Risk Management of a DB Underpin Pension Plan

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Outline

- Introduction and Background
- Risk Management
  - Hedging Strategies
  - Numerical Results
- Comments and Future Work

Introduction of Current Pension Systems

- Two basic kinds of pension plans

  Defined Benefit (DB) and Defined Contribution (DC)

- Pension Reforms

  Three-dimensional classification(Lindbeck, 2003):

  - Actuarial vs Non-Actuarial
  - Funded vs Unfunded
  - Defined Benefit vs Defined Contribution

Introduction and Background

- “Greater of” benefit a DB and DC hybrid plan
  - DC plus DB guaranteed minimum

- The payoff of the guarantee is same as an exchange option payoff.
  - Eg. McGill University, WLU offer a hybrid pension plan with minimum DB guarantee.

Description of Benefits (DB)

- The DB benefits depend on the salary at exit age xr:

$$D_{xe,xr}(t) = \alpha \times S_{xr} \times n \times {}_{xr-xe-t|}\ddot{a}^{(12)}_{xe+t}$$

$D_{xe,xr}(t)$:The actuarial value at age xe+t

α :The accrual rate

S_{xr} :The salary at age xr

n :The years of service

${}_{xr-xe-t|}\ddot{a}^{(12)}_{xe+t}$:The annuity factor

Description of Benefits (DC)

- The DC benefits depend on the monthly contribution:

$$A_{xe,xr}(t+h) = A_{xe,xr}(t)\cdot(1+hf_t)+c\cdot h\cdot S_{xe+t}$$

Where

c :The contribution rate

h :The time interval

Risk Management

- Our problem
  - Valuation and risk management of Max(D-A,0) at exit.
- Assume no exits
  - Given retirement time T=xr-xe, the value of guarantee at entry age xe is,

$$e^{-r(xr-xe)}Max(D_{xe,xr}(T)-A_{xe,xr}(T),0)$$

Exchange Option

- Rates of return of two assets

$$dS_i = S_i[\alpha_i dt + \sigma_i dZ_i], i = 1,2$$

- A European-type option with maturity time T
  - Payoff at maturity time T

$$w(S_1, S_2, T) = Max(S_1 - S_2, 0)$$

  - Price at any time t

$$w(S_1, S_2, t) = S_1 N(d_1) - S_2 N(d_2)$$

$$d_1 = \frac{\ln(S_1 / S_2) + 1/2 \cdot \sigma^2 (T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$\sigma^2 = \sigma_1^2 - 2\sigma_1\sigma_2\rho + \sigma_2^2$$

Monte Carlo Hedging

- At any time t,

change the value of DB and DC accounts

$\Downarrow \quad \Downarrow \quad \Downarrow$

the projected value of guarantee

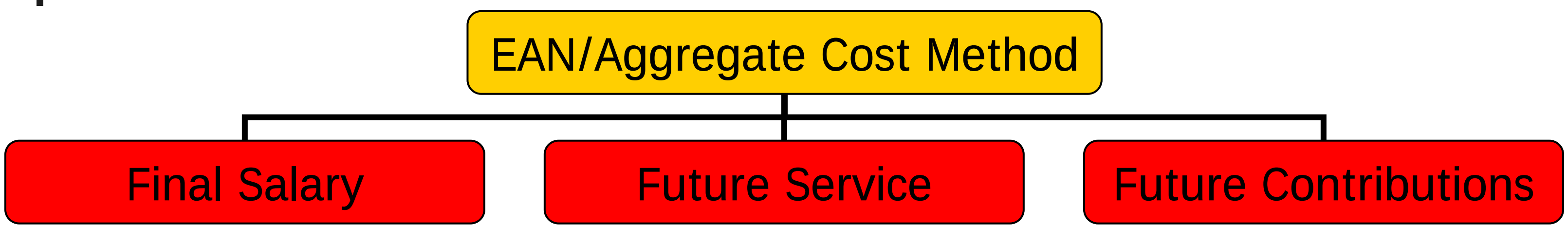
$\Downarrow \quad \Downarrow \quad \Downarrow$

