Using Cohort Change Ratios to Estimate Life Expectancy in Populations Closed to Migration: A New Approach

David A. Swanson
Department of Sociology
University of California Riverside
Riverside, CA 92521 USA
David.Swanson@ucr.edu

Lucky Tedrow
Department of Sociology
Western Washington University
Bellingham, WA 98225 USA
Lucky.Tedrow@wwu.edu

Abstract

For populations closed to migration with at least two successive census counts of the population by age, we show that life expectancy at birth can be computed by using cohort change ratios in combination with an identity whereby the radix of a life table is equal to 1 (l_0 = 1.00). We empirically test our approach against empirical data and find it works reasonably well. As well as some nuances and cautions, we discuss benefits in using this approach to estimating life expectancy, including the ability to develop estimates of average remaining life at any age. We find that the technique is worthy of consideration for use in estimating life expectancy in populations closed to migration, given the cautions we discuss.

As noted in the UN Manual, Methods for Estimating Adult Mortality from Census Data (United Nations (2002: 5), "Census survival methods are the oldest and most widely applicable methods of estimating adult mortality…(and can) provide excellent results (for) populations that experience negligible migration…"

Census survival methods require two age distributions for a population at two points in time, which generally are two successive census enumerations. Ideally, the interval between the census enumerations (e.g., 10 years) is either equal to the width of the age groups (e.g., the age groups are given in ten year increments, 0-9, 10-19,…75-84, 85+) or a whole number multiple thereof (e.g., the age groups are given in five year increments, 0-4, 5-9,…,80-84, 85+), through the final open-ended age group (e.g., 85+).

The UN (2002: 6) shows that using the census survival method, that the expectation of life at age x can be computed as

$$e_x = (T_x/l_{(n/2)})/(l_x/l_{(n/2)}) \quad = \quad T_x / l_x \qquad [1]$$

where

x = age

n = the width of the age groups (up to, but not including the terminal, open-ended age group)

e_x = life expectancy (average years remaining) at age x

T_x = Total person years remaining to persons age x

l_x = number reaching age x

$l_{(n/2)}$ = people aged x to x+n are assumed to be concentrated at the mid-point of the age group, i.e., they are all aged n/2 years exactly.

and $l_{(x+2n/2)}/l_{(x-n/2)} = P2_{(x,n)}/P1_{(x-n,n)}$ [2]

where

$P2_{(x,n)}$ = the number of persons counted in the second census in age group x to x+n

$P1_{(x-n,n)}$ = the number of persons counted in the first census In age group x-n to x

In general, then, the life-table probability of surviving from the mid-point of one age group to the next ($l_{(x+2n/2)}/l_{(x-n/2)}$) is approximated by the census survival ratio ($P2_{(x,n)}/P1_{(x-n,n)}$). Continuing, the UN (2002: 5-6) shows that the cumulative multiplication of the probabilities shown in [2] gives the conditional survival schedule $l_x/l_{(n/2)}$. From the conditional l_x values given by [2] the conditional estimates of the number of person years lived in each age group (${}_nL_x$) can be calculated as

${}_nL_x/l_{(n/2)} = (n/2) *[(l_x/l_{(n/2)} + l_{(x+n)}/l_{(n/2)}]$ [3]

where

${}_nL_x$ = number of person years lived in each age group

Given a value of $T_x/l_{(n/2)}$ for some initial old age x, the UN shows that total remaining years expected at age x (T_x) values can be calculated as:

$$T_{(x-n)}/l_{(n/2)} = T_x/l_{(n/2)} + {}_n L_{(x-n)}/l_{(n/2)} \qquad [4]$$

This leads us back to equation [1], so that the expectation of life at age x using the UN approach is

$$e_x = (T_x/l_{(n/2)})/(l_x/l_{(n/2)}) \quad = \quad T_x / l_x$$

In our alternative approach, we start by noting that when the radix of a life table is equal to 1 (l_0 = 1.00) life expectancy at birth can be computed directly from the expression:

$$e_0 = S_0 + (S_0{*}S_1) + (S_0{*}S_1{*}S_2) +,...,+(S_0{*}S_1{*}S_2,...,{*}S_x) \qquad [5]$$

where

e_0 = life expectancy at birth

S_0 = survivorship from t=0 (e.g., birth) to t=1(e.g., age 1)

S_1 = survivorship from t=1 (e.g., age 1) to t=2(e.g., age 2)

and so on through S_x

and $S_x = {}_1L_x/ {}_1L_{(x-n)}$

Equation [5] is set up for single year age groups. However, we can generalize it to other age groups: ${}_nS_x = {}_nL_x/ {}_nL_{(x-n)}$, so that

$$e_0 = {}_nS_0 + ({}_nS_0{*}{}_nS_1) + ({}_nS_0{*}{}_nS_1{*}{}_nS_2) +,...,+({}_nS_0{*}{}_nS_1{*}{}_nS_2,...,{*}\,{}_nS_x) \qquad [5.a]$$

We also apply census survival rates, although we prefer to use the more general term “cohort change ratios (CCRs). CCRs enter into a range of measures and applications, such as the "Hamilton-Perry Method," which is often used for doing short term population forecasts of small areas such census tracts (Smith, Tayman, and Swanson, 2001:153-158).

