Predictive Analytics and Machine Learning – Practical Applications for Actuarial Modeling (Nested Stochastic) - Addendum

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Practical Applications for Actuarial Modeling (Nested Stochastic) - Addendum

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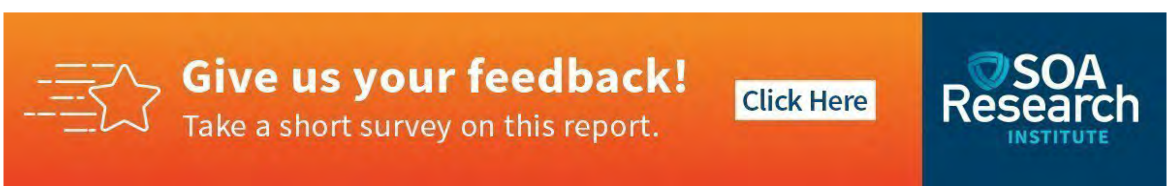

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Predictive Analytics and Machine Learning

Practical Applications for Actuarial Modeling (Nested Stochastic) - Addendum

Foreword

This document reports on the third case study following the principles laid out in the report Predictive Analytics and Machine Learning – Practical Applications for Actuarial Modeling (Nested Stochastic) published in May of 2023 by the SOA Research Institute, authored by the same team and supported by the same individuals in the project oversight group.

Section 1: Case Study 3 - Variable Annuity Capital

1.1 INTRODUCTION

In this section, we explore the use of AIML to identify scenarios most likely to be above a certain percentile for the calculation of the Value at Risk (VaR) and Conditional Tail Expectation (CTE) for variable annuities under VM-21 (NAIC, 2021)[1].

This case study uses the same variable annuity (VA) products as case study 2 where we developed an AIML algorithm to identify which inner loop scenarios should be ran to calculate the capital at future projection points. This case study assumes a VA pricing context with pricing based on the statutory balance sheet.

Note that the methodology explored in this section is not limited to variable annuities and could be applied to other products and use cases that require the calculation of a tail measure such as VaR and CTE.

We will focus on CTE for this paper.

1.1.1 BACKGROUND

Valuation and Capital Requirements of Variable Annuities

VM-21 was effective as of January 1, 2020 and applies to all VA contracts issued on or after January 1, 1981. VM-21 replaced AG43 as the standard for VA statutory reserves. For VA capital requirement, C-3 Phase II was updated to be consistent with VM-21.

There are two components of VM-21:

- The first one is the stochastic reserve, which is based on conditional tail expectation 70 (CTE 70). CTE 70 is defined as the average over the worst 30% of the scenario reserves. Liability assumptions are based on a company's prudent estimate assumptions. Asset assumptions are mostly prescribed to include default rates and credit spreads. Reinvestment strategies should be modeled in alignment with company investment policies but subject to guardrails. Treatment of hedges depends on whether companies have

[1] VM-21 (NAIC, 2021) is section 21 of the NAIC valuation manual, which specifies requirements for principles-based reserves and capital requirements for variable annuities contracts.
NAIC. Valuation Manual. NAIC.org 2021. URL: https://content.naic.org/sites/default/files/pbr_data_valuation_manual_2021_edition.pdf

Future Hedging Strategy (FHS) supporting contracts covered under VM-21.

- The other component is the standard projection amount, which is similar to the stochastic reserve calculation but with certain assumptions prescribed, and is used as a floor in VM-21.

The stochastic amount is based on the distribution of Greatest Present Value of Accumulated Deficiencies (GPVAD). The actuarial model needs to produce first principal cashflow projections on a stochastic basis, and those cash flows are used to calculate GPVAD with and without consideration for future hedging. CTE "best efforts" must account for hedging if the company has a FHS. CTE "adjusted" is produced without any future hedges (existing hedges will run off). The stochastic amount is the weighted average of the CTE "best efforts" and CTE "adjusted," and the weighting is a function of how robust the hedging program is. For reserves, CTE 70 is used to compare against the standard amount to determine the final reserve. For the capital perspective, CTE 98 is used to compare against the standard projection amount.

In this section, our objective is to develop an AIML model that can proxy the VA capital calculation (CTE 98) for the VA products introduced in case study 2.

Runtime Challenge for Variable Annuity Capital

The calculation of VA reserves and capital is cumbersome as it requires running an entire VA block, its supporting assets and reinvestment through 1,000 stochastic scenarios. VA reserves and capital are particularly onerous to calculate in a projected or nested-stochastic setting.

Actuaries often rely on scenario reduction (e.g., running 200 instead of 1,000 scenarios) and/or policy clustering (grouping policies into clusters) to help manage this runtime. However, these techniques may affect the accuracy of the calculation and/or not substantially reduce the runtime.

Solution Proposed by this Case Study

In this case study, we develop an AIML model that can indicate either (A) the rank of any inner loop scenario or (B) whether a given inner loop scenario is likely in the top X% GPVAD.

This model would be valuable for the following reasons:

1. While it would not provide the capital value directly as we did for fair value in case studies 1 and 2, it would allow reducing the number of inner loop scenarios, which would directly reduce runtime.
2. This technique should substantially reduce the amount of training data needed to achieve a desired accuracy relative to a direct proxy model. This is due to being able to generate more data points, all things being equal, as each inner loop scenario would now be a training record.

We will limit this case study to a model for VA capital at CTE 98%.

1.2 ACTUARIAL METHODOLOGY SPECIFICATIONS

Valuation Methodology

For the VA capital perspective, we are focusing on CTE 98 in this section. CTE 98 is calculated as the average over the worst 2% of the scenario results. The CTE in this section reflects the following:

- **Product features** will be reflected as-is, as in most standard actuarial projections. Product features were introduced in the second case study and further detail can be found in appendix A in the original paper.
- **Prudent best-estimate actuarial assumptions** will be used to project the cash flows of the variable annuity product. The actuarial assumptions are detailed further down in this section.

