INV 201 Quantitative Finance Formula Package
Nov 2025/March 2026 /July 2026

The exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

In sources where some equations are numbered and others are not, the page number is provided instead.

Options, Futures, and Other Derivatives, Hull, John C., Pearson, 11th Edition
Chapter 5

(5.1) $F_0 = S_0 e^{rT}$

(5.2) $F_0 = (S_0 - I)e^{rT}$

(5.3) $F_0 = S_0 e^{(r-q)T}$

(5.4) $f = (F_0 - K)e^{-rT}$

(5.5) $f = S_0 - Ke^{-rT}$

(5.6) $f = S_0 - I - Ke^{-rT}$

(5.7) $f = S_0 e^{-qT} - Ke^{-rT}$

(5.8) $F_0 = S_0 e^{(r-q)T}$

(5.9) $F_0 = S_0 e^{(r-r_f)T}$

(5.10) $F_0 = S_0 e^{rT}$

(5.11) $F_0 = (S_0 + U)e^{rT}$

(5.12) $F_0 = S_0 e^{(r+u)T}$

(5.13) $F_0 > (S_0 + U)e^{rT}$

(5.14) $F_0 < (S_0 + U)e^{rT}$

(5.15) $F_0 \leq (S_0 + U)e^{rT}$

(5.16) $F_0 \leq S_0 e^{(r+u)T}$

(5.17) $F_0 = S_0 e^{(r+u-y)T}$

(5.18) $F_0 = S_0 e^{cT}$

(5.19) $F_0 = S_0 e^{(c-y)T}$

(5.20) $F_0 = E[S_T]e^{(r-k)T}$

Chapter 11

(11.1) $c \leq S_0 \quad \text{and} \quad C \leq S_0$

(11.2) $P \leq K$

(11.3) $p \leq Ke^{-rT}$

(11.4) $c \geq \max(S_0 - Ke^{-rT}, 0)$

(11.5) $p \geq \max(Ke^{-rT} - S_0, 0)$

(11.6) $c + Ke^{-rT} = p + S_0$

(11.7) $S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$

(11.8) $c \geq \max(S_0 - D - Ke^{-rT}, 0)$

(11.9) $p \geq \max(D + Ke^{-rT} - S_0, 0)$

(11.10) $c + D + Ke^{-rT} = p + S_0$

(11.11) $S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$

Chapter 13

(13.1) $$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

(13.2) $f = e^{-rT}\left[p f_u + (1-p) f_d\right]$

(13.3) $$p = \frac{e^{rT} - d}{u - d}$$

(13.4) $\mathbb{E}[S_T] = S_0 e^{rT}$

(13.5) $f = e^{-r\Delta t}\left[p f_u + (1-p) f_d\right]$

(13.6) $$p = \frac{e^{r\Delta t} - d}{u - d}$$

(13.7) $f_u = e^{-r\Delta t}\left[p f_{uu} + (1-p) f_{ud}\right]$

(13.8) $f_d = e^{-r\Delta t}\left[p f_{ud} + (1-p) f_{dd}\right]$

(13.9) $f = e^{-r\Delta t}\left[p f_u + (1-p) f_d\right]$

(13.10) $f = e^{-2r\Delta t}\left[p^2 f_{uu} + 2p(1-p) f_{ud} + (1-p)^2 f_{dd}\right]$

Chapter 14

(14.1) $$\Delta z = \varepsilon\sqrt{\Delta t}$$

(14.2) $$z(T) - z(0) = \sum_{i=1}^{N} \varepsilon_i \sqrt{\Delta t}$$

(14.3) $$dx = a\,dt + b\,dz$$

(14.4) $$dx = a(x,t)\,dt + b(x,t)\,dz$$

(14.5) $$S_T = S_0 e^{\mu T}$$

(14.6) $$\frac{dS}{S} = \mu\,dt + \sigma\,dz$$

(14.7) $$\frac{\Delta S}{S} = \mu \Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

(14.8) $$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

(14.9) $$\frac{\Delta S}{S} \sim \phi(\mu\Delta t, \sigma^2 \Delta t)$$

(14.10) $$\Delta S = 0.00288S + 0.0416S\varepsilon$$

(14.11) $$dx = a(x,t)\,dt + b(x,t)\,dz$$

(14.12) $$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}b\,dz$$

(14.13) $$dS = \mu S\,dt + \sigma S\,dz$$

(14.14) $$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial G}{\partial S}\sigma S\,dz$$

(14.15) $$F = Se^{r(T-t)}$$

(14.16) $$dF = (\mu - r)F\,dt + \sigma F\,dz$$

(14.17) $$d(\ln S) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\,dz$$

(14.18) $$\ln S_T - \ln S_0 \sim \phi\left[\left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right]$$

(14.19) $$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right]$$

(14.20) $$\mathrm{Corr}(X(t), X(s)) = \frac{0.5\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)}{t^H s^H}$$

Chapter 15

(15.1) $\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma^2 \Delta t)$

(15.2) $\ln\left(\frac{S_T}{S_0}\right) \sim \phi\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)T, \sigma^2 T\right]$

(15.3) $\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \tfrac{1}{2}\sigma^2\right)T, \sigma^2 T\right]$

(15.4) $E[S_T] = S_0 e^{\mu T}$

(15.5) $\text{Var}(S_T) = S_0^2 e^{2\mu T}\left(e^{\sigma^2 T} - 1\right)$

(15.6) $x = \frac{1}{T}\ln\left(\frac{S_T}{S_0}\right)$

(15.7) $x \sim \phi\left(\mu - \frac{1}{2}\sigma^2, \frac{\sigma^2}{T}\right)$

(15.8) $f = S - Ke^{-r(T-t)}$

(15.8) $dS = \mu S\, dt + \sigma S\, dz$

(15.9) $df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S\, dz$

(15.10) $\Delta S = \mu S \Delta t + \sigma S \Delta z$

(15.11) $\Delta f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial f}{\partial S}\sigma S\, \Delta z$

