate claims is PY subject premium × % Unreported at PY development months.
4. The Actual Loss Ratio (ALR) is (the sum of claim total for PYs 2021 to 2023 + the sum of adjustment to reflect ultimate claims for PYs 2021 to 2023) ÷ the sum of subject premium for PYs 2021 to 2023.
5. Credibility, Z, is based on the sum of subject premium for PYs 2021 to 2023. In the provided credibility table this is in a range where linear interpolation between two values is required.
6. Calculate the experience modification as Z × (ALR – AELR) ÷ AELR

3. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question tested a candidate's understanding of the Clark's stochastic LDF model. This question included data and results from a completed analysis (using Clark's stochastic reserving model) in Excel. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Determine the distribution selected by the likelihood ratio test. Show all calculations.

The loglikelihood for the exponential distribution model is the sum of cells Z7 to Z42. The loglikelihood for the Weibull distribution model is the sum of cells AG7 to AG42. The likelihood ratio test is twice the difference of these two amounts. This equals 10,300. This is greater than the amount for any reasonable level of significance for a chi-square distribution with 1 degree of freedom. Therefore, the null hypothesis is rejected and the distribution with the greater logli8kleihood is selected. This is the Weibull distribution.

(b) Estimate the scale factor, σ^2.

Commentary on Question:

This is calculated in column AH of the Excel solutions spreadsheet tab Q03.

The scale factor is the 1/(n-p) times the sum of (data point paid amount minus the mu-hat for the data point)^2 divided by mu-hat for the data point, where n is the number of data points and p is the number of parameters. Here, there are 36 data points and 10 parameters (8 for years plus omega and theta).

(c) Estimate the process standard deviation of the loss reserve for all accident years combined.

3. Continued

- The total reserve amount is the sum of ULT values from the Weibull analysis that was provided minus the sum of the paid amounts.
- The process variance is the scale factor times the total reserve amount.
- The process standard deviation is the square root of the process variance.

(d) State what this assumption means in terms of the LDF method for loss development.

The same distribution and parameters apply to each accident year.

(e) State one reason why this assumption is unlikely to be true.

Commentary on Question:
There is more than one reason. The model solution is an example of a full credit solution.

There may be different mixes of business from year to year.

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question tested a candidate's understanding of modeling the chain ladder method. It included data and results from a completed analysis in Excel tab Q04. Candidates did not need to perform any calculations. Responses to the questions were in Word using their observations of the analysis provided.

Solution:

(a) State the two aspects that Mack is referencing in this statement.

The same age-to-age factor is used for all accident years and only the most recent observed value is used to project the next value.

(b) Determine if the data support Mack's assumption. Justify your response based on both the graph and the regression output.

The graph indicates that a straight line fits the data very well, but may not go through the origin (the intercept is 20,445). The hypothesis that the intercept is 0 should be rejected based on the regression analysis. Hence, the Mack assumption does not hold.

(c) Describe this test, providing a verbal description, not formulas.

For a fixed development year k, rank the ratios from 1 (smallest) to $I - k$. Do the same with the preceding development factors, leaving out the last one.

4. Continued

Calculate the rank correlation coefficient for these pairs. Then do the same for each value of *k*.

Next, take a weighted average of the $I - 3$ coefficients. Divide this test statistic by its standard deviation. If the absolute value is smaller than a normal distribution significance cutoff, accept the null hypothesis of no correlation.

(d) Describe how the sum of squared errors (SSE) is calculated as presented in Venter.

For each development year (DY) starting with DY2, predict the value by applying the development factor to the previous observed value. Calculate the squared difference of the predicted and actual values and then add them up over all observations.

(e) Explain why the number of parameters must be taken into account when ranking the accuracy of models.

A model with more parameters is likely to fit better due to its complexity. Hence, models with more parameters should be penalized when comparing SSE.

(f) Determine the values of *n* and *p* for the fitted chain ladder model.

$n = 45$ because the calculations start with DY2, so $9 + 8 + .. + 1 = 45$ observations. The number of parameters for the chain ladder method are the number of development factors, which is $p = 9$.