Note that we used a representative, but small number of pricing cells to manage actuarial model runtime.

Scenarios Used to Calculate Capital

We used the SOA scenario generator with a **fixed seed** to calculate the capital. The scenario generator can be found in section 6.3.2 in the original paper.

Using a fixed seed for each capital calculation is critical for this application. The fixed seed result is the same underlying random numbers being generated in the inner loop calculations, resulting in each inner loop scenario ID having a similar shape or pattern, which then allows us to apply the proposed solution for this case study. This solution would not work without a fixed seed.

Best-estimate Actuarial Assumptions

The same assumption was used as the second case study. The 2015 VBT mortality table was used for the mortality assumption, a simple dynamic function was used for lapse, and stochastic withdrawal paths were used as actuarial assumptions.

Readers can refer to appendix B in the original paper for detailed information on the best-estimate assumptions.

1.3 AIML MODEL DEVELOPMENT

Overview

This section details the methodology used to calibrate and test AIML models to identify the tail scenarios for calculating VA capital. We designed an approach to address the problem of VA (variable annuity) capital using the use cases discussed in the first part of the paper. We explore two approaches:

1. Train a model to predict the rank directly through regression; and
2. Identify the worst 5% of scenarios through classification.

The following table summarizes the steps used for this case study and associated references from the subsections below.

Table 1

SUMMARY OF STEPS AND REFERENCES FOR THE CASE STUDY

#	Step	Paper Section	Description
1	Preparation	1.3.1. Preparation	Prepare the environment, load external packages and define the various functions that will support the steps below.
2	Data Generation	1.3.2. Data Generation	Define methodology to generate sample results to train and test the AIML models for the VA Capital case study.
3	Feature Engineering and Selection	1.3.3. Feature Engineering, Feature Selection and Output Definition	Develop additional inputs for the AIML model derived from the original VA Capital inputs. Perform data exploration and feature importance analysis based on data generation with added feature engineering to identify which features should be used as inputs to the AIML model.
4	Model Testing and Selection	1.3.4. Model Development, Testing and Selection	Test performance of various AIML models. Test and control for overfitting. Select the top model based on analyzing performance of sample data and judgment.

The methodology to develop such AIML models generally requires iterating between the steps outlined above. In this section, we focus on the final methodology and analysis that was developed by the researchers.

With the problem statement defined, we designed two different approaches to achieve the goal of predicting scenarios in the CTE 98, i.e., 2% of the tail scenarios. Having the case study set up with 250 inner loop scenarios (we used 250 instead of 1,000 to manage runtime) translates to predict the five top-ranking scenarios given a block of business and the corresponding outer loop scenarios.

One utilizes the classification method, while the other utilizes the regression method. The classification method considers the model to be a binary problem (in the top five or not), whereas the regression method predicts the rank directly.

[The two methods utilized to obtain results for the model are explained below:

1. Identifying 2% scenarios through classification:
    - The GPVAD rank variable is binary transformed to classify the top five of ranked scenarios, thereby facilitating the creation of a model that identifies the top 2% of ranked scenarios
2. Direct rank through regression:
    - The GPVAD rank is directly predicted in the model using the top 5% ranked scenarios. To provide confidence in the results, the top 2% of scenarios are identified from the predicted GPVAD rank above.]

The rest of this section walks the reader through each step highlighted above.

1.3.1. PREPARATION

The first step consists of preparing the environment by loading any external packages and defining the user functions and user inputs.

Python was used to develop the AIML model and produce the analysis provided in this section. We used the following Python packages:

- **NumPy:** Adds support for large, multi-dimensional arrays and matrices, along with a large collection of high-level mathematical functions to operate on these arrays.
- **Pandas:** Used for data transformation and analysis.
- **Matplotlib, Sweetviz and seaborn:** Used to produce various visualizations.
- **Scikit-learn (sklearn) and Keras:** Both of these packages provide a library of machine learning models.

1.3.2. DATA GENERATION

Overview and General Methodology

We used the same economic scenarios to forecast the liabilities over a period of 30 years as the second case study and then performed capital calculations along those paths to generate data.

The steps are outlined in the following table:

Table 2
SUMMARY OF STEPS TO GENERATE DATA

#	Step	Description
1	Economic scenario generator selection	Similar to the second case study, a real-world economic scenario generator (ESG) was used to generate the economic scenarios. Again, we used the Academy Interest Rate Generator (AIRG) (SOA, 2022).
2	Scenario selection	Similar to the second case study, we used a scenario selection technique to convert the original market distributions into a more uniform distribution. Otherwise, the model would likely prioritize the fit towards the mode of the distribution at the expense of poor fit in the tail. This is important as we want the model to perform as well in the tail as in the center of the distribution.
3	New business cell selection	Given this case study is in a pricing context, we used pricing cells to model the liability. The cells are split by gender, age bracket and product (one GMDB and one GMWB). We opted to model both products together to avoid having to duplicate the exercise for both products.
4	Generation	For each scenario and business cell selection combination, perform an actuarial projection of each new business cell and aggregate. Along that projection, produce results for the pivot points identified. For this use case, we used at issue, every year for the first five years, and then every five years from there on out until year 30.

Once the data is generated, we separated it between training and testing. The separation is done by outer projection scenario ID (not inner scenarios used for the GPVAD and CTE calculation).