(15.12) $\Pi = -f + \frac{\partial f}{\partial S}S$

(15.13) $\Delta \Pi = -\Delta f + \frac{\partial f}{\partial S}\Delta S$

(15.14) $\Delta \Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t$

(15.15) $\Delta \Pi = r\Pi \Delta t$

(15.16) $\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$

(15.17) $f = Q\left(\frac{S}{H}\right)^{-2r/\sigma^2}$

(15.18) $f = e^{-rT}\hat{\mathbb{E}}[S_T] - Ke^{-rT}$

(15.19) $\hat{\mathbb{E}}[S_T] = S_0 e^{rT}$

(15.20) $c = S_0 N(d_1) - Ke^{-rT}N(d_2)$

(15.21) $p = Ke^{-rT}N(-d_2) - S_0 N(-d_1)$

(15.22) $c = e^{-rT}\hat{\mathbb{E}}[\max(S_T - K, 0)]$

(15.23) $D_n \le K\left(1 - e^{-r(T-t_n)}\right)$

(15.24) $D_n > K\left(1 - e^{-r(T-t_n)}\right)$

(15.25) $D_i \le K\left(1 - e^{-r(t_{i+1}-t_i)}\right)$

Chapter 19

Table 19.6 Greek letters for European options on an asset providing a yield at rate q

Greek Letter	**Call Option**	**Put Option**
Delta	$e^{-qT}N(d_1)$	$e^{-qT}(N(d_1)-1)$
Gamma	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$
Theta	$-\frac{S_0N'(d_1)\sigma e^{-qT}}{2\sqrt{T}}$ $+\ qS_0N(d_1)e^{-qT}-rKe^{-rT}N(d_2)$	$-\frac{S_0N'(d_1)\sigma e^{-qT}}{2\sqrt{T}}$ $-qS_0N(-d_1)e^{-qT}+rKe^{-rT}N(-d_2)$
Vega	$S_0\sqrt{T}N'(d_1)e^{-qT}$	$S_0\sqrt{T}N'(d_1)e^{-qT}$
Rho	$KTe^{-rT}N(d_2)$	$-KTe^{-rT}N(-d_2)$

Chapter 20

(20.1) $p+S_0e^{-qT}=c+Ke^{-rT}$

(20.2) $p_{\text{BS}}-p_{\text{mkt}}=c_{\text{BS}}-c_{\text{mkt}}$

Chapter 27

(27.1) $dS=(r-q)Sdt+\sigma(t)Sdz$

(27.2) $\frac{dS}{S}=(r-q)\,dt+\sqrt{V}\,dz_S$

(27.3) $dV=\alpha(V_L-V)\,dt+\xi V^a\,dz_V$

(27.4) $[\sigma(K,T)]^2=2\cdot\frac{\partial c_{\text{mkt}}/\partial T+q(T)c_{\text{mkt}}+K[r(T)-q(T)]\,\partial c_{\text{mkt}}/\partial K}{K^2\,(\partial^2 c_{\text{mkt}}/\partial K^2)}$

Chapter 28

(28.1) $$\frac{d\theta}{\theta} = m dt + s dz$$

(28.2) $$\Delta f_1 = \mu_1 f_1 \, \Delta t + \sigma_1 f_1 \, \Delta z$$

(28.3) $$\Delta f_2 = \mu_2 f_2 \, \Delta t + \sigma_2 f_2 \, \Delta z$$

(28.4) $$\Pi = \sigma_2 f_2 f_1 - \sigma_1 f_1 f_2$$

(28.5) $$\Delta \Pi = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2) \, \Delta t$$

(28.6) $$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2}$$

(28.7) $$\frac{df}{f} = \mu \, dt + \sigma \, dz$$

(28.8) $$\frac{\mu - r}{\sigma} = \lambda$$

(28.9) $$\mu - r = \lambda \sigma$$

(28.10) $$df = (r + \lambda \sigma) f \, dt + \sigma f \, dz$$

(28.11) $$\frac{d\theta_i}{\theta_i} = m_i dt + s_i dz_i \quad \text{for } i = 1, 2, \ldots, n$$

(28.12) $$\frac{df}{f} = \mu \, dt + \sum_{i=1}^{n} \sigma_i \, dz_i$$

(28.13) $$\mu - r = \sum_{i=1}^{n} \lambda_i \sigma_i$$

(28.14) $$d\left(\frac{f}{g}\right) = (\sigma_f - \sigma_g) \frac{f}{g} \, dz$$

(28.15) $$f_0 = g_0 \mathbb{E}_g \left[\frac{f_T}{g_T} \right]$$

(28.16) $$dg = r g \, dt$$

(28.17) $f_0 = g_0 \,\hat{\mathbb{E}}\left[\frac{f_T}{g_T}\right]$

(28.18) $f_0 = \hat{\mathbb{E}}\left[e^{-\int_0^T r\,dt} f_T\right]$

(28.19) $f_0 = \hat{\mathbb{E}}\left[e^{-rT} f_T\right]$

(28.20) $f_0 = P(0,T)\,\mathbb{E}_T[f_T]$

(28.21) $F = \mathbb{E}_T[u_T]$

(28.22) $\mathbb{E}_{T^*}[R] = F(t)$

(28.23) $s(t) = \frac{V(t)}{A(t)}$

(28.24) $s(t) = \mathbb{E}_A[s(T)]$

(28.25) $V(0) = A(0)\,\mathbb{E}_A\left[\frac{V(T)}{A(T)}\right]$

(28.30) $V_0 = U_0\,\mathbb{E}_U\left[\frac{V_T}{U_T}\right]$

(28.31) $f_0 = U_0\,\mathbb{E}_U\left[\max\left(\frac{V_T}{U_T} - 1, 0\right)\right]$