(g) State the formula for one of the adjustments.

Commentary on Question:
The model solution is an example of a full credit solution.

Divide by $(n - p)^2$; multiply by $\exp(2p/n)$; multiply by $n^{p/n}$

5. Learning Objectives:

7. The candidate will understand the application of game theory to the allocation of risk loads.

Learning Outcomes:

(7a) Allocate a risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question tested a candidate's ability to calculate property catastrophe risk loads based upon Mango's approach. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

RL = risk load, RLM = RL multiplier, MS = marginal surplus, MV = marginal variance, RLM_{MS} = RLM for MS, RLM_{MV} = RLM for MV, VAR(x) =variance of x, COV(x ,y) = covariance of x and y SD(x) = standard deviation of x, p = probability, L = loss

Solution:

(a) Calculate the renewal risk load for each treaty using the Marginal Variance method.

1. Copy the table and then create an additional column in the table for Treaties P and Q combined.
2. Calculate the mean and VAR for each of P, Q and P+Q. Note that we have five scenarios of which only one may occur. So, VAR = $\Sigma\, pL^2 - (\Sigma pL)^2$. Note that scenario V has L=0 so it can be left out of the calculations.
3. Using Kreps formula, the RLM_{MS} is return on MS × standard normal multiplier ÷ [1 + return on MS]
4. RLM_{MV} is equal to: $RLM_{MS} / (VAR(P+Q))^{0.5}$
5. Calculate the MV for P as VAR(P+Q) – VAR(Q). Calculate the same statistic for Q in a similar manner.
6. Calculate the RL for P and Q as RL(P) = MV(P) × RLM_{MV} and RL(Q) = MV(Q) × RLM_{MV}.

(b) Calculate the renewal risk load for each treaty using the Shapley method.

1. Calculate COV(P, Q) = [VAR(P+Q) – VAR(P) – VAR(Q)] ÷ 2.
2. Calculate the Shapley values
Shapley value P = VAR(P) + COV(P, Q) and
Shapley value Q = VAR(Q) + COV(P, Q)
3. For the Shapley method,
RL(P) = Shapley value(P) × RLM_{MV} and
RL(Q) = Shapley value(Q) × RLM_{MV}.

5. Continued

(c) Describe the relationship between the value of the total risk load for the combined portfolio of P and Q using the Marginal Surplus method versus each of the Marginal Variance and Shapley methods.

Commentary on Question:
The model answer is an example of a full credit solution. Note that the relation in the solution occurs because Marginal Surplus is sub-additive while Marginal Variance is super additive and Shapley is additive.

The Marginal Surplus combined portfolio total risk load would be lower than the combined portfolio total risk load under both the Marginal Variance and Shapley methods.

6. **Learning Objectives:**

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4a) Explain the mathematics of excess of loss coverages in graphical terms.

(4c) Explain and calculate the effect of economic and social inflationary trends on first dollar and excess of loss coverages.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question tested a candidate's knowledge of Lee's approach to excess of loss coverage.

Solution:

(a) Show the formula for each of the following using the above notation:

(i) Increased limits factor (ILF) for losses defined as being limited to an amount K, in which the basic limit is an amount B

(ii) Derivative of the ILF from (i) with respect to changes in K

(iii) Expected loss payment for losses defined as being limited to an amount K in which all losses increase by 5%

(iv) Percent reduction of loss frequency for losses defined as being limited to an amount K when the loss is above a specified amount that changes from J to $J + V$, where $V > 0$

(i) $\mathrm{E}\{h(X;\, 0,\, K)\} \,/\, E\{h(X;\, 0,\, B)\}$

(ii) $G(K) \,/\, E\{h(X;\, 0,\, B)\}$

(iii) $1.05 \times E\{h(X;\, 0,\, K/1.05)\}$

(iv) $100\% \times (1 - E\{N_{J+V})\}/\, E\{N_J)\})$

(b) The pure premium from a policy covering losses above a deductible d may be given by $E[N_d]$•$E[h(X)]$. An assumption must be made for this formula to be true.