calculate delta

$\Downarrow \quad \Downarrow \quad \Downarrow$

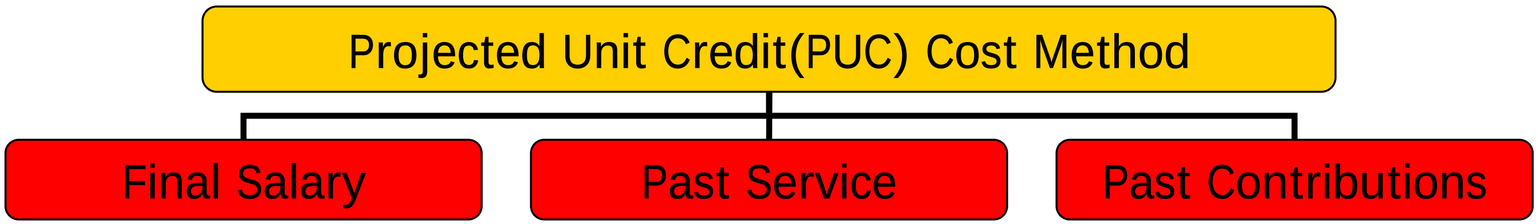
rebalance at time t+h

Strategy 1



- Too Volatile!

Strategy 2



- Eg. Entry age 35, Current age 40, Normal Retirement Age 65

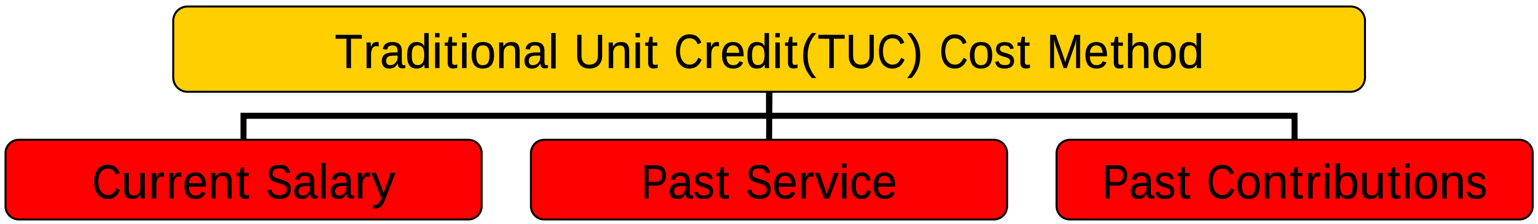
  Projected value of DB: $\alpha \cdot 5 \cdot {}_{25|}\ddot{a}_{40}^{(12)} \cdot S_{40} e^{\int_{40}^{65} s(u)\,du}$

  Projected value of DC: $A_{40} e^{\int_{40}^{65} f(u)\,du}$

  Projected value of guarantee:

$$Max\left(0, \alpha \cdot 5 \cdot {}_{25|}\ddot{a}_{40}^{(12)} \cdot S_{40} e^{\int_{40}^{65} s(u)\,du} - A_{40} e^{\int_{40}^{65} f(u)\,du}\right)$$

Strategy 3



- Eg. Entry age 35, Current age 40, Normal Retirement Age 65

  Projected value of DB: $\alpha \cdot 5 \cdot {}_{25|}\ddot{a}_{40}^{(12)} \cdot S_{40}$

  Projected value of DC: $A_{40} e^{\int_{40}^{65} f(u)du}$

  Projected value of guarantee:

$$Max(0, \alpha \cdot 5 \cdot {}_{25|}\ddot{a}_{40}^{(12)} \cdot S_{40} - A_{40} e^{\int_{40}^{65} f(u)du})$$

Hedging Costs

- At time t, assume delta hedging shares are $\Delta_{1,t}$ and $\Delta_{2,t}$ The value of hedging fund is

$$\Delta_{1,t} \cdot S_{1,t} + \Delta_{2,t} \cdot S_{2,t}$$

- At time $(t+1)^-$, the value of hedging fund is

$$\Delta_{1,t} \cdot S_{1,t+1} + \Delta_{2,t} \cdot S_{2,t+1}$$

- At time (t+1), the value of hedging fund is

$$\Delta_{1,t+1} \cdot S_{1,t+1} + \Delta_{2,t+1} \cdot S_{2,t+1}$$

- At time (t+1), the hedging cash flow CF_{t+1} is

$$(\Delta_{1,t+1} - \Delta_{1,t}) \cdot S_{1,t+1} + (\Delta_{2,t+1} - \Delta_{2,t}) \cdot S_{2,t+1}$$

Two ways to calculate hedging costs

- Discount all hedging cash flows and amortize by the salary-related annuity

$$\frac{\sum_j e^{-rj} CF_j}{\sum_j e^{-rj} S_j}$$

- At time t, calculate the proportion of the hedging cash flows of the salary at time t, then calculate the average

$$\frac{1}{n}\sum_j CF_j / S_j$$

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Parameters

- Base Projections
  - Mean of Salary Growth Rate: 0.04
  - Std of Salary Growth Rate : 0.02
  - Mean of Crediting Rate : 0.10
  - Std of Crediting Rate : 0.20
  - Contribution Rate : 0.125
  - Discount Rate : 0.05

