Model To Develop A Provision For Adverse Deviation (PAD) For The Longevity Risk for Impaired Lives

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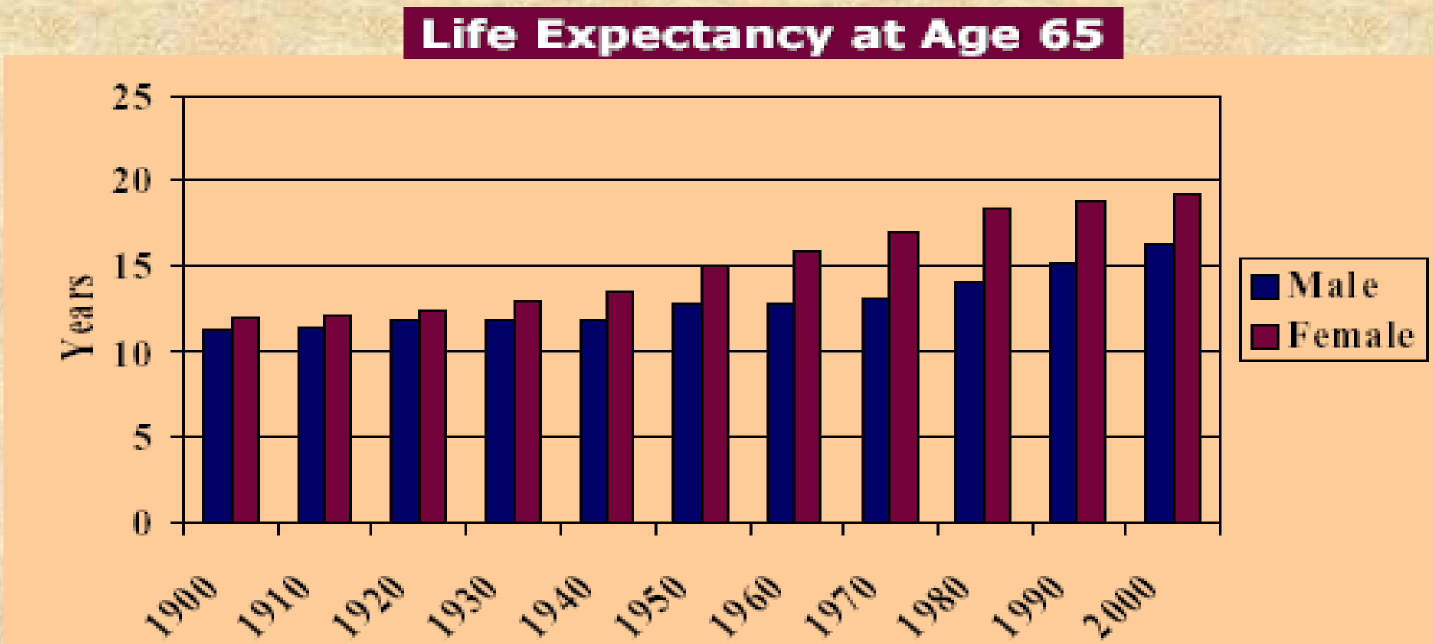
Recent Mortality Trend

- Recent trends in life expectancies over the past century show dramatic improvement mostly at later ages.
- Mortality is improving due to recent medical advances, improvement in healthcare and health education, genetic research, therapeutic advances etc.,
- Recent trends in mortality improvement call for updated survival models when pricing and reserving life annuities and other LTC benefits.