State this assumption.

6. Continued

The loss severity X is distributed independently of the loss frequency N_d

(c) Express each of the following quantities using the labels for the nine areas on the graph:

(i) Increased limit factor for $x2$ before trend

(ii) Increased limit factor for $x2$ after trend

(iii) Trend factor for basic limit losses

(iv) Trend factor for losses limited to $x2$

(v) $x2 - x1$

(vi) Expected ground up losses after trend

(i) (F + I) / I

(ii) (E + F + H + I) / (H + I)

(iii) (H + I) / I

(iv) (E + F+H + I) / (F + I)

(v) D + E + F

(vi) B + C + E + F + H + I

7. **Learning Objectives:**

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4g) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Discussion of Estimating the Premium Asset on Retrospectively Rated Policies, Feldblum

Commentary on Question:

This question tested a candidate's understanding of the methods to calculate the premium asset on retrospectively rated policies.

Solution:

(a) Define the following as used in Fitzgibbon's approach:

(i) Ultimate premium deviation

(ii) Retro reserve

(i) Difference between the ultimate premium and the standard premium
(ii) Difference between the premium deviation using premium to date and the ultimate premium deviation

(b) Describe the method used by Fitzgibbon to estimate the ultimate premium deviation.

Analyze the historical relationship between the loss ratio and the premium deviation using statistical techniques and then apply this relationship to the projected loss ratio.

(c) Describe Berry's approach to estimate the retro reserve.

Estimate ultimate premium using the historical premium emergence pattern and then subtract current premium.

7. Continued

(d) Teng and Perkins developed their approach because they believed there was an issue with the approaches presented by Fitzgibbon and Berry.

Describe this issue.

These approaches lack intuitive appeal as they do not reflect how the retro rating formula works.

(e) Provide an argument in favor of the empirical approach over the formula approach.

Commentary on Question:
More than one argument in favor can be made. The model solution is an example of a full credit solution.

The formula approach represents how the retro rating parameters affect the premium development with loss development.

(f) State the formula from Teng and Perkins for the premium asset for policy year X as of the valuation date, using the notation above.

((ULT – RL-Prior) × CPLD + PB-Prior) – PB-Val

8. **Learning Objectives:**

3. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(3a) Describe a risk margin analysis framework.

(3b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(3c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question tested a candidate's understanding of risk margins as set out in Marshall. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why it may <u>not</u> be preferable to split the claims portfolio for risk margin analysis at the same granular level as used for central estimate valuation purposes.

The valuation portfolio allocation may be at a more granular level than makes practical sense.

(b) Calculate the Total Independent Risk Coefficient of Variation (CoV) for both lines combined.

Commentary on Question:

PL = premium liabilities, OSC =outstanding claims liabilities,
IL = insurance liabilities (comprised of PL and OSC),
LT = liability type (PL or OSC) , LOB = line of business (Liability or Home)
CE = central estimate, SD = standard deviation
IR = independent risk

1. Calculate the Independent Risk CoV for each of the four combinations of liability type and line of business.

   $CoV_{IR}(i, j) = SD_{IR}(i, j) \div CE_{IR}(i, j)$

2. Total Independent Risk CoV is calculated as follows:

8. **Continued**

$$\sqrt{\sum_{i(\text{LOB})}\sum_{j(\text{LT})}\left[\text{CoV}_{\text{IR}}(i,j)\times\text{CE}(i,j)\right]^2} \Big/ \sum_{i(\text{LOB})}\sum_{j(\text{LT})}\text{CE}(i,j)$$

(c) Calculate the total internal systemic risk CoV for both lines combined.

Commentary on Question:
ISR = internal systemic risk, ILW = IL weight, IL% = percentage of IL, CVAR = covariance

1. Calculate IL% for each of the four combinations of liability type and line of business.

$$\text{IL\%}(i,j) = \text{CE}(i,j) \Big/ \sum_{i(\text{LOB})}\sum_{j(\text{LT})}\text{CE}(i,j)$$

2. Create a symmetric matrix (IL,LOB by IL,LOB) of ILWs from IL%.

$\text{ILW}[k,l] = \text{IL\%}(k)\times\text{IL\%}(l)$ where
both *k* and *l* represent a LOB-IL combination creating a 4 by 4 matrix.

