GI ADV Model Solutions Spring 2021

1. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) The underlying losses have the following characteristics:

• Mean	300
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• Standard Deviation 1,200

The estimated parameters of the lognormal distribution based on the method of moments are:

- mu (µ) 4.287
- sigma (σ) 1.683

Demonstrate that this is true.

Commentary on Question:

The appropriate formulas for the lognormal distribution are sigma = $(ln[(sd/mean)^2+1])^0.5$ and $mu = ln(mean) - (sigma^2)/2$. Using these formulas one can demonstrate that mu and sigma are as shown. An alternative solution is to use the estimated parameters to calculate the mean and standard deviation.

sigma = $\ln[(1,200/300)^2+1]^0.5 = 1.683$ mu = $\ln(300) - (1.683^2)/2 = 4.287$

(b) Demonstrate that the ILF at policy limit 1,500 is 1.44.

Commentary on Question:

The appropriate formula for the ILF[L, U] is E[x; U] / E[x; L] where E[x; A] in Excel is $EXP(mu + (sigma^2)/2)$ $\times NORM.S.DIST((LN(A) - mu - sigma^2)/sigma, TRUE)$ $+ A \times (1 - NORM.S.DIST((LN(A) - mu)/sigma, TRUE)).$ The model solution in the Excel solutions spreadsheet uses the values of sigma and mu as presented in the question (i.e., rounded to 3 decimal places) to calculate the ILF. It was equally acceptable to use the mu and sigma values calculated in part (a) for this calculation.

E[x; 500] = 151.59 and E[x; 1,500] = 217.78 ILF[500; 1,500] = 217.78 / 151.59 = 1.44

(c) Calculate the expected losses in the layer using an exposure rating approach.

Expected losses in reinsured layer calculation

- Calculate reinsurance exposure factors for each of the four UL/PL combinations
 - (E[x; 1,000] E[x; 500]) / (E[x; 1,000] E[x; 0]) = (1.28 1.00) / (1.28 0) = 0.219
 - (E[x; 1,500] E[x; 500]) / (E[x; 1,500] E[x; 0]) = (1.44 1.00) / (1.44 0) = 0.306
 - (E[x; 1,500] E[x; 1,000]) / (E[x; 1,500] E[x; 500]) = (1.44 1.28) / (1.44 1.00) = 0.364
 - (E[x; 2,000] E[x; 1,000]) / (E[x; 2,000] E[x; 500]) = (1.53 1.28) / (1.53 1.00) = 0.472
- Calculate reinsurance exposure factor times premium for each of the four UL/PL combinations
 - $0.219 \times 2,000 = 438$
 - $0.306 \times 2,500 = 764$
 - $0.364 \times 4,000 = 1,455$
 - $0.472 \times 4,500 = 2,123$
- Total exposed premium is 4,779 (= 438 + 764 + 1,455 + 2,123)
- Calculate expected loss as expected loss ratio times the total exposed premium: $4,779 \times 55\% = 2,628$

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) and (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) and (c) are for explanatory purposes only.

Solution:

(a) Describe the problem with the IRR method.

The IRR equation is solved by trial and error. However, if the cash flows change sign more than once, then multiple solutions for the IRR can occur.

- (b) You are calculating the underwriting profit margin (UPM) for a one-year policy using the Risk Adjusted Discount Technique with the following assumptions:
 - The premium will be collected at policy inception.
 - Expenses of 24 will be paid as follows:
 - \circ 35% paid six months before policy inception; and
 - 65% paid at policy inception.
 - Losses are expected to be 120 and will be paid as follows:
 - 40% paid nine months after policy inception; and
 - o 60% paid at policy expiration.
 - The tax rate on all income is 25% and taxes will be paid at policy expiration.
 - Equity of 90 supports the policy from policy inception to policy expiration.
 - The risk-free rate is 3.4%.
 - The risk-adjusted rate for losses is 0.6%.

Calculate the premium for this policy.

Commentary on Question:

In this solution, the following abbreviations are used:

- PV = present value
- L = losses
- E = expenses
- C = equity
- *TUW* = *tax on underwriting income*
- *TII* = tax on investment income

The question did not state whether or not discounting was required for tax purposes. As such, the tax on underwriting income could be based on either undiscounted or discounted amounts. The model solution presented here assumes that the tax is on undiscounted underwriting income.

$$\begin{split} P &= PV(E) + PV(L) + PV(TUW) + PV(TII) \\ PV(E) &= 24 \times (0.35 / 1.034^{-0.5} + 0.65) = 24.1416 \\ PV(L) &= 120 \times (0.4 / 1.006^{0.75} + 0.6 / 1.006) = 119.3557 \\ PV(TUW) &= (0.25) (P - E) / 1.034 - (0.25) L / 1.006 = 0.2418P - 35.6238 \\ PV(TII) &= (C + P - E - (0.4)L / 4)(0.034)(0.25) / 1.034 = 0.0082P + 0.4439 \\ P &= 108.3174 + 0.25P \\ \rightarrow P &= 144.42 \end{split}$$

(c) Calculate the UPM for this policy.

