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Using Statistical Analysis and Actuarial Judgment in the Calculation of Medical Liability

For the calculation of the incurred but not reported (IBNR) reserves in health insurance, practicing health actuaries often rely on a two-step process: first, they calculate IBNR using completion factors or some other deterministic measure of IBNR, and then they complete the IBNR reserve calculation with a regression analysis or actuarial judgment, or both, on the most recent experience. In this work, we discuss areas where actuarial judgment needs to be used, even with the application of statistical methods for the calculation of IBNR. Actuarial judgment needs to be used since the prediction of IBNR for months with the most unpaid claims has the most uncertainty. We give a detailed example of combining statistical analysis and actuarial judgment in the calculation of IBNR.

Introduction

This document is based on research of the authors for the Society of Actuaries on statistical methods for health actuaries. It is also based on a Case Study for the Society of Actuaries' Course 7.

Estimates of unknown values that actuaries make include:

- unpaid claims liability;
- premiums.

In this document, we show how to use statistical methods to estimate unknown values and create prediction intervals in the context of setting reserves for health (other than disability) insurance claims that have been incurred but not reported (IBNR).

Two general approaches used in developing estimates and prediction intervals are:

- Statistical methods - For instance, a regression analysis might be applied to estimate a range for the value of claims liability.

- Additional knowledge of factors exogenous to the statistical method - For instance, the range calculated statistically might be adjusted to reflect a known change in benefit plan design.

It is possible to use both approaches on the same problem. It is important that an actuary have a framework for combining both approaches – calculating the statistically "best" estimate under a given set of circumstances and recognizing the limitations to the statistical method selected, and adjusting the estimate accordingly based upon specific knowledge of elements outside the scope of the statistical methods.

This document is organized into the following sections:
- Section 1: Overview of Health Care Liabilities and Introduction to the Completion Factor Method
- Section 2: The Completion Factor Method Using Medical Insurance Data
- Section 3: Regression Analysis for Recent Months

Section 1 - Overview of Health Care Liabilities and Introduction to the Completion Factor Method

In this section, we give an overview of health care liabilities and a description of one of the well-known methods for computing the health plan liability, the completion factor method.

This section is organized as follows:

1. Overview of health care liabilities
2. Introduction to the completion factor method

1.1 Overview of Health Care Liabilities

1.1.1 Definitions

We will refer to the insured under a health insurance contract under consideration as the *member* of the health insurance plan. Members seeking medical care incur a cost that may be reimbursable by the insurance company (a *claim*). The month in which a member sees a provider for medical care is called the *incurred month*.

The amount ultimately paid for claims incurred in a given month is modeled by a process called *development*. One common method of modeling the development process is called the *chain ladder* method. This method is parameterized by estimating the ratios of amounts paid in consecutive months (*development factors*) or percentages of ultimate cost paid up to a given date (*completion factors*). These factors are related to one another in a way that will be illustrated in the examples below.

After a claim is incurred, it is submitted to the insurance company. The month during which the claim is reported to the insurance company is called the *reported* month, and the month in which the insurance company pays the claim is called the *paid month*. In this document, we are only concerned with the incurred and paid months.

The paid month comes after the incurred month, and it can, of course, occur no earlier than the incurred month. (In this document, we are not considering the issue of pre-paid medical care, or capitation.) In a report of paid claims by incurred month, each incurred month is given a row and each paid month is given a column. Only cells in which the paid month is equal to or later than the incurred month are non-zero. Since the non-zero values form a triangle, this report is typically called a *triangle report.*

Lag is the measure of the difference between incurred month and paid month. For instance, a claim that is incurred in July 2005 and paid in July 2005 is defined as being paid at lag 0. A claim that is incurred in July 2005 and paid in August 2005 has a lag of 1.

The process of calculating insurance liabilities is referred to as *valuation*. Unless stated otherwise, when we are using a month as a valuation date, we are referring to the last day of the month, which is the date as of which valuations are usually performed.

1.1.2 Types of Health Reserves

Health (other than disability) insurance reserves may be classified broadly into the following categories:

- *Policy reserves*: amounts necessary for contract obligations created by future claims.

- *Expense liabilities*: amounts needed to pay loss adjustment expenses, taxes and other expenses related to liabilities incurred by the insurer from operations prior to the statement date.

- *Claims reserves*: amounts needed to pay claims already incurred but not yet reported or paid.

In this document, we will be concerned with claims reserves only.

U.S. statutory reserving practices for health insurance in general are governed by the National Association of Insurance Commissioners' (*NAIC) Model Minimum Reserve Standards for Individual and Group Health Insurance.*

1.1.3 Claims Reserves

Claims reserves represent the amounts that the insurer expects to pay in the future on claims that have been incurred prior to the end of the reporting period. Claims reserves are estimates of the amounts yet to be paid. Three major types of claims reserves are: claims due and unpaid, claims in the course of settlement, and claims that have been incurred but not reported. Claims due and unpaid are usually small and known with reasonable precision. The two other types of claims reserves, either because the amount due to the provider of medical care (or policyholder) has not been determined, or because the insurance company has no knowledge of the claims yet, produce a significant level of uncertainty requiring actuarial analysis. The classic methodology for estimating health claims reserves is similar to the methodology used in property/casualty insurance loss reserving. There are, however, notable differences in the data. Key differences include the following:

- In property/casualty insurance, the policyholder rarely makes a claim. In many types of health insurance, there is a high probability that an individual policyholder will make a claim. Consequently, the number of data points that a health insurer must process and manage is staggering compared to the number processed and managed by a property/casualty insurer.

- Due to the enormous volume of claims, specific details, helpful in determining the expected amount of individual claims, are usually not known by the health valuation actuary. The exception to this may be large catastrophic claims, such as claims for severe burns and for low weight premature infants.

- An actuary sometimes has knowledge of hospital contracts, which would help him/her derive expected payment rates for specific known claims.

- Health insurance claims develop much more quickly than many types of casualty claims, such as liability and workers' compensation. The development of health claims from unknown to known (to the insurer) and paid is measured in months. For some casualty products, the development from unknown to known takes years.

1.2 Introduction to the Completion Factor Method

Suppose we know the following about claims incurred in August 2005:

Exhibit 1.1
Claims Incurred, August 2005

Paid Month	Lag	Amount Paid
August 2005	0	$2,000
September 2005	1	1,000
October 2005	2	1,000
November 2005	3	400
December 2005	4	1,100
Total Incurred		$5,500

In addition, we know that all claims incurred in August 2005 have been paid by the end of December 2005. Based on this information, we can derive development factors and completion factors for the month of August 2005 as follows:

Exhibit 1.2
Derivation of Development and Completion Factors
August 2005

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Lag	Amount Paid	Cumulative Paid	Development Factor	Derivation of (4)	Completion Factor	Derivation of (6)
0	2,000	2,000			36.4% [1]	(3) Lag (0)/(3) Lag (4)
1	1,000	3,000	1.50 [2]	(3) Lag 1 ÷ (3) Lag 0	54.5%	(3) Lag (1)/(3) Lag (4)
2	1,000	4,000	1.33	(3) Lag 2 ÷ (3) Lag 1	72.7%	(3) Lag (2)/(3) Lag (4)
3	400	4,400	1.10	(3) Lag 3 ÷ (3) Lag 2	80.0%	(3) Lag (3)/(3) Lag (4)
4	$1,100	$5,500	1.25	(3) Lag 4 ÷ (3) Lag 3	100.0%	Assumed

(1) 36.4% = $2,000 / $5,500
(2) 1.50 = $3,000/$2,000

Exhibit 1.2 shows the relationship between development factors and completion factors. In short, given an assumption about when claims are complete and a set of development factors, one can easily calculate the related completion factors.

Now suppose we have the following information about claims incurred between August and December 2005 and paid through the end of December 2005.

Exhibit 1.3a
Claims Incurred August – December 2005, Paid Through December 2005

Incurred Month	Paid Month					Total
	Aug-05	Sep-05	Oct-05	Nov-05	Dec-05	
Aug-05	$2,000	$1,000	$1,000	$400	$1,100	$5,500
Sep-05		$2,000	1,800	1,400	800	$6,000
Oct-05			$3,000	3,000	2,000	$8,000
Nov-05				900	600	$1,500
Dec-05					$5,000	$5,000

Rather than use the row to indicate the incurred month and the column to indicate the paid month, it is possible to do the opposite, as in Exhibit 1.3b. The particular choice between the two depends on which seems more intuitive and on the further calculations to be performed.

Exhibit 1.3b
Claims Incurred August – December 2005, Paid Through December 2005

Paid Month	Incurred Month				
	Aug-05	Sep-05	Oct-05	Nov-05	Dec-05
Aug-05	$2,000				
Sep-05	1,000	$2,000			
Oct-05	1,000	1,800	$3,000		
Nov-05	400	1,400	3,000	$ 900	
Dec-05	1,100	800	2,000	600	$5,000
Total	$5,500	$6,000	$8,000	$1,500	$5,000

Exhibit 1.4 below shows how we can apply the completion factors from Exhibit 1.2 to the claims paid to date in Exhibit 1.3a in order to estimate claims reserves for IBNR as of the end of December 2005.

Exhibit 1.4
IBNR Claims Reserves
As of December 2005

(1) Incurred Month	(2) Lag Through Dec-05	(3) Claims Paid Through Dec-05	(4) Completion Factor	(5) = (3) ÷ (4) Estimated Incurred	(6) = (5) – (3) IBNR
Aug-05	4	$ 5,500	100.0%	$ 5,500	$ -
Sep-05	3	6,000	80.0%	7,500	1,500
Oct-05	2	8,000	72.7%	11,000	3,000
Nov-05	1	1,500	54.5%	2,750	1,250
Dec-05	0	5,000	36.4%	13,750	8,750
Total		$26,000		$40,500	$14,500

The completion factors used to calculate IBNR in Exhibit 1.4 are based solely on claims incurred in August 2005. It is clear, however, that every month does not develop in the same way. For example, the lag 1 development factor based on September 2005 data is 1.90, not 1.50 as we derived for August 2005.

As an additional example, if we were to calculate the development factors using data from September 2005 only, we would obtain the following development factors, based on the data from Exhibit 1.3b:

Lag 1 Development Factor = ($2,000 + $1,800) ÷$2,000 = 1.90
Lag 2 Development Factor = ($2,000 + $1,800 + $1,400) ÷$3,800 = 1.37
Lag 3 Development Factor = $6,000÷$5,200 = 1.15

Moreover, there is a great deal of information about the development process implicit in the development observed through December 2005 of claims incurred during September, October November, and December 2005. The chain ladder method enables us to take advantage of this additional information.