(28.32) $f_0 = V_0 N(d_1) - U_0 N(d_2)$

(28.33) $a_v = \sum_{i=1}^{n} (\lambda_i^* - \lambda_i)\,\sigma_{v,i}$

(28.34) $a_v = \sum_{i=1}^{n} \sigma_{w,i}\sigma_{v,i}$

(28.35) $a_v = \rho\,\sigma_v \sigma_w$

Chapter 29

(29.5) $L\delta_k \max(R_k - R_K, 0)$

(29.6) $\max\left[cL - \frac{L(1 + R_K d_k)}{1 + R_k d_k}, 0\right]$

(29.7) $L\delta_k P(0, t_{k+1})\,[F_k N(d_1) - R_K N(d_2)]$

(29.8) $L\delta_k P(0, t_{k+1})\,[R_K N(-d_2) - F_k N(-d_1)]$

(29.9) $L\delta_k P(0, t_{k+1}) E_{k+1}\,[\max(R_k - R_K, 0)]$

(29.10) $LA\,[s_F N(d_1) - s_K N(d_2)]$

(29.11) $LA\,[s_K N(-d_2) - s_F N(-d_1)]$

The Volatility Smile, Derman and Miller
Chapter 3

(3.3) $$C(S,t) - P(S,t) = S - Ke^{-r(T-t)}$$

(3.7) $$V(t) = Ie^{-r(T-t)} + \lambda_0 S_t + (\lambda_1 - \lambda_0)C(K_0) + (\lambda_2 - \lambda_1)C(K_1) + \ldots$$

(3.8) $$V(T) = I + \lambda_0 S_T + (\lambda_1 - \lambda_0)(S_T - K_0) + (\lambda_2 - \lambda_1)(S_T - K_1)$$

(3.8) $$V(T) = I + \lambda_0 K_0 + \lambda_1(K_1 - K_0) + \lambda_2(S_T - K_1)$$

(3.9) $$C(S + dS, t + dt) = C(S,t) + \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \cdots$$

(3.11) $$C(S + dS, t + dt) = C(S,t) + \Theta dt + \Delta dS + \frac{1}{2}\Gamma dS^2$$

(3.16) $$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

(3.17) $$\frac{\partial C}{\partial t} + \frac{1}{2}\Gamma\Sigma^2 S^2 = 0$$

Chapter 5

(5.1) $dS = \mu_S S dt + \sigma_S S dZ, \quad dB = Brdt$

(5.2)
$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2 dt$$
$$= \left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}dt + \frac{\partial C}{\partial S}\sigma_S S dZ$$
$$= \mu_C C dt + \sigma_C C dZ$$

(5.3)
$$\mu_C = \frac{1}{C}\left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}, \quad \sigma_C = \frac{S}{C}\frac{\partial C}{\partial S}\sigma_S = \frac{\partial \ln C}{\partial \ln S}\sigma_S$$

(5.10)
$$\frac{(\mu_C - r)}{\sigma_C} = \frac{(\mu_S - r)}{\sigma_S}$$

(5.12)
$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

(5.13)
$$C(S,K,t,T,\sigma,r) = e^{-r(T-t)}[S_F N(d_1) - KN(d_2)], \quad S_F = e^{r(T-t)}S$$
$$d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln\left(\frac{S_F}{K}\right) - \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

(5.19)
$$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)[dS_x - S_x r dx]e^{-rx}$$

(5.20)
$$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)\sigma S_x e^{-rx} dZ_x$$

(5.21) $E[C_0] = E[C_T]e^{-rT}$

(5.22) $\pi(I,R) = V_I - \Delta_R S$

(5.23) $\mathrm{PV}[\mathrm{P\&L}(I,R)] = V(S,\tau,\sigma_R) - V(S,\tau,\Sigma)$

(5.25)
$$\Delta_R = e^{-D\tau}N(d_1), \quad d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \frac{1}{2}\sigma_R^2\tau}{\sigma_R\sqrt{\tau}}, \quad S_F = Se^{(r-D)\tau}$$

(5.27) $d\mathrm{P\&L}(I,R) = dV_I - rV_I dt - \Delta_R[dS - (r-D)Sdt]$

(5.28) $d\mathrm{P\&L}(R,R) = 0 = dV_R - V_R r dt - \Delta_R[dS - (r-D)Sdt]$

(5.34) $\mathrm{PV}[\mathrm{P\&L}(I,R)] = e^{rt_0}\left[e^{-rT}\cdot 0 - e^{-rt_0}(V_{I,t} - V_{R,t})\right] = V_{R,t} - V_{I,t}$

(5.38)
$$d\mathrm{P\&L}(I,R) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt + (\Delta_I - \Delta_R)[(\mu - r + D)Sdt + \sigma_R S dZ]$$

(page 100) The upper bound of the P&L is ... $(\mathrm{V}_{R,0} - V_{I,0})$

(5.41) $\quad \mathrm{PV}[\pi(I,R)]_L = (V_{R,0} - V_{I,0}) - 2Ke^{-2r\tau}\left[N\left(\frac{1}{2}(\sigma_R - \Sigma)\sqrt{\tau}\right) - \frac{1}{2}\right]$

(This is a correction to the text formula)

(5.42) $\quad d\mathrm{P\&L}(I,I) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$

(5.43) $\quad \mathrm{PV}[\mathrm{P\&L}(I,I)] = \frac{1}{2}\int_{t_0}^{T} e^{-r(t-t_0)}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$

(page 103, problem 5-4) $\quad \mathrm{PV}[\mathrm{P\&L}(I,H)] = V_h - V_I + \frac{1}{2}\int_{t_0}^{T} e^{-r(t-t_0)}\Gamma_h S^2(\sigma_R^2 - \sigma_h^2)dt$

Chapter 6

(6.2) $\quad \pi = C - \frac{\partial C}{\partial S}S$

(6.6) $\quad HE \approx \sum_{i=1}^{n}\frac{1}{2}\Gamma_i\sigma_i^2 S_i^2(Z_i^2 - 1)dt$

(6.7) $\quad \sigma_{HE}^2 \approx E\left[\sum_{i=1}^{n}\frac{1}{2}(\Gamma_i S_i^2)^2(\sigma_i^2 dt)^2\right]$

(6.12) $\quad \sigma_{HE} \approx \sqrt{\frac{\pi}{4}}\frac{\sigma}{\sqrt{n}}\frac{\partial C}{\partial \sigma}$

(6.14) $\quad \sigma_{HE} \approx dC \approx \frac{\partial C}{\partial \sigma}d\sigma \approx \frac{\sigma}{\sqrt{n}}\frac{\partial C}{\partial \sigma}$