1.3.3 FEATURE ENGINEERING, FEATURE SELECTION AND OUTPUT DEFINITION

Feature Engineering and Feature Selection

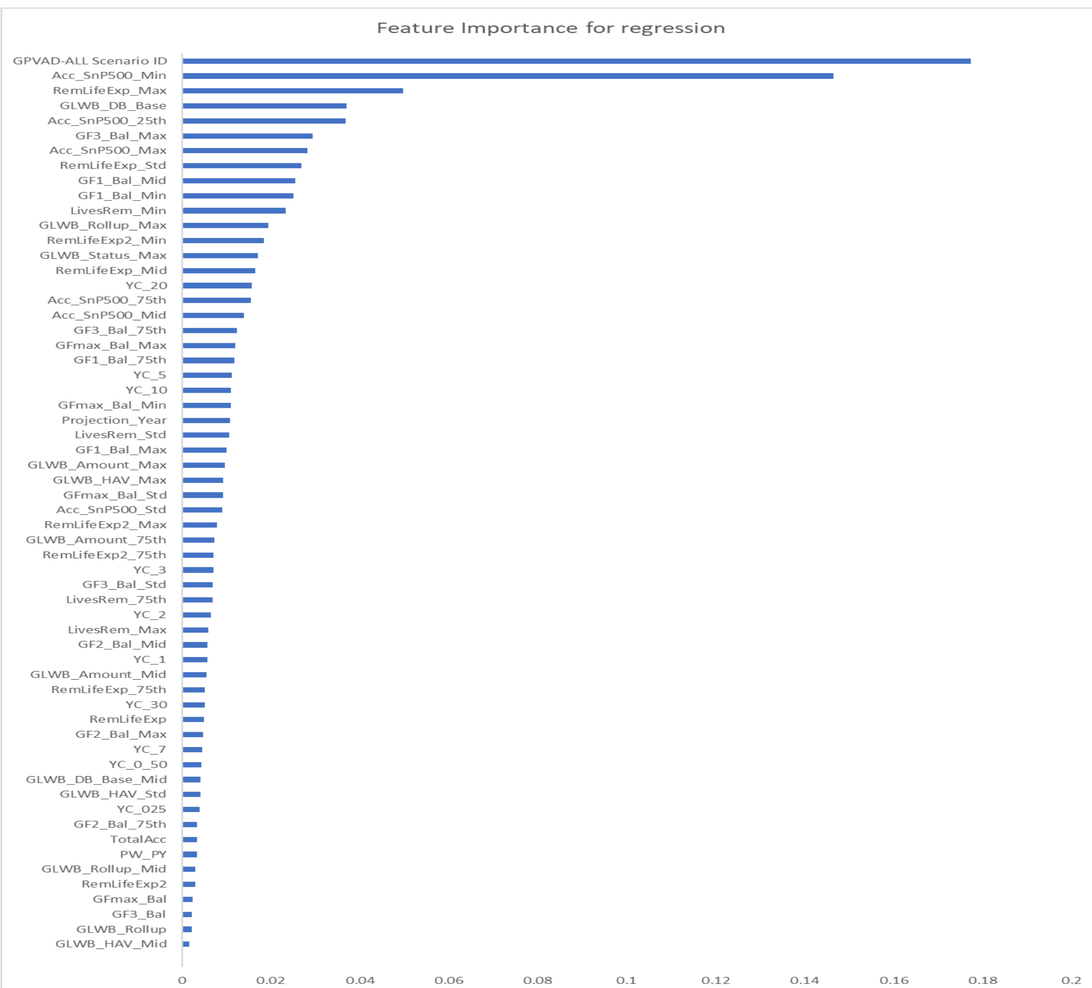
A set of potential features are selected, which combines the inputs with engineered additional features. Various additional characteristics were derived from policy-level calculations to represent the characteristics of aggregated VA blocks and summarized.