Exhibit 1.5 rearranges and calculates the cumulative claims by incurred month and lag from the information found in Exhibit 1.3b above.

Exhibit 1.5
Cumulative Paid Claims by Incurred Month and Lag
August − December 2005

Lag	Incurred Month				
---	Aug-05	Sep-05	Oct-05	Nov-05	Dec-05
0	$2,000	$2,000	$3,000	$ 900	$5,000
1	3,000	3,800	6,000	1,500	
2	4,000	5,200	8,000		
3	4,400	6,000			
4	5,500				

Exhibit 1.6 below shows an intermediate step in the chain ladder method. It calculates cumulative sums across different combinations of lags and incurred months.

Exhibit 1.6
Chain Ladder Method
Intermediate Cumulative Sums

Lag	Incurred Months			
---	Aug-05	Aug-05 - Sep-05	Aug-05 - Oct-05	Aug-05 - Nov-05
0				$ 7,900
1			$12,800	$14,300
2		$ 9,200 [(1)]	$17,200	
3	$4,400	$10,400 [(2)]		
4	$5,500			

(1) $9,200 = $4,000 + $5,200
(2) $10,400 = $4,400 + $6,000

Exhibit 1.7 uses the intermediate sums found in Exhibit 1.6 to calculate estimates of the development factors based upon the development observed through December 2005 of claims incurred between August and December 2005.

Exhibit 1.7
Development Factor Estimates
Claims Incurred August – December 2005, Paid Through December 2005

| Lag | Development Factors Based on Incurred Months | | | | Derivation |
	Aug-05	Aug-05 - Sep-05	Aug-05 - Oct-05	Aug-05 - Nov-05	
1				1.81	Lag 1 ÷ Lag 0
2			1.34		Lag 2 ÷ Lag 1
3		1.13			Lag 3 ÷ Lag 2
4	1.25				Lag 4 ÷ Lag 3

Finally, Exhibit 1.8 translates the estimated development factors from Exhibit 1.7 into estimated completion factors using the method shown in Exhibit 1.2 above.

Exhibit 1.8
Estimated Completion Factors

(1) Lag	(2) Development Factor	(3) Completion Factor	(4) Derivation of Column (3)
0		29.1%	(3) for Lag 1 ÷ (2) for Lag 1
1	1.81	52.7%	(3) for Lag 2 ÷ (2) for Lag 2
2	1.34	70.8%	(3) for Lag 3 ÷ (2) for Lag 3
3	1.13	80.0%	(3) for Lag 4 ÷ (2) for Lag 4
4	1.25	100.0%	Assumed

Based on the completion factors we just derived and our calculation method in Exhibit 1.8, we estimate claim reserves (IBNR) as follows:

Exhibit 1.9
Estimated IBNR

Incurred Month	Lag Through Dec-05	Claims Paid Through Dec-05	Completion Factor	Estimated Incurred	IBNR
Aug-05	4	$ 5,500	100.0%	$ 5,500	$ -
Sep-05	3	6,000	80.0%	7,500	1,500
Oct-05	2	8,000	70.8%	11,304	3,304
Nov-05	1	1,500	52.7%	2,848	1,348
Dec-05	0	5,000	29.1%	17,185	12,185
Total		$26,000		$44,338	$ 18,338

We have derived one value for the completion factor for each lag. This method is deterministic because it provides a single IBNR value for each month as opposed to a probability distribution of possible values. In addition, there is no probability model underlying the development of these factors. The completion factor method, which is a chain ladder method, does not lend itself to the derivation of statistical measures such as confidence intervals. While we can take completion factors from individual months to derive means and variances, this method is not recommended since we are not deriving an underlying distribution. This is referred to as descriptive statistics. The preferred method is inferential statistics, where we derive the underlying distribution.

For business and regulatory reasons, an actuary would like to set an IBNR that is sufficient with high probability. A sufficient IBNR is a liability calculated such that, once all of the claims are paid, the IBNR was greater than the actual outstanding claims paid. Therefore, the actuary needs to derive a confidence interval or a prediction interval. The following are ways to calculate confidence intervals for IBNR:

- As we show in Section 2, the completion factor method, when applied to actual medical insurance data, often produces IBNR estimates for the two most recent months (lags 0 and 1) with relatively large variance. In addition, often over 70% of the liabilities are in lags 0 and 1. Therefore, as we show in Section 3, regression methods can be applied to derive estimates for lags 0 and 1, after deriving estimates for lags greater than 1 using the completion factor method. The regression methods rely on underlying distributions from which we derive our confidence intervals. We go from the more specific problem of deriving a statistical model for deriving IBNR estimates for lags 0 and 1 to using a parametric statistical model to derive estimates for all lags. Using such a model is discussed in Section 4.

In this section, we analyzed 5 months of data. In Sections 2 and 3, we will develop IBNR with 36 months of data.

Section 2 – The Completion Factor Method Using Medical Insurance Data

In Section 1, we gave a simplified example of a deterministic completion factor method. In this section, we expand that example to a 36 month data set.

This section is organized as follows:

1. We describe the data set.
2. We discuss the determination of outliers.
3. We derive completion factors using the completion factor method.

2.1 The Data Set

Our data set is expanded from the example in Section 1 in the following ways:

- Instead of having 5 months of data, we have 36 months of data.
- In addition to paid claims, we have membership data by incurred month.

Our data set consists of claims for medical coverage for the time period January 2001 through December 2003. The claims data:

- Is summarized in a paid vs. incurred month grid.
- Represents medical coverage with no deductibles and coinsurance and relatively low copayments such as $10 per office visit. Thus we assume that the same percentage of claims is paid by the insurer in each month.
- Is exclusive of prescription drug coverage. Prescription drug claims usually have a shorter lag pattern than other medical claims.

The data set can be found in the Appendix.

The observation in each cell of the paid by incurred month grid is the sum of many individual transactions. Each transaction can be thought of as arising from one of at least three processes:

- Non-Catastrophic Initial Claims – Claims of smaller amounts as they are first paid by the claims department. Such claims are often auto-adjudicated (that is, no human intervention), so there is relatively little lag between the report date and the paid date. (This relatively quick administrative process, though, is offset by the so-called "shoe-box" effect, which is the tendency for people to accumulate claims before reporting them to the insurer.)
- Catastrophic Initial Claims – Larger amount claims reported that have on average longer lag and may cause significant changes in total IBNR.
- Adjustments – Claims as they are adjusted for any new information or administrative errors.

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With sufficient data, each of these processes could be modeled separately. Because we cannot observe this level of detail, we will treat the claims as though they come from a single distribution. Our inability to model at this level of detail may limit to some extent the usefulness of our model of the claims development process.

The lag at which initial claims are paid is dependent on:
- how quickly providers (e.g. hospitals, physicians) submit claims; and
- how fast the insurer processes and pays claims.

An adjustment occurs when a claim is paid and later the insurance company learns that it should have paid a different amount. Two of the key reasons for adjustments are:

- Claims paid in error – For instance, a claim was paid twice.
- Third party liability – For instance, Leslie is in an automobile accident caused by Cory. Leslie seeks medical care, and Leslie's medical insurance company pays for the care Leslie receives. Later, Leslie's insurance company recovers the cost of the care from Cory's automobile insurance company.

Because it is possible for the absolute value of negative adjustments to exceed positive adjustments and initial claims, it is possible that an incurred/paid month cell will show a negative amount.

As the processes underlying initial claims and adjustments differ, their underlying statistical distributions are different; thus we are adding some additional variation to the process we will model by combining these processes. This is particularly true at the longer lags, such as lags 9, 10, and 11 in our sample data.

The variation in adjustments is more important at later durations because:

- adjustments cannot be made until after initial claims are paid

  and

- initial claims payments become significantly less frequent at later durations.

For each month, we have corresponding membership information. We truncated the data set to assume that all claims are paid within 12 months (e.g., all claims incurred in January 2001 are paid by January 31, 2002). We truncated the data set because the vast majority of medical claims are paid within the first 12 months following incurral; thus, the applicability of our models to real life IBNR calculations is not significantly impacted by this truncation.

2.2 Outliers

An outlier is an observation that is far removed from the general pattern of the data. Before analyzing the data and selecting our method for evaluating IBNR, we do not start with rules such as all claims over $x are to be considered outliers. The determination of an outlier depends on the model used to estimate IBNR. An outlier with respect to one model may not be an outlier relative to another model. We can determine outliers as follows:

First we graphically view the data to see which points may be considered as candidates for outliers. Then, as our next step, we do one of the following for each potential outlier:

- Use a statistical test such as the six-sigma rule to determine outliers. This rule states that if a point is within three standard deviations of the mean, it is not considered an outlier. Statistical computing packages routinely produce diagnostic measures for determining outliers. Such diagnostics are not available in Excel.

- Remove the point from the data if we are sure, based on your knowledge of claims history, that the point is an outlier – one needs to be careful in removing data points as outliers based on observation without further analysis.

- To understand the impact of the outlier, determine IBNR with and without the outlier included to determine the total effect of the outlier on IBNR. If we take out claims as outliers, we may need to make an adjustment at the end of the IBNR calculation to adjust for the claims taken out.

Two key issues to keep in mind in determining outliers are the following:

- As mentioned above, an outlier in one model may not be an outlier in another model. When using a test like the six sigma test, a given point may be within three standard deviations of the mean in one model and more than three standard deviations from the mean in another model.

- You should be able to explain why a point is an outlier. For instance, if the paid claims for each of 23 of the last 24 months amount to between $180 and $200 per member per month (PMPM) and, for one of the 24 months, they amount to $250 PMPM, that highest month may be an outlier. As a check of the reasonableness of the $180 to $200 PMPM range, the pricing actuary who set the premium rates may be able to verify expected claims levels.

2.2.1 Screening Our Data for Outliers

In analyzing our data for potential outliers, we plot our data as in Exhibit 2.1 (found on next page).

In that graph, t is the lag and i is the incurred month. "Original Y" is the claims amount incurred in month i, $0 \leq i \leq 35$, and paid at lag t, $0 \leq t \leq 12$. This plot shows that there is a potential outlier (see the arrow) in the middle of the data set. We have identified this as the observation corresponding to $i = 12$ and $t = 7$. This point corresponds to the $756,000 that was incurred in January 2002 and paid in August 2002.

