41st Actuarial Research Conference

August 10-12, 2006

Pension Valuation using Conditional Tail Expectation

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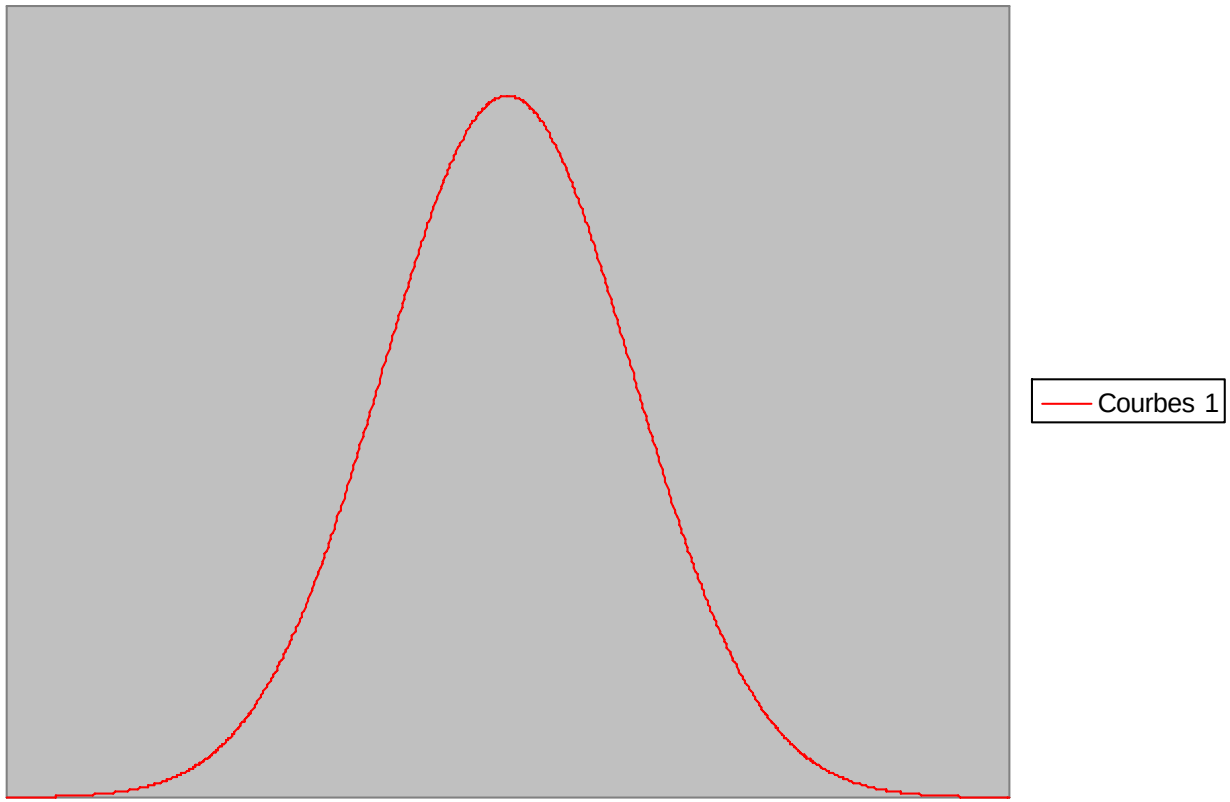
Courbes 1

Legal requirements

Pension plan funding has to follow legal guidelines

Defined contributions plans:

- **Liabilities = Assets**
- **No problem to calculate liabilities.**

Defined benefits plans: more complex calculations.

Context

Defined benefits plans

- Present value of benefits cannot be evaluated precisely.
- Assumptions required: mortality, withdrawal, disability, retirement age and interest rate.
- Interest rate assumption is the most difficult to ascertain and has the biggest influence on the value of the liabilities.

Method used to evaluate liabilities

Method 1: Economic value of benefits

Definition: **Value of benefits calculated using the rate of return of high grade bonds.**

Advantages:

- **Value similar to guaranteed value obtained from an insurance company.**
- **No use of risk premium of other type of assets.**

Disadvantages:

- **Conservative.**
- **Can lead to over funding.**
- **Gains used by future generation of participants.**

Method used to evaluate liabilities

Method 2: Present value of benefits

Definition: Present value of benefits calculated by anticipating a significant portion of the expected risk premium of underlying assets.

Advantages:

- **Better estimate of long term cost.**

Disadvantages:

- **Can be too optimistic.**
- **Can lead to under funding.**
- **Losses paid by future participants.**
- **Can produce unjustified benefits improvements.**

Suggested method for valuation

Definition:

Pension valuation anticipating a portion of the expected risk premium using conditional tail expectation of the projected return distribution of the asset portfolio.

Suggested method

Detailed outline of the suggested method

Asset value at valuation date= Market value

Expected return based on market values for each asset class.

Construction of the matrix of correlation between asset classes.

Valuation rate calculated using CTE.

Suggested method

No smoothing of asset values.

Margin for adverse deviation calculated using the difference between the best estimate of the returns and the rate used for valuation.

Margin will be small if risk is low and large if risk is high.

Determination of the hypotheses

Expected return, volatilities and correlation and duration.

Duration impacts volatility; short duration is associated with high volatility.

Volatility of Canadian Equities for the period 1924-2004

Holding period	Return volatility
1 year	18.4%
5 years	7.9%
10 years	4.0%
15 years	3.3%

Calculation of the distribution of returns of assets

Normal distribution used.

Could use other distributions, copulas, etc...

Simulation of the curve of returns of assets

Return used for valuation purposes

Mean of the returns below a certain percentage of the returns:

$$E(\text{Return} \mid \text{Return} < \text{Return}(x))$$

$$P(\text{Return} < \text{Return}(x)) = x$$

Example

Assets: **60% Equities** **40% Bonds**

Liabilities: **A single payment of $ 1000 at t=15**

Assumptions:

	Expected return	Volatility
Equities	**8%**	**3.3%**
Bonds	**4%**	**1.0%**

Flat yield curve and 30% correlation between asset classes.

Example

Valuation using Conditional Tail Expectation (CTE) of the 40^{e} percentile of lower returns of simulated returns.

Approximately equal to the 20^{e} percentile of lower returns.

Results

Expected return:	**6.4%**
CTE at 40^{e} percentile:	**Between 4.3% and 4.4%**
Return at 20^{e} percentile:	**Between 4.5% and 4.6%**
Hypothesis used:	**4.5%**
Value of liabilities CTE40:	**$516.72 = 1000*(1.045)$^{-15}**
Value at 6.4%:	**$394.34 = 1000*(1.064)$^{-15}**
Perfect matching :	**$555.26 = 1000*(1.04)$^{-15}**

Thank you for your attention

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