Weather Derivatives and Short-Period Rainfall Indices

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Presentation Outline:

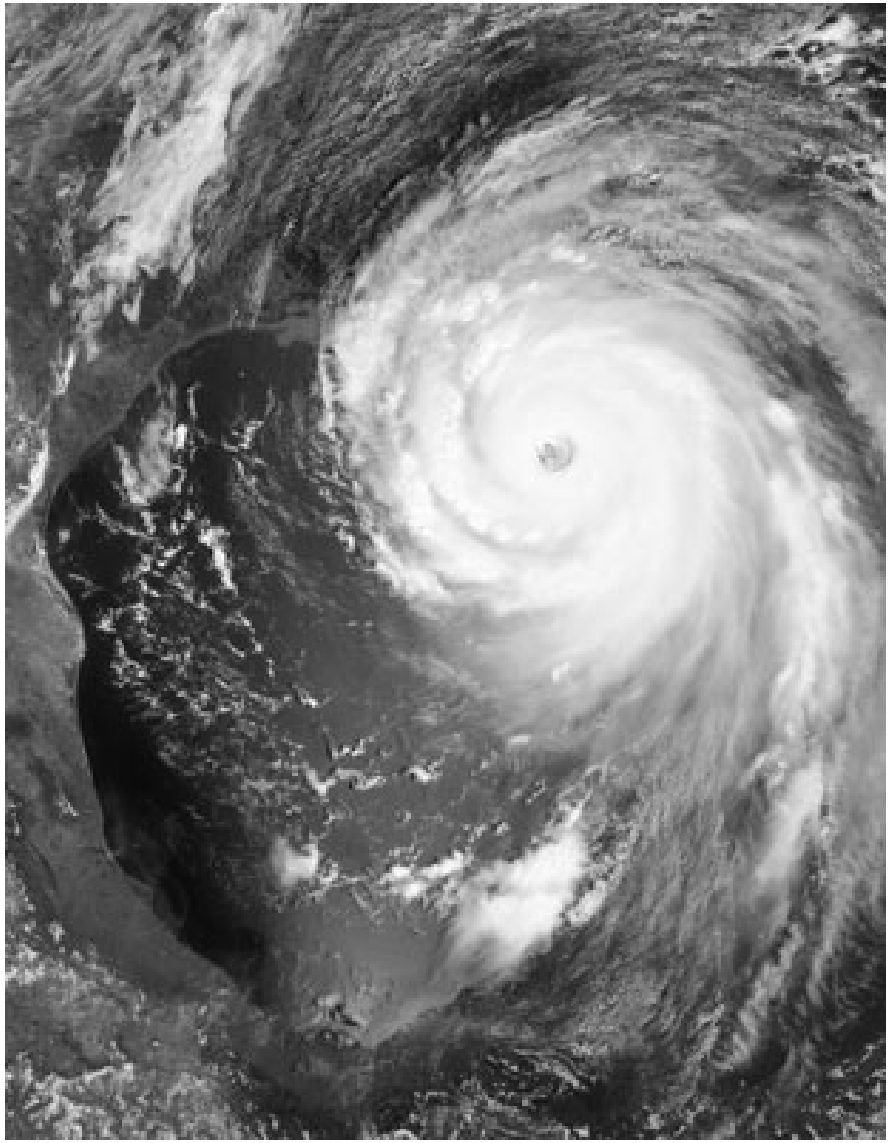
- Weather and financial risk
- Catastrophic and non-catastrophic events
- Weather derivatives and useful indices

- Rainfall time series for derivative valuation
- Modeling daily rainfall accumulations
- Modeling hourly rainfall accumulations

Weather and Financial Risk

- Financial risk due to weather uncertainty
  - Agriculture
  - Natural disasters (hurricanes)
  - Energy (both supply and demand)
- Weather insurance
  - For high risk, low probability events
- Weather derivatives
  - For low risk, high probability events

Catastrophic Weather Event: Hurricane Katrina



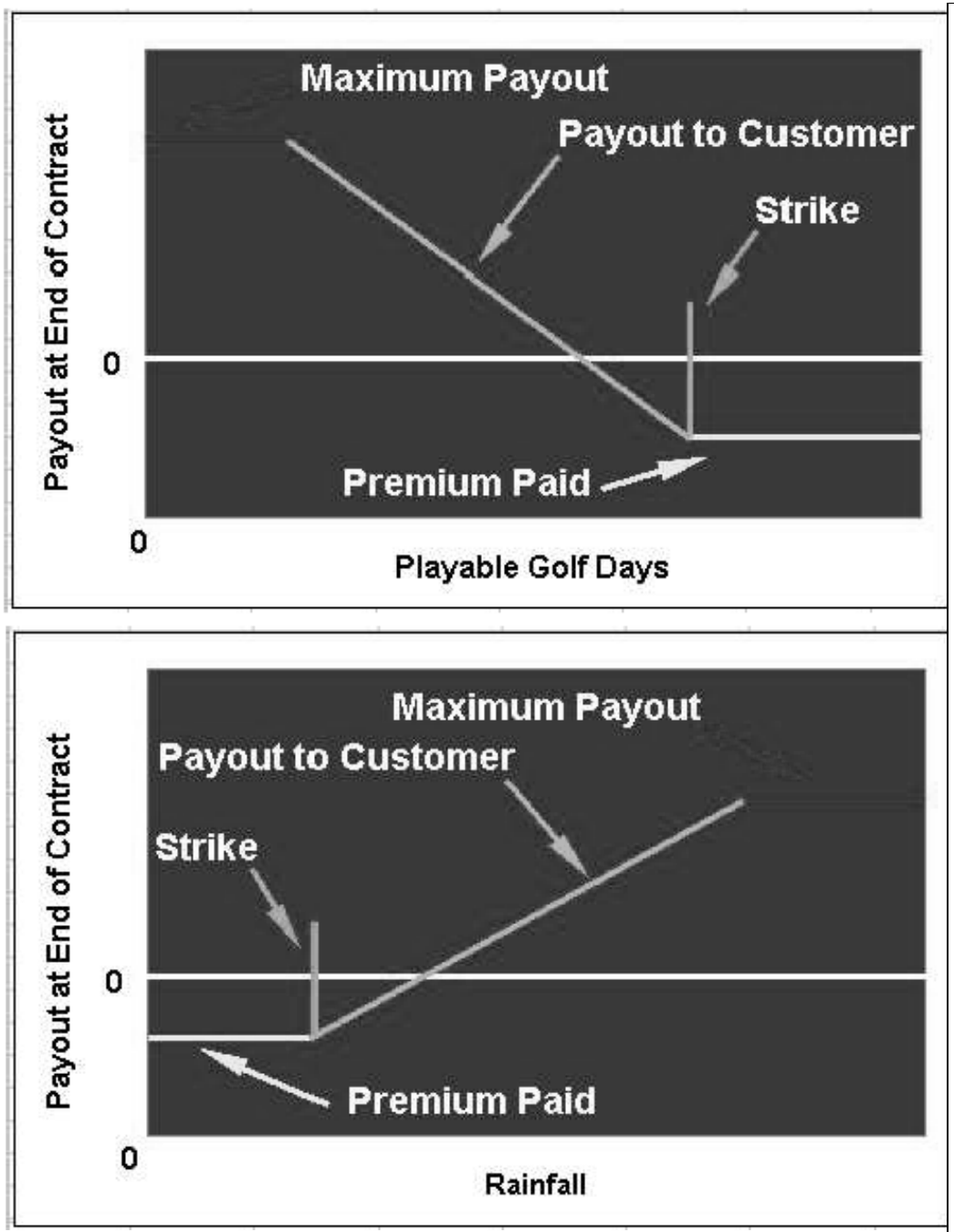
www.spatiallyadjusted.com/2005/08/30/satellite-image-of-hurricane-katrina-approaching-the-gulf-coast/

http://www.katrinahelp.com/hurricane-katrina-pictures.html

Non- Catastrophic Weather Events

- Wind – reduction in rounds played
- Rain – reduction in rounds played, course damage or reduced playability
- Heat – reduction in rounds played, increased irrigation costs
- Cold – reduction in rounds played,