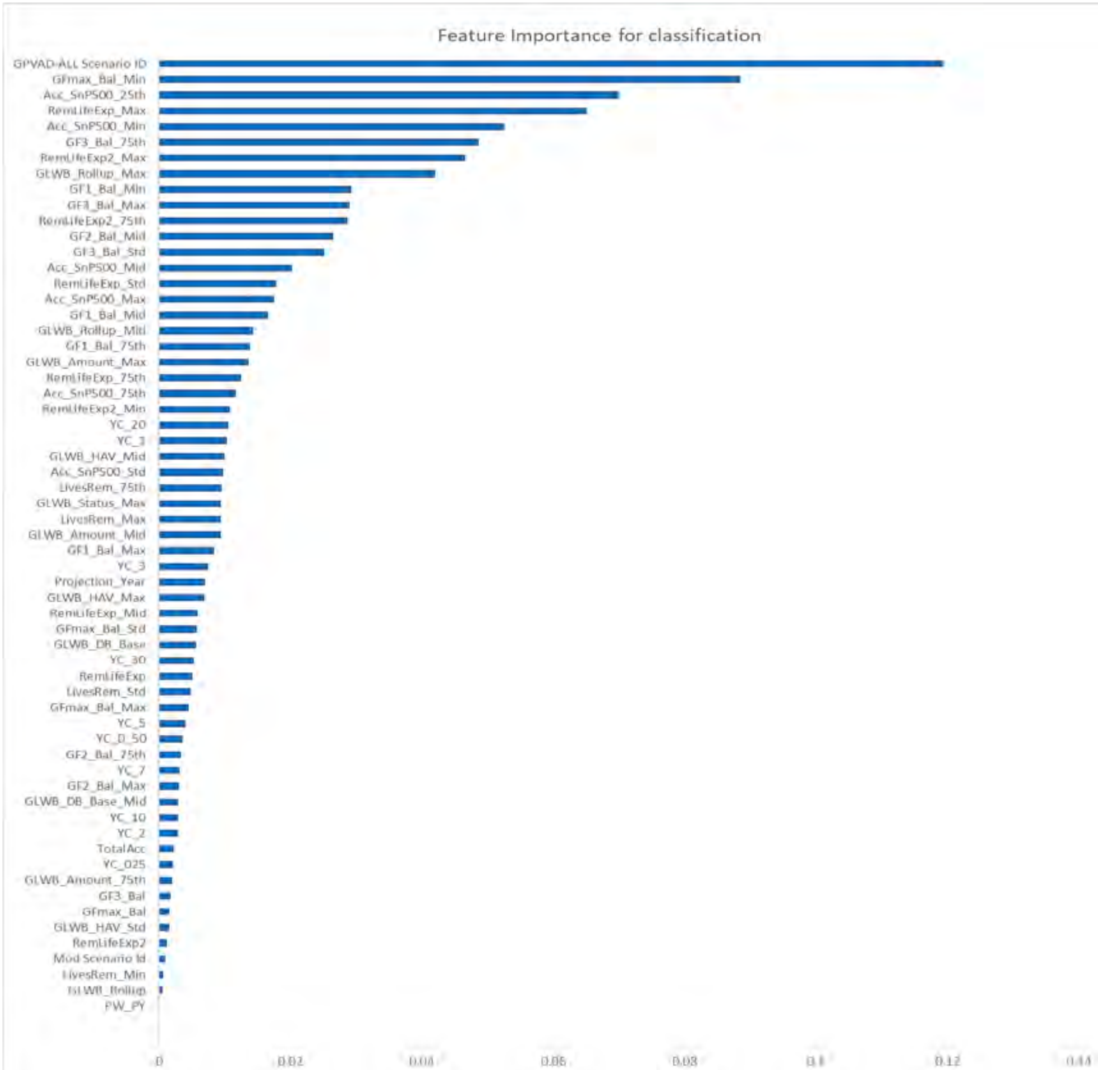
Appendix A in this paper contains a table that summarizes the complete set of potential features used for feature engineering.

We used XGBoost to develop a view of the relative importance of each feature candidate. When XGBoost was implemented, the random function used the default value of seed to minimize the fluctuation in the ranking variation of the features. Seed value remained unchanged across all model results for consistency.

The tables of feature importance for both methodologies are provided below:

Figure 1
RANKING SUMMARY TABLE





Other features with an importance score of zero were eliminated, which resulted in 61 features for both tables above.

According to the preceding table for regression, the GPVAD-ALL Scenario ID (inner loop scenarios) and Acc_SnP_Min (minimum account value of S&P) have the greatest impact on the rank and, according to the preceding table for classification, the GPVAD-ALL Scenario ID (inner loop scenarios) and GFmax_Bal_Min (maximum guaranteed fund balance at minimum) have the greatest impact on the rank. However, as the illustration shows, many of the features showed insignificant predictive power to target. Therefore, we performed further analysis based on a feature importance score above 25bps.

See appendix B for the feature set summary table after selecting the most high-ranking features from the analysis above.

1.3.4 MODEL DEVELOPMENT, TESTING AND SELECTION

In order to generate a training data point, we would have to perform a full VA capital calculation. Given those can take hours to complete and we would likely have to generate many thousands, if not hundreds of thousands of training points to achieve a reliable proxy, the runtime and associated cost would be prohibitive.

Here, instead of predicting capital values, we train the AIML model to predict the ranking of given inner loop scenarios based on the attributes of the blocks.

The purpose of this alternative approach is twofold:

1. More lenient construct for the margin of error between actual and predicted rankings: By focusing on the ranking of scenarios rather than the precise capital values, we allow for a more flexible margin of error in the model's predictions. This flexibility can be beneficial in practice, considering the complexity and uncertainty associated with variable annuity capital calculations.

2. More efficient training data cost: Since the model is trained to predict rankings rather than capital values, it requires less data for training. This reduces the overall cost of data generation and allows for more efficient use of computational resources.

By adopting this alternative problem statement, we aim to address the limitations of the conventional approach and provide a more cost-effective and insightful solution for predicting variable annuity capital values.

With the features and model outputs identified, we calibrated three commonly used models for predicting variable annuity capital values. The three models we selected for calibration are as follows:

1. Multivariate Regression: We employed a multivariate regression model to capture the linear relationships between the input features and the target variable, which is the variable annuity capital value. This model assumes a linear relationship between the features and the target and estimates the coefficients for each feature through regression analysis.

2. XGBoost: This is an optimized gradient boosting algorithm that excels in handling complex non-linear relationships and capturing interactions among features. We utilized XGBoost to build a boosted ensemble model that combines multiple decision trees to predict the variable annuity capital value. XGBoost's ability to handle non-linear relationships can help capture more complex patterns in the data.

3. Neural Network: We employed a neural network model, specifically a feedforward neural network, to capture intricate non-linear relationships between the input features and the target variable. Neural networks are capable of learning complex patterns and relationships in the data by utilizing multiple layers of interconnected neurons. This model architecture allows for the extraction of high-level features and nonlinear transformations, potentially leading to improved accuracy.

Output Definition

The table below summarizes the outputs that were considered for both the classification and regression models:

Table 3
SUMMARY OF OUTPUTS FOR CLASSIFICATION OUTPUT

Variable	Source and Description
Top 5% Ranked Scenarios	**Engineered Output** Considered top 5% ranked scenarios as 1 and others ranked scenarios as 0

The classification output is scaled and selected as the model output for the following reasons:

1. Classification output reduces the variance between actual rank and predicted rank as values are considered to be 1 or 0, which helps in identifying the prediction accuracy better.