(6.18) $\quad \frac{\sigma_{HE}}{C} \approx \sqrt{\frac{\pi}{4n}} \approx \frac{0.89}{\sqrt{n}}$

Chapter 7

(7.14) $\quad \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial t} - \sqrt{\frac{2}{\pi dt}}\left|\frac{\partial^2 C}{\partial S^2}\right|\sigma S^2 k = r\left(C - S\frac{\partial C}{\partial S}\right)$

(7.18) $\quad \breve{\sigma}^2 = \sigma^2 + 2\sigma k\sqrt{\frac{2}{\pi dt}}$

(7.19) $\quad \breve{\sigma} \approx \sigma \pm k\sqrt{\frac{2}{\pi dt}}$

Chapter 8

(8.3) $\quad P[\ln(S_T) > \ln(K)] = P\left[Z > \frac{-\ln\left(\frac{S_t}{K}\right) - \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right] = N(d_2)$

(8.6) $\quad \Delta_{\mathrm{ATM}} \approx \frac{1}{2} + \frac{d_1}{\sqrt{2\pi}} \approx \frac{1}{2} + \frac{\sigma\sqrt{\tau}}{2\sqrt{2\pi}}$

(8.9) $\quad \Delta \approx \Delta_{\mathrm{ATM}} - \frac{1}{\sqrt{2\pi}}\frac{J}{\nu}$

Chapter 10

(10.3) $S = V - B,\ \dfrac{dS}{S} = \dfrac{dV}{S} = \dfrac{V\sigma dZ}{S} = \sigma\dfrac{S+B}{S}dZ,\ \sigma_S = \sigma\left(1 + \dfrac{B}{S}\right)$

(10.4) $\dfrac{dS}{S} = \mu(S,t)dt + \sigma S^{\beta-1}dZ$

(10.5) $dS = \mu S dt + \sigma S dZ,\ d\sigma = p\sigma dt + q\sigma dW,\ E[dWdZ] = \rho dt$

(10.10) $\text{Profit} = \dfrac{1}{2}\Gamma S^2(\sigma^2 - \Sigma^2)dt = \dfrac{1}{2}\Gamma(dS)^2 - \dfrac{1}{2}\Gamma S^2\Sigma^2 dt$

(10.15) $D = -\dfrac{\partial C_{\text{BSM}}}{\partial K} - \dfrac{\partial C_{\text{BSM}}}{\partial \Sigma}\dfrac{\partial \Sigma}{\partial K}$

INV201-100-25: Chapter 5 of Financial Mathematics A Comprehensive Treatment by Campolieti

(5.4)
$$\begin{cases} \varphi_1 S_T^1(\omega^1) + \varphi_2 S_T^2(\omega^1) + \cdots + \varphi_N S_T^N(\omega^1) = X(\omega^1) \\ \varphi_1 S_T^1(\omega^2) + \varphi_2 S_T^2(\omega^2) + \cdots + \varphi_N S_T^N(\omega^2) = X(\omega^2) \\ \qquad\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\ \varphi_1 S_T^1(\omega^M) + \varphi_2 S_T^2(\omega^M) + \cdots + \varphi_N S_T^N(\omega^M) = X(\omega^M) \end{cases}$$

(??) $\boldsymbol{\varphi}\mathbf{D} = \mathbf{X}.$

Theorem 5.9 *(The first FTAP). There are no arbitrage portfolios iff there exists a strictly positive solution* $\mathbf{\Psi} \in \mathbb{R}^M$ *to the linear system of equations*

(5.10) $\mathbf{D\Psi} = \mathbf{S}_0.$

That is, there exists $\mathbf{\Psi} = \left[\Psi_1, \Psi_2, \cdots, \Psi_M\right]^\top \gg 0$ *such that*

$$\sum_{j=1}^{M} S_T^i(\omega^j)\Psi_j = S_0^i, \quad \forall i = 1, 2, \ldots, N.$$

(5.15) $\tilde{p}_j := \dfrac{\Psi_j}{\sum_{i=1}^M \Psi_i}, \quad j = 1, 2, \ldots, M.$

Theorem 5.10 *(The first FTAP—the 2nd version). There are no arbitrage portfolios in a single-period* N*-by-*M *model iff there exist probabilities* $\{\tilde{p}_j > 0 : j = 1, 2, \ldots, M\}$ *such that the discounted asset price processes*

$$\left\{\bar{S}_t^i\right\}_{t\in\{0,T\}}, \quad i = 1, 2, \ldots, N,$$

are all martingales with respect to the probability measure $\tilde{\mathbb{P}}$. *Such a probability measure is called the risk-neutral probability measure.*

Theorem 5.11 *(The second FTAP). Assuming absence of arbitrage, there exists a unique solution to the state-price equation,* $\mathbf{\Psi} \gg 0$, *iff the market is complete.*

Theorem 5.12 *(The second FTAP—the 2nd version). Assuming absence of arbitrage, there exists a unique set of risk-neutral probabilities* $\{\tilde{p}_j > 0 \ : \ j = 1, 2, \ldots, M\}$ *iff the market is complete.*

$$\begin{cases} \Psi_1 + \Psi_2 + \Psi_3 = (1+r)^{-1} \\ u\Psi_1 + m\Psi_2 + d\Psi_3 = 1 \end{cases} \iff \begin{cases} \Psi_1 = \dfrac{(1+r)-d}{(1+r)(u-d)} - \dfrac{m-d}{u-d}c \\ \Psi_2 = c \\ \Psi_3 = \dfrac{u-(1+r)}{(1+r)(u-d)} - \dfrac{u-m}{u-d}c \end{cases} \tag{5.30}$$

Theorem 5.14 *(The first and second FTAPs—the 3rd version)*

(1) *There are no arbitrage opportunities iff there exists an equivalent martingale measure with respect to a given numéraire asset.*

(2) *Assuming absence of arbitrage, there exists a unique equivalent martingale measure with respect to a given numéraire asset iff the market is complete.*

INV-201-108-25: Mitigating Interest Rate Risk in Variable Annuities: An Analysis of Hedging Effectiveness under Model Risk

