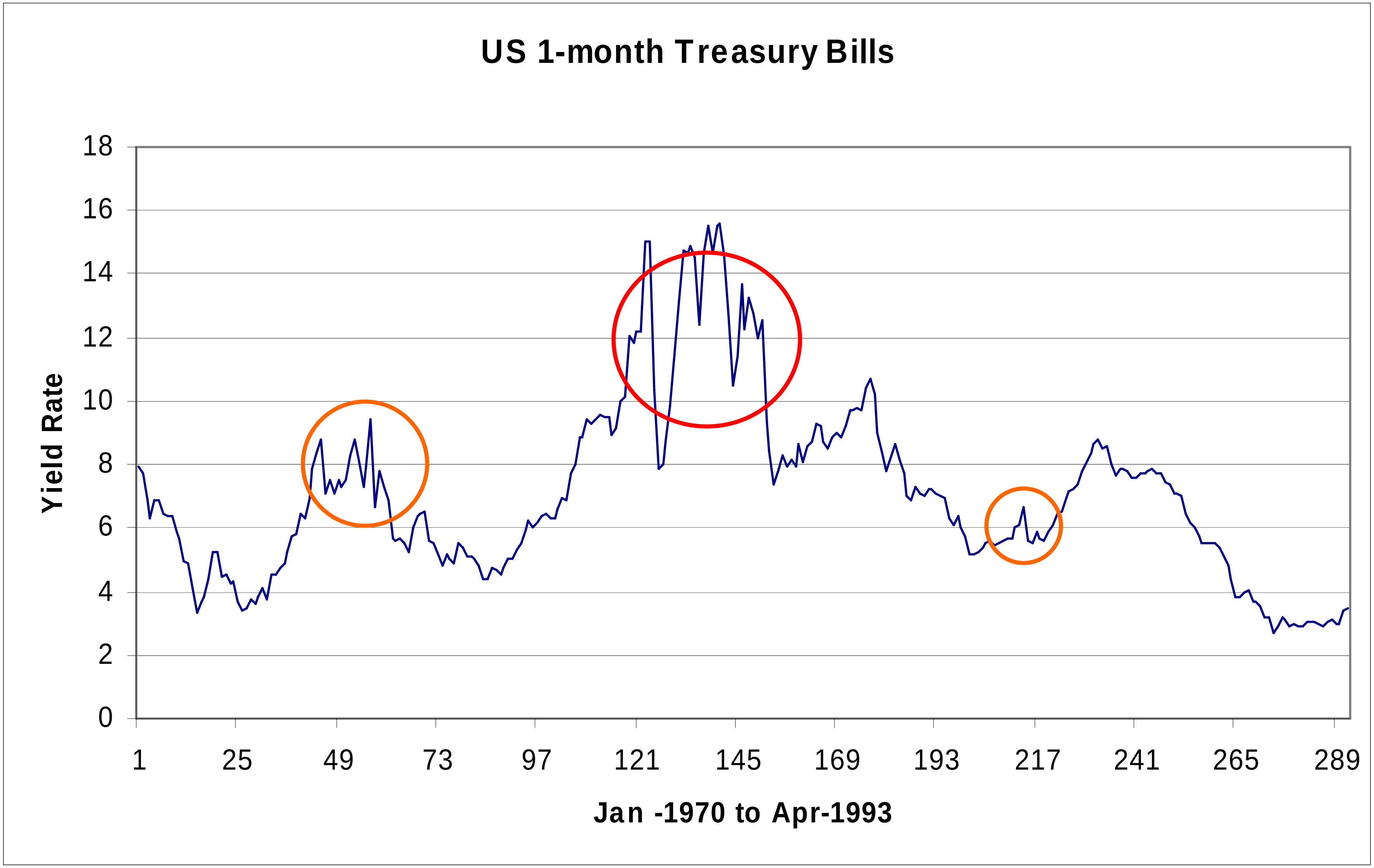
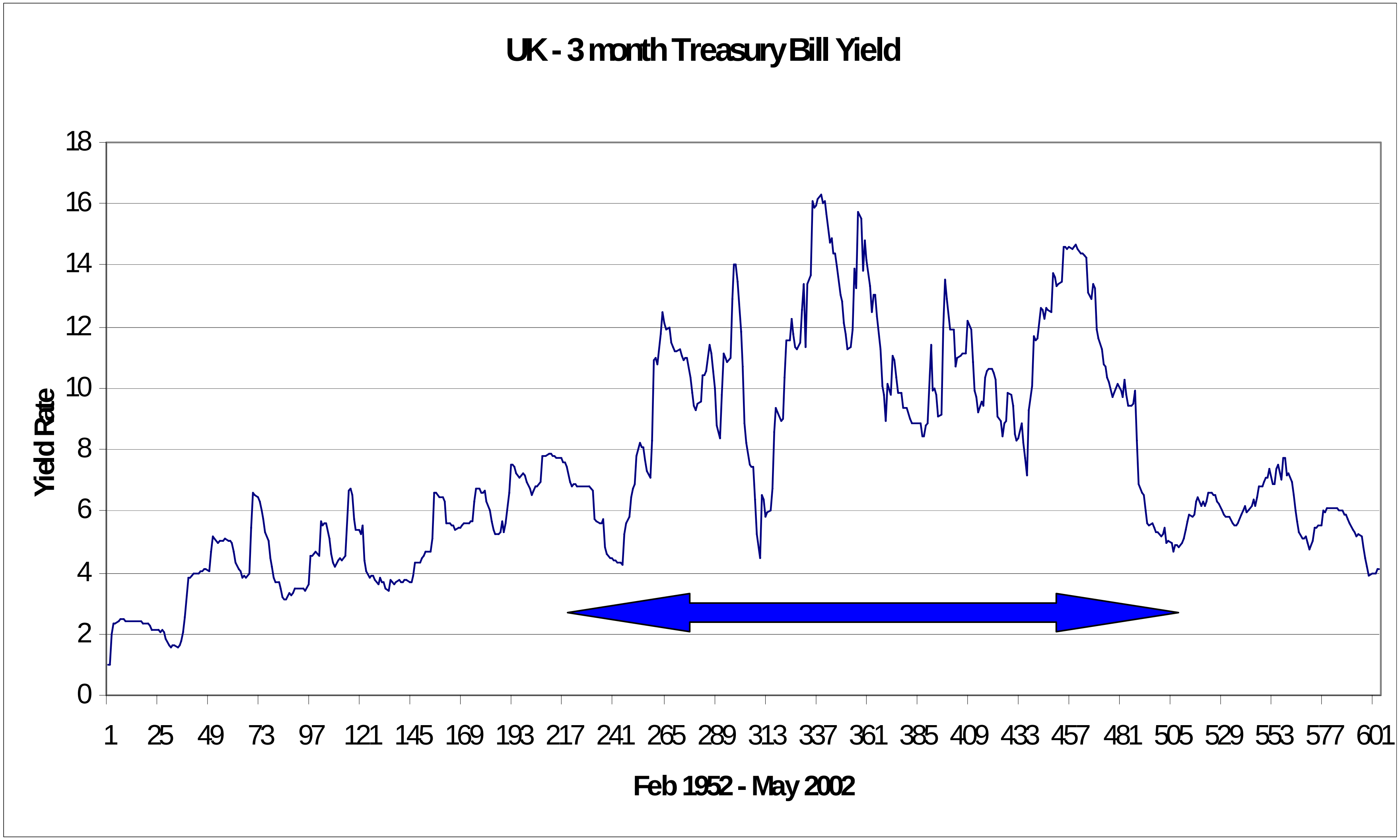
Econometric Models for Interest Rates