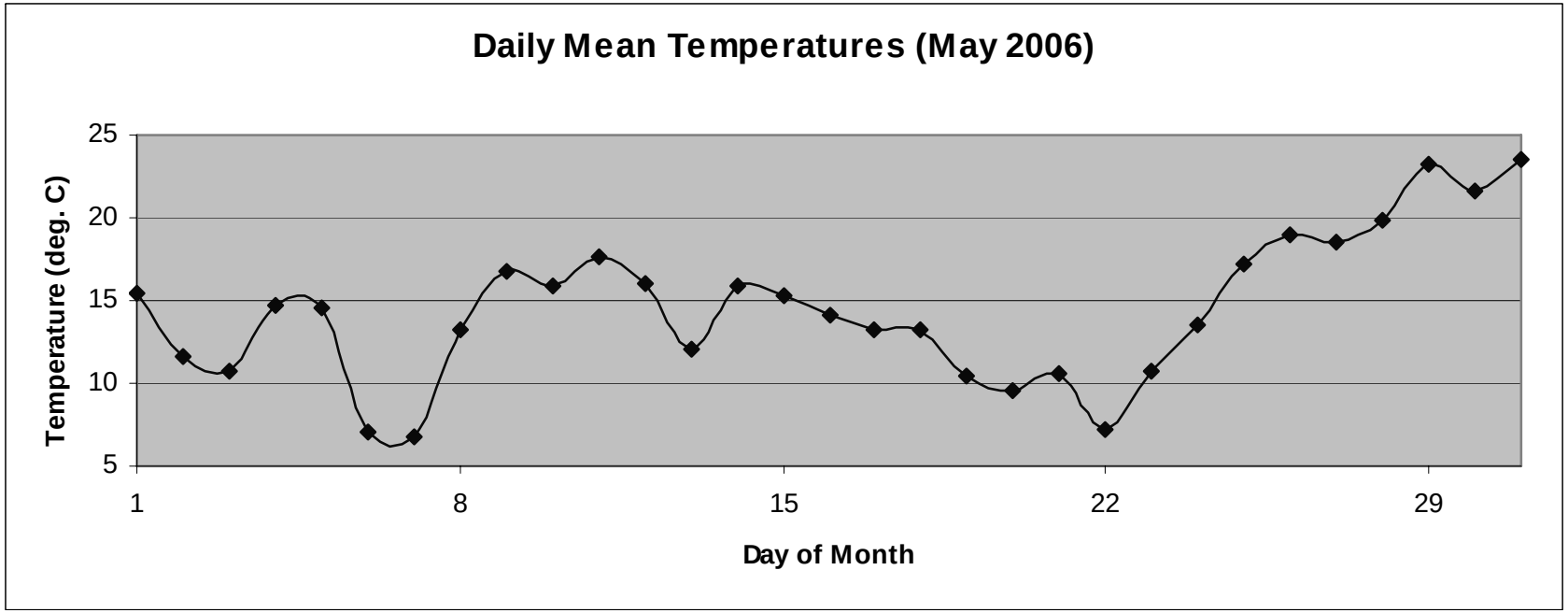
http://www.guaranteedweather.com/display_file.php?file=254

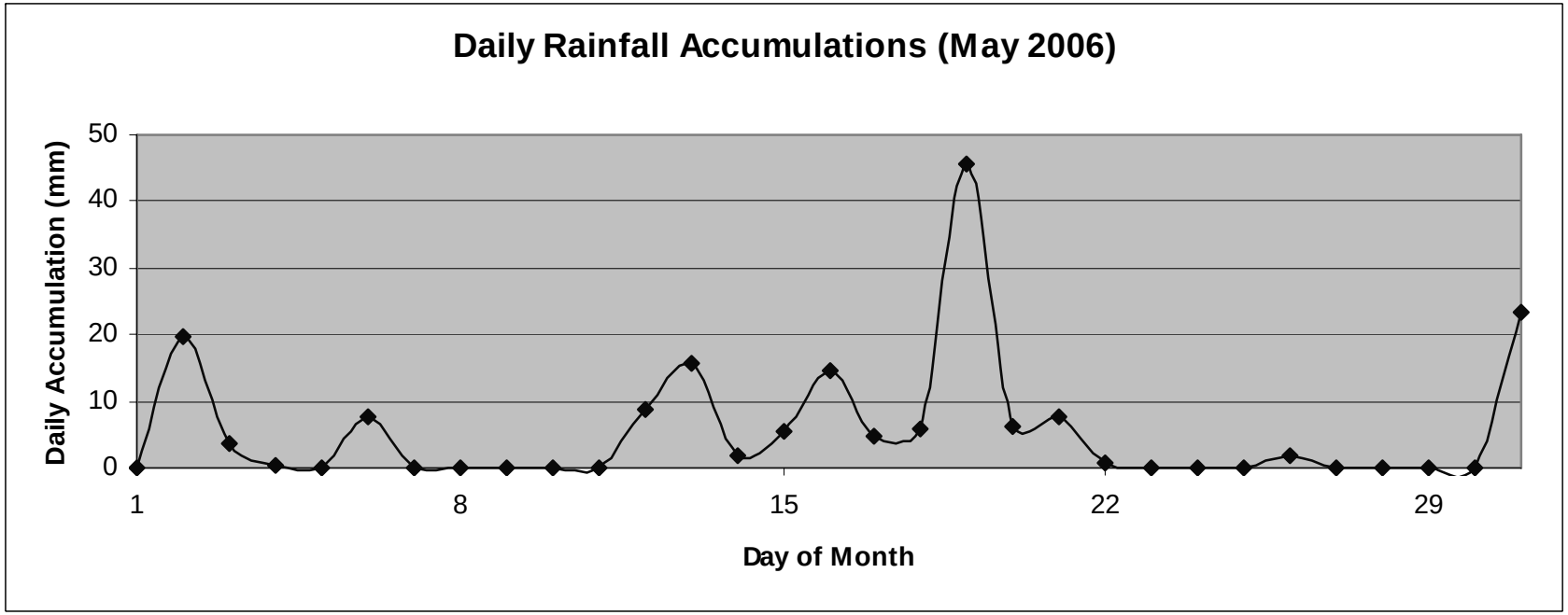


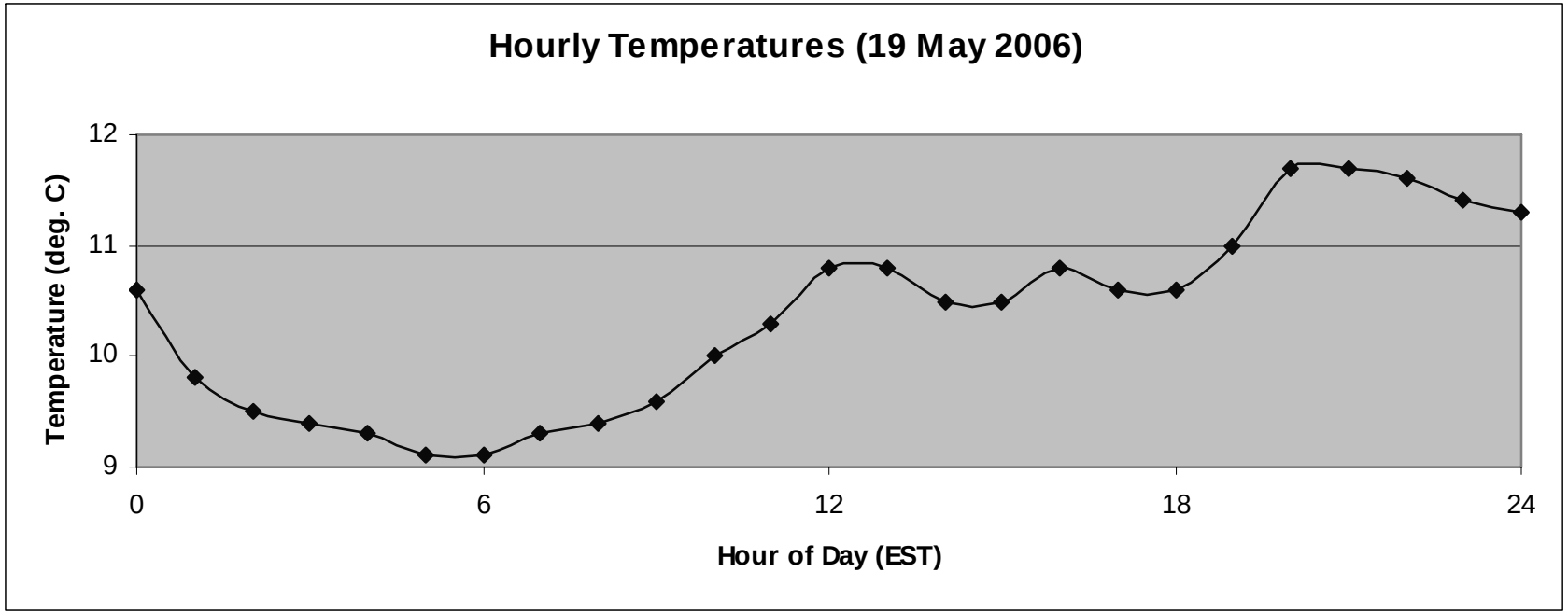
http://www.guaranteedweather.com/display_file.php?file=254