Source: National Vital Statistics

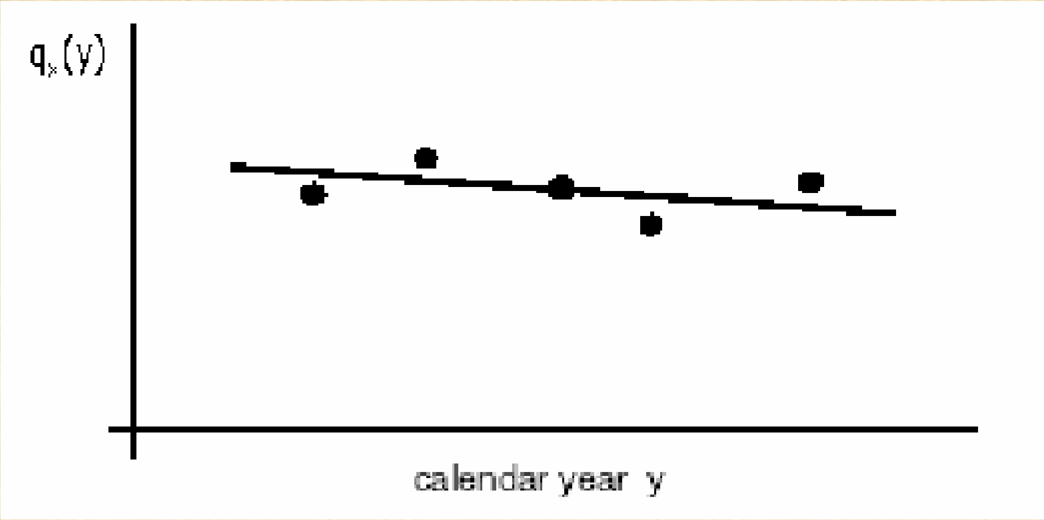
Projecting Mortality Improvements

- Mortality Improvement Projection procedures or models
  - Should reduce inconsistencies that may emerge as a result of the extrapolation.
  - Should recognize the current mortality trend.
  - Should be able to minimize Random mortality fluctuations and systematic deviations.

Mortality Risk

There are two types of risks:

1. Risk of Random Fluctuation (Statistical Volatility)



• Future mortality experience

— Projected mortality

➢Well-known type of risk in the insurance business, in both life and non-life areas.

➢Fundamental results in risk theory state that the severity of this risk decreases as the portfolio size increases.

2. **Longevity Risk (Systematic Risk)**

- The risk exists as the result of an actual mortality trend different from the forecasted one.
- The systematic deviations can be thought of as a “model risk” or “parameter risk”, referring to the model used for projecting mortality and the relevant parameters.
- The risk cannot be hedged by increasing the portfolio size. On the contrary, its financial impact increases as the portfolio size increases.

Analyzing Mortality Risk in Deterministic and Stochastic Framework

- The survival function S(x) ($S(x) = P\{T_0 > x\}$ where T_0 is a random lifetime of a newborn) can be obtained from the past data or projections.

- The random present value of benefits, Y is given by

$$Y = \sum_{k=1}^{K_x} R_k v^k$$

Where
K_x = the curtate residual lifetime of the insured age x
R_k = payment made by the insurer in the k^{th} year

- The random present value of benefits for the portfolio, $\mathbf{Y_{Tot}}$

$$Y_{Tot} = \sum_{i=1}^{N} Y_i$$

Where Y_i = random present value of the i^{th} insured

- Under the hypothesis of homogenous and independent risks, we can obtain the followings for fixed S:

$E[Y_{Tot}] = N\,E[Y_i]$ $\qquad$ $Var(Y_{Tot}) = N\,Var(Y_i)$

- The mortality risk can be measured by the coefficient of variation of Y_{Tot} (Risk Index)

$$\text{Risk Index} = r = \frac{\sqrt{Var(Y_{Tot})}}{E[Y_{Tot}]} = \frac{1}{\sqrt{N}} \cdot \frac{\sqrt{Var(Y_i)}}{E[Y_i]}$$

Analyzing mortality risk in deterministic and Stochastic framework (Cont'd)

- In a stochastic framework, a finite set of survival functions, S will be adopted and assigned to them probability distribution P, where $P = \{p_1, p_2, \ldots\ldots p_k\}$ with $\sum p_i = 1$

- The probability distribution, P is assigned according to the degree of confidence in corresponding projection.

- The risk index of the portfolio can be obtained as follows:

  $$\boldsymbol{E[Y_{Tot}]} = E_P[E[Y_{Tot}|S]] = N\, E_P[E[Y \mid S]]$$

  $$\boldsymbol{Var(Y_{Tot})} = E_P[Var(Y_{Tot}|S)] + Var_P(E[Y_{Tot} \mid S]) = N\, E_P[Var(Y \mid S)] + N^2\, Var_P(E[Y \mid S])$$

  $$\boldsymbol{r} = \frac{\sqrt{\mathbf{Var(Y_{Tot} \mid S)}}}{\mathbf{E[Y_{Tot} \mid S]}} = \mathrm{Sqrt}\left(\frac{1}{N} \cdot \frac{E_P[Var(Y \mid S)]}{E^2[Y|S]} + \frac{Var_P(E[Y \mid S])}{E^2\,[Y|S]} \right)$$

- The 1st term of ***r*** shows the ***random fluctuation risk*** as in the deterministic case. The 2nd term is the ***longevity risk*** *which is independent of N*.