The graph in Exhibit 2.1 helps us identify positive outliers, whereas we show all of the negative values in the graph in Exhibit 2.2, also on next page.

Exhibit 2.1
Initial Scatter Plot of the Data
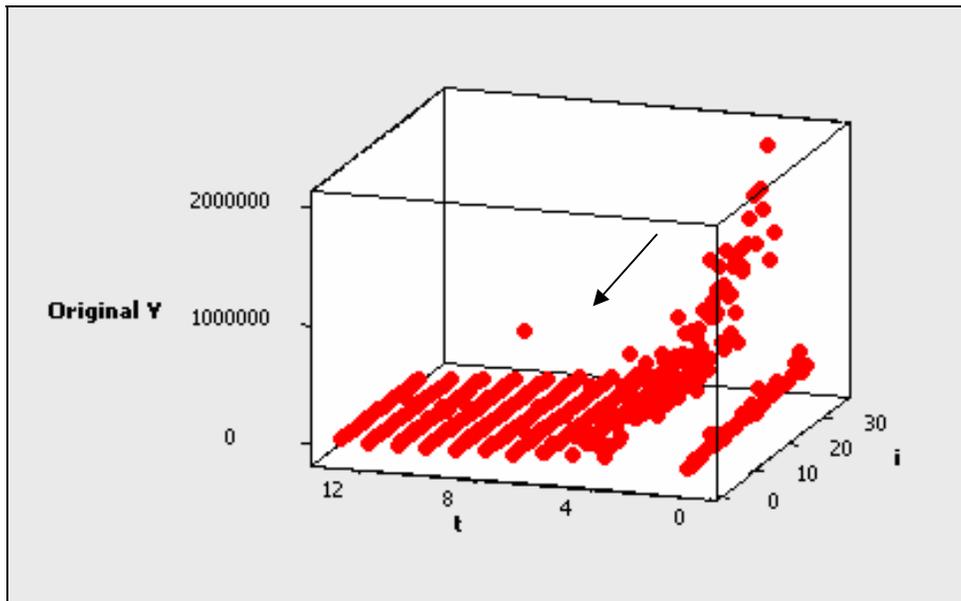
Exhibit 2.2
Data Points with Negative Values
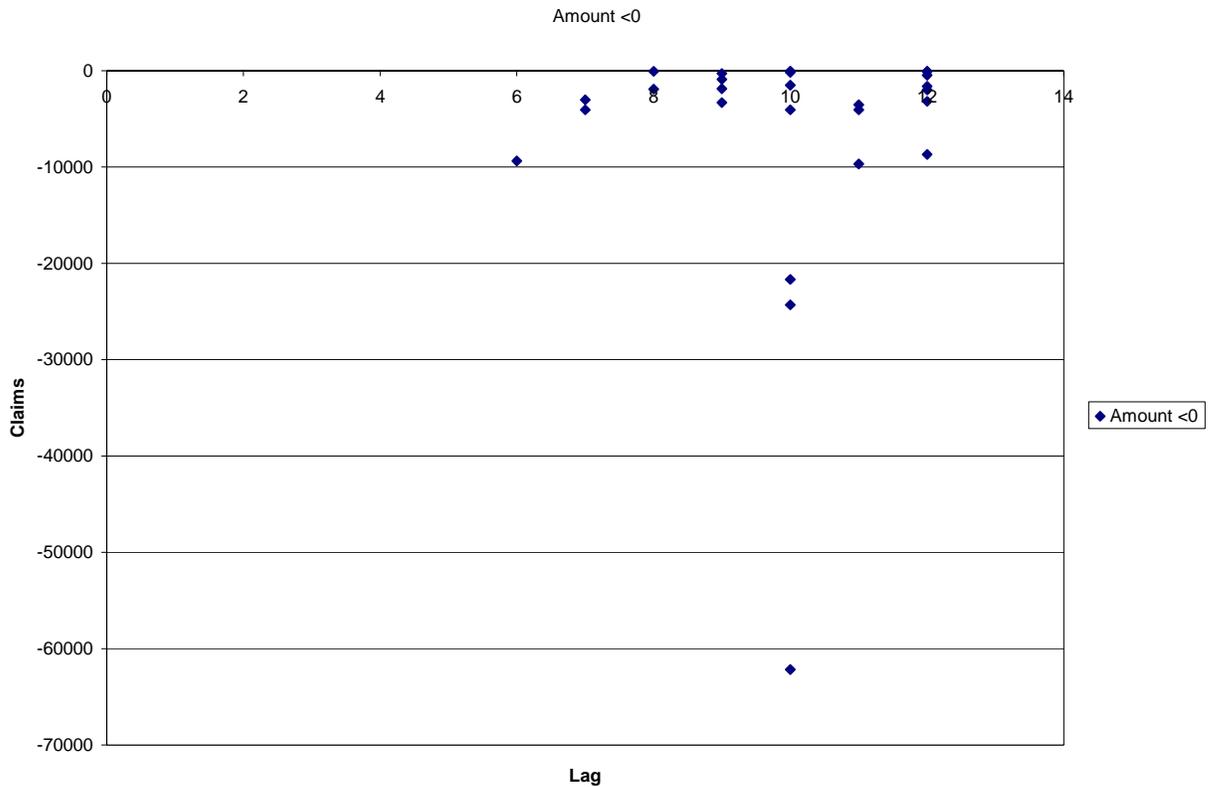
The graph in Exhibit 2.2 shows negative data points by lag. The graph shows no negative data points in lags 0 through 5.

Exhibit 2.2 shows a potential outlier at lag 10. The actual value is -$62,165, which occurs at $i = 19$ (month). This point is the sum of all claims transactions that were incurred in August 2002 and paid in June 2003.

From the claims processing department, the actuary learns about the following two claims:

- There was a $750,000 severe burn case that was incurred in January 2002 and paid in August 2002.
- The medical insurance company paid $60,000 for an auto accident claim that was incurred in August 2002. An auto insurance company reimbursed the medical insurance company $60,000 for this claim in June 2003.

Based on this knowledge, if we remove these claims as outliers, the new values in the triangle would be the following:

Exhibit 2.3
Potential Outliers

Incurred Month	Paid Month	Lag	(1) Current Value in Claims Triangle	(2) Potential Outlier Claim	(3) = (1) – (2) New Value in Claims Triangle	Cause of Claim
Jan-02	Aug-02	7	$756,000	$750,000	$6,000	Severe Burn Case
Aug-02	Jun-03	10	($62,165)	($60,000)	($2,165)	Auto Accident – Third party payment received after claim paid

When we use the completion factor method, these are the two data points we will assess to determine whether or not they are outliers.

2.3 The Completion Factor Method

In Exhibit 2.4, we calculate the completion factors as we did in Exhibit 1.8 for the example in Section 1. Note that we still work with total claims, not claims per member per month. The calculation is as follows:

Exhibit 2.4
Calculation of Deterministic Completion Factors
36 Month Sample Data

Completion Factor – Lag12	1.00000	(1) Due to data truncation as discussed above
Lag 12 claims (January 2001 through December 2002 – **24 months**)	$43,464,941	(2)
Lag 11 claims (January 2001 through December 2002 – **24 months**)	$43,428,968	(3)
Completion Factor – Lag 11	0.99917	(4) = (3) ÷ (2)
Lag 11 claims (January 2001 through January 2003 – **25 months**)	$45,977,288	(5)
Lag 10 claims (January 2001 through January 2003 – **25 months**)	$45,861,250	(6)
Lag 10 claims divided by Lag 11 Claims	0.99748	(7) = (6) ÷ (5)
Completion Factor – Lag 10	0.99665	(8) = (4) × (7)

In a manner similar to Exhibit 1.4 in Section 1, we calculate IBNR by lag as follows:

Exhibit 2.5
Calculation of IBNR – Sample Data
Lags 2 Through 12

Month	Lag	(1) Cumulative Paid Through December '03	(2) Completion Factor	(3)=(1)÷(2) Incurred Claims	(4)=(3)-(1) IBNR
Dec-03	0	$ 96,378	0.03936	$2,448,572	$2,352,193
Nov-03	1	1,283,817	0.59821	2,146,086	862,269
Oct-03	2	2,193,388	0.84701	2,589,571	396,184
Sep-03	3	2,688,921	0.91445	2,940,472	251,551
Aug-03	4	2,466,086	0.94775	2,602,055	135,969
Jul-03	5	2,502,042	0.96126	2,602,876	100,834
Jun-03	6	2,196,919	0.97130	2,261,842	64,923
May-03	7	2,385,024	0.99008	2,408,921	23,897
Apr-03	8	2,237,437	0.99275	2,253,778	16,341
Mar-03	9	2,361,919	0.99583	2,371,810	9,891
Feb-03	10	2,187,349	0.99665	2,194,699	7,351
Jan-03	11	2,548,319	0.99917	2,550,430	2,111
Dec-02	12	1,860,925	1.00000	1,860,925	0
Total – Lags 0 Through 12		$27,008,524		$31,232,037	$4,223,513
Total – Lags 2 Through 12		25,628,328		26,637,379	1,009,051

All of the incurred estimates for lags 2 through 11 are between $2 million and $3 million. The incurred claims estimates for lags 0 and 1, $2,448,572 and $2,146,085, respectively, seem to be reasonable estimates at first glance. However, as we shall see shortly, the variation in these estimates is relatively large.

When discussing outliers, we identified two potential outliers:
- the $750,000 claim incurred in January 2002 and paid in August 2002;
- the ($60,000) claim adjustment incurred in August 2002 and paid in June 2003.

We show how these potential outliers affect IBNR for lags 2 through 12.

Exhibit 2.6
Effect of Potential Outliers on IBNR for Lags 2 through 12
Completion Factor Method

	With Both Outliers Included	With $750,000 Claim Excluded	With ($60,000) Claim Adjustment Excluded
IBNR	$1,009,051	$833,796	$1,033,963
Change in IBNR Due to Outlier	N/A	(17.4%)	2.5%

For this document, we assume that a change of 5% or more in IBNR is significant.

Based on that criterion, in analyzing the IBNR for lags 2 through 12, we conclude that the $750,000 claim is an outlier and the ($60,000) claim adjustment is not an outlier when using the completion factor method. In our software, we have removed the $750,000 claim which we consider catastrophic.