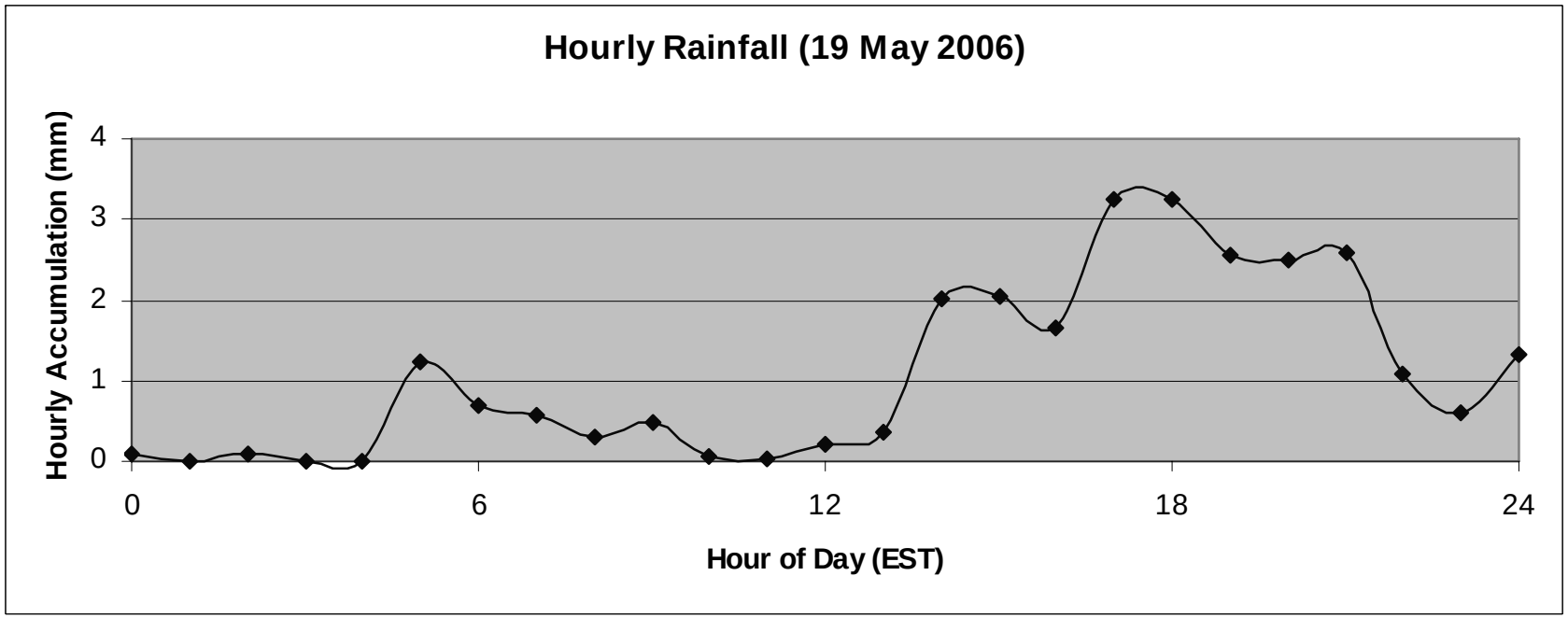
Useful Indices for Weather Derivatives

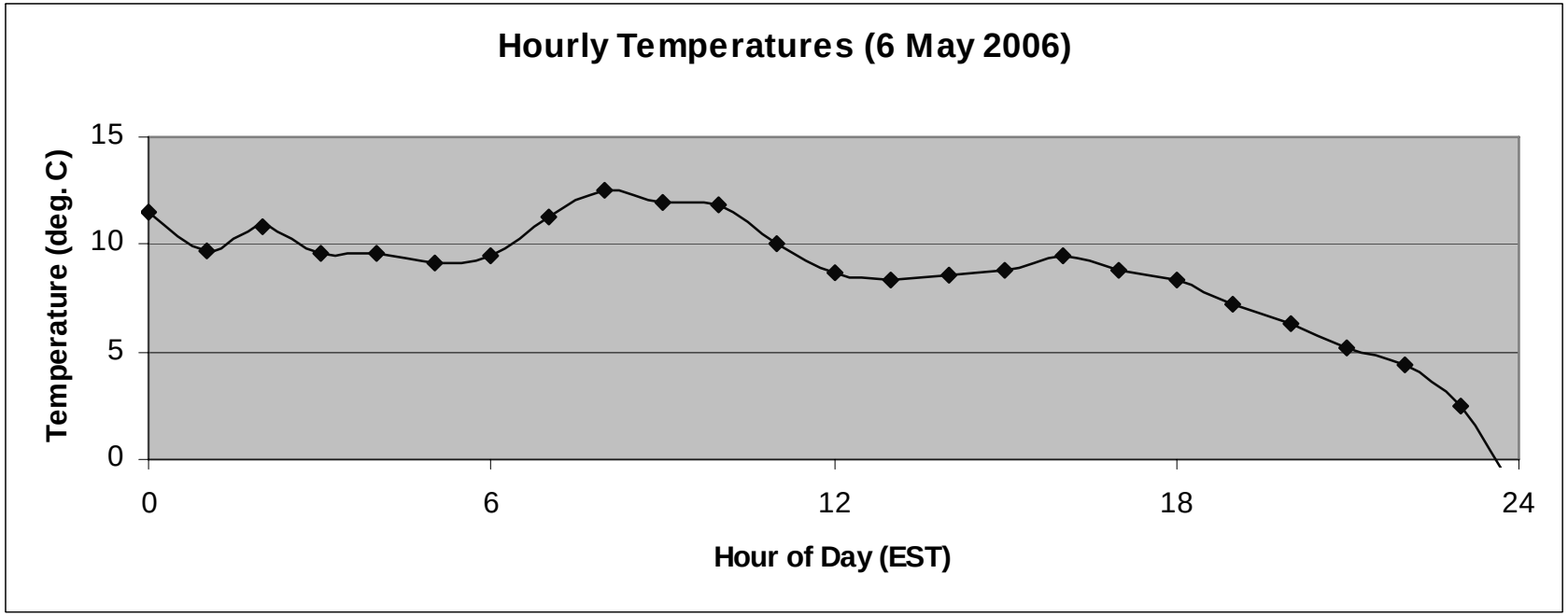
- Temperature-based
  - Heating Degree Days (HDD)
  - Cooling Degree Days (CDD)
  - Relatively continuous in space and time
- Precipitation based
  - Intermittent, mixed distribution
  - Statistics depend on averaging in time and space