- The deterministic approach can only address the ***random fluctuation risk***.

The Mortality risk can be addressed by:

- Establishing an adequate solvency margin
- Reinsuring
- Investing in Longevity Bonds
- Developing a model to calculate a Provision for Adverse Deviation (PAD)

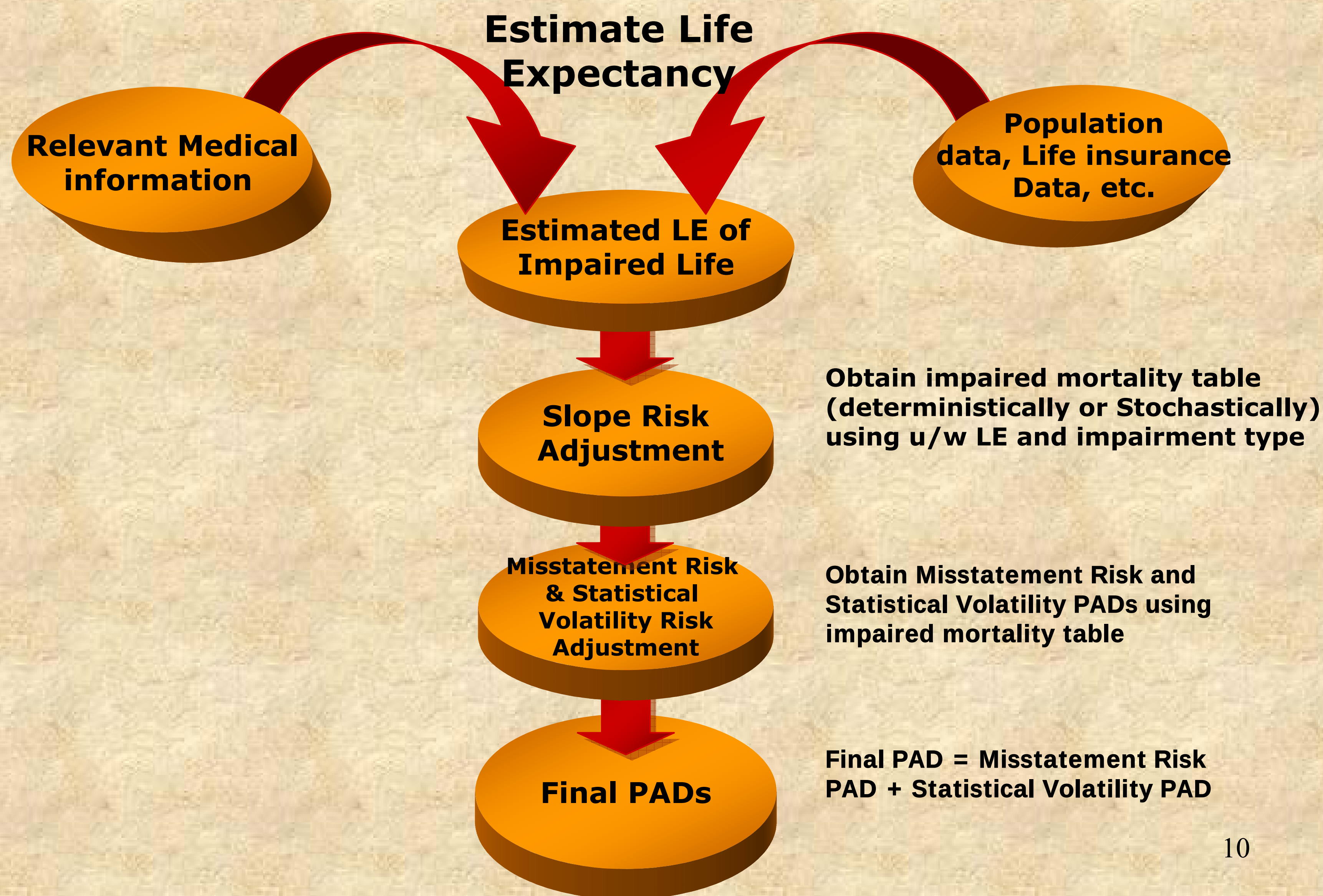
The current study only focuses on developing a PAD model

PAD Model

- Two components of the longevity :
  - Slope risk – Risk of **under pricing** given benefit because of failure to capture the correct impaired mortality slope (impaired mortality pattern) of the policyholder;
  - Misstatement risk – Risk that the underwriter will **understate** true life expectancy (LE) based on medical information obtained at underwriting;
- Statistical volatility risk – Risk that **actual** future years lived will exceed “true” LE;

Statistical volatility and slope risks exist even if there is **no** misstatement risk.