R. Keith Freeland

Summary of Talk

- Data
  - U.K. Interest Rates
- Single Factor Models
- Regime Switching Models
- Results
- Conclusions


US 1-month Treasury Bills
Yield Rate
18
16
14
12
10
8
6
4
2
0
1
25
49
73
97
121
145
169
193
217
241
265
289
Jan -1970 to Apr-1993



UK - 3 month Treasury Bill Yield
Yield Rate
0
2
4
6
8
10
12
14
16
18
1
25
49
73
97
121
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169
193
217
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265
289
313
337
361
385
409
433
457
481
505
529
553
577
601
Feb 1952 - May 2002

Single Factor Models

- $dr_t = \mu_t + \sigma_t \, dW_t$

Model	μ_t	σ_t
Merton (1973)	μ	σ
Dothan (1978)	$\mu \, r_t$	$\sigma \, r_t$
Vasicek (1977)	$\alpha \, (\mu - r_t)$	σ
CIR (1985)	$\alpha \, (\mu - r_t)$	$\sigma \, r_t^{1/2}$
Pearson-Sun (1994)	$\alpha \, (\mu - r_t)$	$\sigma \, (r_t - \beta)^{1/2}$
Brennan-Schwartz (1979)	$\alpha \, (\mu - r_t)$	$\sigma \, r_t$
Black-Karasinski (1991)	$\alpha r_t - \gamma r_t \log r_t$	$\sigma \, r_t$

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CIR Discrete Model

- $r_t - r_{t-1} = \alpha\,(\mu - r_{t-1}) + \sigma\, r_{t-1}^{1/2}\, \varepsilon_t$,
  - where $\varepsilon_t \sim N(0,1)$

- Exact Conditional Distn
  - $r_t | r_{t-1}$ has a noncentral chi-square distn

Regime Switching Models

$$r_t - r_{t-1} | \rho_t \sim \begin{cases} N(\mu_{1,t}, h_{1,t}) & \text{w.p. } p_{1,t} \\ N(\mu_{2,t}, h_{2,t}) & \text{w.p. } p_{2,t} \end{cases}$$