2. Top 5% are selected to identify the prediction accuracy in the tail (which has significance) instead of the whole set of ranks, and also help to reduce the variance of middle-ranked scenarios, which are subjective.
3. Therefore, the engineered output "Top 5% Classification Output" was selected for this case study.

Table 4

SUMMARY OF OUTPUTS FOR REGRESSION OUTPUT

Variable	Source and Description
GPVAD Rank	**Capital Pricing Output** GPVAD Scenario ranked from 1 to 250

The regression output is scaled and selected as the model output for the following reasons:

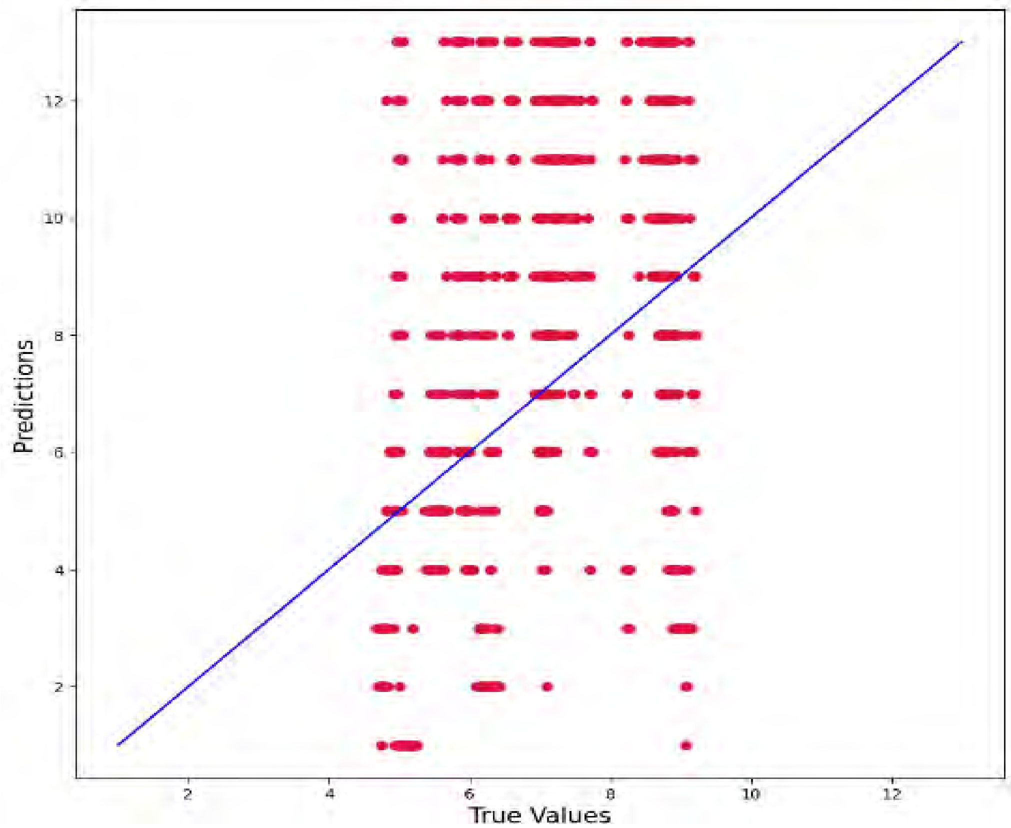
1. GPVAD Rank provides direct and very accurate proxy to actuarial calculations, which can help us identify the rank of each scenario. GPVAD Rank can be used to identify the top 5% ranked scenarios to measure the prediction accuracy in the tail instead of the whole set of ranks, and also help to reduce the variance of middle-ranked scenarios, which are generally subjective.
2. Therefore, the engineered output "GPVAD Rank" was selected for this case study.

Multivariate Regression

For the regression output model, we began by testing a multivariate regression model using feature set. This model achieved a R^2 of 15.76% and a mean absolute error (MAE) of 2.88. This is clearly expected as the tail scenarios cannot be predicted with a linear relationship.

The graph below illustrates the actual against predicted across the test cases:

Figure 2
ACTUAL TO PREDICTED GRAPH



As shown in the graph above, the findings of Multivariate Linear Regression (MLR) are inconclusive, hence alternative models should be used. Because the prediction value is a binary classification (0,1) and MLR models are implemented on continuous target values, the classification output model was not implemented for MLR. However, it will be considered for the XGBoost.

XGBoost

We then tested the performance of XGBoost with feature set selected. This model is often a prime candidate for proxy actuarial models. Again, we used the feature list established in the previous step and the same training and test data. We implemented XGBoost for both the classification and regression output models. The regression output model was hyper parametrized before the classification transformation.

XGBoost produced a much better proxy model than the multivariate regression and neural network.

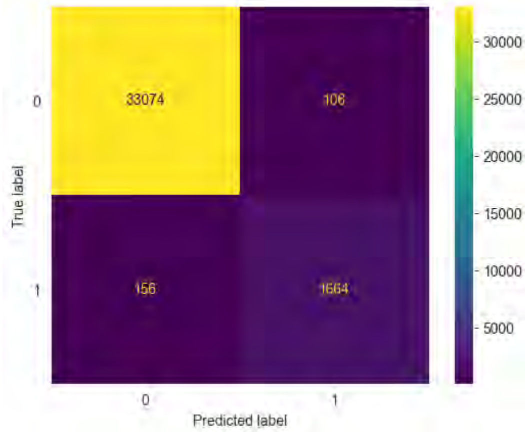
When comparing against previous case studies, a slightly different approach was considered. A confusion matrix was used to compare the model results. To have an apples-to-apples comparison between the two models, the regression output model was transformed to classification to calculate the accuracy and precision scores.