When excluding the $750,000 burn case, our IBNR by month is the following:

Exhibit 2.7
IBNR with Catastrophic Burn Case Excluded

		(1)	(2)	(3) = (1) ÷ (2)	(4) = (3) − (1)
		Cumulative Paid through December '03	Completion Factor	Incurred Claims	IBNR
Month	Lag				
Dec-03	0	$ 96,378	0.03990	$2,415,373	$ 2,318,994
Nov-03	1	1,283,817	0.60644	2,116,988	833,171
Oct-03	2	2,193,388	0.85865	2,554,461	361,073
Sep-03	3	2,688,921	0.92702	2,900,603	211,682
Aug-03	4	2,466,086	0.96077	2,566,775	100,689
Jul-03	5	2,502,042	0.97447	2,567,585	65,543
Jun-03	6	2,196,919	0.98465	2,231,175	34,255
May-03	7	2,385,024	0.98993	2,409,296	24,272
Apr-03	8	2,237,437	0.99263	2,254,040	16,603
Mar-03	9	2,361,919	0.99576	2,371,975	10,056
Feb-03	10	2,187,349	0.99659	2,194,823	7,475
Jan-03	11	2,548,319	0.99916	2,550,467	2,148
Dec-02	12	1,860,925	1.00000	1,860,925	-
		IBNR			
		Dec-02 – Oct-03 (Lags 2 through 12)			$ 833,796
		Nov-03 – Dec-03 (Lags 0 and 1)			3,152,165
		Total IBNR Dec-02 – Dec-03			$3,985,962

2.3.1 Analysis of the Completion Factor Method

The completion factor method is deterministic since there are no probabilistic methods are used in the procedure.

The method is also cumulative; thus if incurred month *a* has twice as many claims as incurred month *b,* then the claims payment pattern of month *a* will have twice the weight in determining the completion factors as the claims payment pattern of month *b.*

As noted in Section 1, we are interested in setting confidence intervals for reserves, but because the completion factor method derives one point value for each completion factor, we cannot obtain error estimates from that method.

To test the completion factors, we analyze the completion factors derived for each of the completed months, January 2001 through December 2002. Thus we have 24 data points on which to analyze the 13 completion factors developed (for lags 0 through 12, inclusive).

When comparing our cumulative completion factors with the mean unweighted monthly completion factors (after eliminating the $750,000 outlier, as described before), we obtain the following comparison:

Exhibit 2.8
Comparison of Completion Factors

Lag	Cumulative Factors	Mean of Unweighted Individual Months
0	0.03990	0.03215
1	0.60644	0.58789
2	0.85865	0.84632
3	0.92702	0.92366
4	0.96077	0.95917
5	0.97447	0.97211
6	0.98465	0.98437
7	0.98993	0.98975
8	0.99263	0.99223
9	0.99576	0.99551
10	0.99659	0.99645
11	0.99916	0.99904
12	1.00000	1.00000

The mean of unweighted individual months is derived by averaging the completion factors for each incurred month separately. Because the completion factors in both columns above are relatively close, we conclude that we can use the data from individual

months to obtain some knowledge about our completion factors derived from using weighted averages.

Note that it might not always be the case that the monthly unweighted completion factors give us good information about the weighted average completion factors. We first need to compare the two sets of factors (as in Exhibit 2.8) before continuing with the analysis. We use actuarial judgment to determine if this comparison is acceptable.

In comparing our monthly completion factors, a good measure of the stability of our monthly completion factors is the standard deviation of the observed completion factors by lag divided by their mean. This ratio is analogous to a statistical measure known as the coefficient of variation. We know that completion factors are not random data from the same probability distribution, so their descriptive statistics do not provide the information we would normally expect.

Using the completion factor data from our 24 completed months, we obtain the results in Exhibit 2.9.

Exhibit 2.9
Completion Factor Method
Calculation of the Coefficient of Variation

	Completion Factor		
	Mean of	Standard	Coefficient of variation
Lag	Individual Months	Deviation	(Std. Dev./Mean)
0	0.03215	0.02548	0.79237
1	0.58789	0.12032	0.20467
2	0.84632	0.06114	0.07225
3	0.92366	0.04414	0.04778
4	0.95917	0.02483	0.02588
5	0.97211	0.02271	0.02336
6	0.98437	0.01471	0.01494
7	0.98975	0.01410	0.01425
8	0.99223	0.01444	0.01455
9	0.99551	0.01460	0.01467
10	0.99645	0.01025	0.01028
11	0.99904	0.00293	0.00294
12	1.00000	-	-

Note that the coefficient of variation at Lag 12 is zero since we assumed that all claims are paid by lag 12, which means that zero claims are paid after lag 12 with 100% probability. Therefore, the standard deviation of the amount paid after lag 12 is zero.

For this document, we define a relatively stable completion factor as one with a coefficient of variation less than 0.1. This definition may seem arbitrary, but it is based on our objective for stability of the completion factor estimate. The exhibit above indicates that, based on our definition, we need to look for other methods for lags 0 and 1. If we had selected 0.05, we would have concluded that we need to look for additional methods for lags 0, 1 and 2.

Another way to see why lags 0 and 1 have relatively high coefficients of variation is to ask the following question: For every $100 of claims already paid, how much is the corresponding IBNR?

If the completion factor is 0.75, the calculation of the IBNR is the following:

Exhibit 2.10
Example of How Claims Paid and
Completion Factor Determine IBNR

Claims Paid	$100	(1)
Completion Factor	0.75	(2)
Incurred Claims	$133.33	(3) = (1) ÷ (2)
IBNR = Incurred Claims Minus Claims Paid	33.33	(4) = (3) –(1)

For our completion factors by lag, the values are the following:

Exhibit 2.11
IBNR per $100 of Paid Claims
for the Calculated Completion Factors

Lag	(1) Completion Factor[*]	(2) = $100÷(1) - $100 IBNR for $100 in Paid Claims	(3) Coefficient of Variation
0	0.03990	$2,406.14	0.79237
1	0.60644	64.90	0.20467
2	0.85865	16.46	0.07225
3	0.92702	7.87	0.04778
4	0.96077	4.08	0.02588
5	0.97447	2.62	0.02336
6	0.98465	1.56	0.01494
7	0.98993	1.02	0.01425
8	0.99263	0.74	0.01455
9	0.99576	0.43	0.01467
10	0.99659	0.34	0.01028
11	0.99916	0.08	0.00294
12	1.00000	-	-

* With catastrophic case excluded

As the exhibit above shows, as the "IBNR for $100 in Paid Claims" decreases, the coefficient of variation decreases. Thus we have two measurements showing that the variation in the IBNR calculations is concentrated in lags 0 and 1.

In summary, we have used descriptive statistics to show the reasonableness of the deterministic completion factor method for lags 2 through 12 in our example. We recommend that, before running the completion factor method, an actuary check for outliers. Any identified outliers should be removed from the data before applying the completion factor method. After running the method, we recommend that descriptive

statistics, such as the coefficient of variation, be used to determine the reasonableness of the model.

Going forward with the completion factor method, we recommend the following:

- Use a different method to estimate IBNR for lags 0 and 1.
- Use $833,796 as the estimate of IBNR using the completion factor method for lags 2 through 12.
- Add an adjustment for unknown catastrophic claims after completing the calculation of IBNR.

In Section 3, we will show how the deterministic completion factor method can be combined with another method in order to add an estimate of error. There, we develop regression models for lags 0 and 1 (incurred months 34 and 35). To develop these estimates, we will use the PMPM incurred claims estimates developed by the completion factor method for months 0 through 33. Using PMPM incurred claim values is common industry practice since PMPM values remove the variation in claims due to changes in enrollment.

These PMPM values are the following:

Exhibit 2.12
PMPM Incurred Claim Estimates from Completion Factor Method
Data for Regression in Section 3

(1)	(2)	(3)	(4)	(5)	(6)=(4)÷(5)
Lag	Month	Months in Regression	Estimated Incurred Claims	Membership	PMPM Estimate
2	Oct-03	33	$2,554,461	11,843	$215.69
3	Sep-03	32	2,900,603	11,731	247.26
4	Aug-03	31	2,566,775	11,689	219.59
5	Jul-03	30	2,567,585	11,787	217.83
6	Jun-03	29	2,231,175	11,814	188.86
7	May-03	28	2,409,296	11,927	202.00
8	Apr-03	27	2,254,040	11,986	188.06
9	Mar-03	26	2,371,975	12,130	195.55
10	Feb-03	25	2,194,823	12,201	179.89
11	Jan-03	24	2,550,467	12,227	208.59
12	Dec-02	23	1,860,925	12,132	153.39
13	Nov-02	22	1,762,655	11,951	147.49
14	Oct-02	21	2,034,275	11,889	171.11
15	Sep-02	20	1,699,016	11,735	144.78
16	Aug-02	19	1,859,121	11,655	159.51
17	Jul-02	18	2,103,032	11,577	181.66
18	Jun-02	17	1,872,651	11,580	161.71
19	May-02	16	1,933,155	11,703	165.18
20	Apr-02	15	1,974,315	11,654	169.41
21	Mar-02	14	1,589,754	11,753	135.26
22	Feb-02	13	1,715,552	11,823	145.10
23	Jan-02	12	1,843,543	11,705	157.50
24	Dec-01	11	1,410,154	11,555	122.04
25	Nov-01	10	1,673,063	11,444	146.20
26	Oct-01	9	1,852,522	11,456	161.71
27	Sep-01	8	1,587,443	11,400	139.25
28	Aug-01	7	1,915,574	11,420	167.74
29	Jul-01	6	1,722,416	11,180	154.06
30	Jun-01	5	1,755,594	11,174	157.11
31	May-01	4	1,602,252	11,130	143.96
32	Apr-01	3	1,610,332	11,069	145.48
33	Mar-01	2	1,962,246	11,070	177.26
34	Feb-01	1	1,765,964	11,118	158.84
35	Jan-01	0	1,609,389	11,154	144.29

Section 3 – Regression Analysis for Recent Months and a Description of the Spreadsheet

We ended Section 2 with per member per month incurred claims (point) estimates for lags 2 through 35. In this section, we derive estimates for lags 0 and 1 and derive confidence intervals for those point estimates.

