Guaranteed Minimum Withdrawal Benefit in Variable Annuities

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Contents

Variable Annuities

- Popular savings instruments

- Guaranteed Minimum Withdrawal Benefit (GMWB) is the most popular VA rider.

Guaranteed Minimum Withdrawal Benefit

- Downside income protection against investment risk

- Potential upside equity gain

  If the investment performance is strong, policyholders will benefit.

GMWB Example

- GMWB is elected at the issuance of the variable annuity.
- Single premium: $ 100,000
- Initial Guaranteed Withdrawal Balance(GWB): $ 100,000
- Maximum Annual Withdrawal Amount(MAWA):
  $7,000 = 7% of initial GWB

A GMWB Example

Contract year	Investment return	Fund before withdrawal	Annual withdrawal	Fund after withdrawal	Remaining benefit
1	10%	110,000	7,000	103,000	93,000
2	10%	113,300	7,000	106,300	86,000
3	-10%	95,670	7,000	88,670	79,000
4	-60%	35,468	7,000	28,468	72,000
5	-60%	11,387	7,000	4,387	65,000
6	10%	4,826	7,000	0	58,000
7	r	0	7,000	0	51,000
⋮	⋮	⋮	⋮	⋮	⋮
14	r	0	7,000	0	2,000
15	r	0	2,000	0	0

GMWB charge

- The charge is deducted from the account value.

  Example:

  - 50 basis points of average daily account value

Financial view of GMWB

- GMWB is a put option attached to an equity-like insurance product.
- If the account value is always higher than withdrawal amount, there is no liability under the GMWB.
- If the account balance reaches zero, GMWB guarantees all remaining periodic payments.
- This put option has a random exercise time.
- In theory market value of the put option should equal market value of the premium contributions.

Pricing a Simple GMWB

- Initial investment: $100
- Contract term: 15 years
- Annual withdrawal amount: $6.67
- Fund volatility: 20%
- Risk-free rate: 5%
- Charge as a percentage q of account value continuously
- All policyholder fully utilize GMWB from the first contract year.
- All policyholders have the same entry age.
- No lapses, no deaths, no dynamic behavior.

Pricing a Simple GMWB

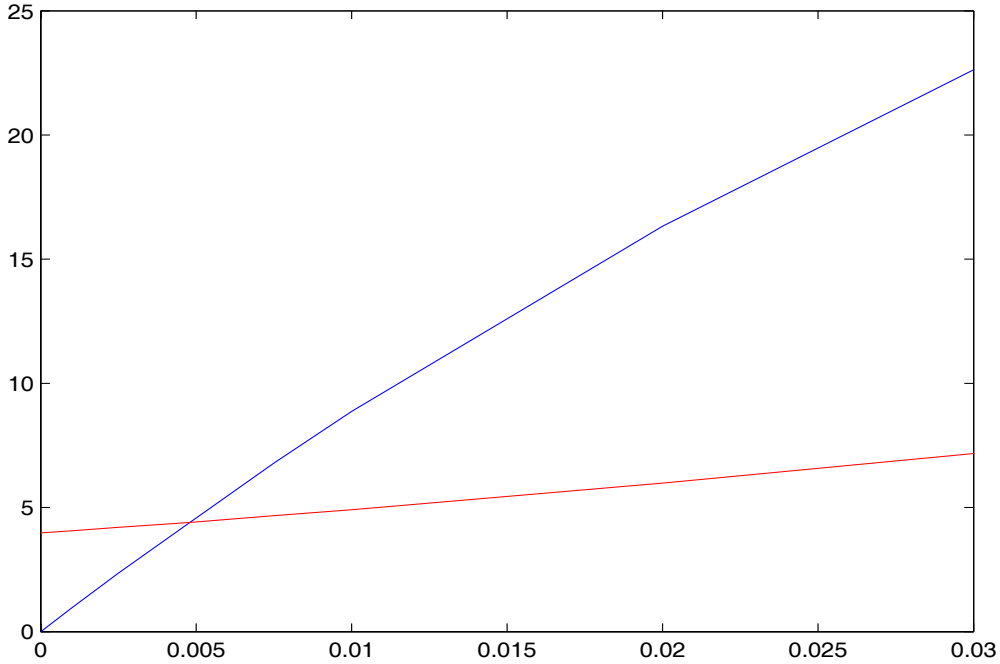


Figure 7: Plot of present value of contributions against value of put option. Lines cross when q= .0048 when put value =4.40

Policyholder Behavior

Policyholder behavior dramatically affects the cost of GMWB in the real world.