When training the model, we incorporated the 5% tail scenario to account for model margin of error.

The graph (confusion matrix) below illustrates the actual against predicted across the test cases for the top 5% ranked scenarios:

Figure 3

CONFUSION MATRIX OF TOP 5% SCENARIOS – CLASSIFICATION METHOD



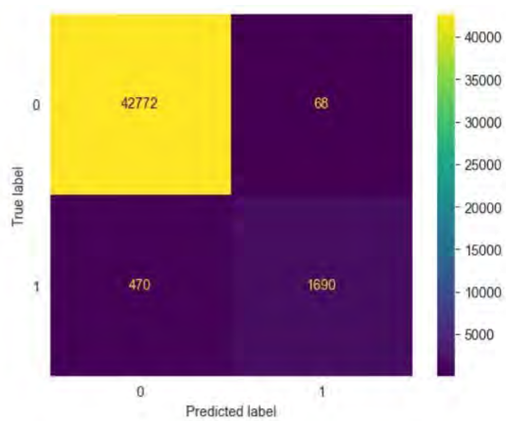
The accuracy and precision scores of the classification output model are as follows:

$$Accuracy\ score\ of\ xgboost\ classification\ model = \frac{33074 + 1664}{35000} = 99.14\%$$

$$Precision\ score\ of\ xgboost\ classification\ model = \frac{1664}{1664 + 156} = 91.42\%$$

Figure 4

CONFUSION MATRIX OF TOP 5% SCENARIOS – REGRESSION METHOD



The accuracy and precision scores of the regression output (binary transformation) model are as follows:

$$Accuracy\ score\ of\ xgboost\ regression\ model = \frac{42772 + 1690}{45000} = 98.82\%$$

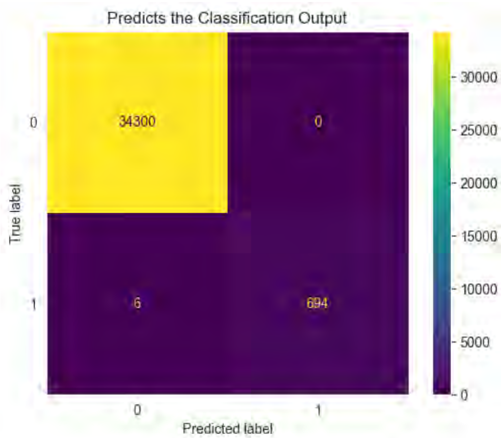
$$Precision\ score\ of\ xgboost\ regression\ model = \frac{1690}{1690 + 470} = 78.24\%$$

As illustrated above from the accuracy and precision scores, the classification output model performed slightly better than the regression output model. To improve the model's capabilities and provide more context for the model's significance with respect to actuarial applications, the top 2% of scenarios in the top 5% scenario-ranked model are examined. For analyzing the top 2% ranked scenarios, instead of accuracy and precision scores, an actuarial opinion-based formula is used to identify the model's performance.

The graph (confusion matrix) below illustrates the actual against predicted across the test cases for the top 2% ranked scenarios :

Figure 5

CONFUSION MATRIX OF TOP 2% SCENARIOS – CLASSIFICATION METHOD

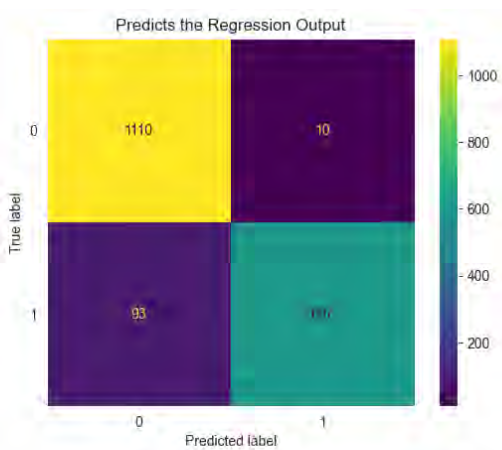


The below calculation is for classification output:

$$\frac{Top\ 2\%\ Scenarios\ accurately\ predicted\ in\ the\ top\ 5\%\ prediction\ model}{Total\ 2\%\ Scenarios} = \frac{694}{700} = \mathbf{99.14\%}$$

Figure 6

CONFUSION MATRIX OF TOP 2% SCENARIOS – REGRESSION METHOD



The below calculation is for regression output:

$$\frac{Top\ 2\%\ Scenarios\ accurately\ predicted\ in\ the\ top\ 5\%\ prediction\ model}{Total\ 2\%\ Scenarios} = \frac{607}{700} = \mathbf{86.71\%}$$

In the graph above, the classification output model significantly outperformed the regression output model. The inclusion of non-top ranked scenarios in the classification output model that is considered as negative input (classification value of 0) added complexity to the training data, allowing it to surpass the regression output model.

The following graph illustrates the distribution of the predictions made against the actual prediction for the classification and regression output:

Figure 7

DISTRIBUTION OF PREDICTIONS OUTPUT VS ACTUAL OUTPUT FOR CLASSIFICATION AND REGRESSION OUTPUT

As illustrated in the above graphs, classification output (top graph) has better overlapping between the actual and predicted values than regression output (bottom graph). The regression output distribution shows that the variance between actual and predicted values is greater for 10 to 12 ranked scenarios and less significant for 13 to 15 ranked scenarios. The bump on the top and slight increase in the width on the right side for the regression output model reduces the predicted value of the top 2 to 5 ranked scenarios.

The XGBoost model with classification output appears to be performing adequately as a proxy for the VA Capital. However, we decided to continue testing additional models to see if we could further refine the performance of the model.