3. Create the covariance matrix (IL,LOB by IL,LOB) of CoV_{ISR} values.

$\text{CVAR}_{\text{ISR}}[k,l] = \text{CoV}_{\text{ISR}}(k)\times\text{CoV}_{\text{ISR}}(l)$ where
both *k* and *l* represent a LOB-IL combination creating a 4 by 4 matrix.

4. Total Independent Risk CoV is calculated as follows:

$$\sqrt{\sum_k\sum_l \text{COR}_{\text{ISR}}[k,l]\times\text{ILW}[k,l]\times\text{CVAR}_{\text{ISR}}[k,l]}$$

(d) Marshall et al. identify several additional analyses that may be conducted to give an actuary further comfort regarding this approach for calculating risk margins.

Describe two of the additional analyses identified.

Commentary on Question:
There are more than two additional analyses. The model solution is an example of a full credit solution.

8. **Continued**

Sensitivity testing to determine how key assumptions affect final outcomes by varying each of the assumptions.

External benchmarking which involves comparing adopted CoVs against industry benchmarks.

9. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5k) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Commentary on Question:

This question tested a candidate's understanding of methods to measure the existence of risk transfer.

Solution:

(a) Describe what must be shown for a contract to be identified as transferring "substantially all of the insurance risk" of the primary contract.

The downside risk assumed by the reinsurer is essentially the same as that faced by the cedant with respect to the original portfolio without reinsurance.

(b) Identify two types of reinsurance contracts in which risk transfer is "reasonably self-evident."

Commentary on Question:

There are more than two types. The model solution is an example of a full credit solution.

- Standard catastrophe excess of loss contracts,
- Per risk excess of loss contracts without any loss sensitive features

(c) Ruhm and Brehm present one rule and several risk metrics for determining risk transfer.

Describe the rule.

10-10 rule: Threshold for risk transfer requires a 10% chance of a 10% net loss.

(d) Describe two of the risk metrics.

Commentary on Question:

There are more than two risk metrics. The model solution is an example of a full credit solution.

9. Continued

Expected reinsurer deficit metric is based on defining risk as the probability of net economic loss times the average loss severity, measured against expected premium as the base.

Value at Risk metric defines risk by a percentile, such as the 95th percentile of annual loss.

(e) Identify a type of reinsurance that doesn't meet the rule from part (c) but clearly transfers significant insurance risk.

Catastrophe excess of loss reinsurance (low-probability and high severity).

10. **Learning Objectives:**

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) Understand the difference in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question tested a candidate's knowledge regarding the development of excess limits and layers. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total IBNR as of December 31, 2024 for the layer 500,000 excess of 500,000, using Siewert's formula.

Commentary on Question:

There is more than one way to do this calculation using Siewert's formula. The model solution is an example of a full credit solution that is laid out so that it is easy to follow.

CDF(x, K) = cumulative development factor at x months of development for a limit amount K where K is in thousands, ULT(AY, K) = ultimate claims for AY at limit amount K, C(AY, x, K) = reported claims at x months of development for AY at limit amount K.

1. Calculate CDFs at total limits [CDF(x, total)] using the incremental development factors at total limits.

2. Calculate CDFs for 500,000 and 1,000,000 limit using Siewert's formula.

For a K limit as of Dec. 31, 2024, for m=12, 24, …72:

$\text{CDF}(m, K) = \text{CDF}(m, \text{total}) \times R_{84}(500) / R_m(K)$

10. **Continued**

3. Calculate ultimate claims (ULT) by AY for 500,000 limit, 1,000,000 limit and total limits.

   For a K limit, as of Dec. 31, 2024, AY 20XX is at
   $m = 12 \times (2024 + 1 - 20XX)$ months of development and
   ULT(20XX, K) = CDF(m, K) × C(20XX, m, K), AY=2018 to 2024