Model	$\mu_{i,t}$	$h_{i,t}$
Lognormal	$\alpha_i (\mu_i - r_{t-1})$	σ^2_i
GARCH	$\alpha_i (\mu_i - r_{t-1})$	$b_{0i} + b_{1i} \varepsilon^2_{t-1} + b_{0i} h_{t-1}$
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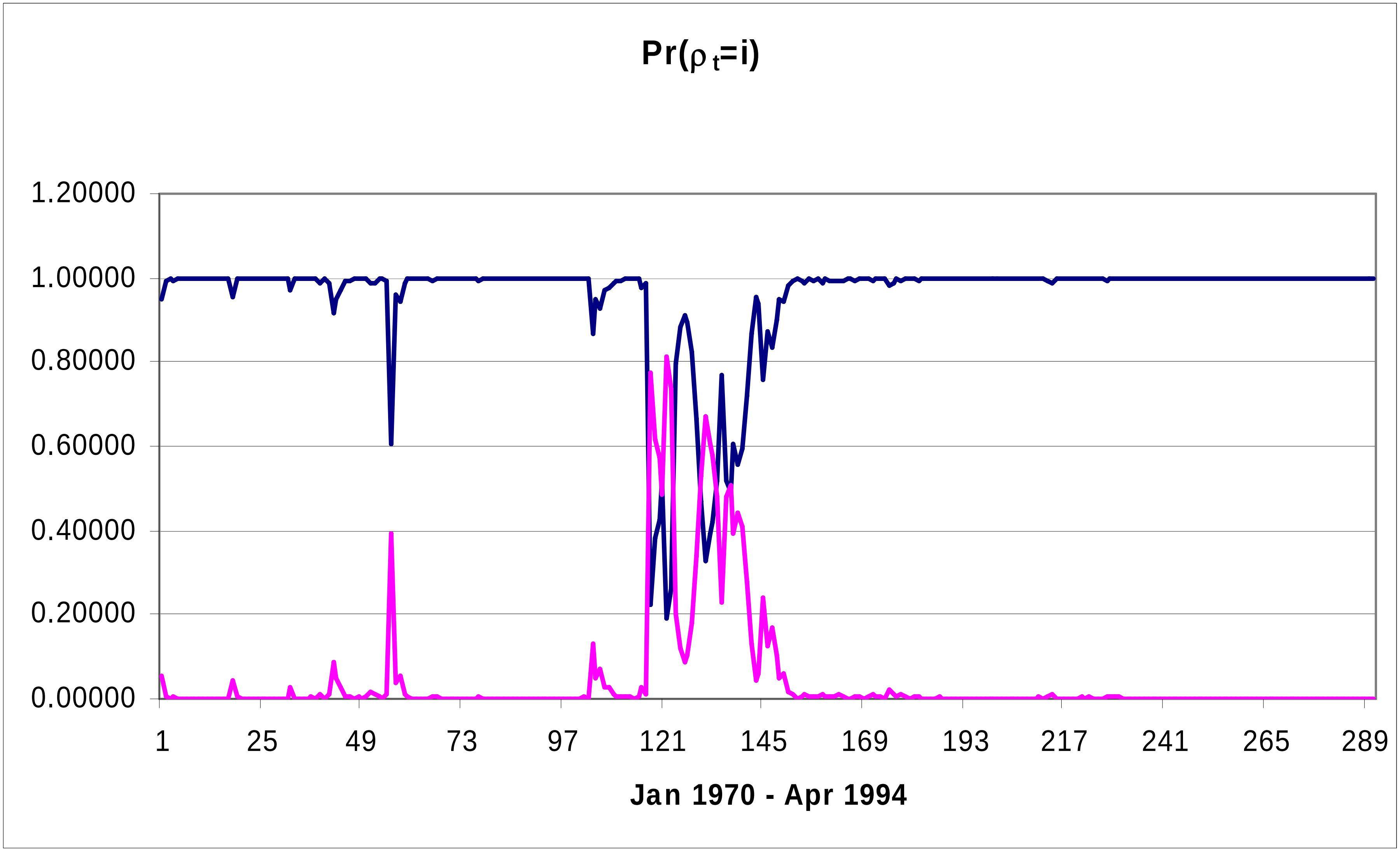
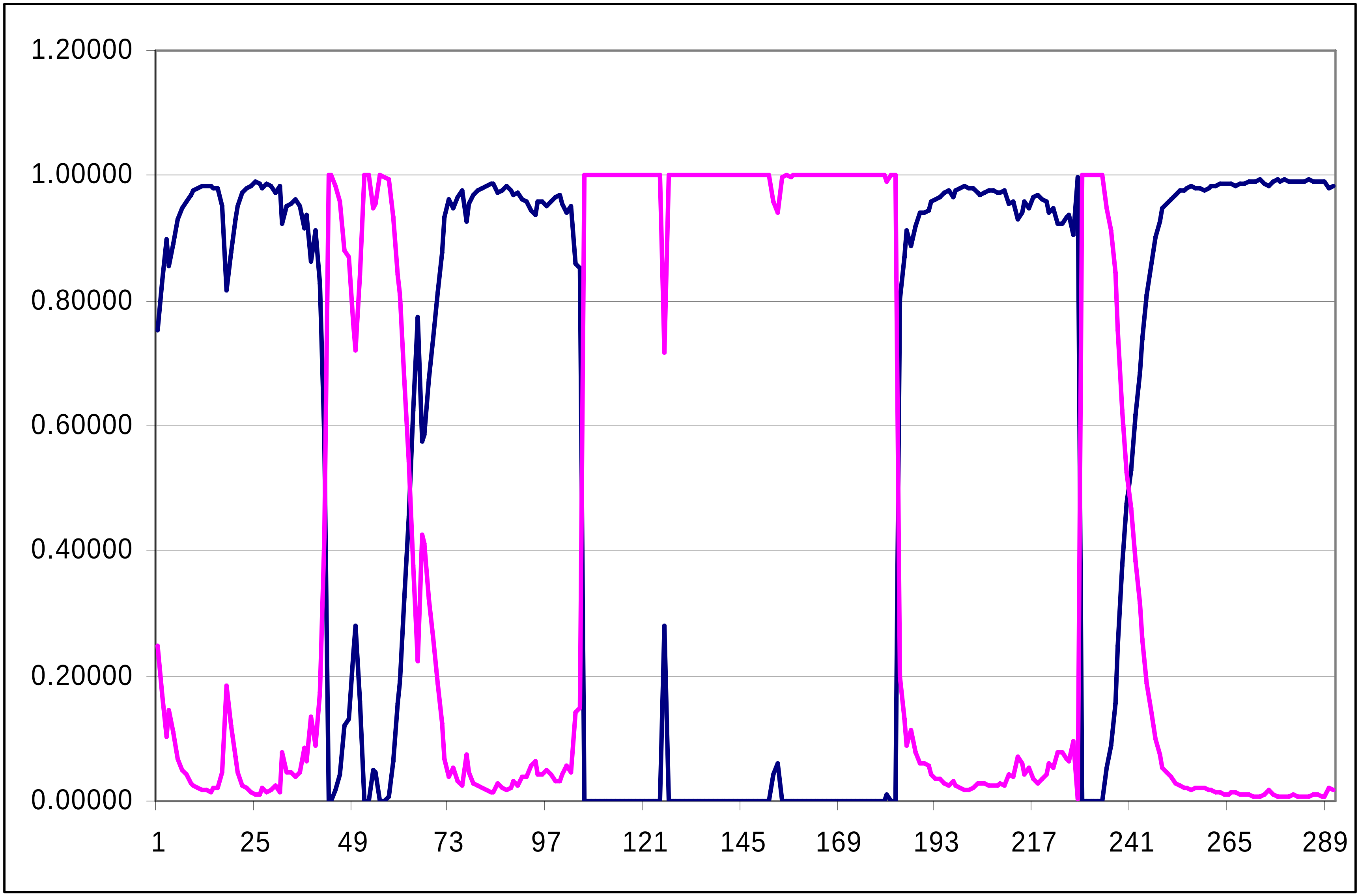
Generalize RS