Neural Network

The final model we tested was a neural network (NN). The model chosen was a three-layer Dense NN. The first layer is an input layer with 100 nodes, with one hidden layer of 200 nodes and an output layer of one node. The NN was optimized by calculating weights that minimize the mean squared error between the observed and estimated GPVAD Scenario Rank. For training, the data was split into 100 batches and each batch was passed through the nodes 100 times.

We tested the model and all the Neural Network (NN) results were inconclusive and had a low R^2 value of 2.27%. As a result of the NN model's low performance in terms of R^2 value, no further testing was performed.

Model Comparison

Below is a summary of the results of the models tested and formulas used to compare them:

Accuracy Score: The proportion of true predictions to the total number of predictions. For this scenario, the accuracy score will be the model's prediction accuracy in correctly predicting the Top 5% ranked and non-Top 5% ranked values. Following is the formula for accuracy score:

$$\frac{Top\ 5\%\ scenarios\ ranked\ prediction\ rightly + Non\ - Top\ 5\%\ scenarios\ ranked\ prediction\ rightly}{Total\ scenarios\ ranked\ prediction}$$

Precision Score: The proportion of positive true predictions to the total number of positive predictions. For this scenario, a positive feature is predicting Top 5% ranked values right as this value is considered to be of significant importance. Following is the formula for precision score:

$$\frac{Top\ 5\%\ scenarios\ ranked\ prediction\ rightly}{Total\ Top\ 5\%\ scenarios\ ranked\ prediction}$$

Formula Engineered: The formula considering the top 2% scenarios predicting right in the top 5% ranked model. Following is the formula:

$$\frac{Top\ 2\%\ scenarios\ accurately\ predicted\ in\ the\ top\ 5\%\ prediction\ model}{Total\ Top\ 2\%\ scenarios}$$

Coefficient of Determination (R^2): The proportion of the variance in the target variable predicted by the independent variables. For this scenario, GPVAD Rank is considered the target variable and independent variables are described in appendix B “Summary of Feature Set.”

Table 5
SUMMARY OF THE RESULTS (MODEL PREDICTION)

Model	Method	Accuracy Score	Precision Score	Formula Engineered	R^2
Multiple Least-Squares Regression (MLS)	Regression	N/A	N/A	N/A	15.76%
XGBoost	Classification	99.14%	91.42%	99.14%	N/A
XGBoost	Regression	98.82%	78.24%	86.71%	92.20%
Neural Network	Regression	N/A	N/A	N/A	2.27%

As illustrated above, XGBoost (classification output model) performed better than all the other models in the table and, therefore, will be selected to evaluate the model performance against the actuarial method for ranking scenarios.

1.4 CONCLUSION

This paper demonstrates that AIML can be a useful tool to identify which scenarios are most likely to contribute to the tail in a CTE calculation.

While case studies 1 and 2 demonstrated that AIML can be used as a direct and very accurate proxy to actuarial calculations, it may not always be possible to develop such proxies when the computing cost to generate the training data to achieve the desired accuracy is too significant.

In those cases, actuaries may still be able to restructure the problem and find significant runtime reduction as we did in this case study. Actuaries may find many opportunities to design solutions where AIML is a component of the solution like we did here.

Lastly, we remind readers that there were important assumptions made in this case study; in particular, we assumed that the use case used a seed for the stochastic simulation. The design used in this case study would not be feasible without the use of a seed.

Section 2: Acknowledgments

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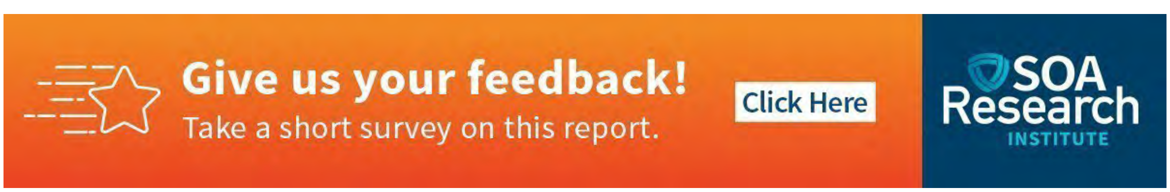

Appendix A: Summary of Potential Features

Variable	Source and Description	Candidate?
Account Value of S&P	Information	No *Information only*
Account Value of S&P at 25th percentile	Engineered feature	Yes
Account Value of S&P at 75th percentile	Engineered feature	Yes
Maximum Account Value of S&P	Engineered feature	Yes
Account Value of S&P at 50th Percentile	Engineered feature	Yes
Minimum Account Value of S&P	Engineered feature	Yes
Account Value of S&P standard deviation	Engineered feature	Yes
Guaranteed Fund 1 balance	Information	No *Information only*
Guaranteed Fund 1 balance at 75th Percentile	Engineered feature	Yes
Maximum Guaranteed Fund 1 balance	Engineered feature	Yes
Guaranteed Fund 1 balance at 50th Percentile	Engineered feature	Yes
Guaranteed Fund 1 balance minimum	Engineered feature	Yes
Guaranteed Fund 1 balance	Information	No *Information only*
Guaranteed Fund 2 balance at 75th percentile	Engineered feature	Yes
Maximum Guaranteed Fund 2 balance	Engineered feature	Yes
Guaranteed Fund 2 balance 50th percentile	Engineered feature	Yes
Guaranteed Fund 3 balance	Capital Pricing input	Yes
Guaranteed Fund 3 balance at 75th Percentile	Engineered feature	Yes
Maximum Guaranteed Fund 3 balance	Engineered feature	Yes
Guaranteed Fund 3 balance standard deviation	Engineered feature	Yes
Maximum Guaranteed Fund balance	Engineered feature	Yes
Maximum Guaranteed Fund balance	Engineered feature	Yes
Maximum Guaranteed Fund balance at minimum	Engineered feature	Yes

