Optimal Reversible Annuities to Minimize the Probability of Lifetime Ruin

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Abstract

We find the minimum probability of lifetime ruin of an investor who can invest in a market with a risky and a riskless asset and who can purchase a reversible life annuity. The surrender charge of a life annuity is a proportion of its value. Ruin occurs when the total of the value of the risky and riskless assets and the surrender value of the life annuity reaches zero. We find the optimal investment strategy and optimal annuity purchase and surrender strategies in two situations: (i) the value of the risky and riskless assets is allowed to be negative, with the imputed surrender value of the life annuity keeping the total positive; or (ii) the value of the risky and riskless assets is required to be non-negative. In the first case, although the individual has the flexiblity to buy or sell at any time, we find that the individual will not buy a life annuity unless she can cover all her consumption via the annuity and she will never sell her annuity. In the second case, the individual surrenders just enough annuity income to keep her total assets positive. However, in this second case, the individual's annuity purchasing strategy depends on the size of the proportional surrender charge. When the charge is large enough, the individual will not buy a life annuity unless she can cover all her consumption, the so-called safe level. When the charge is small enough, the individual will buy a life annuity at a wealth lower than this safe level.

Key words. Life annuities, retirement, optimal investment, stochastic control, free-boundary problem

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