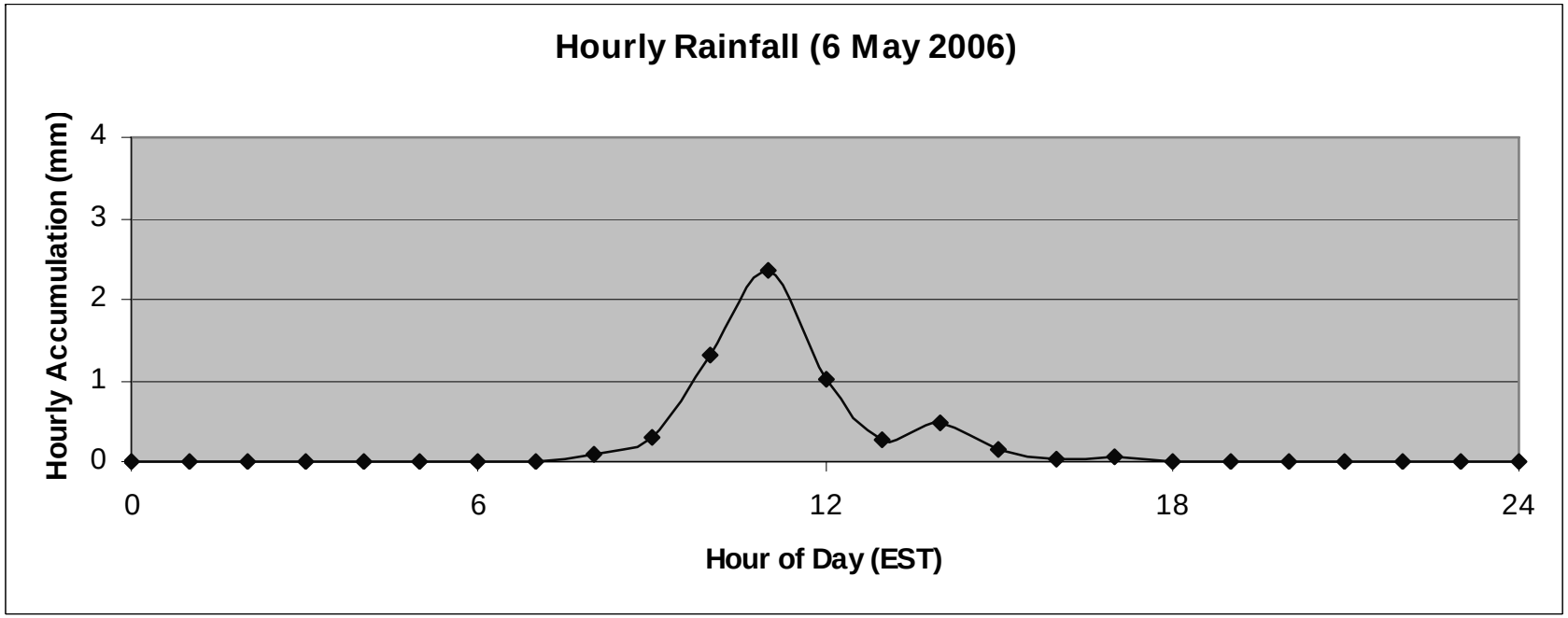

Daily Mean Temperatures (May 2006)
Temperature (deg. C)
25
20
15
10
5
1
8
15
22
29
Day of Month


Daily Rainfall Accumulations (May 2006)
Daily Accumulation (mm)
50
40
30
20
10
0
1
8
15
22
29
Day of Month


Hourly Temperatures (19 May 2006)
Temperature (deg. C)
12
11
10
9
0
6
12
18
24
Hour of Day (EST)


Hourly Rainfall (19 May 2006)
Hourly Accumulation (mm)
4
3
2
1
0
0
6
12
18
24
Hour of Day (EST)


Hourly Temperatures (6 May 2006)
Temperature (deg. C)
15
10
5
0
0
6
12
18
24
Hour of Day (EST)


Hourly Rainfall (6 May 2006)
Hourly Accumulation (mm)
4
3
2
1
0
0
6
12
18
24
Hour of Day (EST)

Multiplicative Cascade

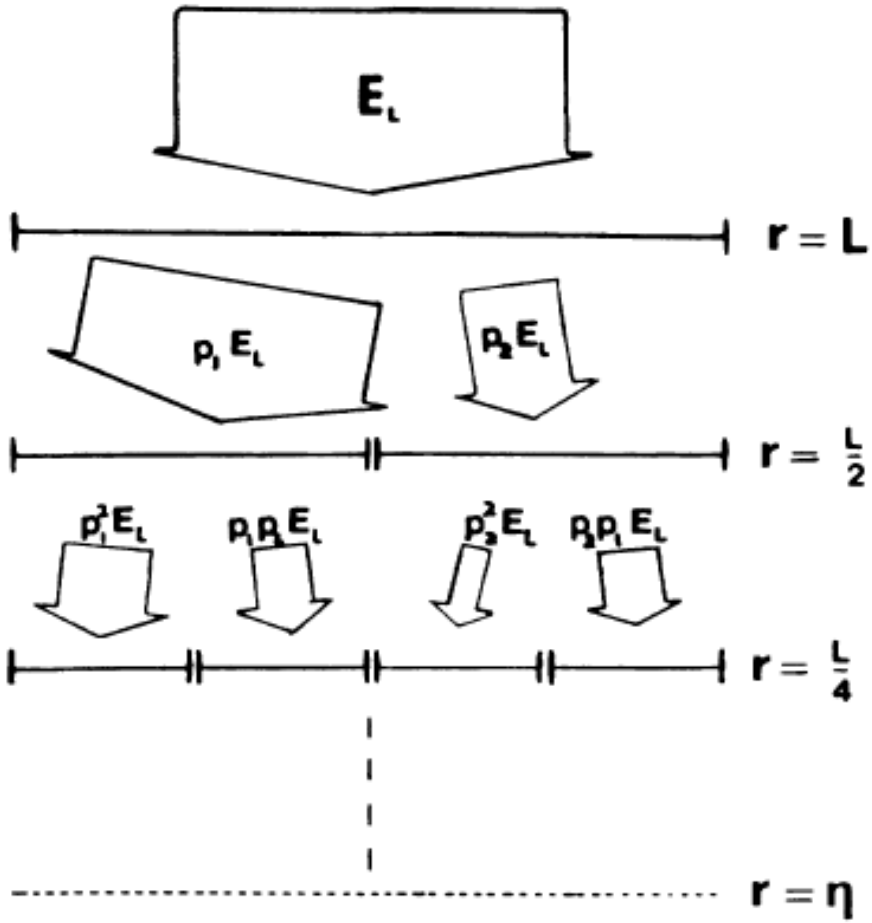


FIG. 3. One-dimensional version of a cascade model of eddies, each breaking down into two new ones. The flux of kinetic energy to smaller scales is divided into nonequal fractions p_1 and p_2. This cascade terminates when the eddies are of the size of the Kolmogorov scale, η.

(From Meneveau and Sreenivasan, 1987. Phys.Rev. Letters, **59**, 1424-1427)

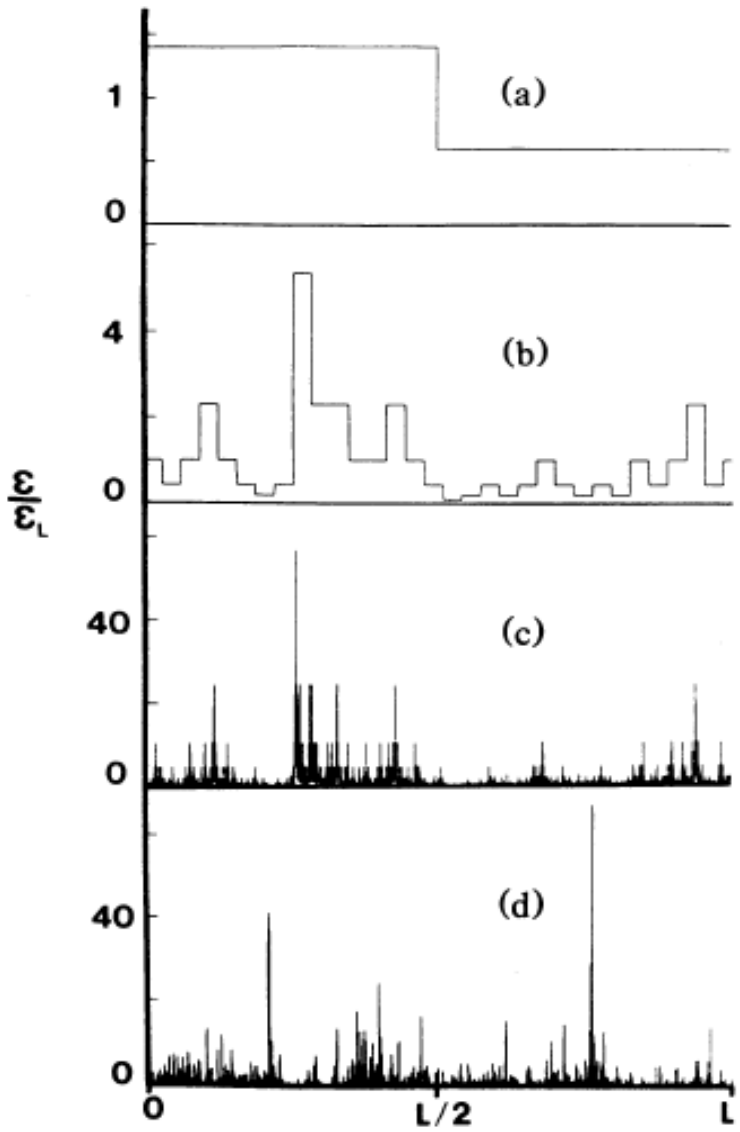


FIG. 4. Different stages during the construction of the proposed p model of the dissipation field [(a) first stage, (b) fifth stage, (c) twelfth stage], and (d) an experimental signal of ϵ corresponding to the twelfth stage of construction (see text).