Variable	Source and Description	Candidate?
Maximum Guaranteed Fund balance standard deviation	Engineered feature	Yes
Yield Curve	Information	No *Information only*
Yield Curve at Time 1	Engineered feature	Yes
Yield Curve at Time 2	Engineered feature	Yes
Yield Curve at Time 3	Engineered feature	Yes
Yield Curve at Time 5	Engineered feature	Yes
Yield Curve at Time 7	Engineered feature	Yes
Yield Curve at Time 10	Engineered feature	Yes
Yield Curve at Time 20	Engineered feature	Yes
Yield Curve at Time 30	Engineered feature	Yes
Yield Curve at Time 50	Engineered feature	Yes
Benefit base (GLWB) – Amount	Information	No *Information only*
Benefit base (GLWB) – Amount at 75th Percentile	Engineered feature	Yes
Maximum Benefit base (GLWB) – Amount	Engineered feature	Yes
Benefit base (GLWB) – Amount at 50th Percentile	Engineered feature	Yes
Benefit base (GLWB) – Death value	Capital Pricing input	Yes
Benefit base (GLWB) – Death value at 50th percentile	Feature engineering	Yes
Maximum Benefit base (GLWB) – Ratchet	Feature engineering	Yes
Benefit base (GLWB) – Ratchet at 50th Percentile	Feature engineering	Yes
Maximum Benefit base (GLWB) – Ratchet Standard Deviation	Feature engineering	Yes
Benefit base (GLWB) – Rollup	Capital Pricing input	Yes
Maximum Benefit base (GLWB) – Rollup	Feature engineering	Yes
Maximum Benefit base (GLWB) – Rollup at 50th Percentile	Feature engineering	Yes
Maximum Benefit base (GLWB) – Status	Feature engineering	Yes

Variable	Source and Description	Candidate?
Inner Loop scenario	Capital Pricing input	Yes
Lives Remaining	Information	No *Information only*
Lives Remaining at 75th Percentile	Feature engineering	Yes
Maximum Lives Remaining	Feature engineering	Yes
Minimum Lives Remaining	Feature engineering	Yes
Lives Remaining Standard Deviation	Feature engineering	Yes
Outer loop Scenario	Information	No *Information only*
Projection Year	Feature engineering	Yes
Remaining Life Expectancy	Capital Pricing input	Yes
Remaining Life Expectancy at 75th percentile	Feature engineering	Yes
Maximum Remaining Life Expectancy	Feature engineering	Yes
Remaining Life Expectancy at 50th percentile	Feature engineering	Yes
Remaining Life Expectancy standard deviation	Feature engineering	Yes
Remaining Life Expectancy 2	Capital Pricing input	Yes
Remaining Life Expectancy 2 at 75th percentile	Feature engineering	Yes
Maximum Remaining Life Expectancy	Feature engineering	Yes
Minimum Remaining Life Expectancy 2	Feature engineering	Yes
Total Account Value	Capital Pricing input	Yes

Appendix B: Summary of Feature Set

Variable	Selected
Account Value of S&P at 25th percentile	No
Account Value of S&P at 75th percentile	No
Maximum Account Value of S&P	No
Account Value of S&P at 50th Percentile	No
Minimum Account Value of S&P	Yes
Account Value of S&P standard deviation	Yes
Guaranteed Fund 1 balance	No
Guaranteed Fund 1 balance at 75th Percentile	Yes
Maximum Guaranteed Fund 1 balance	No
Guaranteed Fund 1 balance at 50th Percentile	Yes
Guaranteed Fund 1 balance minimum	Yes
Maximum Guaranteed Fund 2 balance	Yes
Guaranteed Fund 2 balance 50th percentile	No
Guaranteed Fund 3 balance	No
Guaranteed Fund 3 balance at 75th Percentile	Yes
Maximum Guaranteed Fund balance	No
Maximum Guaranteed Fund balance at minimum	Yes
Yield Curve at Time 1	Yes
Yield Curve at Time 3	Yes
Yield Curve at Time 20	Yes
Benefit base (GLWB) – Amount at 75th Percentile	Yes
Maximum Benefit base (GLWB) – Amount	Yes
Benefit base (GLWB) – Amount at 50th Percentile	Yes
Maximum Benefit base (GLWB) – Ratchet	Yes
Benefit base (GLWB) – Ratchet at 50th Percentile	Yes
Maximum Benefit base (GLWB) – Ratchet Standard Deviation	Yes

Variable	Selected
Benefit base (GLWB) – Rollup	Yes
Maximum Benefit base (GLWB) – Rollup	No
Maximum Benefit base (GLWB) – Rollup at 50th Percentile	Yes
Maximum Benefit base (GLWB) – Status	Yes
Inner Loop scenario	Yes
Lives Remaining	No
Lives Remaining at 75th Percentile	Yes
Maximum Lives Remaining	Yes
Minimum Lives Remaining	Yes
Lives Remaining Standard Deviation	Yes
Outer loop Scenario	No
Projection Year	Yes
Remaining Life Expectancy	No
Remaining Life Expectancy at 75th percentile	No
Maximum Remaining Life Expectancy	No
Remaining Life Expectancy at 50th percentile	No
Remaining Life Expectancy standard deviation	Yes
Remaining Life Expectancy 2	Yes
Maximum Remaining Life Expectancy	No
Minimum Remaining Life Expectancy 2	Yes
Total Account Value	Yes

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