Arbitrage-Free Perspective on Economic Capital Calibration

by David Wang

A stockbroker comes into the office in the morning, logs on to his computer, and sees two different price quotes for the same stock. Naturally, he puts in buy orders on the lower quote and sell orders on the higher quote. He can make money out of it until the stock is listed with just a single price quote.

This is a simple example of arbitrage opportunity. In reality, arbitrage opportunities rarely exist, and, when they do, market participants (especially hedge funds) jump on them fast and they disappear quickly. Therefore, "arbitrage-free" is an important assumption in finance. At any time, a given asset should only have a single price. That assumption further leads to risk-neutral valuation techniques. Because there can be only one price on the asset, market participants with different risk tolerance levels will have to reach the same price. Removing risk premium and assuming risk-neutral thus provides a consistent pricing framework for all investors.

Insurance products are, of course, nontradable, and thus do not have an observable market price. However, market-consistent reporting, such as market-consistent embedded value (MCEV), Solvency II or International Financial Reporting Standards (IFRS), attempts to put a price to insurance products using market-consistent principles.

A company typically determines the market-consistent value of its products using the risk-neutral valuation techniques, particularly if those products include embedded guarantees. There can sometimes be debate on how risk-neutral parameters can be calibrated, particularly for long-term liabilities. For the purposes of this essay, we are going to ignore such debate and instead assume that a final price has been agreed on, at least internally by the company, as a fair market price for the products.

Let us further assume that this price is determined in accordance with CFO Forum MCEV principles.* If we simplify the MCEV calculations, then the price can be determined as follows:

Formula 1:

Price = Risk-Neutral Net Cash Flows (RNNCF)

– Cost of Non-Hedgeable Risks (CNHR)

– Frictional Cost (FC)

RNNCF calculates the average of the present values of net cash flows related to the insurance products across risk-neutral scenarios. Because risk-neutral valuation is used, it essentially captures all the market risks that can be hedged.

Risk-neutral valuation assumes investment returns that are the same as the discount rates. Thus, the emergence of earnings and the timing of regulatory reserves and capital have no impact on the results. In other words, the increase in reserve and capital is offset by the interest earned on reserve and capital. The only cost of capital captured in the calculation is the cost of non-hedgeable risk capital through CNHR and the taxation/investment expense through FC.

Now let us pause here and think about the "arbitrage-free" assumption at the beginning. At any time, there can be only one price on any asset. If the company considers the price calculated above as the fair price for its products, then it must hold true that the same price has to be arrived at if the company uses a real-world pricing approach instead of a risk-neutral approach.

This gives us a very good basis to calibrate the appropriate economic capital.

* Please refer to http://www.cfoforum.nl/embedded_value.html for details.

Arbitrage-Free Perspective ... *by David Wang*

In real-world pricing, the company would replace all economic scenarios and assumptions with those reflecting realistic probabilities. Risk premiums are allowed to be assumed in the projection. And if the present values are discounted at the earned rate, the impact of reserve and capital is neutral, just as it is in risk-neutral pricing.

Let us denote real-world net cash flows (RWNCF) to be the average of the present values of net cash flows related to the insurance products across real-world scenarios. Because risk premiums are explicitly allowed in the scenarios, RWNCF benefits from the higher expected return without proper allowance for the higher market risk. Therefore, to reach the same price, RWNCF has to be reduced by a cost of capital that includes both CNHR and the cost of hedgeable market risks, or the cost of the entire economic capital.

Formula 2:

Price = Real-World Net Cash Flows (RWNCF)

– Cost of Total Economic Capital (CTEC)

– Real-World Frictional Cost (RWFC)

If we combine Formula 1 and Formula 2, we get

Equation 1:

$$\text{Price} = \text{RNNCF} - \text{CNHR} - \text{FC} = \text{RWNCF} - \text{CTEC} - \text{RWFC}$$

This equation provides a very useful guideline for the company in its economic capital calibration. In particular, it helps the company define the economic capital tail event that corresponds to the degree of risk the company takes on. For example, the European Solvency II sets the tail event to be 1 over 200, and the U.S. C3 Phase II sets the tail event to be a conditional tail event of 90 (CTE90). But in reality, companies vary significantly in all respects, including product mix, investment strategy and experience monitoring, and therefore the degree of risk each company is exposed to should vary significantly too. Having the same tail event is certainly recommended for regulatory capital such as Solvency II and C3 Phase II, but each company should still determine an economic capital that really matches its own risk.

Equation 1 suggests that the appropriate economic capital tail event should be set such that the equation will hold. In other words, real-world pricing will not overstate the price of the products as long as the economic capital considered matches all the risks that the products expose the company to.

One often-debated issue in economic capital calculation is whether it should be a runoff approach as with the C3 Phase II or a one-year shock approach as with Solvency II. Equation 1 suggests that it probably does not matter because there can only be one price and therefore results from different economic capital models should be the same. Thus the selection of the economic capital calculation approach becomes more a modeling decision.

Another debate in actuarial work is whether pricing should be done on a risk-neutral basis or a real-world basis. Equation 1 suggests that both should provide the same answer as long as the correctly calibrated economic capital is recognized in real-world pricing. Typically in the United States, however, real-world pricing only recognizes the regulatory capital. Companies need to realize that the resultant price may not fully reflect all the risks companies are exposed to.

Arbitrage-Free Perspective ... *by David Wang*

The application of Equation 1 can range from one product, to a product line, to the entire corporation. The corporate-level application is probably more meaningful because it allows for diversifications across different products, and the market capitalization of the company can be directly used as the price instead of having to perform a risk-neutral valuation and a real-world valuation.

In summary, Equation 1 suggests a clean and conclusive way to calibrate the economic capital. However, a lot of the details still need to be studied when we apply Equation 1 in the real world. One of the biggest challenges is perhaps how a company can arrive at the market-consistent price for a long-term product with complicated guarantees. We will not discuss it in this essay, but will continue our research and discussions in a separate paper.

David Wang, FSA, FIA, MAAA, is a principal in the Life Practice at Milliman Inc., in Seattle, Wash. He can be contacted at *david.wang@milliman.com.*