(From Meneveau and Sreenivasan, 1987. Phys.Rev. Letters, **59**, 1424-1427)

Daily Rainfall Accumulation Modeling

- Occurrence and severity of precipitation
- Each day may have rain (R) or no rain (NR)
- For raining days, accumulation varies

- Occurrence: First-order Markov model
- Severity: Gamma distribution(s)
  ~ Independent day-to-day

1971-2000 Daily Total Precipitation Records by Day of Year, Dorval Trudeau Airporth, May 1 – June 19

		###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###
April	30	MD	MD	MD	MW	MD	MD	MD	T	T	MD	MD	MD	MW	MW	MD	MD	MW	MW	MD	MD	MW	MW	MD	MD	MD	MW	MD	MD	MD	MD
May	1	T	0	1.3	4.8	0	9.7	0	1.2	0.4	0	0	T	0.2	1.2	0.6	7	0.2	0.2	T	0	8.8	3	0	18	0	0.5	10	0	0	1
May	2	T	1	5.3	0	T	2	10.3	T	0	0	0	0	16.1	0	0	0.4	0	T	15.8	0	4.2	5.4	0	T	0	T	T	T	0	0
May	3	4.8	10.9	10.2	7.9	0	2.3	0	T	14.8	0	0	0	2.6	0	0	T	0	0	4	0	T	0.4	MW	0	0	T	28.5	5.5	0	0
May	4	14	12.7	T	T	1.5	T	0	0	T	0	0	0	4.2	15.1	1.2	1.2	0	0	0	0	0	0	T	0	0	T	0	8.5	T	0.5
May	5	T	3.6	1	0	0.8	T	3.4	0	1	0.8	0	0	2.4	T	12.6	5.2	T	0	1.6	12.8	0	0	7.4	0	1.4	T	0	4	T	0
May	6	T	4.1	T	19.8	0	3	T	0	0	5.4	7	0	0	0	2	1	4	0	19.2	3.8	17	0	22.8	T	1	0	1.5	3.5	0	1.5
May	7	0	0	0	3.8	0	23.1	0	0	0	4	0	0	0.4	0	3	11.6	0.2	0	2	0.4	3	0	0	0	0	0	0	0.5	0	5.5
May	8	T	0	3.3	T	0	0.8	0	2.4	0	0	0	4	21.2	32.8	0	4.6	0	0	3.6	T	0	0	0	1.2	0	0	0	0	7.5	4
May	9	0	0	19.8	21.6	0	0	0	12.8	0	4.4	0	3.4	9	0.6	T	0	0.2	0	0.2	0.4	0	2.4	0	1.2	0	0	4	T	1.5	21.5
May	10	0	0	2.5	3.6	0	0	0	T	0	T	1.6	0	2	0	0.8	0.2	0	0	0	11.2	0	1	T	0.8	0.2	15	0.5	T	0	29.5
May	11	0	0.5	9.4	0	0	10.2	0	0	0	1.2	23.4	0	0.2	9	1.4	0	10.8	1	6.2	5.6	0	0	T	0.8	10	13.5	0	0	0	1.5
May	12	1.8	T	4.1	14.2	T	T	0.7	T	T	0	16.4	1	0	2.6	0.2	0	1.6	T	9.6	T	0	0	2.4	2.6	0.8	29.5	7.5	0	0	0
May	13	0	0	T	T	T	0	0	T	4.6	T	1.8	0	0	0.2	T	0	0	7.2	2.6	0.8	0	3.8	0.6	0	T	0	T	0	0	2.5
May	14	T	3.6	0	T	0	4.6	0	0	0	0.4	0	0	0	1.2	T	0	2.8	0	0.2	0	T	0	MW	0	7	T	0	0	0	T
May	15	0	T	T	1.3	10.7	0	0	7	4.2	0	3.8	0	10	2.8	0	0	11.4	T	0	0.8	0	0	3.6	T	4	T	5.5	0	0	T
May	16	0.8	8.6	0	T	13.7	0.5	0	3.8	2.6	0	1.4	0.2	0	T	1.6	8.4	T	8	0	0.2	0	0.8	MW	33.4	0.4	0.5	1	0	0	0
May	17	1.8	T	9.4	T	0	1	0	0.6	0	0	0	0	0	0	0	1.8	T	6	0	34.4	19.8	15.6	MW	3	15.8	0	0	0.5	0	0
May	18	T	1.3	11.2	0	0	6.6	0	1.4	0	28.4	T	T	0	0	0	0	T	0.4	0	1	0	0	0	T	0	13.5	0	0	0	34.5
May	19	0	0	8.9	0	0	31	0	0	0	0	0	T	0.2	T	T	10.8	0	T	0	0.6	0	0	4.4	0	0	1.5	0.5	0	11.5	0
May	20	8.9	0	1	0	0	2	0	0.2	T	0	0	2.6	1.8	T	1	5.6	0	8.6	0	5.6	0	0	4	0	0	10.5	T	4.5	0	0
May	21	0.3	0	3.8	T	0	0.3	0	3.4	5.6	0	1.4	0	0	0.4	0.4	2.4	T	7.6	3.2	1.6	0	0	MW	0	T	0.5	0.5	0.5	0	0
May	22	0	0	T	6.4	0	1.3	0	0	T	0	0	0	2.4	T	0	6.6	8.2	0	1.8	T	0	0	T	0	0	0.5	T	0	T	0
May	23	0	0	0	5.8	0	1.5	0	0	1.2	T	0	T	7	26.8	0	9.6	6.2	0	0	0	0	3.8	0	1.4	2.8	T	0	0	0	1
May	24	T	0	0	6.1	0	0.5	0	0	T	0	0	0.8	0.2	1.2	0	1.6	0.6	0	3	0	0	T	18.7	T	2	0	0.5	0	16	22
May	25	4.3	0	0	T	0	0	0	0	8	0	0.6	0	0.2	0.2	T	0	0	T	T	0	1.8	0	0.4	4.6	0	0	0.5	0	1	6
May	26	0.8	0	0	2.8	11.7	0.3	0	0	10.8	0	1.8	0	14.4	0.2	1	0	T	T	4	0	31	T	0.6	24.8	0.6	0	0	T	5	1.5
May	27	1.8	0	0	T	17.5	0.5	0	0	9.3	0	14.2	0	8	0	7.4	0	0.6	0	1.6	0	0.4	T	0.2	0	0	0	0	0	1.5	0
May	28	0	0	19.8	0.3	0	0	10	0	3.6	0	10	1.4	0	T	0	0	3.4	0	0	0	0	0	0.2	1	9	T	0	0.5	0.2	0
May	29	0	0	2.3	1.5	0	0	0	0	6.4	0	0.8	0	T	21.4	0	0	2	0	0	T	1	0	1	1.4	25.5	6	0	2.5	0	0
May	30	0	15.5	0	T	16.3	0	0	0	9.7	1.5	4.4	0	26.4	2.4	T	3.8	T	11.2	4.6	0	3.4	0	0	3.6	1	0.5	15.5	0	0	0
May	31	0	6.4	7.1	5.1	1	18.3	0	10.2	4.8	0.4	0	1.8	8.2	0	8.4	0.6	2.8	0	T	0	0.8	20.2	17	2.6	0	0	0	20	0	T
June	1	0	17	T	0	6.6	0.3	0.2	0	0	0.2	0	8.4	T	0	0.2	12.4	2.4	T	0.2	0	T	2.2	MW	2	0	0	0	0	1.5	0
June	2	3	17.5	0	0	0	0	T	3.5	0	T	0	1.4	0.6	0	0	0	T	0.2	2.2	T	0	0	0	5	4.5	0	0	9	14.5	0.5
June	3	1	0	0	5.6	T	2.5	T	0	0	0.8	4.2	0	T	T	0	0	2.4	0	0.2	14.6	T	0	0.2	0	49.2	1.5	0	0.5	11.5	1
June	4	T	10.2	1	0	1.8	0	0.2	0	0	2.4	10.6	0.8	3.4	0.8	0	T	0	0	15.8	3.2	T	0	0	0	0	3.5	0	0.5	0	1
June	5	0	0	T	T	20.1	0	2.8	4.8	2.6	0	0	1.2	0.2	0	11.4	0.2	T	0.4	0	T	T	0.8	3.4	0	0	1	0	T	0	0
June	6	0.3	0	0	0	6.6	0.5	T	0	0	0	18	1.4	5.8	5.6	3.8	0	0	0	0	0.2	0	18	1.2	12	0	0	0	0	0.5	0.5
June	7	7.4	0	2	0	3.8	T	12.5	5	0	0	0.4	0	1	14.2	0	1.4	T	0	0	T	0	13.6	T	5.2	T	11	0	T	1.5	0
June	8	1.5	T	T	0	0.5	6.4	2.5	2	6.6	0.8	9.8	0	2.4	T	T	5	61.6	0	0	T	0	0	0	0	T	2	0	T	T	3.5
June	9	0	18.8	T	T	0	0	T	4.8	0	5.8	3.8	0	0	0	1	0	19.6	0	4.8	4.2	0	0	0.8	0	0	17	0	0	0	T
June	10	0	9.1	2.5	24.6	0	0	0	0	T	0.2	0.2	0	1.6	T	0	0	0	0.4	38.4	2.6	0	0	0	0	0	2	0	0	0	3
June	11	0	0	11.9	1.5	0	14	T	0	6.4	0	T	3.2	0	0	0	6	0.2	T	0.6	1	0.2	0	0.4	0.4	2.5	6	0	T	0	13
June	12	0	0	24.9	T	4.8	0	T	0	2.4	0	1.4	0	3.6	0	9.4	8.6	4.6	0	0	0	5	3.2	0	3.8	0	0.5	0	10.5	0	0