 $\begin{array}{l} UPM = 1 - L/P - E/P \\ UPM = 1 - 120 \; / \; 144.42 - 24 \; / \; 144.42 = 0.00291 = 0.291\% \end{array}$

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

- (1e) Apply a parametric model of loss development.
- (1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Estimate the scale factor, σ^2 .

Commentary on Question:

The table of data included two columns with values for the cumulative distribution function, G, at the beginning and ending of the interval for each row (Col J and Col K). CDF values were required to calculate the scale factor. The table of data also included two empty columns, Col H for sigma-squared and Col I for the expected increment. The amounts for the expected increment were required to calculate sigma-squared in Col H.

For each row in the table, the expected increment (Col I) is calculated as the onlevel premium times the ELR times [G at the end of the interval minus G at the beginning of the interval].

Then, for each row, sigma-squared (Col H) is calculated as the square of the difference between the increment and the expected increment divided by the expected increment.

The sum of Col H, sigma-squared, divided by 8 is equal to 105.066236. This is the value of the scale factor. Note that the value of 8 in the formula is the number of rows (10) less the number of estimated parameters (2).

(b) Estimate the process standard deviation of the loss reserve for all accident years combined.

For each accident year, we need to compute the loss reserve at the end of calendar year 2020. The loss reserve is the onlevel premium for the year times the ELR times [1 minus G at the end of the interval]. For accident year 2017, the end of the interval is 48 months, for accident year 2018, the end of the interval is 36 months, and so on. The loss reserve for all accident years combined is 5,730.13.

The process standard deviation of the loss reserve for all accident years combined is 775.91. This is equal to the square root of [the loss reserve for all accident years combined times the scale factor].

(c) Estimate the expected loss for 2022.

Expected loss = expected premium times ELR. $20,000 \times 0.5424 = 10,848$.

(d) Estimate the coefficient of variation due to process variance for the 2022 loss.

The coefficient of variation due to process variance for the 2022 loss is the standard deviation due to process variance for the 2022 loss divided by the expected loss for 2022.

The standard deviation due to process variance for the 2022 loss is 1,067.59. This is the square root of [the expected loss for 2022 times the scale factor].

The coefficient of variation due to process variance for the 2022 loss is 1,067.59 / 10,848 = 0.0984.

(e) Estimate the coefficient of variation due to parameter variance for the 2022 loss.

The coefficient of variation due to parameter variance for the 2022 loss is the standard deviation of the ELR divided by the ELR.

The standard deviation of the ELR is the square root of the variance of the ELR which is 0.03834 (= $0.00147^{0.5}$).

The coefficient of variation due to parameter variance for the 2022 loss is 0.07069 (= 0.03834 / 0.5424).

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

- (1b) Test for the validity of these assumptions.
- (1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Compute the sum of squared errors (SSE).

Commentary on Question:

SSE is equal to the sum of the squared differences between the incremental loss data and the fitted loss data for development years 2 through 6, accident years 1 through 6.

SSE = $(3,257 - 4,987)^2$ + ... + $(4,932 - 5,619)^2$ = 38,402,687

- (b) You now wish to use regression to fit the following alternative models:
 - (i) Constant only
 - (ii) Factor only
 - (iii) Factor only, with each observation weighted by the reciprocal of the previous cumulative losses

Construct the fitted triangle and compute the SSE, for each of the three alternative models.

Commentary on Question:

The SSEs for each of the alternative models are computed in the same manner as for the model in part (a).

(i) Constant only

The constant values for each development period are the averages of the incremental loss data.

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
a	5,387	3,822	3,484	2,481	2,230

These constant values are used to construct the fitted triangle as follows:

			Devel	opment	t Year		
AY	1	2	3	4	5	6	7
1		5,387	3,822	3,484	2,481	2,230	
2		5,387	3,822	3,484	2,481	2,230	
3		5,387	3,822	3,484	2,481		
4		5,387	3,822	3,484			
5		5,387	3,822				
6		5,387					
7							

SSE for (i) = 48,783,977

(ii) Factor only

The factor values for each development period are the sum of the product of the cumulative loss data at the beginning of the development period and the incremental loss data during the development period all divided by the sum of the squares of the cumulative loss data at the beginning of the development period.