US 1-month T-bills



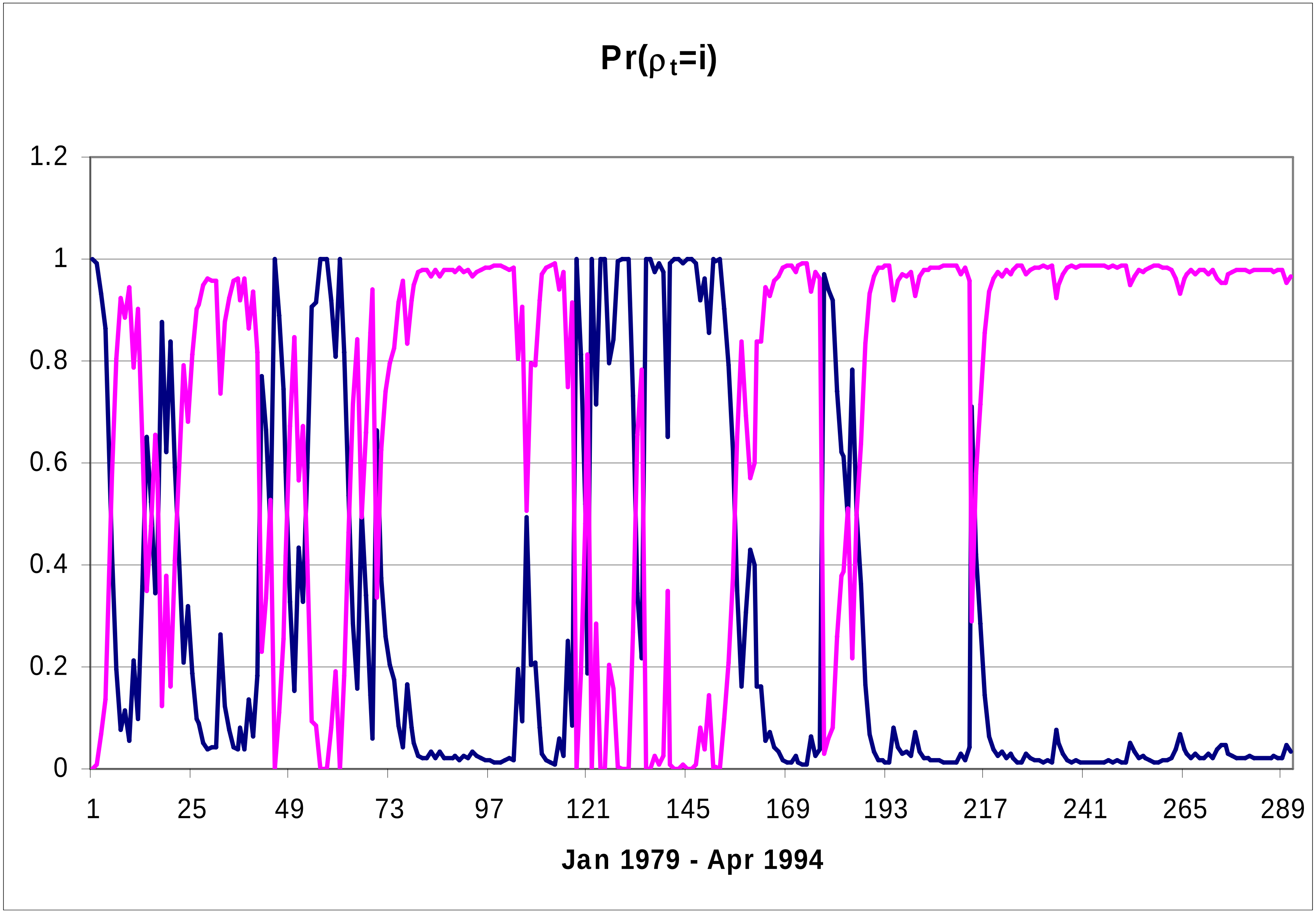
RS-CIR (non-central chi-sq)