This section is organized as follows:

1. Description of the problem and variables.
2. Discussion of key statistical concepts used.
3. Description of the models we have in our spreadsheet which we use in analyzing our data.
4. Derivation of IBNR using the spreadsheet.
5. Explanation of how to calculate IBNR.
6. Summary.

3.1 Our Problem and Variables

We have 34 points for months 0 through 33 (January 2001 through October 2003), which we derived in Section 2. These points are shown in Exhibit 2.12 at the end of Section 2. Thus, our main independent (predictor) variable is time. Usually, health care costs increase over time.

Our dependent (response) variable is monthly incurred claims cost, measured on a per member per month (PMPM) basis. Our problem is to find PMPM claims cost estimates for incurred months 34 and 35, which correspond to November 2003 and December 2003, respectively. For these two months, completion factors from lags 1 and 0, respectively, are needed.

We also have two other independent variables which we analyze:

- Weekday/weekend (Day Factor) - Many medical offices are closed (or have limited hours) on weekends and holidays while hospitals and other facilities remain open for emergency and urgent care. We assume that an average weekend/holiday has 35% of the utilization of an average weekday. We call this the "Day Factor". We model the weekday/weekend variable by dividing our independent variable y by w_t. Thus we are normalizing the number of weekdays per month.
- Additional time variable (Time 2) – We have the capacity in our spreadsheet to add an additional variable to each month. In this section of the spreadsheet, we assign values of 0 or 1 for this second variable in our search for appropriate models. We would use this variable if we have knowledge about our data that we think could be modelled using this extra variable. For instance, if we knew that all health care providers increased their rates by 10% on January 1, 2002, we could set this variable equal to 0 for months in 2001 and 1 for months in 2002

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and 2003. This is a one-time increase which is different from the continual increase in our main independent variable.

3.2 Key Statistical Concepts

Key statistical concepts we review here are the following:

- R-square and adjusted R-square values;
- Number of data points;
- p-values in a regression output;
- Interval estimation;
- Interval estimates for simple linear regression.
- Residuals

3.2.1 R-Square and Adjusted R-Square Values

When we run regression models, among the first results we review are the R-square and adjusted R-square values.

We define R-square and adjusted R-square as follows:

R-square is the coefficient of determination of the regression model. This is also interpreted as the percentage of the variation in the observed values of the response that is explained by the regression model. The larger the R-square value, the greater is the indication that the model is satisfactory. R-square is defined as

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST},$$

where SSR is the sum of squares accounted for in the regression model, SST is the total sum of squares of the deviations of the response values from their mean, and $SSE = SST - SSR$. One drawback of R-square is that adding more independent variables causes the R-square value to increase even though there is no improvement to the model. In order to correct for this anomaly, the adjusted R-square is defined to be

$$R^2(adj) = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)},$$

where k is the number of predictors in the regression model and n is the total number of observations on the response variable. Larger adjusted R-square values indicate a better fit of the model.

3.2.2 Number of Data Points

We have 34 data points. Let's say that the best model we can derive with 34 points has an adjusted R-square value of x. Let's say that if we use 30 data points, we can derive an adjusted R-square value of y, with y being 1% greater than x. Should we use the model

with 30 data points? If we take this to the extreme, R-square and adjusted R-square would equal 1 if we had only two data points.

We follow the principle that unless we have a compelling reason to exclude points, such as with outliers, we do not eliminate data points just to get a model with a slightly better fit. However, we often transform variables or add additional variables to get a better fit.

3.2.3 p-Values in a Regression Output

In a regression analysis output, there are several test statistics and corresponding p-values reported. First there is the F-statistic and the corresponding p-value. This tests if the overall model is statistically significant. A $p\text{-value} < 0.05$ is an indicator that we most probably have an overall model that is statistically significant when compared to a model using none of the independent variables. Then there are t-statistics and corresponding p-values for testing each individual regression coefficient (beta value). The independent (predictor) variables with $p\text{-values} < 0.05$ are considered to be useful in describing the relationship between the predictor variables and the dependent (response) variable.

3.2.4 Interval Estimation

When using a regression model to predict the IBNR (or incurred claims PMPM values), the actuary may wish to construct confidence and/or prediction intervals. Is it more appropriate to use confidence or prediction intervals? It depends on our answer to the following questions:

- Do we want the expected (average) value of the incurred claims PMPM corresponding to new values of the predictor variables (Lag 0 and 1)? We then need confidence intervals.

- Do we want the value of the incurred claims PMPM corresponding to new values for Lags 0 and 1? We then need prediction intervals.

The actuary would ordinarily be interested in the potential range of values of IBNR, which is the prediction interval. The range within which the average value of IBNR would be likely to fall is usually not of interest. We note that prediction intervals are larger than confidence intervals.

3.2.5 Interval Estimates for Simple Linear Regression

We now develop interval estimates in the context of simple linear regression.

We will limit our exposition to that simple model.

Suppose that we want to find the relationship between a dependent variable Y and an independent variable X based on a set of sample observations collected on the two variables, given in Exhibit 3.1 on next page.

Exhibit 3.1
Sample Data for Linear Regression Example

x	y
10	8.03
8	6.90
13	7.58
9	8.81
10	8.33
14	9.96
6	7.24
4	4.26
12	10.85
7	4.82

Let $x_1, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ be these observed values (with $n = 10$).

Using simple linear regression, we will estimate the linear function that best fits this data. The model is written as $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma^2); i = 1, \cdots, n$. Here the idea is to find a model to predict the values of the dependent variable Y for given values of the independent variable X. Hence the *x*-values are considered to be fixed and the Y-values are considered to be random.

Following are the least squares estimates of the parameters:

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}, \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 ,$$

where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i, S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2, S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}), \text{ and}$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i, \ i = 1, \cdots n.$$

Suppose that we want to predict the value of the response variable corresponding to a new observation on the independent variable X, say *x**.

The point estimate of the mean of the new observation is $\hat{y}^* = \hat{\beta}_0 + \hat{\beta}_1 x^*$.

A $100(1-\alpha)\%$ prediction interval for the new observation is

$$\left[\hat{y}^* - t_{\alpha/2}(n-2)\sqrt{\hat{\sigma}^2\left(1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right)} \ , \ \hat{y}^* + t_{\alpha/2}(n-2)\sqrt{\hat{\sigma}^2\left(1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right)}\right],$$

where $t_{\alpha/2}(n-2)$ is the upper tail $\alpha/2$ value of the student t-distribution with $n-2$ degrees of freedom.

Note that the estimate is most accurate at the mean of the observed values of the independent variable $\bar{x}$ and the prediction interval is narrowest at $x^* = \bar{x}$. The further away you move from $\bar{x}$, the wider the prediction interval gets.

The numerical values we obtain in applying linear regression to our sample data in Exhibit 3.1 are the following:

The regression equation derived is $y = 2.885 + 0.5154\,x + \varepsilon$.

Exhibit 3.2
Regression Output
Linear Regression Example

	Coefficient	SE Coef	t	p-Value (p)
Constant	2.885	1.353	2.13	0.066
X	0.5154	0.1385	3.72	0.006

Standard deviation (s) = 1.31457
R-square = 63.4%

The analysis of variance shows the following:

Exhibit 3.3
Analysis of Variance
Linear Regression Example

Source	Degrees of Freedom (df)	Sum of Squares (SS)	Mean Square (MS)	F-Value (F)	p-Value (p)
Regression	1	23.932	23.932	13.85	0.006
Residual Error	8	13.825	1.728		
Total	9	37.757			

The mean of our data $\bar{x}$ is 9.30. We choose three points on which to calculate prediction intervals 11, 15 and 18. The response variables that we obtain are the following (with $\alpha = 0.05$):

Exhibit 3.4
Predicting Values
Linear Regression Example

Independent Variable	Dependent Variable	Prediction Interval		Length
11	8.554	5.329	11.780	6.451
15	10.616	6.952	14.279	7.327
18	12.162	7.939	16.384	8.445

This example illustrates that, as the independent variable moves further away from the mean of 9.30, the length of the prediction interval increases.

When calculating IBNR, because our main predictor is time, moving further away from the mean is equivalent to projecting further out into the future. Because the width of the prediction interval is increasing as we get further away from $\bar{x}$, we generally restrict ourselves to using regression to project as few time periods into the future as is deemed really necessary. Hence, we might use data from months 0 through 32 to project the PMPM values for months 33 to 35, but we would not use that same data to project the PMPM value for month 44 unless we really needed to.

As we discussed in Section 2, prediction intervals are derived using statistical methods, and not using deterministic methods (e.g. the completion factor method). We have two statistical methods on which to derive prediction intervals:

- Prediction intervals based on lags 0 and 1 only, as derived from the regression models run in this section.
- A parametric method which derives prediction intervals for all IBNR estimates. This is discussed in Section 4.

Because over 70% of the IBNR is typically in lags 0 and 1, prediction intervals based on lags 0 and 1 should be reasonable for many applications.

3.2.6 Residuals

In regression analysis the assumption is that the dependent variable is random and the independent variable is not random and there is a relationship between the dependent variable and the independent variables and this relationship is fixed except for a random factor. Once a set of observations on the dependent variable and the independent variables is available one can get a best estimate of the relationship and this is called the regression equation. This equation can be used to predict the value of the dependent variable for any set of values of the independent variables. The difference between the predicted value and the observed value of the independent variable is called the residual and that in fact gives an estimate of the random variation. When the model is formulated the independent variable is assumed to be the sum of the fixed relationship between the

dependent variable and the independent variable and an error term. This error term is assumed to reflect the randomness of the dependent variable and, in the usual regression models, it is assumed that these error terms are normally distributed with mean 0 and an unknown variance. Once a set of independent observations on the dependent variable is available, then there is a set of independent error terms.

We test the assumption that the model is a good model using the following:

- Residual Plot
- Histogram
- Shapiro-Wilk Test

The plot of these residual values is called the residual plot. The residual plot should not show any discernable pattern. If the picture is expanding or contracting in one direction or another, then the assumption of constant variance is not supported. If our model fits the data well, the histogram of residuals should show a normal, or bell-shaped, pattern. In our software, we ask how many bins or intervals you want to use in the histogram. You can choose between 7 and 12 bins. With a histogram, one visually observes the normality pattern. A more rigorous test for normality is the Shapiro-Wilk Test.

We analyze the residuals further with such tests as the Shapiro-Wilk Test for normality in Section 4. In this section, we will only analyze residuals based on observations.

