Pricing and hedging with discontinuous functions: quasi-Monte Carlo methods and dimension reduction

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Quasi-Monte Carlo (QMC) methods have become important numerical tools in computational finance. Many studies have demonstrated the greater efficiency of QMC relative to Monte Carlo (MC) methods, even for pricing high-dimensional exotic derivative securities. Some of these studies have argued the importance of effective dimension in determining the efficiency of QMC. Consequently, dimension reduction methods based on Brownian bridge (BB) and principal component analysis (PCA) have been proposed to enhance QMC. While the notion of effective dimension is an important factor for affecting the performance of QMC methods, in this paper we demonstrate that the adverse effect from the discontinuity is even more pronounced. This calls for a significant concern as discontinuities occur naturally in pricing and hedging exotic derivative securities. The estimation of Greeks using pathwise approach is another example which induces discontinuity. The purpose of this paper is twofold. The first is to establish the relationship among the dimension reduction methods, the feature of discontinuity and the performance of QMC methods, with special interest in understanding why BB and PCA do not offer a consistent advantage in QMC. The key is that dimension reduction methods could change the structure of discontinuity and thus could adversely affect the performance of QMC methods. In particular, we demonstrate that for digital options, BB and PCA make the problems harder for QMC, since they implicitly introduce irregular discontinuities and increase the effective dimension comparing with the standard discretization. The second purpose of the paper is to overcome the obstacle of discontinuity. We develop a new and novel way of handling discontinuity by ensuring the inherent discontinuity to be "QMC-friendly" in the sense that the discontinuity occurs in the axis-parallel hyperplanes. By making the discontinuity "QMC-friendly", we recover the greater efficiency of QMC. Extensive numerical experiments are conducted to demonstrate the efficiency and robustness of the proposed method for pricing exotic options with discontinuous payoffs and for estimating Greeks. It is also of interest to note that while our proposed method significantly outperforms other competitive approaches such as BB and PCA, the resulting problem also has a strong degree of additivity as well as low effective dimension. All these evidences support the overwhelming success of our proposed method.