US 1-month B-bills



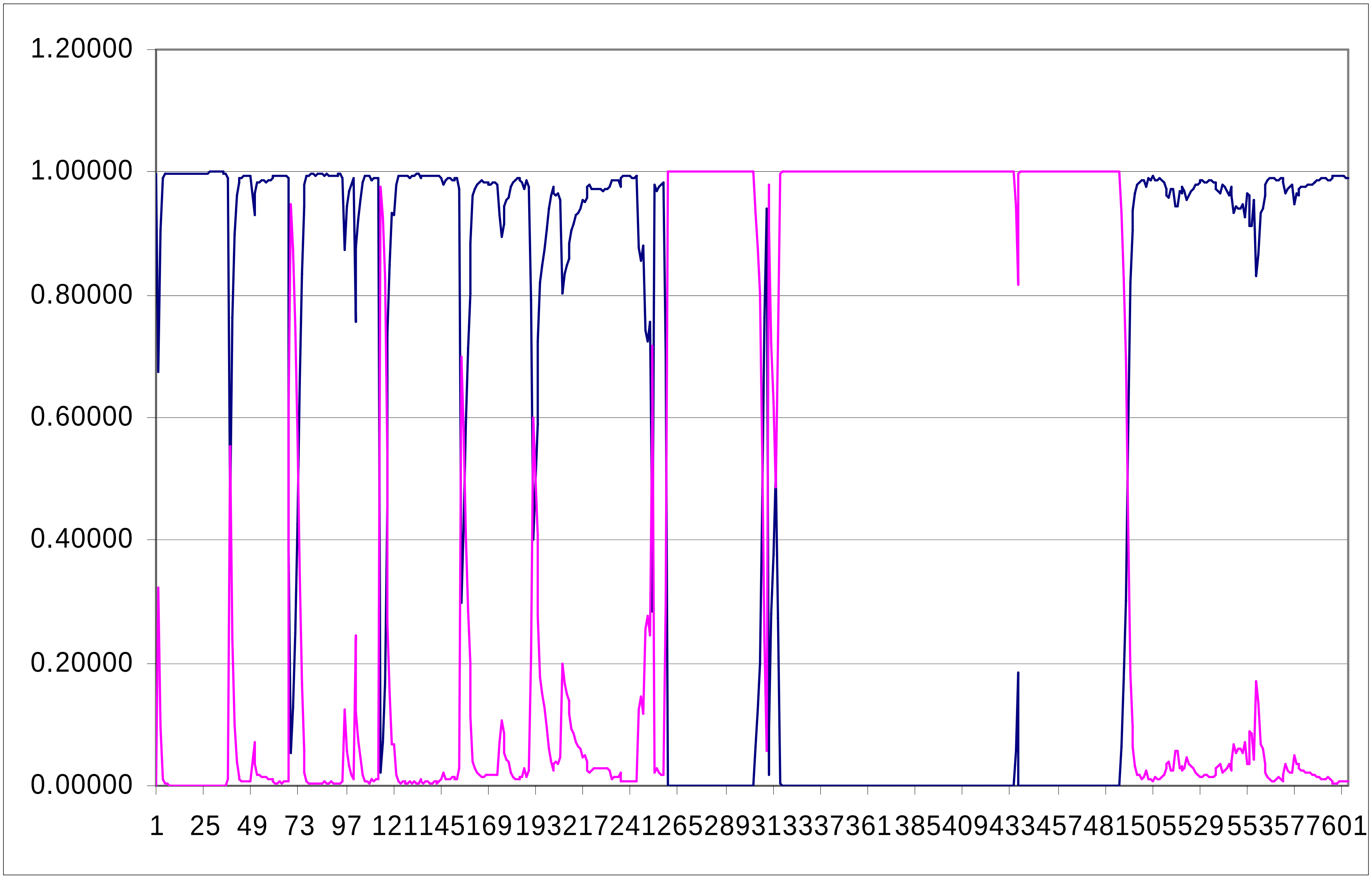
CIR (non-central chi-sq) & LN

US 1-month T-bills



RS-CIR (non-central chi-sq)

UK 1-month B-bills



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UK 3-month T-bills