(1) $dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S$

(2) $d\nu_t = \kappa(\theta - \nu_t)dt + \sigma_\nu \sqrt{\nu_t} dW_t^\nu$

(page 505) $dr_t^{(j)} = a_j(b_j - r_t^{(j)})dt + \sigma_{t,j}\sqrt{r_t^{(j)}} dW_{t,j}^r$

(page 506) $\Pi_{t+h} = (\Pi_t - \Delta_t S_t - n_t P_{t,t+T^B})B_{t+h}/B_t + \Delta_t S_{t+h} + n_t P_{t+h,t+T^B}$

(page 506) $dS_t = r_t S_t dt + \sigma_S S_t dZ_t^S$

(page 506) $dr_t = (\upsilon(t) - ar_t)dt + \sigma_r dZ_t^r$

(page 508) $dA_t = (\mu - \alpha)A_t dt - \omega_t dt + \sqrt{\nu_t} A_t dW_t^S$

(page 508) $L_T = L_T^{(D)} + L_T^{(A)}$

(page 508) $L_T^{(D)} = \int_0^T \left[\max(G_t - A_t, 0)B_{t,T} - \int_0^t \alpha A_s B_{s,T} ds\right] {}_t p_x u_{x+t} dt$

(page 509) $L_T^{(A)} = \left[\max(G_T - A_T, 0) - \int_0^T \alpha A_t B_{t,T} dt\right] {}_T p_x$

(3) $\Omega_t = \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \beta_{t,v} \max(A_v, G_v)\ {}_{v-t}p_{x+t} u_{x+v} dv + \beta_{t,T} \max(A_T, G_T)\ {}_{T-t}p_{x+t} | \mathcal{F}_t\right]$

(page 510) $L_T = \frac{A_0}{T} \int_\tau^T B_{t,T} dt - \int_0^\tau \alpha A_t B_{t,T} dt$

(5) $\Omega_t = \frac{A_0}{T} \int_t^T P_{t,v} dv + \mathbb{E}^{\mathbb{Q}}[\beta_{t,T} A_T | \mathcal{F}_t]$

(page 512) $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(HL_T^{(i)}\right)^2}$

(page 512) $CTE(1-p)\% = \frac{1}{N_p} \sum_{i=1}^{N_p} HL_T^{(i)}$

(page 522) $P_{t,T} = \mathcal{A}(t,T) e^{-\mathcal{B}(t,T) r_t}$

(page 522) $\mathcal{B}(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]$

(page 522) $\rho_t^B = \frac{\partial P_{t,t+T^B}}{\partial r_t} = -\mathcal{B}(t, t+T^B) P_{t,t+T^B}$

(page 523) $$\Delta_t = \frac{\partial \mathcal{L}_t}{\partial A_t} \times \frac{\partial A_t}{\partial S_t} = \left[\int_t^T \frac{\partial \Psi(t,v,A_t,G_v)}{\partial A_t}\ {}_{v-t}p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t,T,A_t,G_T)}{\partial A_t}\ {}_{T-t}p_{x+t} - \left(1 - \int_t^T e^{-\alpha(v-t)}\ {}_{v-t}p_{x+t} u_{x+v} dv - e^{-\alpha(T-t)}\ {}_{T-t}p_{x+t}\right)\right] \times \frac{A_t}{S_t}$$

(This is a correction to the text formula)

(page 523) $\rho_t = \int_t^T \frac{\partial \Psi(t,v,A_t,G_v)}{\partial r_t}\ {}_{v-t}p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t,T,A_t,G_T)}{\partial r_t}\ {}_{T-t}p_{x+t}$

(page 525) $\Delta_t = \left(e^{-\alpha(T-t)}\Phi(z) - 1\right) \times \frac{A_t}{S_t}$

(page 525) $\rho_t = -\frac{A_0}{T}\int_t^T \mathcal{B}(t,v)P_{t,v}dv + e^{-\alpha(T-t)}\frac{A_0}{T}\int_t^T e^{\alpha(v-t)}\mathcal{B}(t,v)P_{t,v}\Phi(z-m_v)dv$

(page 525) $dv_t = \tilde{\kappa}(\tilde{\theta} - v_t)dt + \sigma_v\sqrt{v_t}d\tilde{W}_t^v$

(page 525) $\text{1-year}VIX_t = \sqrt{\mathbb{E}^{\mathbb{Q}^{\mathbb{P}}}\left[\int_t^{t+1} v_s ds|\mathcal{F}_t\right]} = \sqrt{A + Bv_t}$

(page 525) $A = \tilde{\theta}\left(1 - \frac{1-e^{-\tilde{\kappa}}}{\tilde{\kappa}}\right), \quad B = \frac{1-e^{-\tilde{\kappa}}}{\tilde{\kappa}}$

INV201-109-25: Investment Instruments with Volatility Target Mechanism, Albeverio, Steblovskaya, and Wallbaum

(2) $\mathcal{O}_0 = \frac{1}{B_T}\mathbb{E}^*(f(S))$

(3) $V_t = \beta_k S_t + \gamma_k B_t$

(5) $\tilde{V}_t = \beta_k \tilde{S}_t + \gamma_k$

(7) $\hat{\mathcal{O}}_0 = \frac{1}{B_T}\mathbb{E}^*(f(V))$

(8) $dS_t = S_t(rdt + \sigma dW_t)$

(10) $P_{dp} = K + p_{dp}\cdot K\max\left\{\frac{S_T}{S_0} - 1, 0\right\}$

(11) $RB = K(1 - e^{-rT})$

(12) $g_{dp}(x) = \max\left\{\frac{K}{S_0}x - K, 0\right\}$

(17) $p_{dp} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dp})}$

(18) $P_{dp} = K + p_{dp}\cdot K\max\left\{\frac{S_T}{S_0} - 1, 0\right\}$

(19) $P_{dpa} = \max\left\{K; K\cdot\left(1 + p_{dpa}\cdot\frac{1}{S_0}\left(\frac{1}{n}\sum_{i=0}^{n} S_{t_i} - S_0\right)\right)\right\}$