Results Comparison (% of Salary)

Table 1: Discount Rate: 5%, Strategy 2

Entry Age	Hedging Cost 1	Hedging Cost 2
20	4.69	4.33
	(0.0086)	(0.0086)
25	4.55	4.25
	(0.0082)	(0.0080)
30	4.38	4.14
	(0.0078)	(0.0076)
35	4.20	4.01
	(0.0074)	(0.0073)
40	4.03	3.87
	(0.0069)	(0.0068)
45	3.80	3.68
	(0.0063)	(0.0062)
50	3.56	3.48
	(0.0056)	(0.0055)
55	3.36	3.20
	(0.0049)	(0.0048)

Table 2: Hedging Cost 1, Strategy 2

Entry Age	Discount Rate:5%	Discount Rate:3%
20	4.69	4.01
	(0.0086)	(0.0094)
25	4.55	3.98
	(0.0082)	(0.0091)
30	4.38	3.92
	(0.0078)	(0.0085)
35	4.20	3.84
	(0.0074)	(0.0081)
40	4.03	3.74
	(0.0069)	(0.0073)
45	3.80	3.61
	(0.0063)	(0.0068)
50	3.56	3.43
	(0.0056)	(0.0060)
55	3.36	3.19
	(0.0049)	(0.0051)

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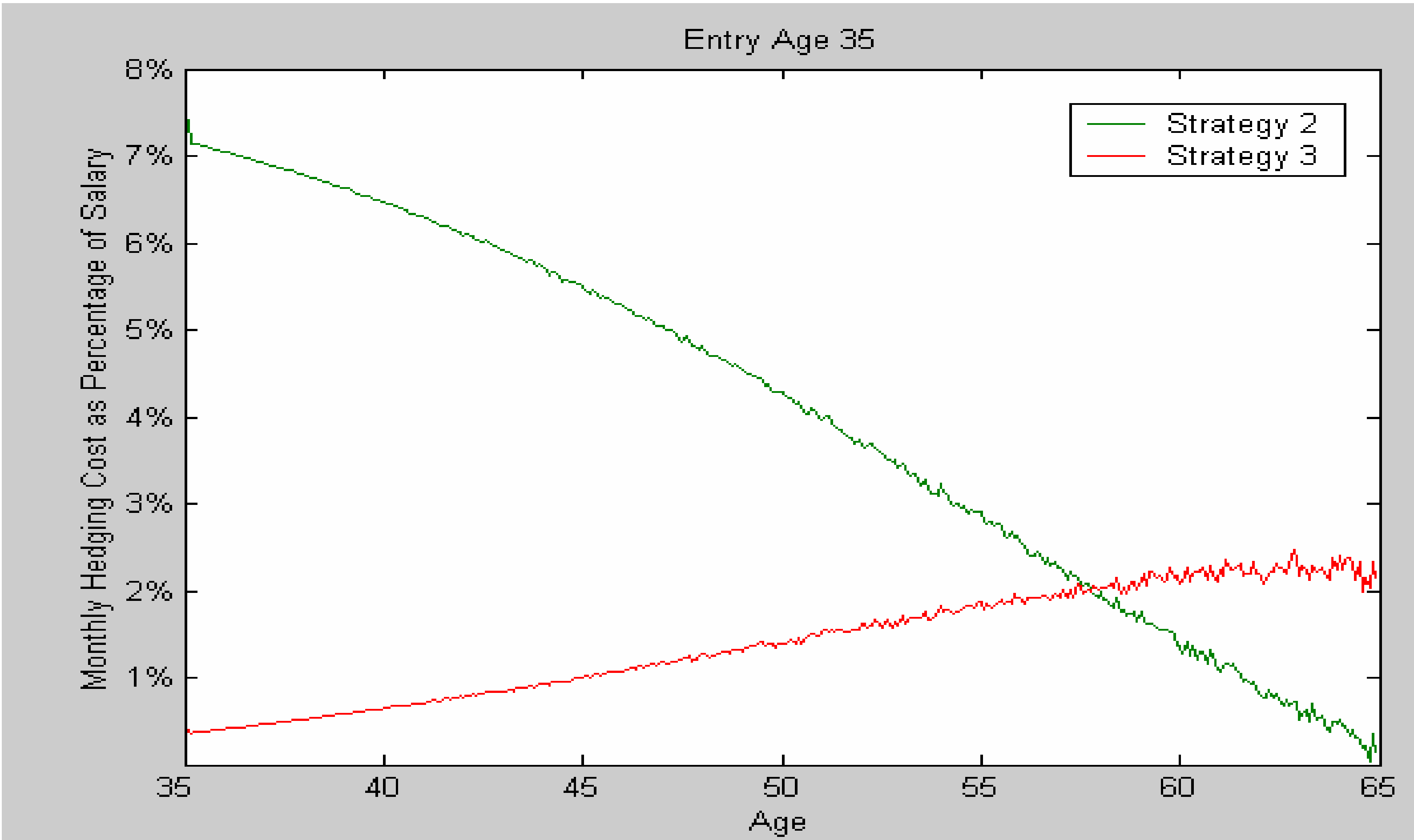
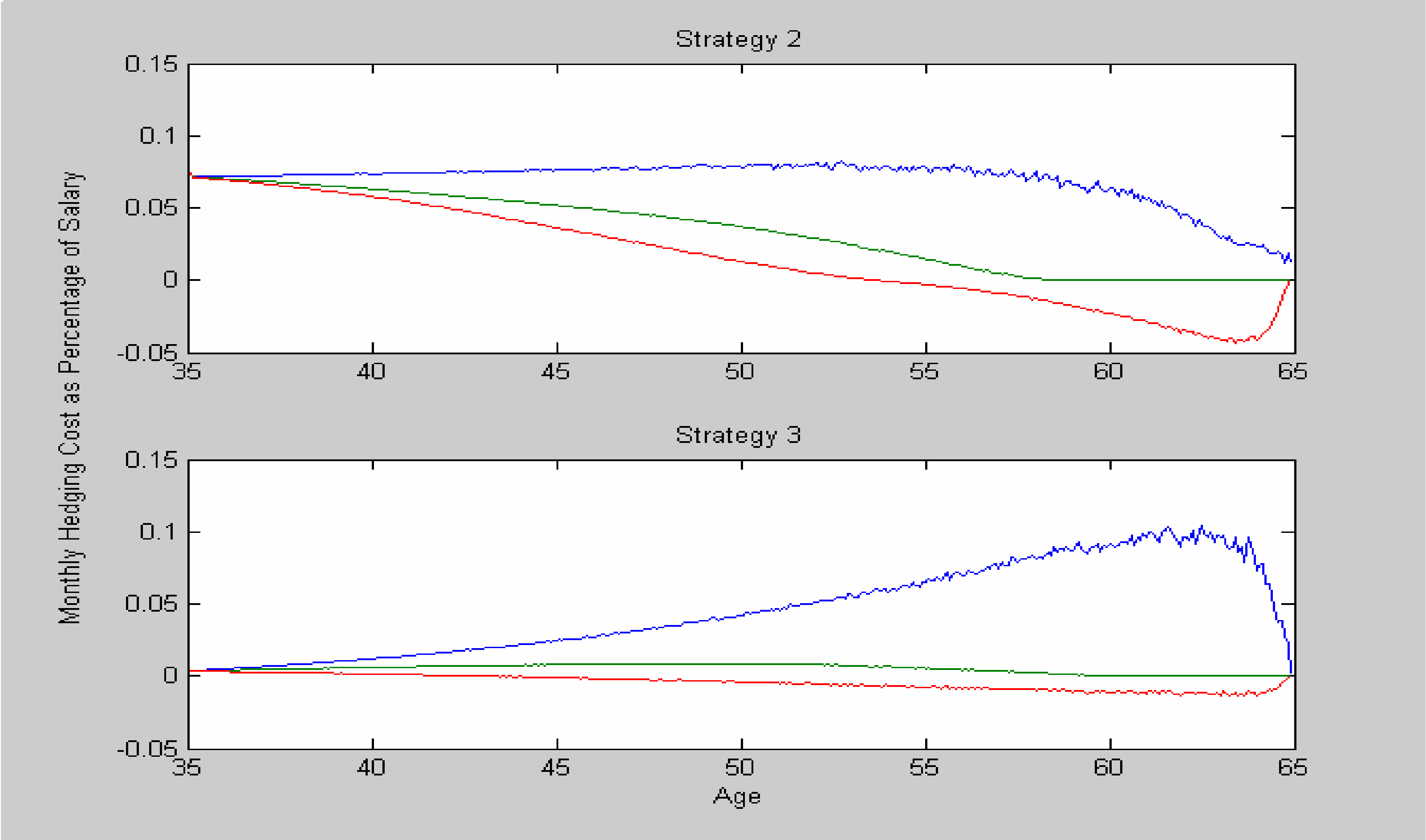
Monthly Hedging Cost