4. Calculate IBNR by AY for the layer 500,000 excess of 500,000:

   For each AY 20XX:
   ULT(20XX, 500 excess 500) = ULT(20XX, 1,000) − ULT(20XX, 500)
   IBNR (20XX, 500 excess 500) =
   ULT(20XX, 500 excess 500) − (C(20XX, m, 1,000) – C(20XX, m, 500)

5. Total IBNR for the layer 500,000 excess of 500,000 is the sum of IBNR by AY for AY = 2018 to 2024.

(b) Calculate the total IBNR as of December 31, 2024 for losses excess of 1,000,000, using Siewert's formula.

Commentary on Question:
There is more than one way to do this calculation using Siewert's formula. The model solution is an example of a full credit solution that is laid out so that it is easy to follow.

1. Calculate CDFs for excess 1,000,000 limit using Siewert's formula.

   For *excess* 1,000,000 as of Dec. 31, 2024, for m=12, 24, …72:
   CDF(m, *excess* 1,000) = CDF(m, total) × $(1 - R_{84}$ (1,000)) / $(1 - R_m$(1,000))

2. Calculate reported claims for excess 1,000,000.

   For AY 20XX = 2018 to 2024:
   $m = 12 \times (2024 + 1 - 20XX)$ months of development and
   C(20XX, m, *excess* 1,000) = C(20XX, m, Total) − C(20XX, m, 1,000)

3. Calculate IBNR by AY for excess of 1,000,000:

   For each AY 20XX:
   IBNR(20XX, *excess* 1,000) =
   [CDF(m, *excess* 1,000) – 1] × C(20XX, m, *excess* 1,000)

4. Total IBNR for excess of 1,000,000 is the sum of the IBNR by AY for excess 1,000,000 for all AYs.

10. Continued

(c) Provide one advantage and one disadvantage of estimating development using the theoretical approach.

Advantage: The approach can be used when complete data triangles at alternative limits are not available or when the available data are insufficient or unreliable.

Disadvantage: The formulas are very sensitive to the estimated relativities which require professional judgement.

(d) Calculate the ultimate losses for AY 2024 as of December 31, 2024 for losses excess of 1,000,000, using the ILF method.

Commentary on Question:
The model solution is an example of a full credit solution that is laid out so that it is easy to follow.

1. Calculate the trended ILF for use on AY 2024.

   The ILFs are for annual policies effective Jan. 1, 2025. The average accident date for these policies is July 1, 2025.
   For AY 2024, the average accident date is July 1, 2024. Therefore, we need to trend the given ILFs by −1 year. Trend is 4%.
   Th ILF for total relative to 1,000,000 is 1.082.
   The trended ILF total relative to 1,000,000 is $1.082\times(1.04^{-1}) = 1.04$.

2. Calculate ultimate claims for excess of 1,000,000 using the ILF method.

   IBNR(2024, 1,000) is given as 1,320,000
   ILF(total relative to 1,000) for AY 2024 = 1.04
   ULT(2024, total) =
   ILF(total relative to 1,000) × [C(2024, 12, 1000) + 1,320,000]
   ULT(2024, 1,000) = C(2024, 12, 1000) + 1,320,000
   ULT(2024, *excess* 1,000) = ULT(2024, total) − ULT(2024, 1,000)

11. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6b) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question tested a candidate's understanding of the reasons for claims-made policies and the principles of claims-made ratemaking.

Solution:

(a) Compare claims-made coverages to occurrence coverages for the following features:

(i) Investment income earned on insurance funds

(ii) Cost in an inflationary environment affecting both frequency and severity

(iii) Pricing accuracy when there are sudden unpredictable changes in trend

(i) Investment income earned from claims-made policies is substantially less than under occurrence policies.

(ii) A claims-made policy should always cost less than an occurrence policy, as long as claim costs are increasing.

(iii) Claims-made policies priced on the basis of the prior trend will be closer to the correct price than occurrence policies priced in the same way

(b) Provide two scenarios that show how this coverage gap can occur.