(20) $g_{dpa}(S) = \max\left\{\frac{K}{S_0}\frac{1}{n}\sum_{i=0}^{n} S_{t_i} - K, 0\right\}$

(25) $p_{dpa} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dpa})}$

INV201-105-25: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng Chapter 1

(page 15) $$G_{(k+1)/n} = G_{k/n}\left(1+\frac{\rho}{n}\right), \quad \text{for } k=0,1,\cdots$$

(1.15) $$G_{(k+1)/n} = \max\left\{G_{k/n}, F_{(k+1)/n}\right\}, \quad \text{for } k=0,1,\cdots$$

(page 15) $$G_{(k+1)/n} = \frac{G_{k/n}}{F_{k/n}}\max\left\{F_{k/n}, F_{(k+1)/n}\right\}, \quad \text{for } k=0,1,\cdots$$

(1.16) $$\frac{G_{(k+1)/n}-G_{k/n}}{G_{k/n}} = \frac{(F_{(k+1)/n}-F_{k/n})_+}{F_{k/n}}$$

(page 16) $$G_{(k+1)/n} = G_0\prod_{j=0}^{k-1}\max\left\{1, \frac{F_{(j+1)/n}}{F_{j/n}}\right\}$$

(1.17) $$G_{(k+1)/n} = \max\left\{G_{k/n}\left(1+\frac{\rho}{n}\right), F_{(k+1)/n}\right\}, \quad \text{for } k=0,\ldots$$

(1.18) $$G_{k/n} = \left(1+\frac{\rho}{n}\right)^k \max_{j=0,\ldots,k}\left\{\left(1+\frac{\rho}{n}\right)^{-j}F_{j/n}\right\}$$

(1.19) $$G_t = \sup_{0\le s\le t}\{F_s\}$$

(1.23) $$L_e^{(\infty)}(T_x) = e^{-rT}(G-F_T)_+ I(T_x>T) - \int_0^{T\wedge T_x} e^{-rs}m_eF_s ds$$

(1.24) $$L_d^{(\infty)}(T_x) = e^{-rT_x}(Ge^{\rho T_x}-F_{T_x})_+ I(T_x\le T) - \int_0^{T\wedge T_x} e^{-rs}m_dF_s ds$$

(1.28) $$L_w^{(n)} := \sum_{k=n\tau}^{(n\tau-1)\vee\lceil nT\rceil} e^{-rk/n}\frac{w}{n} - \sum_{k=1}^{(n\tau-1)\wedge\lceil nT\rceil} e^{-r(k-1)/n}F_{(k-1)/n}\frac{m_w}{n}$$

(page 24) $$L_w^{(\infty)} := \int_\tau^{\tau\vee T} e^{-rt}w dt - \int_0^{t\wedge T} e^{-rt}m_wF_t dt, \quad \tau := \inf\{t>0 : F_t\le 0\}$$

(page 27) $$L_{lw}^{(n)} := \sum_{k=n\tau}^{(n\tau-1)\vee\lfloor nT_x\rfloor} e^{-r(k+1)/n}G_{k/n}\frac{h}{n} - \sum_{k=0}^{(n\tau-1)\wedge\lfloor nT_x\rfloor} e^{-rk/n}G_{k/n}\frac{m_w}{n}$$

(page 27) $$L_{lw}^{(\infty)} := \int_\tau^{\tau\vee T_x} e^{-rt}G_th dt - \int_0^{t\wedge T_x} e^{-rt}G_tm_w dt$$

(1.39) $$P\prod_{k=1}^{T}\max\left(\min\left(1+\alpha\frac{S_k-S_{k-1}}{S_{k-1}}, e^c\right), e^g\right)$$

(1.41) $$P\prod_{k=1}^{T}\max\left(\min\left(\left(\frac{S_k}{S_{k-1}}\right)^\alpha, e^c\right), e^g\right)$$

(1.42) $$\max\left(P\left(\frac{\max\{S_k : k=1,\cdots,T\}}{S_0}\right)^\alpha, G_T\right)$$

$$(1.43)\qquad \max\left(P\left(\frac{\sup_{0\le t\le T}\{S_t\}}{S_0}\right)^{\alpha}, G_T\right)$$

Chapter 4

$$(4.52)\qquad dF_t = (r-m)F_t dt + \sigma F_t dW_t, \quad 0 < t < T$$

$$(4.53)\qquad B_e(t,F) = {}_Tp_x\left[Ge^{-r(T-t)}\Phi\left(-d_2\left(T-t,\frac{F}{G}\right)\right) - Fe^{-m(T-t)}\Phi\left(-d_1\left(T-t,\frac{F}{G}\right)\right)\right]$$

$$(4.54)\qquad d_1(t,u) = \frac{\ln u + (r-m+\sigma^2/2)t}{\sigma\sqrt{t}}$$

$$(4.55)\qquad d_2(t,u) = d_1(t,u) - \sigma\sqrt{t}$$

$$(\text{page 153})\qquad P_e(t,F) = m_e\, {}_tp_x F\int_0^{T-t} e^{-ms}\, {}_sp_{x+t}\, ds = m_e\, {}_tp_x F\, \bar{a}_{x+t:\overline{T-t}|m}$$

$$(\text{page 153})\qquad {}_Tp_x\left[e^{(\rho-r)T}\Phi\left(\frac{\rho-(r-m-\sigma^2/2)}{\sigma}\sqrt{T}\right)\right.$$
$$\left. -e^{-mT}\Phi\left(\frac{\rho-(r-m+\sigma^2/2)}{\sigma}\sqrt{T}\right)\right] = m_e\bar{a}_{x:\overline{T}|m}$$

$$(\text{page 154})\qquad \tilde{\mathbb{E}}\left[e^{-r(T_2-t)}(G_{T_1}-F_{T_2})_+ I(T_x > T_2)|\mathcal{F}_t\right]$$
$$= {}_{T_2}p_x e^{-r(T_2-t)}\left[G_{T_1}\Phi(-d_2(T_2-t, F_t/G_{T_1})) - F_t e^{(r-m)(T_2-t)}\Phi(-d_1(T_2-t, F_t/G_{T_1}))\right]$$