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
b	1.1892	0.4562	0.2655	0.1861	0.1629

The fitted triangle is constructed as the product of the factor for a development period and the cumulative loss data at the beginning of the development period as follows:

			Devel	lopment	Year		
AY	1	2	3	4	5	6	7
1		5,960	3,772	2,896	2,197	2,206	
2		126	1,955	1,433	1,985	2,245	
3		4,055	4,102	3,683	3,003		
4		6,725	5,271	4,186			
5		1,299	4,364				
6		1,799					
7							

SSE for (ii) = 119,881,690

(iii) Factor only, with each observation weighted by the reciprocal of the previous cumulative losses

The factor values for each development period are the sum of the incremental loss data during the development period divided by the sum of the cumulative loss data at the beginning of the development period.

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
b	1.9254	0.4479	0.3033	0.1928	0.1632

The fitted triangle is constructed as the product of the factor for a development period and the cumulative loss data at the beginning of the development period as follows:

			Devel	opment	Year		
AY	1	2	3	4	5	6	7
1		9,650	3,704	3,309	2,276	2,210	
2		204	1,919	1,637	2,056	2,249	
3		6,565	4,028	4,208	3,112		
4		10,888	5,176	4,782			
5		2,102	4,285				
6		2,913					
7							

SSE for (iii) = 159,918,807

(c) Compute one test statistic, based on the SSE, for each model.

Commentary on Question:

There were several test statistics that could have been computed. Only one was required for full credit. The model solution in the Excel solutions spreadsheet shows the computation of three different test statistics (Venter, AIC and BIC).

	Base	(i)	(ii)	(iii)
Venter	384,027	216,818	532,808	710,750
AIC	104,389,326	80,431,181	197,651,492	263,661,539
BIC	171,742,037	103,165,551	253,518,908	338,187,103

(d) Identify the best model based on the value of this test statistic for each model.

The best model is (i) as it has the lowest value for the test statistic.

(e) Describe the correlation structure of the best model.

All observations are mutually independent of one another, so all correlations are zero.

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel for part (c). An example of a full credit solution for part (c) is in the Excel solutions spreadsheet. The model solution in this file for part (c) is for explanatory purposes only.

Solution:

(a) Identify which risk load is larger.

Under the Marginal Variance method, the renewal risk load for account X is greater than the risk load for account X during build-up.

(b) Explain why there is this difference.

During build-up, the Marginal Variance risk load is a factor times the variance of account X, i.e., Var(X). On renewal, the Marginal Variance risk load is a factor times the variance of the combined accounts X and Y, less the variance of account Y. That is, Var(X+Y) - Var(Y).

Var(X+Y) - Var(Y) = Var(X) + 2Cov(X,Y) > Var(X) since Cov(X,Y) is greater than 0.

- (c) Calculate the renewal risk load for each account using the following methods:
 - (i) Marginal Variance
 - (ii) Shapley

(i)

/			
	Х	Y	X+Y
Variance	5,764,994	2,883,138	11,719,844
Change in variance	8,836,706	5,954,850	
Marginal Variance -			
Risk Load	397.65	267.97	

(ii)

Covariances	Х	Y
Х	5,764,994	1,535,856
Y	1,535,856	2,883,138
	Х	Y
Shapley value	7,300,850	4,418,994
Shapley - Risk Load	328.54	198.85

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) through (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) through (c) are for explanatory purposes only.

Solution:

(a) Verify that the internal systemic risk coefficient of variation is 5.0% (rounded to one decimal place).

Commentary on Question:

In the solution that follows, claim liabilities are shown in thousands.

 $[(0.05^{2})(8 / 12)^{2} + (0.09^{2})(4 / 12)^{2} + (2)(0.05)(0.09)(0.25)(8)(4) / (12^{2})]^{0.5} = 0.05011$

(b) Calculate the aggregate coefficient of variation for both lines combined.

 $[0.052^2 + 0.050^2 + 0.033^2]^{0.5} = 0.07933$

(c) Calculate the amount of the risk margin at the 80% adequacy level.

(0.07933)(12,000)(0.8416) = 801

(d) Provide one argument in favor of and one argument against assuming the lognormal distribution for claims in this situation.

In favor is the fact that the distribution is likely to be positively skewed and this is better represented by the lognormal distribution.

Against is the fact that at lower adequacy levels the lognormal distribution provides smaller margins and/or lognormal decreases (relative to the CoV) as the CoV increases.

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3b) Calculate the expected value premium for increased limits coverage and excess of loss coverage.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the expected payment per loss for a policy with a limit of 100.