- Dynamic GMWB utilization

  Possible influencing factors:

  1. in-the-moneyness of GMWB
  2. contract duration
  3. age
  4. different benefit features

- Lapse
- Mortality

Model Policyholder Behavior using Three-state Model

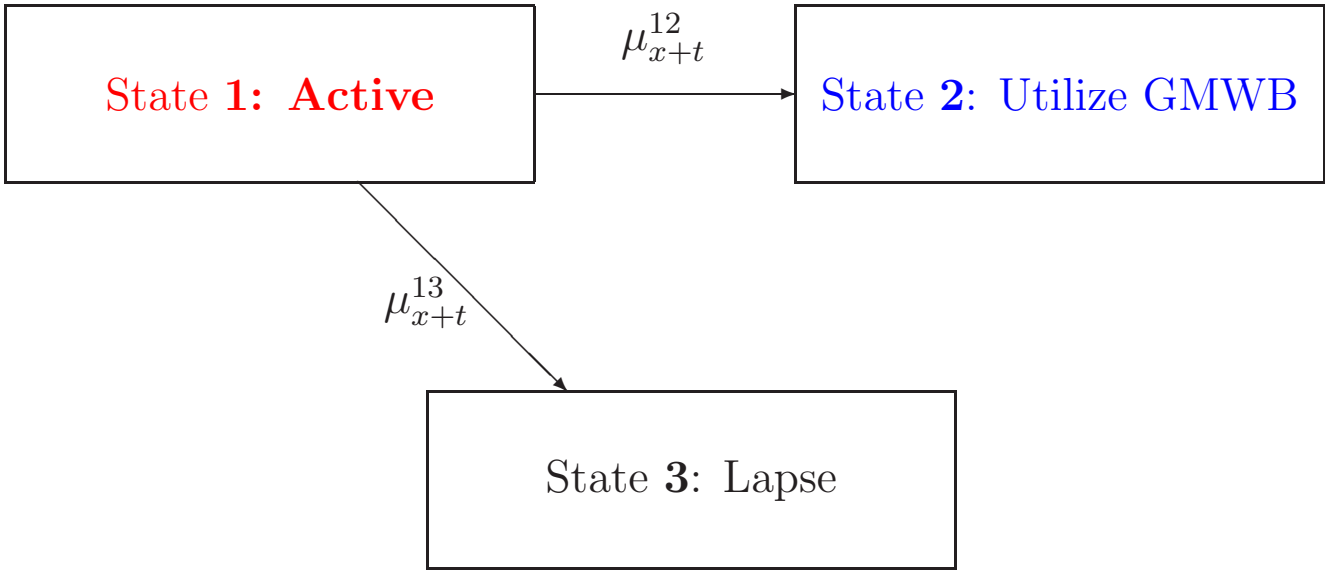


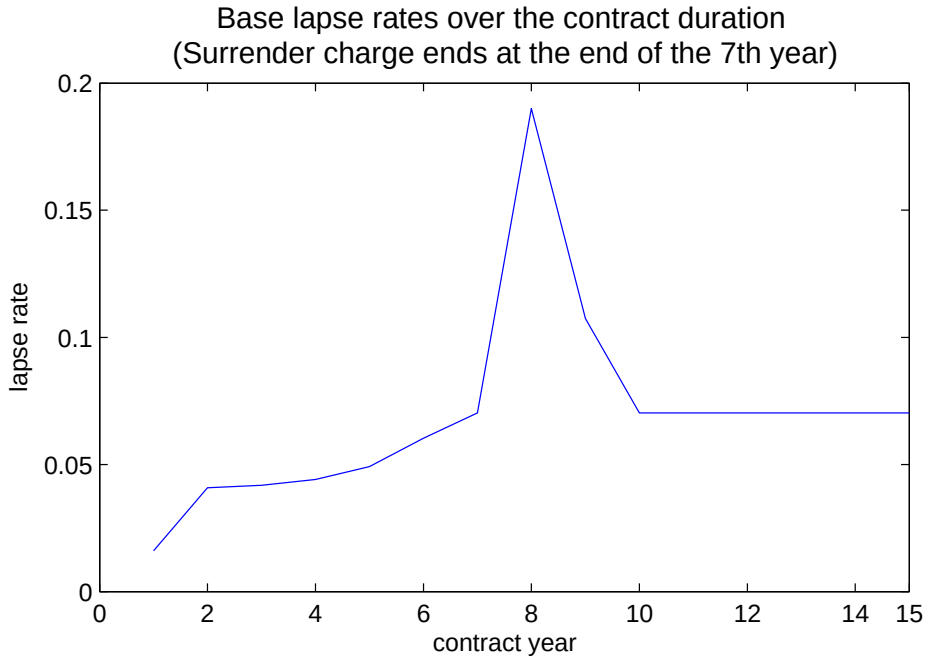
Figure 1: State Transition Diagram For Policyholders

Base Transition Intensities

- Utilization rate:

$$\bar{\mu}^{12} = 0.2$$

- Lapse rate $\bar{\mu}^{13}(t)$ changes over time because of decreasing surrender charges.