(This is a correction to the text formula)

$$(4.60)\qquad N_d(0,F_0) := B_d(0,F_0) - P_d(0,F_0) = \tilde{\mathbb{E}}\left[\int_0^T e^{-rt}\, {}_tp_x \mu_{x+t}(G_t-F_t)_+ dt - \int_0^T e^{-rt} m_d\, {}_tp_x F_t dt\right]$$

$$(4.61)\qquad B_w(t,F) = \tilde{\mathbb{E}}[e^{-r(T-t)}F_{T-t}I(F_{T-t}>0)|F_0=F] + \frac{w}{r}(1-e^{-r(T-t)})$$

$$(4.63)\qquad N_w(t,F) = \tilde{\mathbb{E}}\left[w\int_{\tau-t}^{(\tau\vee T)-t} e^{-rs}ds - m_w\int_{(\tau\wedge t)-t}^{(\tau\wedge T)-t} e^{-rs}\tilde{F}_s ds \,\middle|\, \tilde{F}_0 = F\right]$$
$$= \tilde{\mathbb{E}}\left[w\int_{\tau}^{\tau\vee(T-t)} e^{-rs}ds - m_w\int_0^{\tau\wedge(T-t)} e^{-rs}F_s ds\,\middle|\, F_0 = F\right]$$

$$(\text{page 158})\qquad e^{-rT}F_T I(\tau > T) + \frac{w}{r}(1-e^{-rT}) - F_0$$

$$(\text{page 158})\qquad w\int_{\tau\wedge T}^{T} e^{-rs}ds - m_w\int_0^{t\wedge T} e^{-rs}F_s ds$$

$$(\text{page 159})\qquad F_0 - \mathbb{E}\left[\int_0^{\tau\wedge T} e^{-rs}(mF_s+w)ds\right] = \mathbb{E}[e^{-r(\tau\wedge T)}F_{\tau\wedge T}] = \mathbb{E}[e^{-rT}F_T I(\tau > T)]$$

$$(\text{page 160})\qquad B_{lw}(F_0) := \tilde{\mathbb{E}}\left[\int_0^{T_x} we^{-rs}ds + e^{-rT_x}F_{T_x}I(F_{T_x}>0)\right]$$

$$= \frac{w}{r} - \frac{\delta w}{r(\delta + r)} + \delta\tilde{\mathbb{E}}\left[\int_0^\tau e^{-(\delta+r)t} F_t dT\right]$$

(4.75) $$N_{lw}(F_0) = \frac{w}{r+\delta}\tilde{\mathbb{E}}[e^{-(\delta+r)\tau}] + m_w\tilde{\mathbb{E}}\left[\int_0^\tau e^{-(\delta+r)u} F_u du\right]$$

(page 162) $$B_{lw}(t,F) = \frac{w}{r}(1 - \tilde{q}_{x+t}(r)) + \tilde{\mathbb{E}}\left[\int_0^\tau e^{-rs} F_s\, q_{x+t}(s) ds\right]$$

(page 163) $$N_{lw}(t,F) = \frac{w}{r}\tilde{\mathbb{E}}[e^{-r\tau}\bar{Q}_{x+t}(\tau)] - \frac{w}{r}\tilde{\mathbb{E}}\left[\int_\tau^\infty e^{-ru} q_{x+t}(u) du\right] + m_w\tilde{\mathbb{E}}\left[\int_0^\tau e^{-ru}\bar{Q}_{x+t}(u) F_u du\right]$$

(page 166) Investment Income: $I[t] = A[t] \times \left(H[t] + U[t] - \frac{1}{2}L[t]\right)$

(page 167) Credited to Policholder Account: $J[t] = C[t] \times \left(Q[t] - \frac{1}{2} \times L[t]\right)$

(page 167) Risk Charges: $K[t] = B[t] \times \left(Q[t] - \frac{1}{2} \times L[t]\right)$

(page 167) Mortality: $L[t] = Q[t] \times F[t]$

(page 167) Lapses: $M[t] = S[t] \times D[t]$

(page 167) Surrender Charge: $N[t] = M[t] \times G[t]$

(page 167) Annuitization: $O[t] = R[t] \times E[t]$

(page 167) Current Inforce (as % of initial): $P[t] = P[t-1] \times (1 - D[t] - E[t] - F[t])$

(page 168) Policyholder Fund Value (BOY): $Q[t] = T[t-1] + H[t]$

(page 168) Policyholder Fund Value before Lapses & Annuitizations (EOY): $R[t] = Q[t]+J[t]-L[t]$

(page 168) Policyholder Fund Value before Lapses & after Annuitizations (EOY): $S[t] = R[t]-O[t]$

(page 168) Policyhodler Fund Value after Lapses & Annuitizations (EOY): $T[t] = S[t]-M[t]$

(page 168) Statutory Reserve (BOY): $U[t] = V[t-1]$

(page 168) Statutory Reserve (EOY): $V[t] = T[t]$

(page 168) GMDB Benefit (5% – roll-up rate): $W[t] = P[t-1] \times \max(Q[t], H[1] \times (1+5\%)^t)$

(page 168) GMDB Benefits: $X[t] = (W[t] - U[t])_+ \times F[t]$

(page 168) Poicy Fee Income (30 – annual policy fee: $AG[t] = 30 \times AD[t-1]$

(page 168) Total Revenues: $AH[t] = AE[t] + AF[t] + AG[t]$

(page 168) Premium-Based Administrative Expenses: $AO[t] = AE[t] \times AJ[t]$

(page 168) Per Policy Adminstrative Expenses (2% – inflation rate):
$AP[t] = AK[t] \times AM[t] \times (1 + 2\%)^{t-1}$

(page 169) Commissions: $AQ[t] = AE[t] \times AL[t]$

(page 169) GMDB Cost (0.4% of account value – GMDB cost): $AR[t] = T[t] \times 0.4\%$