3.3 Models We Tested

In this section, we are using 34 PMPM (point) estimates from 34 consecutive months. We programmed the spreadsheet using the following models:

Exhibit 3.5
Statistical Models in the Spreadsheet

Name	Type of Model	Dependent Variables	Equation
LinRegr	Linear	Time t [1]	$y = \beta_0 + \beta_1 t + \varepsilon$
QuadRegr	Quadratic	Time t	$y = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon$
ExpRegr	Exponential	Time t	$y = \exp(\beta_0 + \beta_1 t) + \varepsilon$
Lin2Var	Linear	Time t_1, User Input t_2 [2]	$y = \beta_0 + \beta_1 t_1 + \beta_2 t_2 + \varepsilon$
Quad2VarRegr	Quadratic	Time t_1, User Input t_2	$y = \beta_0 + \beta_1 t_1 + \beta_2 (t_1)^2 + \beta_3 t_2 + \varepsilon$
Exp2VarRegr	Exponential	Time t_1, User Input t_2	
AdjLinRegr	Linear	Time t, Weekday/weekend w_t	$y/w_t = \beta_0 + \beta_1 t + \varepsilon$
AdjQuadRegr	Quadratic	Time t, Weekday/weekend w_t	$y/w_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon$
AdjExpRegr	Exponential	Time t, Weekday/weekend w_t	$y/w_t = \exp(\beta_0 + \beta_1 t) + \varepsilon$
AdjLin2Var	Linear	Time t_1, User Input t_2, Weekday/weekend w_t	$y/w_t = \beta_0 + \beta_1 t_1 + \beta_2 t_2 + \varepsilon$
AdjQuad2VarRegr	Quadratic	Time t_1, User Input t_2, Weekday/weekend w_t	$y/w_t = \beta_0 + \beta_1 t_1 + \beta_2 (t_1)^2 + \beta_3 t_2 + \varepsilon$
AdjExp2VarRegr	Exponential	Time t_1, User Input t_2, Weekday/weekend w_t	$y/w_t = \exp(\beta_0 + \beta_1 t_1 + \beta_2 t_2) + \varepsilon$

We note the following:
(1) Time (t or t_1) is measured by the numbers we assign to each month.
(2) The input variable (t_2) is a variable whose value by month is input by the actuary each time the model is run. In this section, we are limiting our use of this variable to 0 and 1. This variable enables the user to treat some months as being different from other months. For example, the actuary may have information about hospital pricing changes that occurred in the middle of the time frame during which the observations were gathered. It may be appropriate to model the data before and after the date of that pricing change differently.

3.3.1 Key Steps to Applying the Methodology Discussed Above to Our Data

The high-level steps we follow in determining our estimates for lags 0 and 1 are as follows:

1. Plot the dependent variable against the independent variables to observe the visual patterns.

2. Based on the plot, determine which of the models programmed in the spreadsheet seems to be an appropriate choice for our data. In addition to the model that the actuary thinks is most appropriate, the actuary may want to run the simple regression model as a baseline for comparison of all models run.

3. Fit the model and review the diagnostics. The key statistical values we can check are the adjusted R-square value as well as the p-value for the F-test. We can also look at the residuals, checking the normality and constant variance assumptions, and spotting outliers.

   In addition to checking that the model makes statistical sense, the model needs to make sense based on knowledge we have of exogenous factors affecting the health care plan or book of business for which we are estimating incurred claims.

4. Run additional models and review the diagnostics. If you change one item at a time in running successive models, you will have an idea of the effect of each change. For instance, let's say your first model is a linear regression model with one independent variable, time. If you want to improve on this model, you might next add either the weekend/weekday variable or the additional time variable, but not both at the same time.

3.4 Numerical Example: Deriving IBNR for Our Data Set

As a first step, we show our data graphically. The plot of our 34 PMPM estimates for the time period January 2001 through October 2003 shows the following:

Exhibit 3.6
Graphical View of the Data for our Regression Analysis

Values from Completion Factor Method
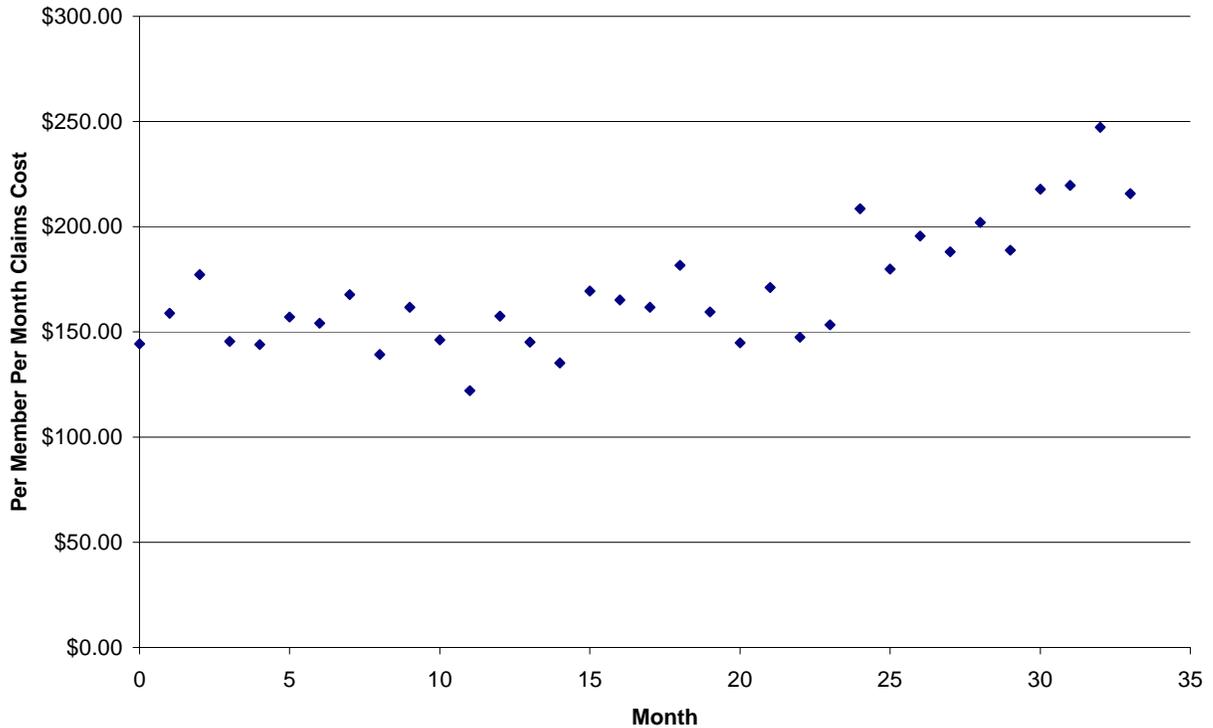
Instead of showing claims costs constantly increasing over time, the graph seems to show two levels of claims costs, with one level for months 0 through 23 and another level for months 24 through 33; therefore, we will assign our (0,1) variable as follows:

- 0 for months 0 through 23;
- 1 for months 24 though 35.

Note that we are assuming that the claims pattern that commenced at Lag 24 will continue in the two lags that we are predicting, Lags 34 and 35.

Based on this observation, the first two models we run are the following:

- LinRegr: Linear regression model as a base
- Lin2Var: Linear regression model with second variable defined as:
  - 0 for months 0 through 23;
  - 1 for months 24 though 35.

The key output values from LinRegr are the following, using our spreadsheet:

Exhibit 3.7
Regression Results
LinRegr Model

Estimators for coefficients		
		p-values
beta_0	135.2215	7.03E-20
beta_1	2.095960	9.19E-07
Standard Error	19.79333	
R^2	0.534173	
Adjusted R^2	0.519616	
p-value for F-test	9.19E-07	

This model has an adjusted R-square of 0.520; we would like to improve on this value. The p-value for the F-test and the p-values for the coefficients are all significantly less than 0.05. Therefore, if we cannot improve on our adjusted R-square value, we would use LinRegr.

The key output values from Lin2Var are the following:

Exhibit 3.8
Regression Results
Lin2Var Model

Estimators for coefficients		**p-values**
beta_0	148.52583	7.83E-22
beta_1	0.52689	2.49E-01
beta_2	42.78974	1.07E-04
Standard Error	15.72873	
R^2	0.71504	
Adjusted R^2	0.69665	
p-value for F-test	0.00000	

With Lin2Var, our adjusted R-square has improved from 0.520 to 0.697, an increase of 0.177. This relatively high increase in adjusted R-square indicates that our Boolean variable is contributing significantly in predicting the value of the dependent variable.

To the Lin2Var model, we add the Day Factor, which is AdjLin2Var; with this model we obtain the following regression results:

Exhibit 3.9
Regression Results
AdjLin2Var Model

Estimators for coefficients		p-values
beta_0	153.58718	1.39E-23
beta_1	0.52399	2.05E-01
beta_2	43.62881	2.09E-05
Standard Error	14.20020	
R^2	0.76025	
Adjusted R^2	0.74478	
p-value for F-test	0.00000	

Our adjusted R-square value is 0.745, which is an improvement over the adjusted R-square value of 0.697 we obtained without the day factor.

To see if our model can be improved, we now test to see if the exponential model, with the day and Time 2 factors (model AdjQuad2VarRegr), gives us an improved R-square value. With model AdjExp2VarRegr, our results are the following:

Exhibit 3.10
Regression Results
AdjExp2VarRegr Model

Estimators for coefficients		p-values
beta_0	5.03637	2.66E-47
beta_1	0.00292	2.02E-01
beta_2	0.23455	3.12E-05
Standard Error	0.07853	
R^2	0.75219	
Adjusted R^2	0.73620	
p-value for F-test	0.00000	

With this model, our adjusted R-square value decreases from 0.745 with AdjLin2Var to 0.736. We do not use this model because it is a more complicated model with a lower adjusted R-square. We now see if using the quadratic model, with the Day Factor and Time 2 Factor (model AdjQuad2VarRegr) will improve the results. Thus we run the AdjQuad2VarRegr model. Our results are the following:

Exhibit 3.11
Regression Results
AdjQuad2VarRegr Model

Estimators for coefficients		p-values
beta_0	163.07512	1.09E-21
beta_1	-1.64617	1.07E-01
beta_2	0.08586	2.46E-02
beta_3	25.54311	2.93E-02
Standard Error	13.25123	
R^2	0.79795	
Adjusted R^2	0.77775	
p-value for F-test	0.00000	

We see that the adjusted R-square value with this model is 0.778, which is the largest adjusted R-square value we have obtained. We select this model for further analysis.