Commentary on Question:

There are more than two scenarios. The model solution is an example of a full credit solution.

1. An insured switches from claims-made coverage to occurrence coverage. There then exists a gap for claims that have not yet been reported but occurred before the effective date of the new occurrence policy.

11. Continued

2. An insured switches insurers for their claims-made coverage. The new insurer's retroactive date is later than the old insurer's retroactive date. There then exists a gap in coverage for claims that occurred between the two retroactive dates and have not yet been reported.

(c) Describe the following:

(i) Step factor

(ii) Tail policy

(iii) Tail factor

Commentary on Question:
These terms were meant to be described in the context of claims-made rating. However, this was not clearly stated. Some candidates described tail factor in the context of claims development. They were given credit for this response.

(i) The relativity of the claims for an immature claims-made policy to the claims for the mature claims-made policy.

(ii) Policy covering claims that are reported after a claims-made policy has expired for claims that occurred while the claims-made policy was in effect.

(iii) The relativity of the claims covered by a tail policy to the claims reported under the claims-made policy.

(d) Describe the risk of reserve inadequacy for claims-made policies relative to occurrence policies.

Claims-made policies incur no liability for pure IBNR claims so the risk of reserve inadequacy is greatly reduced.

12. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6a) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question tested a candidate's understanding of deductibles. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the amount the insurer must pay for each claim using each of the following deductible terms:

(i) Split (MB/BI) Straight 4,000 / 8000

(ii) Disappearing 15,000 to 50,000

(iii) Franchise 20,000

(iv) Split (MB/BI) Franchise 3,000 / Time 1-day

(v) Aggregate 130,000

(vi) Percentage 4%

Commentary on Question:

Note that the solution in Excel shows some cells as generalized formulas to calculate the claim amount after deductible. Candidates were not expected to derive these. It was expected that with only three claims each would be viewed individually. DED = deductible

(i) Split (MB/BI) Straight 4,000 / 8000

C01: MB 18,730 – 4,000 = 14,730, BI 105,000 – 8,000 = 97,000,
→MB&BI = 111,730
C02: MB 8,330 – 4,000 = 4,330, BI 10,000 – 8,000 = 2,000
→MB&BI = 6,330
C03: MB 15,315 – 4,000 = 11,315, BI 60,250 – 8,000 = 52,250
→MB&BI = 63,565
TOTAL: 111,730 + 6,330 + 63,565 = 181,625

12. Continued

(ii) Disappearing deductible (DD) 15,000 to 50,000

C01: loss is 132,730, DD is 0; claim limit applies
→ 120,000
C02: loss is 18,330, DD = 15,000×(50,000 –18,330)÷(50,000–15,000) = 13,753
→ 4,757 [= 18,330 – 13,753]
C03: loss is 75,535, DD is 0
→ 75,565
TOTAL: 120,000 + 4,757 + 75,565 = 200,322

(iii) Franchise deductible (FD) 20,000

C01: loss is 123,730 > FD so no DED applies; claim limit applies
→ 120,000
C02: loss is 18,330 < FD, so DED = 18,330
→ 0
C03: loss is 75,535, > FD so no DED applies
→ 75,565
TOTAL: 120,000 + 0 + 75,565 = 195,565

(iv) Split (MB/BI) Franchise 3,000 / Time 1-day

C01: MB loss is 18,730 > FD so no DED applies, claim is 18,730
BI loss is 25,000 day 1, 80,000 after, time DED is 25,000, claim is 80,000
→ 98,730
C02: MB loss is 8,330 > FD, so no DED applies, claim is 8,330
BI loss is 10,000 day 1 so DED is 10,000, claim is 0
→ 8,330
C03: MB loss is 15,315 > FD so no DED applies, claim is 15,315
BI loss is 20,250 day 1, 40,000 after, claim is 40,000
→ 55,315
TOTAL: 98,730 + 8,330 + 55,315 = 162,375