Methodology



Slope Risk Adjustment

- Hardest risk to quantify because different impaired mortality slopes can have the same LE, but different annuity costs.

- Impaired mortality table can be constructed after obtaining initial estimated LE.

- There are 2 methods will be used to estimate the future mortality, q'_{x+t} for impaired annuitants.
  - For acute or degnerative chronic conditions q'_{x+t} can be represented as a *generic model*: $q'_{x+t} = A_t \, q_{x+t} + b_t$ where

q_{x+t} = Mortality after t years of healthy life who purchased at age x

b_t = Substandard flat extra A_t = Substandard mortality multiple

Time parameter *t* is needed because many chronic condition's extra mortality would be expected to tail off with advancing years.

Slope Risk Adjustment (Cont'd)

- For static chronic medical conditions, $\boldsymbol{q'_{x+t}}$ can be represented by a combination of ***Generic model*** and ***Log-Linear Declining Relative Rate (LDR)*** method.

Log-Linear Declining Relative Rate (LDR) methods

LDR method: $ln[q'_{x+t}/q_{x+t}] = \beta*(\alpha - x)$, where,

q_{x+t} = Mortality after t years of healthy life who purchased at age x

α = Parity age (i.e. estimated mortality rate is equal to q_{x+t} at $x = \alpha$)

$q'_{x+t} = q_{x+t}$ for $x + t > \alpha$

β = Declining rate of log relative risk (depends on the level of impairment)

The parameters α and β are estimated from the observed data.

Example: Consider a Spinal cord injury situation

- The period shortly after spinal cord injury is one of especially high risk.

  $\boldsymbol{q'_{x+} = Aq_{x+t}}$ is appropriate for q'_{x+t}

- After that he has fairly low risk over his life span

  LDR method gives better estimates for q'_{x+t}

Slope Risk (Cont'd)

- Finally, $\boldsymbol{q'_{x+t}}$ will be estimated by using one of the above methods depending on the following disability scenarios: Policyholder has
  - Temporary high risk to period ***n*** and normal health after
  - Permanent impairment
  - Permanent impairment with temporary high risk for period ***n***

PAD For Misstatement Risk

- Requires 2 initial inputs:
  - Level of confidence / reliability of underwriter;
  - Level of tolerance of the **cost** of the misstatement risk.

- Underwriter reliability at level (1-α) is translated into:
  Pr [‘true’ LE <= ‘underwriting’ LE] = 1-α

- Probability distribution assumed on LE understatement satisfies two constraints:
  - Sum of probabilities for each year of LE understatement must equal α
  - Probability **decreases** as level of LE understatement **increases**.

- Probabilities are assigned exponentially for each LE understatement under three degrees of difficulty in estimating LE:
  - ***Low***
  - ***Medium***
  - ***High***

PAD For Misstatement Risk (Cont'd)

- Cost of misstatement risk is normalized to equal (A-B)/B where:

  A = ***annuity cost*** or ***loss function value at issue*** when LE equal to 'true' LE

  B = ***annuity cost*** or ***loss function value at issue*** when LE equal to u/w LE

- PAD is chosen such that expected normalized misstatement cost with PAD is within tolerance level.

Life annuity cost = $\sum_{k=1}^{K_x} R_k \, {}_kp_x \, v^k$ *Loss function at issue* = $\sum_{k=1}^{K_x} R_k v^k - \sum_{k=1}^{K_x} p_k v^k$

Expected Normalized Misstatement Cost = $\sum$(A – B)/B * Pr(LE understatement)

<u>Example</u>: An impaired policyholder needs a lifetime annuity and gives $1M premium to the insurance company.

If estimated LE = 5 years

Annual benefit =$200K (approximately)

If he lives 1 year longer than expected, The company has paid out 20% more in benefits.

If Estimated LE = 10 years

Annual benefits = $100 K (approximately)

If he lives 1 year longer than expected, The company has paid 10% more in benefits.