Environment Canada Daily Total Precipitation Records
Dorval Trudeau Airport (May 1971-2000)

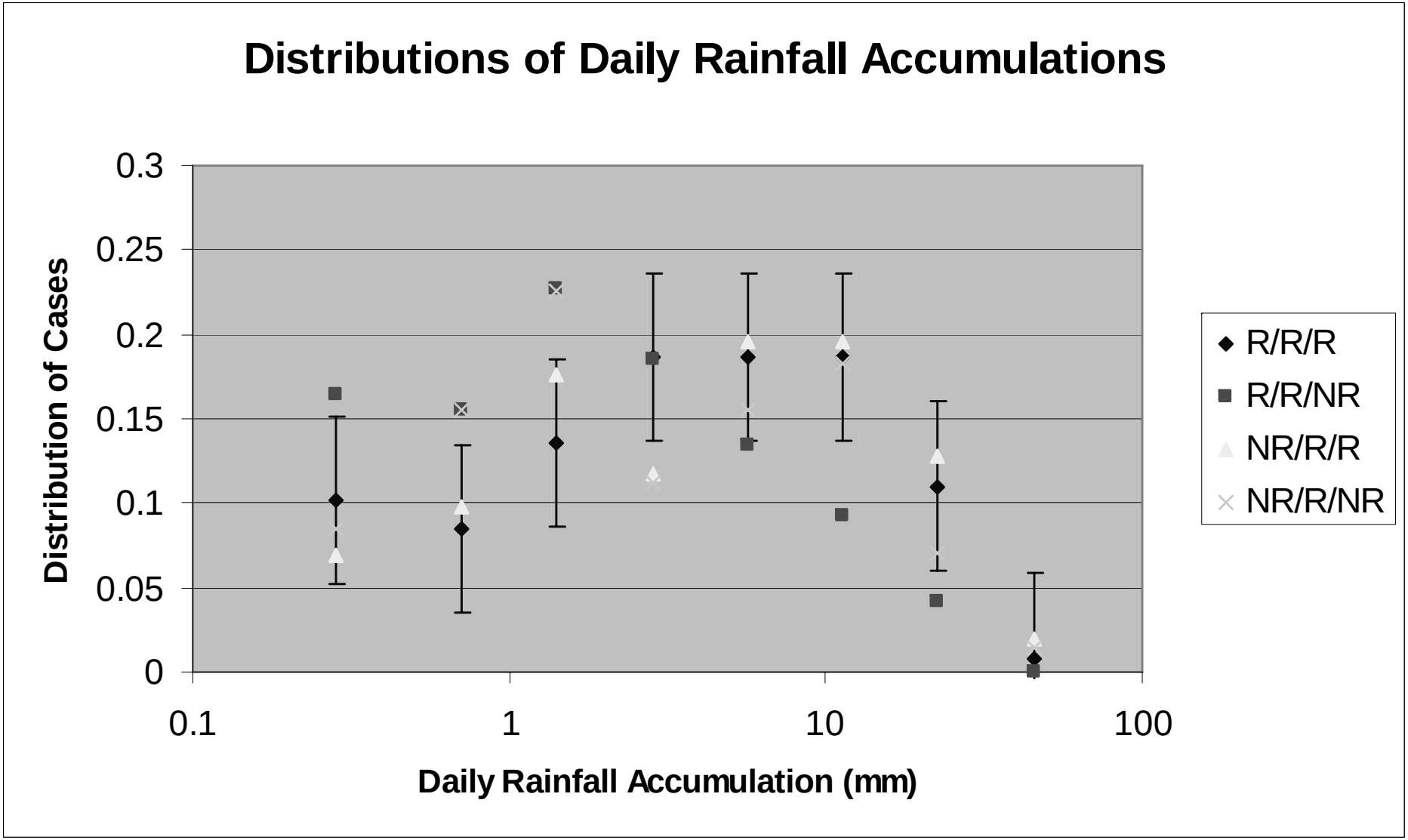
		###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###
April	30	0	1	0	1	1	1	1	1	0	0	1	1	0	0	1	0	1	0	0	0	1	1	1	1	0	0	1	0	0	0
May	1	0	1	1	0	1	0	1	0	0	0	0	0	0	1	0	0	1	0	1	1	1	1	0	0	1	1	1	1	1	0
May	2	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	0	1
May	3	0	0	0	0	0	0	1	1	0	1	0	1	0	0	0	1	0	0	1	1	1	0	0	1	1	0	0	0	1	1
May	4	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	1	0
May	5	1	0	1	1	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	0	1	0	0	1	1	1
May	6	1	0	1	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	1	0	1	1	1	1	1	1	1	0	0	0
May	7	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1
May	8	1	0	1	1	1	0	1	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	1	0	0
May	9	0	0	0	0	0	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	1	1	1	0	0	0	1	0
May	10	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	1	1	0	1	1	1	1
May	11	1	1	0	1	1	0	1	0	1	1	0	0	0	0	0	1	0	0	1	1	1	1	0	1	0	0	0	0	1	0
May	12	1	0	0	1	1	0	1	0	0	0	1	0	1	1	0	1	0	0	1	0	0	0	1	0	1	1	1	0	1	1
May	13	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	1	1	0	1	1	1	1	1	0	0	1	1	0	0	1
May	14	1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1	1	1	0	0	0	1	0	1	0	0	0
May	15	0	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1
May	16	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1
May	17	0	1	1	0	1	1	0	0	1	0	0	1	0	0	1	1	1	0	1	0	0	1	0	0	0	0	0	0	1	0
May	18	0	0	1	0	1	0	1	0	1	0	0	1	1	0	1	0	0	1	0	1	1	0	1	0	0	1	1	0	0	0
May	19	0	1	0	0	0	1	0	0	1	0	0	1	0	1	1	1	0	1	0	0	0	0	1	1	1	0	1	1	1	1
May	20	1	0	1	1	0	0	1	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	1	0	0	1	1	1	0	0
May	21	1	1	0	0	1	0	0	1	1	0	1	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	1	1
May	22	1	1	1	0	0	1	1	0	1	0	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0
May	23	1	1	0	1	1	0	1	0	1	1	1	0	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0	0	1	1
May	24	1	1	1	0	1	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	1	0	1	0	1	0	0	0	0	1
May	25	0	0	0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0
May	26	0	1	0	1	1	0	0	1	0	1	0	0	0	1	1	0	0	1	0	0	1	1	0	1	0	0	0	0	0	0
May	27	1	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	1	1	1	0	1	1	0	1
May	28	0	0	0	0	1	0	0	1	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	1	0	0	0	1	0	0
May	29	0	1	0	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	0	0	0	1	1	0	0	1	1	0	1
May	30	0	1	0	1	0	1	1	0	1	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1	0	1	0	0	1	1
May	31	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	0	1	0	1	1
June	1	0	1	0	1	0	0	1	0	1	0	0	1	0	0	1	1	0	1	0	0	1	1	1	0	0	0	1	0	0	0
June	2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	1	0
June	3	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1	1	0	1	1	1	0	0
June	4	0	1	1	1	0	1	1	1	0	0	0	0	0	1	0	0	1	0	0	1	0	1	1	0	0	0	1	1	0	1
June	5	1	1	0	1	0	0	1	1	1	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0	1	0	0	0
June	6	0	1	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	1
June	7	0	0	0	1	1	0	0	0	1	1	0	0	1	1	0	0	0	1	1	1	0	1	1	1	0	1	0	0	0	0
June	8	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	0	0	1	1	1	0	1	0	1	0	1	0	1	1	1
June	9	0	1	1	0	0	0	0	0	1	1	0	1	0	0	0	1	1	1	1	0	0	0	0	0	1	0	0	0	1	1
June	10	0	0	1	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	1	1	1	0	1	1	0	1	0	0	0
June	11	0	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	1	1	1	0	0	0	1	1
June	12																														