(20 + 30 + 60 + 90 + 6(100)) / 10 = 80

(b) The equivalence between the size mode of summation and the layer mode of summation for the layer from 100 to 200 can be expressed as

$$\int_{100}^{200} x dF(x) + 200G(200) - 100G(100)$$

= $\int_{100}^{110} G(x) dx + \int_{110}^{130} G(x) dx + \int_{130}^{160} G(x) dx + \int_{160}^{200} G(x) dx$

where F(x) is the cumulative distribution function and G(x) = 1 - F(x).

Calculate the value of each term in this equation.

Commentary on Question:

Note that there are seven terms in this equation, three on the left-hand side (terms 1 to 3) and four on the right-hand side (terms 4 to 7).

Term (1) = (110 + 110 + 130 + 160) / 10 = 51Term (2) = (200)(10 - 8) / 10 = 40Term (3) = (100)(10 - 4) / 10 = 60Term (4) = (110 - 100)(6) / 10 = 6Term (5) = (130 - 110)(4) / 10 = 8Term (6) = (160 - 130)(3) / 10 = 9Term (7) = (200 - 160)(2) / 10 = 8

(c) Calculate increased limits factors for 110, 130, 160 and 200 with a basic limit of 100.

Commentary on Question:

The increased limits factor (ILF) for a limit is calculated as the average of the losses capped at the limit divided by the average of the losses capped at the basic limit.

	Limited	
	Average	
Limit	Loss	ILF
100	80	1.0000
110	86	1.0750
130	94	1.1750
160	103	1.2875
200	111	1.3875

(d) Demonstrate that these increased limits factors are consistent.

Commentary on Question:

To demonstrate this, one needs to show that the expected payment per unit of coverage decreases as the limit increases.

Lower limit	Upper limit	Rate of increase
100	110	(1.0750 - 1.0000) / (110 - 100) = 0.00750
110	130	(1.1750 - 1.0750) / (130 - 110) = 0.00500
130	160	(1.2875 - 1.1750) / (160 - 130) = 0.00375
160	200	(1.3875 - 1.2875) / (200 - 160) = 0.00250

The rate of increase is decreasing so they are consistent.

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) through (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) through (d) are for explanatory purposes only.

Solution:

(a) Show that with the expected loss ratio of 54.0%, the 2021 treaty profit is 7.0% of ceded premium.

Commission at a 54% loss ratio is 25% + (60% - 54%) = 31%Profit = 100% - 54% - 31% - 8% = 7%

(b) Show that using the loss distribution above, the expected 2021 treaty profit is 3.6% of ceded premium.

First compute the commission at each loss amount.

Treaty		Loss	
Loss	Probability	Ratio	Commission
0	0.0916	0.0%	45.0%
40,000	0.1465	12.1%	45.0%
110,000	0.1954	33.2%	43.4%
180,000	0.1954	54.4%	30.6%
250,000	0.1563	75.5%	25.0%
320,000	0.1042	96.7%	25.0%
390,000	0.0595	117.8%	25.0%
400,000	0.0511	120.8%	25.0%

The expected loss is 53.95%.

The expected commission is 34.45%. Expected profit = 100% - 53.95% - 34.45% - 8% = 3.59%

(c) State whether or not the sliding scale commission structure is "balanced." Justify your answer.

It is not balanced because the expected loss ratio is close to one end of the commission slide.

(d) Recalculate the expected 2021 treaty profit from (b) as a percentage of ceded premium, allowing for the loss ratio in 2020.

The carryforward loss ratio is that in excess of 60%, the top loss ratio in the commission scale slide. Therefore, the carryforward loss ratio adjustment to the commission slide with a 2020 loss ratio of 75.5% is 15.5%.

Adjusted Loss	Adjusted	
Ratio	Commission	
14.5% or below	45%	
14.5%-34.5%	Sliding 0.5:1	
34.5%-44.5%	Sliding 1:1	
44.5% or above	25%	

Treaty	D 1 1 11	Loss	~ · ·
Loss	Probability	Ratio	Commission
0	0.0916	0.0%	45.0%
40,000	0.1465	12.1%	45.0%
110,000	0.1954	33.2%	35.6%
180,000	0.1954	54.4%	25.0%
250,000	0.1563	75.5%	25.0%
320,000	0.1042	96.7%	25.0%
390,000	0.0595	117.8%	25.0%
400,000	0.0511	120.8%	25.0%

The expected adjusted commission is 31.84%. Expected profit = 100% - 53.95% - 31.84% - 8% = 6.21%

(e) Another approach to assessing the effect of a carryforward provision is to look at the "long run" of the contract.

State two problems with this approach.

One problem is that the contract may not be renewed in which case there is no "long run" of the contract.

Another problem is that an assessment of the variance over a multiyear period may be complex.