(v) Aggregate deductible (AGG) 130,000

C01: loss is 123,730, first claim, fully within AGG; the AGG applies; claim is 0; the remaining AGG is 6,270
→ 0
C02: loss is 18,330; remaining AGG of 6,270 applies; claim is 12,060
AGG is exhausted
→ 12,060
C03: loss is 75,565, third claim, no AGG remains; claim is 75,565
→ 75,565
TOTAL: 0 + 12,060 + 75,565 = 87,625

(vi) Percentage deductible (PD) 4% of total insured value (TIV)
DED = 4% of 120,000 = 4,800

12. Continued

C01: 123,730 – 4,800 = 118,930
C02: 18,330 – 4,800 = 13,530
C03: 75,565 – 4,800 = 70,765
TOTAL: 118,930 + 13,530 + 70,765 = 203,225

(b) Compare the insurer's treatment of a policy deductible for first-party coverages versus third-party coverages.

First-Party: Insurer pays the insured the loss amount reduced for the deductible.

Third-Party: Insurer pays the third party the full amount of the loss and recovers the deductible from the insured.

13. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5h) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to use a collective risk model for catastrophe risks. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Demonstrate that the mean and coefficient of variation of aggregate losses are 2.065 billion and 1.179 billion, respectively.

1. Calculate the expected number of losses (N) and expected loss size (X)

$$\mathrm{E}(N)=\sum_{n=0}^{4} np_{N=n}=1.11 \text{ and } \mathrm{E}(X)=\sum_{x=1}^{4} xp_{X=x}=1.86$$

2. Calculate the expected value of N^2 and X^2

$$\mathrm{E}(N^2)=\sum_{n=0}^{4} n^2 p_{N=n}=2.67 \text{ and } \mathrm{E}(X^2)=\sum_{x=1}^{4} x^2 p_{X=x}=4.32$$

3. Calculate the variance of N and X

$$\mathrm{Var}(N)=\mathrm{E}(N^2)-\mathrm{E}(N)^2=1.4379 \text{ and } \mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2=0.8604$$

4. Calculate mean and coefficient of variation (CoV) of aggregate losses

$$\mathrm{E}(NX)=\mathrm{E}(N)\times\mathrm{E}(X)=1.11\times1.86$$
$$=2.065$$

$$\mathrm{CoV}(NX)=\frac{\left(\mathrm{E}(N)\times\mathrm{Var}(X)+\mathrm{Var}(N)\times\mathrm{E}(X)^2\right)^{0.5}}{\mathrm{E}(NX)}=\frac{(1.11\times0.8604+1.4379\times1.86^2)^{0.5}}{2.065}$$
$$=1.179$$

13. Continued

(b) Demonstrate that the method of moments estimates are $\mu = 0.289$ and $\sigma^2 = 0.872$.

$$\sigma^2 = \log_e(\text{CoV}(NX)^2 + 1) = \log_e(2.39109) = 0.872$$
$$\mu = \log_e(\text{E}(NX)) - \frac{\sigma^2}{2} = \log_e(2.065) - 0.436 = 0.289$$

(c) Calculate the probability of aggregate losses exceeding 4 billion using the lognormal model.

Commentary on Question:
There are several ways to calculate the answer to this question. The Excel spreadsheet shows three different ways (using Excel normal distribution function, using the standard normal table with interpolation and using the standard normal table taking a simple average). For methods using the table, the spreadsheet uses functions to acquire the proper value from the table. This was not expected from candidates. Citing the proper value from a visual inspection of the table was acceptable.

$\mathsf{N}(X, \mu, \sigma)$ is the probability of a random variable <=X using a normal distribution with mean μ and standard deviation σ.

$$P(XN > 4) = 1 - \mathsf{N}(4, \mu, \sigma) = 1 - \mathsf{N}(4, 0.289, 0.934) = 0.12$$

(d) State one advantage and one disadvantage of using a lognormal model.

Commentary on Question:
There are several advantages and disadvantages. The model solution is an example of a full credit solution.

Advantage: It's simple to use, even when source data is limited.
Disadvantage: There is no easy way to reflect changing per occurrence limits.