Following Smith, Tayman, and Swanson (2001: 155) and using notation from equation [2], a CCR can be generally defined as:

$$ {}_nCCR_x = P2_{(x,n)} / P1_{(x-n,n)} \qquad [6] $$

To our knowledge, nobody has yet combined either equation [5] or [5.a] for computing life expectancy with equation [6] in order to estimate e_x. Starting with

$$ {}_nS_x = {}_nL_x / {}_nL_{(x-n)} \approx P2_{(x,n)} / P1_{(x-n,n)} \qquad [7] $$

We have, as shown in equation [5.a]

$$ e_0 = {}_nS_0 + ({}_nS_0 * {}_nS_1) + ({}_nS_0 * {}_nS_1 * {}_nS_2) +,...,+({}_nS_0 * {}_nS_1 * {}_nS_2,...,* {}_nS_x) $$

As is the case with the more involved UN approach, this approach will only work for populations closed to migration, but there are many areas of interest around the world where this is the case, or approximately so. The world as a whole meets this requirement. Countries with populations closed to migration include North Korea and Burma, among others. Other such populations are found in the historical record - the former Soviet Union, Albania from 1950 to 1980, and the Peoples Republic of China from 1950 through 1970, for example. Still others may be defined by race and ethnicity or other 'rules' of membership (e.g., Indigenous Populations in Australia and Canada, Native Hawaiians).

Broadly speaking, the method can be applied to any population subject to renewal through a single increment (birth) and extinction through a single decrement (death), where there are at least two successive census counts that provide the population by age. We also note that unlike the UN method, the approach we take can be used to yield estimates of life expectancy at birth.

We have developed life expectancy estimates directly from cohort change ratios constructed for the world as a whole and selected countries closed to migration. As an empirical test, we compare our life expectancy estimates to estimates for the world as a whole available from the US Census Bureau (2010) for the period 1950-55 to 2045-50. We find rather close agreement between these estimates, as shown in Table 1, particularly in recent years.

(TABLE 1 ABOUT HERE)

Table 2 shows e_0 estimates for Burma for the period 1975-80 to 2005-2010. As was the case for the world as a whole, our estimates are compared with those available from the US Census Bureau (2010). Again, we find reasonably close agreement. However, our estimates of life expectancy at birth remain almost constant at age 60 from 1995-2000 to 2005-10 while those available from the Census Bureau increase from 61 to 65 years.

(TABLE 2 ABOUT HERE)

As well as some nuances (e.g., converting CCRs into survival ratios may require additional refinements) and cautions (e.g., the population data by age may be faulty), we find benefits in using this approach to estimating life expectancy, including the ability to develop estimates of average remaining life at any age (not shown here). We find that the technique is worthy of consideration for use in estimating life expectancy in populations closed to migration, given the cautions we discuss. We suggest that in addition to the current methods used by the UN and others to estimate life expectancy for populations lacking the data needed for life table construction, that this method be considered when such a population is closed to migration, or nearly so.

References

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Table 1. Comparison of Life Expectancy Estimates for the World as Whole calculated from Cohort Change Ratios (during each period), 1950-55 to 2045-50 shown in Equation [7] and Equation[5.a] with estimated values available from the US Census Bureau (2010).

YEARS	e_0 Calculated from UN Census Sx values using [6] and [4]* total e_0	e_0 Calculated from UN Census Sx values using [6] and [4]* male e_0	e_0 Calculated from UN Census Sx values using [6] and [4]* female e_0	UN Life Expectancy(e_0) total e_0	UN Life Expectancy(e_0) male e_0	UN Life Expectancy(e_0) female e_0
1950-55	51.80	50.28	53.38	46.6	45.2	48
1955-60	53.95	52.01	56.01	49.5	48.1	50.9
1960-65	56.00	54.61	57.35	52.4	51	53.7
1965-70	58.43	56.69	60.15	56.1	54.6	57.6
1970-75	60.40	58.89	61.85	58.2	56.6	59.8
1975-80	61.77	59.73	63.81	60.2	58.3	62
1980-85	62.61	60.48	64.77	61.7	59.7	63.7
1985-90	63.49	61.27	65.76	63.2	61.2	65.2
1990-95	64.05	61.55	66.65	64	61.9	66.2
1995-2000	64.90	62.39	67.54	65.2	63	67.4
2000-05	65.80	63.35	68.34	66.4	64.2	68.6
2005-10	66.65	64.30	69.07	67.6	65.4	69.8
2010-15	67.65	65.42	69.93	68.9	66.7	71.1
2015-20	68.57	66.36	70.83	70.1	67.9	72.3
2020-25	69.38	67.17	71.65	71.1	68.9	73.4
2025-30	70.14	67.91	72.42	72.1	69.9	74.4
2030-35	70.81	68.58	73.09	73.1	70.8	75.4
2035-40	71.43	69.19	73.72	73.9	71.7	76.3
2040-45	72.02	69.79	74.31	74.8	72.5	77.1
2045-50	72.59	70.37	74.88	75.5	73.3	77.9

The calculations are found in a worksheet available from the authors.

* e_0 is estimated by summing the products of adjacent S_x values as follows,

$$e_0 = S_1 + S_1*S_2 + S_1*S_2*S_3 + S_1*S_2*S_3 +,\ldots,+S_1*S_2*S_3*,\ldots*S_k$$

Table 2. Table 1. Comparison of Life Expectancy Estimates For Burma Calculated from Cohort Change Ratios (during each period), 1975-80 to 2005-10 as shown in Equation [7] and Equation[5.a] with estimated values available from the US Census Bureau (2010).

Source of Life Expectancy Estimate/Year	1975-80	1980-85	1985-90	1990-95	1995-2000	2000-2005	2005-2010
Life expectancy at birth (US Census, 2010)	54	56	56	59	61	63	65
Estimated Life Expectancy from CCRs	49.99	52.47	55.96	56.97	60.09	60.82	60.99