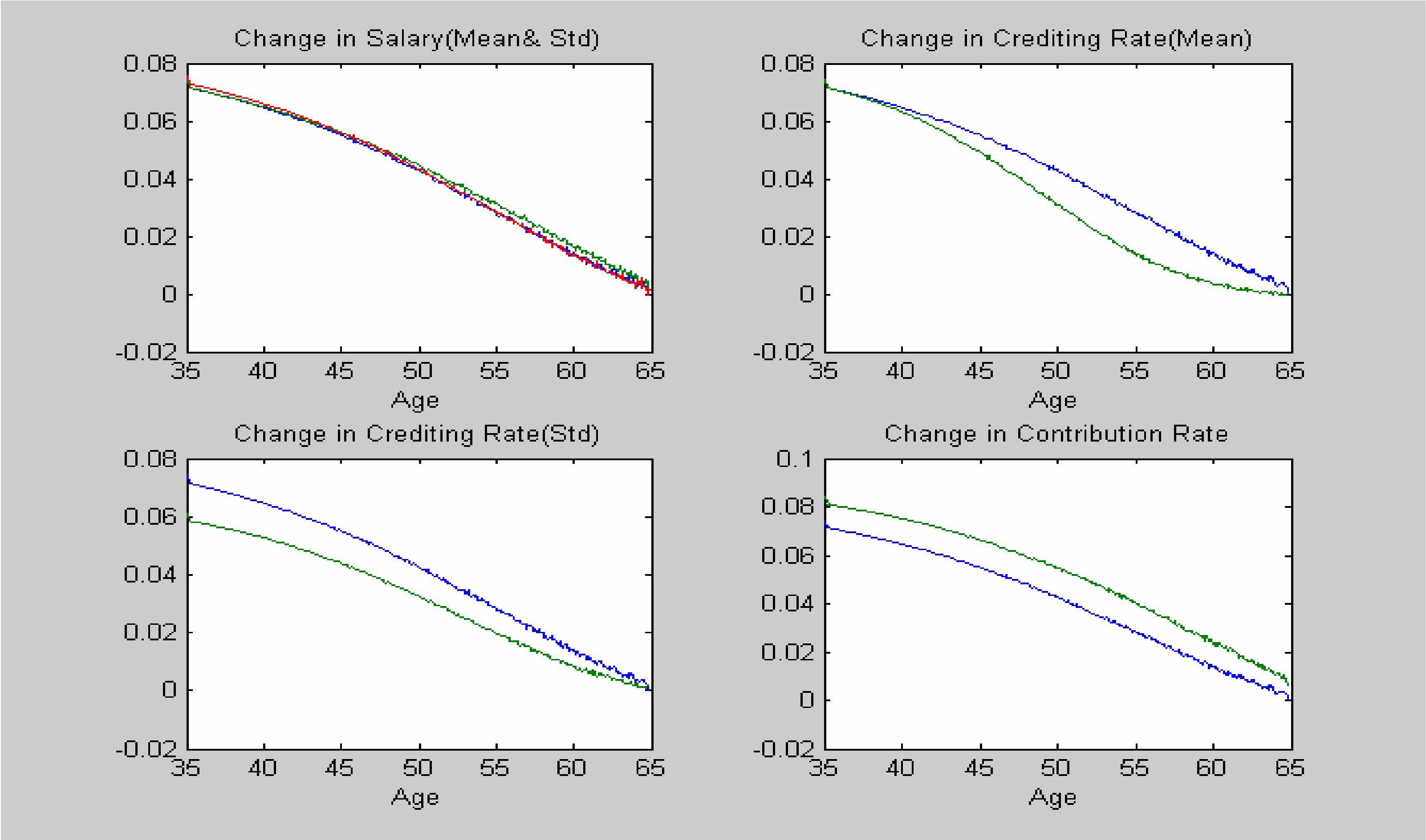


Quantile



Strategy 2
0.15
0.1
0.05
0
-0.05
35
40
45
50
55
60
65
Strategy 3
0.15
0.1
0.05
0
-0.05
35
40
45
50
55
60
65
Age
Monthly Hedging Cost as Percentage of Salary

Scenario Tests
(Strategy 2, Hedging Cost 2)



Conclusions

- Hedging costs depend on many factors, but are not very sensitive to the entry age or the salary growth rate.

- Standard errors depend on the entry age and increase when the entry age increases.

Future Work

- Use inflation-linked bonds hedge salary and explore basis risk.

- Introduce stochastic interest and annuity.

Thank You!