Sometimes adding variables does not increase the accuracy of a model, which happens when we have multicolinearity. The text by Belsey, Kuh, and Welsch discusses the issue of multicolinearity.

As a check for reasonableness of this choice, we compare the R-square value of all of the models in our software as follows:

Exhibit 3.12
Summary of Adjusted R-Square Values

Name	Type of Model	Adjusted R-Square Value
LinRegr	Linear	0.520
QuadRegr	Quadratic	0.721
ExpRegr	Exponential	0.503
Lin2Var	Linear	0.697
Quad2VarRegr*	Quadratic	0.740
Exp2VarRegr	Exponential	0.676
AdjLinRegr	Linear	0.553
AdjQuadRegr	Quadratic	0.747
AdjExpRegr	Exponential	0.549
AdjLin2Var	Linear	0.745
AdjQuad2VarRegr	Quadratic	0.778
AdjExp2VarRegr	Exponential	0.736

* In this analysis, the Time 2 variable is defined as follows:
   ▪   0   1 – 23

▪ 1   24-35

As this exhibit shows, the AdjQuad2VarRegr model has the highest Adjusted R-square value. The residual graph for the AdjQuad2VarRegr model shows the following:

Exhibit 3.13
Residuals
AdjQuad2VarRegr

Graph Of Residuals

Residuals vs **Month No.**
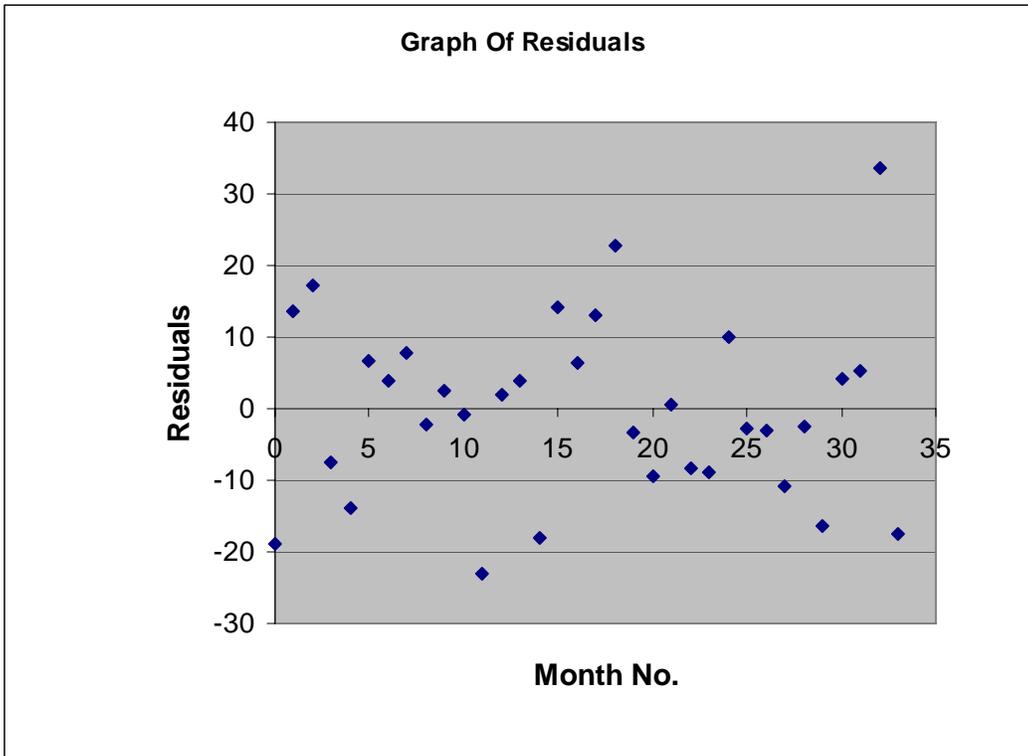
The residuals in this model do not show a pattern. Therefore, we do not reject our model based on our examinations of the residuals.

Our results from the AdjQuad2VarRegr model are PMPM incurred values of $207.70 for November 2003 and $230.08 for December 2003. Before we conclude that we will accept the values from AdjQuad2VarRegr as our estimates, we need to check these values for reasonableness. Are these values reasonable? The PMPM values we have from the completion method are the following:

Exhibit 3.14
Testing for Reasonableness of Results
2003 Per Member Per Month Input Values

Jan-03	$208.59
Feb-03	179.89
Mar-03	195.55
Apr-03	188.06
May-03	202.00
Jun-03	188.86
Jul-03	217.83
Aug-03	219.59
Sep-03	247.26
Oct-03	215.69

Our PMPM values of $207.70 for November 2003 and $230.08 for December 2003 seem to be in a reasonable range based on the PMPM values we have calculated for 2003. Thus we accept the values from AdjQuad2VarRegr as our PMPM estimates.

3.4.1 Adjustment for Catastrophic Cases

We took a $750,000 catastrophic case out of our calculations, so our calculation of IBNR to this point does not include any reserve for catastrophic cases. A general formula for calculating the mean value of IBNR for one catastrophic case is the following:

- expected cost of catastrophic case
  multiplied by the
- expected probability of claim occurring.

We calculate our reserve for catastrophic cases as follows:

With our data, let's assume that the cost of catastrophic case is $750,000. If we use a conservative estimate of the probability of the catastrophic claim occurring in a given month, we would only use our 24 months of completed claims as our base (calendar years 2001 and 2002). Thus our probability is 1/24. The claim was incurred in January 2002 and paid in August 2002 (Lag 7). Thus the $750,000 was in IBNR in 7 out of 24 months. Our catastrophic reserve is calculated as follows:

<div align="center">Exhibit 3.15</div>
<div align="center">Calculation of Additional IBNR for Unknown Catastrophic Cases</div>

Cost Per Catastrophic Claim	$750,000	(1) Cost For Actual Claim in Data
Frequency	1/24	(2) Conservative Estimate – Based on Completed Months
Months in IBNR	7	(3)
IBNR For Catastrophic Cases	$218,750	(4) = (1) × (2) × (3)

A key issue with catastrophic claims reserves is the following:

- If the catastrophic claim has not been incurred, the IBNR is over-stated by $218,750.
- If the catastrophic claim has been incurred, the IBNR is under-stated by $531,250.

This type of catastrophic claim lends itself to a separate financing arrangement. Among the ways an insurer can approach this issue are the following:

- Set up a separate catastrophic reserve for catastrophic claims – In this research, we did not have enough data on catastrophic claims to derive this IBNR.
- Purchase reinsurance from an external source

The prediction intervals that we are developing are designed to cover fluctuations in non-catastrophic claims, while some sort of reinsurance arrangement can provide a margin for catastrophic claims.

Apart from including a very large margin, the actuary needs to document the risk of under-statement due to catastrophic cases. The best way to minimize the issue of unknown catastrophic claims is to attempt to get information on catastrophic claims from case management or other internal company sources; in other words, an actuary can't minimize the occurrence of catastrophic claims, but s/he can minimize the occurrence of *unknown* catastrophic claims.

If a catastrophic case is known, we just add the estimated cost of the case to our IBNR estimate. For instance, if, through the case management department, we know about a $500,000 catastrophic case for which we have not paid any claims, we would add $500,000 to our IBNR.

The estimate of IBNR for catastrophic cases can be refined by using statistical distributions that include a multiple number of claims and multiple claims amounts.

3.5 Calculating IBNR

We have derived PMPM estimates for November and December 2003. We calculate IBNR from these estimates as follows:

IBNR = Incurred Claims – Claims paid to date

where Incurred Claims = PMPM × Membership.

Our calculation of IBNR for the months of November and December 2003 is the following:

Exhibit 3.16
Calculation of IBNR for Lags 0 and 1

	Incurred Month		
	November 2003	December 2003	
PMPM Estimate	$207.70	$230.08	(1)
Members	11,902	11,844	(2)
Total Incurred Claims	$2,472,045	$2,725,068	(3) = (1) * (2)
Claims Paid			
Paid Month			
Nov 2003	$58,510	N/A	(4) *
Dec 2003	$1,225,307	$96,378	(5)
Total Paid	$1,283,817	$96,378	(6) = (4) + (5)
IBNR	$1,188,228	$2,628,690	(7) = (3) – (6)

* By definition, in our data, paid date occurs on or after the incurred date; therefore, the value for claims incurred in December 2003 and paid in November 2003 is not applicable. The value $58,510, claims paid and incurred in November 2003, can be found in bold in the Appendix in Exhibit A1.

Our total IBNR without margin is the following:

Exhibit 3.17
Total IBNR as of December 2003
Without Margin

December 2003 – Lag 0	$2,628,690	(1)
November 2003 – Lag 1	1,188,228	(2)
December 2002 Through October 2003 – Lags 2 through 12	833,796	(3)
Sub-total	4,650,714	(4)=(1)+(2)+(3)
IBNR for Catastrophic Cases	218,750	(5)
Total	$4,869,464	(6)=(4)+(5)

3.6 Confidence Intervals

The upper bound of the 95% confidence interval is calculated as follows:

Exhibit 3.18
Calculation of the Upper Bound of a Prediction Interval (with 95% Confidence Level)

	November 2003	December 2003	Total	
PMPM Value	$207.70	$230.08		(1)
One-Sided Prediction Interval Length	28.82	33.00		(2)
Upper Bound – Prediction Interval	236.52	263.08		(3) = (1) + (2)
Members	11,902	11,844		(4)
One-Sided Interval for All Members (Margin)	$343,016	$390,852		(5) = (2) * (4)
IBNR – Best Estimate			$4,650,714	(6) Line (4) – Exhibit 3.17
Half length of the Prediction Interval			$638,273	(7)*
Upper Bound of IBNR with 95% Prediction Interval			$5,288,987	(8) = (6) + (7)
Margin as Percent of Best Estimate			12.1%	(9) = (7) ÷ (6)

* The confidence was calculated for lag 0 and 1 together, and not separately for lag 0 and lag 1. Thus, from Line (7), $638,273 ≠ $343,016 + $390,852 (from line (5))

Thus, we are 97.5% confident (the upper bound is only one side of the confidence interval) that, once all of the claims incurred on or before December 2003 are paid, the total paid will be less than or equal to $5,288,987, excluding catastrophic claims. The margin is the amount that we are adding to have sufficient IBNR to cover paid claims 97.5% of the time.