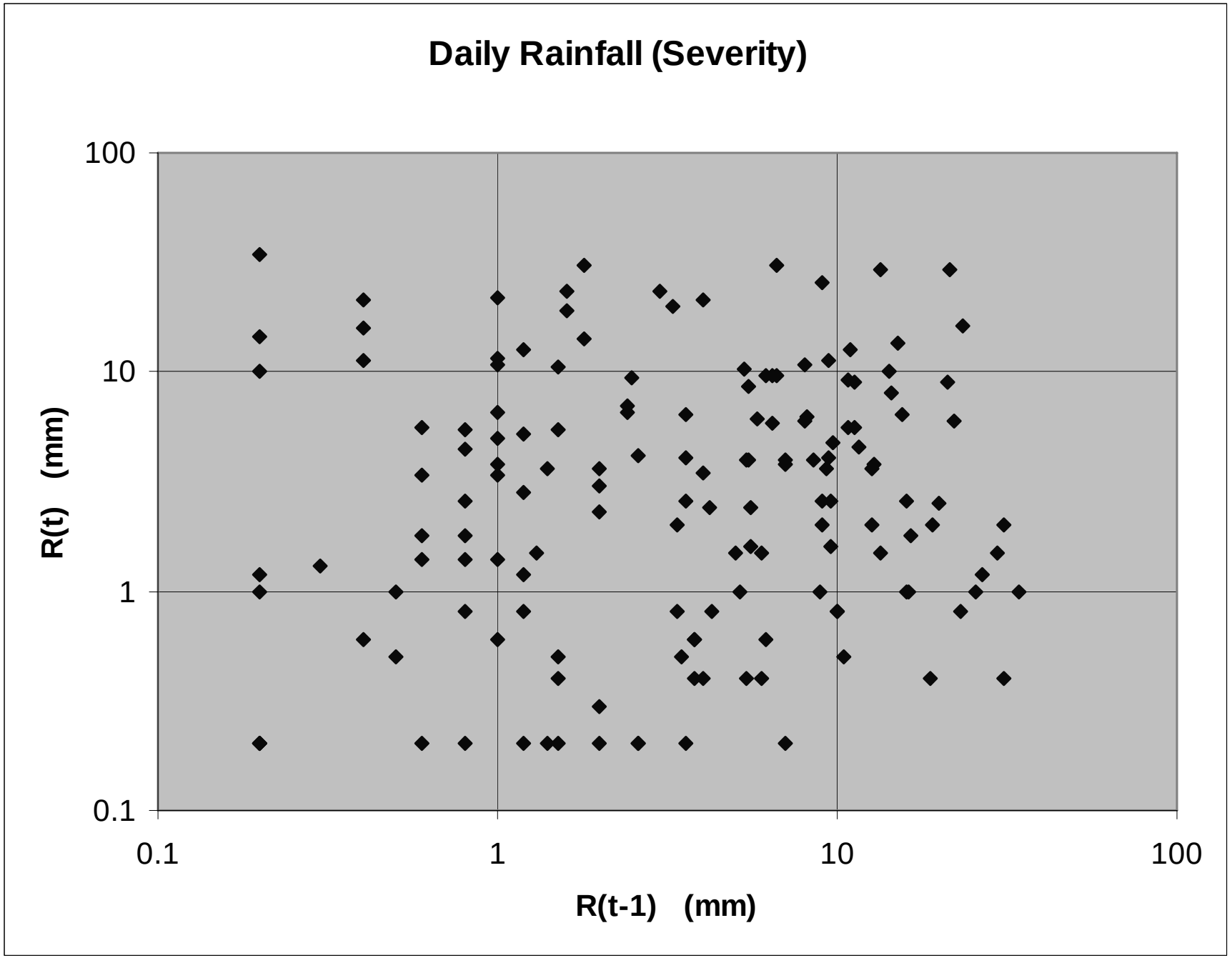
Random Rainfall Occurrence, Daily Occurrences Indedpendent
0 for ‘no rain’ (NR) day and 1 for ‘rain’ (R) day

		###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###
April	30	0	0	1	0	0	0	0	1	0	1	1	1	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	0	0	0
May	1	1	0	1	0	1	0	0	0	1	1	1	1	1	1	0	1	1	0	1	0	0	0	1	0	1	1	0	1	1	0	1
May	2	0	0	0	0	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	0	1	0	0	0
May	3	0	0	0	0	1	0	0	1	0	1	0	1	1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1
May	4	0	0	1	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0
May	5	0	1	1	0	0	0	0	1	1	0	1	1	0	0	0	1	1	0	1	1	0	0	1	0	0	0	1	0	0	0	0
May	6	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0	0
May	7	0	0	0	1	1	0	0	0	0	1	1	1	0	1	0	0	1	1	0	0	1	1	1	1	0	0	1	0	0	0	0
May	8	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	1	1	1	0	1	0	1	0	1	1
May	9	0	1	1	0	1	0	0	0	0	1	0	0	0	1	0	1	1	0	0	1	0	1	1	1	0	0	0	1	0	0	0
May	10	0	1	1	1	0	0	0	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
May	11	0	1	1	1	1	0	1	0	1	1	1	1	1	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1
May	12	0	0	1	1	0	1	1	0	0	1	1	1	1	0	0	0	1	0	1	0	0	0	0	1	1	0	0	1	0	0	1
May	13	1	0	0	0	1	1	1	0	0	1	0	1	1	1	0	0	1	0	1	0	0	0	1	0	1	1	0	1	1	1	0
May	14	0	1	0	0	1	0	1	0	1	1	0	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
May	15	0	1	0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
May	16	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	0	1	0	1	1	1
May	17	0	0	0	1	0	1	0	0	0	0	1	1	0	1	0	1	1	0	1	0	0	0	1	0	0	0	0	1	1	0	0
May	18	1	1	0	0	0	1	1	0	1	1	1	1	0	0	1	0	1	0	1	0	0	0	1	1	0	0	0	1	0	0	0
May	19	1	1	0	0	0	1	1	0	1	0	0	0	1	1	0	1	1	1	0	0	0	1	0	1	0	1	1	1	0	1	1
May	20	1	1	0	1	0	0	1	1	0	0	0	0	0	0	1	0	1	1	0	0	1	1	0	1	1	1	0	1	1	1	1
May	21	0	1	0	1	0	0	1	1	0	1	0	1	1	0	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	1	1
May	22	1	1	0	0	1	0	1	1	1	1	0	1	1	0	1	0	0	0	0	1	1	0	0	0	1	0	0	1	1	1	1
May	23	1	1	0	0	1	0	1	1	1	1	0	1	1	1	1	0	0	0	1	1	1	1	1	0	1	0	0	0	0	0	0
May	24	1	1	0	1	1	1	0	1	1	1	0	0	1	1	1	1	1	0	0	1	1	0	1	1	0	0	0	0	0	0	0
May	25	0	0	0	1	0	0	1	0	1	1	1	0	1	1	1	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	1
May	26	0	0	0	1	0	0	1	0	1	1	1	0	1	1	1	1	1	1	0	0	1	0	0	1	0	0	0	0	0	0	1
May	27	1	0	0	0	1	1	0	0	0	1	0	1	1	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	1
May	28	1	0	0	1	0	0	0	1	0	1	0	0	1	1	0	0	1	1	0	1	0	1	1	0	1	1	0	0	0	0	1
May	29	1	0	0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	0	1	1	0	1	0	1	0	1	1	1	0	0	0
May	30	1	1	0	0	0	0	0	0	1	1	0	1	0	1	1	1	1	1	0	0	0	1	1	0	1	0	1	1	0	0	1
May	31	0	1	1	0	1	1	0	0	1	1	0	0	1	1	0	1	1	0	0	1	0	1	1	0	0	1	1	0	0	0	0
June	1	0	0	0	1	0	1	1	0	1	1	0	0	1	1	1	0	1	1	0	1	0	0	1	1	1	1	1	1	0	0	0
June	2	1	1	1	1	0	0	0	1	1	0	1	0	0	1	1	0	1	0	0	0	1	0	0	0	1	1	1	1	1	1	1
June	3	1	1	0	1	1	1	1	0	1	0	1	1	0	1	0	1	0	0	1	0	1	1	1	1	1	1	0	0	1	0	0
June	4	1	1	0	0	0	1	0	0	1	1	0	1	1	0	0	1	1	1	1	0	1	1	0	1	1	0	0	0	1	0	0
June	5	0	1	0	1	1	1	0	0	1	1	0	0	0	0	0	1	0	1	0	1	1	1	1	0	1	1	0	0	1	0	0
June	6	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	1	1	0	0	1	1	1	0	0	0	1	0
June	7	0	1	0	0	0	0	0	0	1	1	1	0	0	1	0	1	1	1	0	1	0	0	0	1	1	1	0	1	1	1	0
June	8	0	0	1	0	1	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1	0	0	1	1	0
June	9	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	1	0	1	0	1	1	0	1	0	1	0	1	1	1	0
June	10	1	1	1	1	0	0	0	0	1	1	1	1	1	0	0	0	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0
June	11	1	1	1	1	1	1	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	1	1	1	0	1	1	0	0	1	0
June	12																															