Simplified Example of Misstatement Risk PAD

- Assume the following:
  - Impaired u/w LE = 5 years
  - LE of corresponding healthy lives at same issue age = 10 years
  - i = 0
  - Underwriter reliability = 85%
  - Level of tolerance = 5%
- If ‘true’ LE is 6 years, then normalized cost is (6-5)/5 = 1/5
- Assume that the underwriter can recognize the level of difficulty in estimating in LE is “Medium” then the probability distribution of the normalized cost is given by:

TRUE LE	Normalized Cost	Probability	Normalized Cost * Prob
≤ 5	-	0.850	-
6	0.2	0.077	0.155
7	0.4	0.037	0.150
8	0.6	0.018	0.111
9	0.8	0.009	0.007
10	1.0	0.004	0.004
	TOTAL		**0.053**

E[Normalized Cost] > 5%

Simplified **Example** (cont'd)

- Suppose the PAD of 1 year increase in LE is used
  ie pricing LE = 6 years
- Then, normalized cost distribution is as follows:

NO PAD

TRUE LE	**Normalized Cost**	**Probability**	**Normalized Cost * Prob**
≤ 5	-	0.850	-
6	0.2	0.077	0.155
7	0.4	0.037	0.150
8	0.6	0.018	0.111
9	0.8	0.009	0.007
10	1.0	0.004	0.004
	TOTAL		**0.053**

$= (6 - 5)/5$

PAD OF 1 YEAR INCREASE IN LE

TRUE LE	**Normalized Cost**	**Probability**	**Normalized Cost * Prob**
≤ 6	-	0.927	-
7	0.170	0.037	0.006
8	0.330	0.018	0.006
9	0.500	0.009	0.005
10	0.670	0.004	0.003
	TOTAL		**0.019**

$= (7 - 6)/6$

- Since E[Normalized Cost] < 5%, PAD for misstatement risk equals 1 year increase in LE

PAD For Statistical Volatility Risk

- Exists because LE is the expected value of the future lifetime random variable.
- **Actual** future years lived have roughly a 50-50 chance of exceeding the underwriting LE, even if it is correct.
- PAD for statistical volatility risk takes the form:
  $(C * \sigma) / \sqrt{N}$
  where
  C = level of confidence required for PAD
  σ = standard deviation of the future lifetime random variable
  N = average number of policies sold.

Measuring Riskiness of the Portfolio

- Assume the following:
  - A person age 30 is suffering from a spinal cord injuary -Frankel Grade ABC (C1– C4)
  - Impaired u/w LE = 25 years
  - LE of corresponding healthy lives at same issue age = 49.75 years
  - i = 6% Annual benefit = $100
  - Underwriter reliability = 85%, Level of tolerance = 5%
  - The level of difficulty in estimating in LE is “Medium”

Risk Index of Annuity Portfolio

	N = 500		**N = 1000**	
	NO PAD	**PAD = 3**	**NO PAD**	**PAD = 2**
$E[Y_{Tot}\|S]$	*477494.991*	*618040.577*	*904989.982*	*1108419.764*
$Var(Y_{Tot}\|S)$	*43445745.989*	*39129265.057*	*86891491.978*	*84120045.539*
r	*0.01380*	***0.01012***	*0.01030*	***0.00827***

- Riskiness of the portfolio is decreasing after applying the PAD to the initial Estimate.
- Risk index (r) is also decreasing with the size of the annuity portfolio.

Application of the Model

- Applied PAD model for a leading New England insurance company's ***Structured Settlements*** business

> •A financial or insurance arrangement, including periodic payments, that a claimant accepts to resolve a personal injury or to reflect a statutory period payment obligation

Actual To Expected Analysis

- Actual to expected in force deaths for calendar years 2001 through 2004 were compared using our PAD model vs company’s model

Actual to Expected Death Analysis

Calendar Year	Our Model	Company Model
2001	1.44	1.10
2002	1.71	1.02
2003	1.25	0.95
2004	1.51	1.08

Expected deaths based on 1983 IAM table (Adjusted table)

- Actual to expected ratios of death are **higher** using our PADs compared to the company's model

Implications Of PAD Model : Life Settlements

- Model has generated interest by Life Settlements companies as a means to improve on the underwriting information provided by outside underwriting agencies.

- Model lends itself naturally for commercialization to be used for:
  - Impaired Annuity Products;
  - Structured Settlements pricing;
  - Life Settlements pricing;

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