Note that we did not assign a confidence interval around the catastrophic case IBNR.

If we want different levels of confidence for our prediction intervals, we can use our spreadsheet to derive these levels. Other prediction levels with our AdjQuad2VarRegr model are the following:

Exhibit 3.19
IBNR With Different Prediction Intervals

Prediction Interval	One Sided Prediction Interval	IBNR With Margin (Without Catastrophic IBNR)	IBNR With Margin (With $218,750 Catastrophic IBNR)
95%	97.5%	$5,288,987	$5,507,737
50%	75%	4,864,011	5,082,761
90	95%	5,288,902	5,507,652
99%	99.5%	5,510,085	5,728,835

A major limitation of the method that we just described in calculating prediction intervals is that the intervals are just based on our calculations for the most recent two months (Lags 0 and 1). In our other paper in this journal (ARCH, 2007), we discuss methods where prediction intervals are calculated for all months.

3.7 Summary

In this paper, we have shown that just using the completion factor will often not yield a satisfactory result in the calculation of IBNR. One has to use statistical methods and apply knowledge about the product to improve the calculation of IBNR.

References

Key references in the study of statistical methods for computing medical insurance IBNR include the following:

- Barnett Glen, and Ben Zehnwirth, "Best Estimates for Reserves," Proceedings of the Casualty Actuarial Society 87, Part 2, No. 167, November 12-15, 2000, pp. 245-321. Available online at: http://www.casact.org/pubs/proceed/proceed00/00245.pdf
- Belsey, David A., Kuh, Edwin, and Welsch, Roy E., *Regression Diagnostics; Identifying Influential Data and Sources of Collinearity*, John Wiley, New York, 1980
- Black, Kenneth Jr., and Harold D. Skipper, Jr., *Life and Health Insurance,* Thirteenth Edition, Prentice Hall, Upper Saddle River, New Jersey, 2000.
- Bornhuetter, and Ferguson "The Actuary and IBNR" *Proceedings of the Casualty Actuarial Society,* vol. 59, No. 112, 1972.
- Brown, Robert L., *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, (Second Edition), Actex Publications, Winsted, Connecticut, 2001.
- Fearrington, Doug and Robert Lynch, "Approaches to Determining Unpaid Claim Liabilities: Old and New," *The Records of the Society of Actuaries,* Valuation Actuary Symposium, Boston, Massachusetts, September 20-21, 2004, Session 39TS.
- Litow, Mark E., "A Modified Development Method For Deriving Health Claim Reserves", *Transactions of Society of Actuaries*, 1989, Volume 41
- Little, R. J. A. and Rubin, D. B., *Statistical Analysis with Missing Data,* John Wiley, New York, 1987.
-Lloyd, John `C., "Health Reserves", Society of Actuaries Study Note 8GM-305-00, Society of Actuaries, Schaumburg, Illinois
- Lynch, Robert G., "Gerbils on Espresso: A Better Way to Calculate IBNR Reserves with Low Variance," *Contingencies,* January-February 2004, pp. 28-38.
- Mosley, Roosevelt C., "Estimating Claim Settlement Values Using GLM," *Casualty Actuarial Society, Discussion Paper Program - Applying and Evaluating Generalized Linear Models Including Research Papers on the Valuation of P&C Insurance Companies,* 2004. Available online at: http://www.casact.org/pubs/dpp/dpp04/
- Pindyck, R. S. and Rubinfeld, D. L., *Econometric Models and Economic Forecasts, Fourth Edition,* Irwin McGraw-Hill, New York, 1998.
- Rubin, D. B., *Multiple Imputations for Non-Response in Surveys,* John Wiley, New York, 1986
- Sen Ashis and Srivastava Muni, *Regression Analysis; Theory, Methods, and Applications,* Springer-Verlag, New York, 1990.
- Society of Actuaries, Course 7, March 2006, Advance Reading
- Sutton, Harry L. and Sorbo, Allen J., *Actuarial Issues in the Fee-For-Service/Prepaid Medical Group*, Center for Research in Ambulatory Health Care Administration, Denver, Colorado, 1987, pages 62-67
- Zehnwirth, Ben, "Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals, and Risk Based Capital," Casualty Actuarial Society Forum 1994, pp. 510-605. Available online at: http://www.casact.org/pubs/forum/94spforum/94spf447.pdf

Zehnwirth, Ben, "Three Powerful Diagnostic Models for Loss Reserving," University of Melbourne, Centre for Actuarial Studies, Australia, Working Paper Series, 1996. http://www.economics.unimelb.edu.au/actwww/No34.pdf

Appendix

The data set used in Section III is presented below. Each row corresponds to an ascending month while each column corresponds to a lag. Month 0 corresponds to January 2001 in the text. Claim data is the following:

Exhibit A1
Claim Data

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	180	436082	933353	116978	42681	41458.5	5088	22566	4750.5	3280.5	-187.5	1464	1696.5
1	5161.68	940722	561967	21694.4	171659	11007.8	19088	5212.94	4337.08	7843.52	2972.79	4061.21	10235.9
2	42263.5	844293	720302	94633.8	182077	32215.8	12936.9	22815	1754.41	4695.1	1325.66	757.52	2177.11
3	20780.6	762302	394625	78043	157950	46173.2	126254	4838.86	336.5	1572.86	9572.67	1947.42	5936.67
4	20345.9	772404	392330	315888	39196.7	21360.4	8720.76	5452.19	16627.4	2117.83	4119.37	5666.43	-1977.1
5	20490.7	831793	738087	65525.9	27767.9	12184.6	1493.28	11265	1805.47	29278.1	13019.7	2966.56	-83.05
6	37954.5	1126675	360514	89316.7	40126	16576.2	16701.4	2443.54	8265.69	11310.5	8006.03	1403.37	3123.59
7	138558	806362	589304	273117	36912.5	16831.5	19941.2	13310.2	8619.23	4678.52	3094.16	4608.62	236.1
8	28331.7	954543	246571	205528	60060.2	15198.5	42208	17568.2	1685.95	9897.12	3367.21	2062.16	421.49
9	104160	704796	565939	323789	45307.3	32517.9	26226.7	7976.11	3363.65	991.52	33962.8	2199.55	1292.75
10	40746.9	927158	425794	146145	66663.3	31214.1	12807.6	15858.6	373.68	3079.32	411.52	936.57	1874.71
11	10860.8	847338	272165	134798	71804.3	27800.1	17917	3929.79	2793.64	846.17	1961.73	1879.33	16059.9
12	77938.2	896195	544372	173606	41595.4	4209.16	16473.3	756000	-65.62	-1880.5	-4053.9	84232.7	4920.96
13	38041.4	1035439	438153	115587	12488.7	22260.1	13203.1	6394.65	2056.12	-3323.4	33397.4	3478.9	-1624.7
14	39409.5	1022024	255002	169881	35230.1	40307.5	21067.2	5377.81	5508.42	17606.1	-24320	1298.16	1362.02
15	68252.7	1414379	317110	91880.3	53969.6	10887.9	3170.9	11659.6	20860.7	1033.06	-21670	2633.57	148.7
16	124824	1053972	516876	145954	25171.2	12608.9	7704.25	29632.9	4555.18	6203.3	3872.11	1115.66	665.74
17	49725.3	1119099	533444	80181.9	32202.9	23204.7	18806.7	7944.07	4151.88	-910.3	3663.86	608.33	528
18	44317.5	1297335	385789	141155	150726	35075.2	16176.1	8070.24	67.03	14216.6	2325.84	7090.56	687.22
19	134152	1111151	493175	101439	46656.6	22824.1	12818.3	3780.94	1265.42	2466.91	-62165	246.82	-8689.3
20	29968.2	1382043	178587	71030.3	25708.1	15068.3	3145.03	-4058.2	-1919.6	4984.06	-1523	-3538.9	-477.91
21	210377	999963	528880	201410	58003	26174.1	-9371.4	2016.79	9794.64	6688.15	-40.13	453.36	-73.44
22	56654.4	1206370	376504	56321.8	19590.5	12054.8	21076.9	11572.8	4038.96	821.64	6611.88	-9677.9	714.96
23	89180.6	1240938	279553	57163.5	75343.9	12665	71741.4	9048.55	1297.94	12164.4	19615.6	-4603.9	-3183.9
24	131568	1301927	716180	150253	110031	78147.5	4609.85	19855	18447.7	14432.4	118.97	2747.78	
25	76262	1130312	692736	174283	38890.6	41810.9	8834.16	18122.5	4268.11	-290.76	2119.3		
26	159575	1313809	704116	68411.5	30184.6	64401.6	19228.7	-3020.5	3220.03	1994.04			
27	76312.7	1505842	437084	50872.4	116723	18159.6	10974.7	12663.7	8804.83				
28	104028	1667823	360676	153274	37529.2	34840	17479.1	9373.82					
29	79687.9	1235573	776240	65302.8	18722.7	10778.6	10614.6						
30	76394.6	1689354	442965	234171	36806.5	22351.3							
31	110460	1492980	589184	93366.4	180095								
32	196687	2011979	313416	166839									
33	268365	1027925	897097										
34	**58510.1**	1225307											
35	96378.3												

Membership by month is the following:

Exhibit A2
Membership Data

Month	Month Number	Membership
Jan-01	0	11,154
Feb-01	1	11,118
Mar-01	2	11,070
Apr-01	3	11,069
May-01	4	11,130
Jun-01	5	11,174
Jul-01	6	11,180
Aug-01	7	11,420
Sep-01	8	11,400
Oct-01	9	11,456
Nov-01	10	11,444
Dec-01	11	11,555
Jan-02	12	11,705
Feb-02	13	11,823
Mar-02	14	11,753
Apr-02	15	11,654
May-02	16	11,703
Jun-02	17	11,580
Jul-02	18	11,577
Aug-02	19	11,655
Sep-02	20	11,735
Oct-02	21	11,889
Nov-02	22	11,951
Dec-02	23	12,132
Jan-03	24	12,227
Feb-03	25	12,201
Mar-03	26	12,130
Apr-03	27	11,986
May-03	28	11,927
Jun-03	29	11,814
Jul-03	30	11,787
Aug-03	31	11,689
Sep-03	32	11,731
Oct-03	33	11,843
Nov-03	34	11,902
Dec-03	35	11,844