First-Order Markov Model for Daily Rainfall Occurrence
0 for ‘no rain’ (NR) day and 1 for ‘rain’ (R) day

Independent	*Previous Day Dependence*		*Two Previous Days*	
Pr(R(t))	**Previous Day**	**Pr(R(t)|Previous)**	**Previous Days**	**Pr(R(t)|Previous)**
			NR(t-2),NR(t-1)	0.297+/-0.02
	NR(t-1)	0.316+/-0.021		
			R(t-2),NR(t-1)	0.353+/-0.039
0.428+/-0.017				
			NR(t-2), R(t-1)	0.600+/-0.039
	R(t-1)	0.570+/-0.026		
			R(t-2), R(t-1)	0.545+/-0.035


Distributions of Daily Rainfall Accumulations
Distribution of Cases
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
1
10
100
Daily Rainfall Accumulation (mm)
R/R/R
R/R/NR
NR/R/R
NR/R/NR


Daily Rainfall (Severity)
100
10
1
0.1
R(t) (mm)
0.1
1
10
100
R(t-1) (mm)

Hourly Rainfall Accumulation Modeling

- Occurrence and severity of precipitation
- Each hour may have rain (R) or no rain (NR)
- For raining hours, accumulation varies

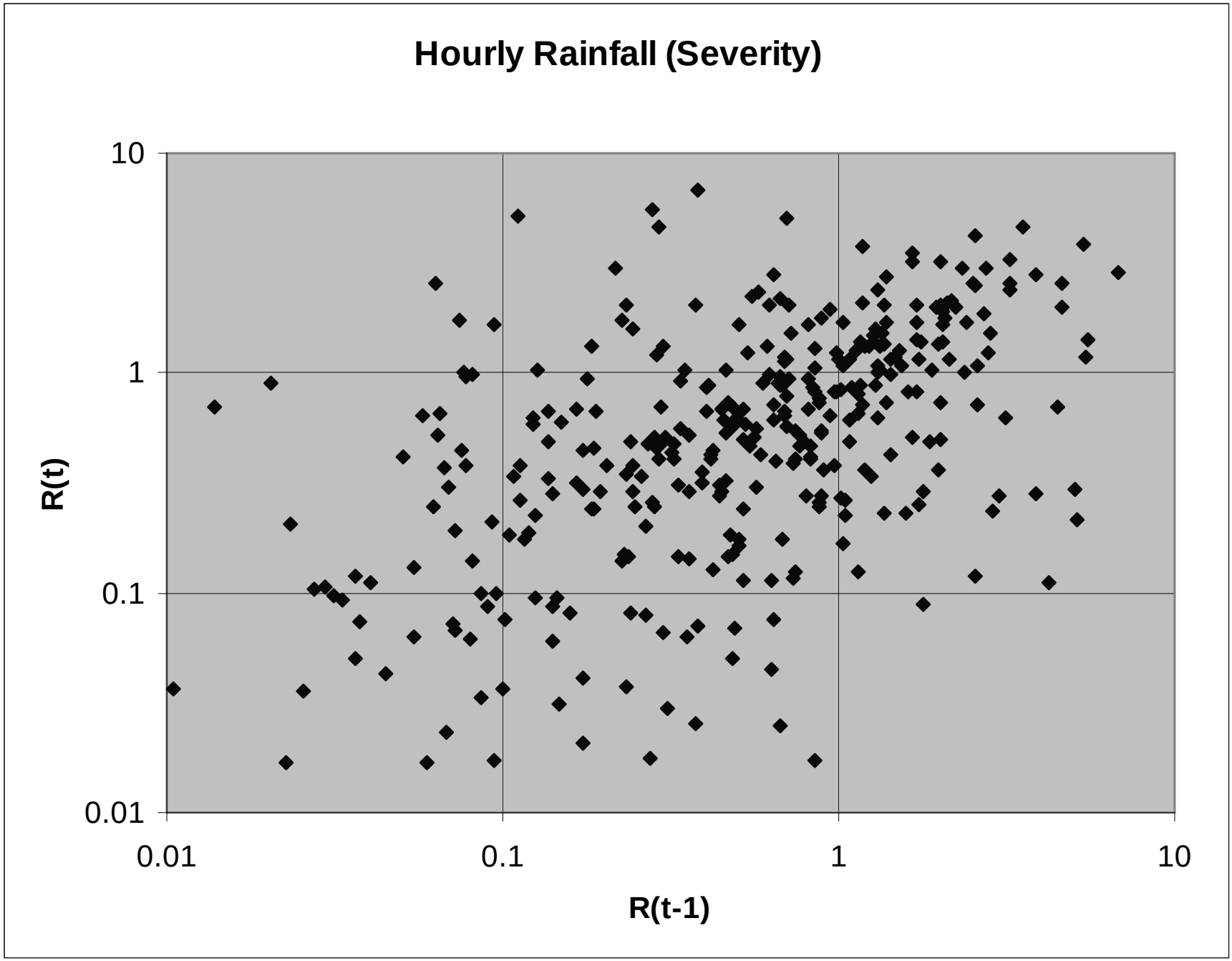
- Occurrence: Higher-order Markov model
- Severity: Lognormal distribution(s)
  Hour-to-hour dependence

Short-Period Rainfall Measurements

http://www.radar.mcgill.ca/dsd.html

- POSS (Precipitation Occurrence Sensor System)
- Low power radar
- Vertically pointing
- Detects raindrops
- Measures drop speeds
- Determines drop sizes
- Calculates rainfall rates

Independent	*Previous Hour Dependence*		*Two Previous Hours*		*Three Previous Hours*	
Pr(R(t))	**Previous Hour**	**Pr(R(t)|Previous)**	**Previous Hours**	**Pr(R(t)|Previous)**	**Previous Hours**	**Pr(R(t)|Previous)**
					NR, NR, NR	0.078+/-0.006
			NR(t-2),NR(t-1)	0.082+/-0.006		
					R , NR, NR	0.131+/-0.025
	NR(t-1)	0.092+/-0.006				
					NR, R , NR	0.114+/-0.031
			R(t-2),NR(t-1)	0.194+/-0.026		
					R , R , NR	0.262+/-0.040
0.248+/-0.007						
					NR, NR, R	0.527+/-0.037
			NR(t-2),R(t-1)	0.539+/-0.033		
					R , NR, R	0.581+/-0.075
	R(t-1)	0.725+/-0.016				
					NR, R , R	0.732+/-0.040
			R(t-2),R(t-1)	0.795+/-0.017		
					R , R , R	0.811+/-0.018


Hourly Rainfall (Severity)
10
1
0.1
0.01
R(t)
0.01
0.1
1
10
R(t-1)

Hourly Rainfall Modelling (continued)

- Occurrence depends on several previous hours
- Severity depends on at least previous hour

$$\log R(t+1) = \log R(t) + \varepsilon(t)$$

- Seasonal variation of time series parameters
- Diurnal variation of time series parameters

Summary

- Rainfall time series modelling
  - Historical observations to fit model parameters
  - Generate time series for derivative valuations
  - Daily rainfall modelling - manageable
  - Hourly rainfall modelling - very difficult
- Future work:
  - Spatial distribution of rainfall and basis risk