Dynamic Transition Intensities

We use an exponential function to model the impact of in-the-moneyness on utilization and lapse intensity. Assume

$$\mu_{12}(t) = \bar{\mu}_{12} \exp\{\lambda(ITM(t))\}$$

and

$$\mu_{13}(t) = \bar{\mu}_{13} \exp\{-\nu(ITM(t))\}$$

where

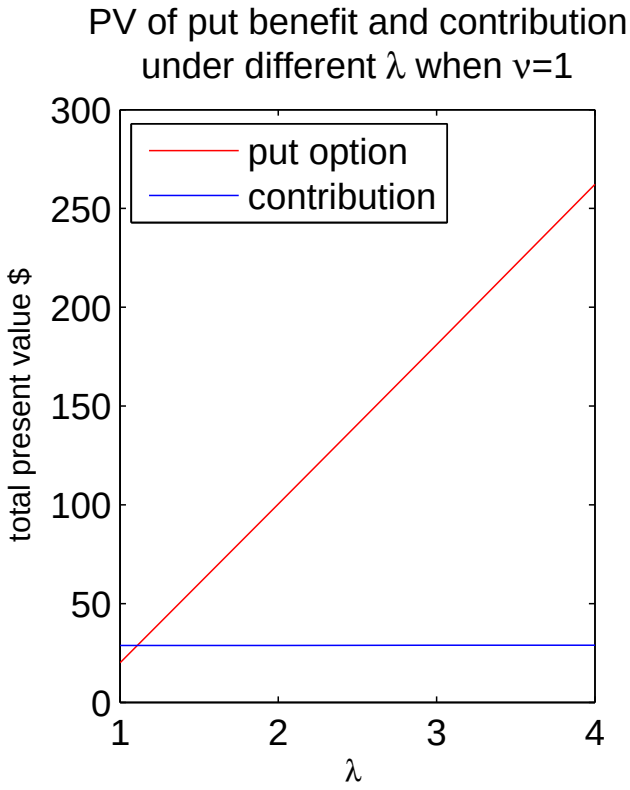
$$ITM(t) = \max\left(0, (\frac{GB(t)}{AV(t)} - 1)\right).$$

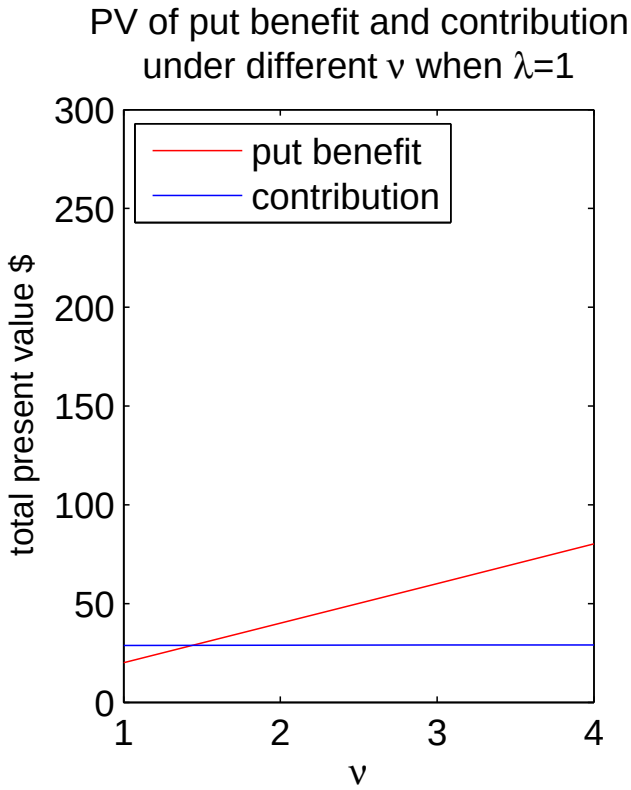
$GB(t)$ is the remaining guaranteed benefit at time t.

$AV(t)$ is the current account value at time t.

Sensitivity Analysis

Assume there are 1,000 policyholders elected GMWB at the same time. Using the previous GMWB example, we can see how the values of put benefit and contributions change with λ and ν.





Summary

1. The value of GMWB depends on policyholder behavior.
2. It's hard to model, because no published data available.
3. My future research will explore pricing, hedging, and risk management of GMWB.