(page 169) Total Expenses: $AS[t] = AO[t] + AP[t] + AQ[t] + AR[t]$

(page 169) Death Claims: $AT[t] = L[t]$

(page 169) Annuitization: $AU[t] = O[t]$

(page 169) Surrender Benefit: $AV[t] = M[t] - N[t]$

(page 169) Increase in Reserve: $AW[t] = V[t] - V[t-1]$

(page 170) GMDB Benefit: $AX[t] = (W[t] - U[t])_+ \times F[t]$

(page 170) Total Benefits: $AY[t] = AT[t] + AU[t] + AV[t] + AW[t] + AX[t]$

(page 170) Book Profit Before Tax: $AZ[t] = AH[t] - AS[t] - AY[t]$

(page 171) Taxes on Book Profit (37% – federal income tax rate): $BF[t] = BE[t] \times 37\%$

(page 171) Book Profits after Tax: $BD[t] = BE[t] - BF[t]$

(page 171) Target Surplus (BOY): $BI[t] = BJ[t-1]$

(page 171) Target Surplus (EOY)(0.85% – target surplus rate): $BI[t] = V[t] \times 0.85\%$

(page 171) Increase in Target Surplus: $BG[t] = BI[t] - BH[t]$

(page 172) Interest on Target Surplus (5% – interest rate on surplus): $BK[t] = BH[t] \times 5\%$

(page 172) Taxes on Interest on Target Surplus: $BL[t] = BK[t] \times 37\%$

(page 172) After Tax Interest on Target Surplus: $BJ[t] = BK[t] - BL[t]$

(page 172) Distributable Earnings: $BM[t] = BD[t] + BJ[t] - BG[t]$

(4.78) $$\mathfrak{B} = \sum_{k=1}^{nT} \left(1 + \frac{r}{n}\right)^k P_{k/n}$$

(4.79) $$\mathfrak{B} = \int_0^T e^{-rt} P_t dt = \int_0^T e^{-rt} k_t \mu^l_{x+t}\,{}_t p_x F_t dt + m \int_0^T e^{-rt}\,{}_t p_x F_t dt - \int_0^T e^{-rt} E_t dt - \int_0^T e^{-rt} \mu^d_{x+t}\,{}_t p_x (G_t - F_t)_+ dt$$

Chapter 6

(page 264) $$B_e(t, F_t) = {}_T p_x \times \left[G e^{-r(T-t)} \Phi\left(-d_2\left(T-t, \frac{F_t}{G}\right)\right) - F_t e^{-m(T-t)} \Phi\left(-d_1\left(T-t, \frac{F_t}{G}\right)\right)\right]$$

(6.16) $$c(t,s) = -\left(\frac{\partial}{\partial t} + rs\frac{\partial}{\partial s} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2}{\partial s^2} - r\right) f(t,s) = m\,{}_t p_x F(t,s)$$

(page 269) $$\Delta_t = \frac{\partial}{\partial s} f(t, S_t) = -\frac{F_t}{S_t}\left[{}_T p_x e^{-m(T-t)} \Phi\left(-d_1(T-t, \frac{F_t}{G})\right) + m \int_t^T e^{-m(s-t)}\,{}_s p_x ds\right]$$

(page 275 Greeks)

$$\Delta_t = -{}_T p_x \frac{F}{s} e^{-m(T-t)} \Phi\left(-d_1(T-t, \frac{F}{G})\right) - \frac{P_e}{s}$$

$$
\begin{aligned}
\Gamma_t &= {}_Tp_x \frac{F}{s^2} e^{-m(T-t)} \frac{\phi\left(d_1(T-t, \frac{F}{G})\right)}{\sigma\sqrt{T-t}} \\
\Theta_t &= {}_Tp_x \Bigg[rGe^{-r(T-t)} \Phi\left(-d_2(T-t, \frac{F}{G})\right) - mFe^{-m(T-t)} \Phi\left(-d_1(T-t, \frac{F}{G})\right) \\
&\qquad -e^{-m(T-t)} \frac{\sigma F \phi\left(d_1(T-t, \frac{F}{G})\right)}{2\sqrt{T-t}} \Bigg] + m\, {}_tp_x F \quad \text{(This is a correction to the text formula)} \\
\mathcal{V}_t &= {}_Tp_x F e^{-m(T-t)} \phi\left(d_1(T-t, \frac{F}{G})\right) \sqrt{T-t}
\end{aligned}
$$

Calibrating Interest Rate Models

(page 12)

$$
P[r_{t+s} < 0 | r_t] \quad = \quad \Phi\left(- \frac{\bar{r} + (r_t - \bar{r}) e^{-\gamma s}}{\sigma \sqrt{\frac{1-e^{-2\gamma s}}{2\gamma}}} \right)
$$

(page 14)

$$
r_{t+s} \quad = \quad \bar{r} + (r_t - \bar{r}) e^{-\gamma s} + \left(\frac{\sigma^2}{2\gamma} (1 - e^{-2\gamma s}) \right)^{\frac{1}{2}} Z,
$$

(page 15)

$$
(5) \qquad f(r_{t+s} | r_t) = c_s \chi^2 (c_s r_{t+s}, \nu, \lambda_{t+s})
$$

where $\chi^2(., \nu, \lambda_{t+s})$ is a non-central χ^2 density function with ν degree of freedom, and non-centrality parameter λ_{t+s}, with:

$$
\begin{aligned}
c_s &= \frac{4\gamma}{\alpha(1 - \exp(-\gamma s))} \\
\nu &= \frac{4\gamma}{\alpha} \bar{r} \\
\lambda_{t+s} &= c_s r_t \exp(-\gamma s)
\end{aligned}
$$

(page 22)

$$
\begin{aligned}
\hat{\beta} &= \frac{\sum_{i=1}^n r_{i\Delta} r_{(i-1)\Delta} - \frac{1}{n} \sum_{i=0}^{n-1} r_{i\Delta} \sum_{i=1}^n r_{i\Delta}}{\sum_{i=0}^{n-1} r_{i\Delta}^2 - \frac{1}{n} \left(\sum_{i=0}^{n-1} r_{i\Delta} \right)^2} \\
\hat{\alpha} &= \frac{1}{n} \left(\sum_{i=1}^n r_{i\Delta} - \hat{\beta} \sum_{i=0}^{n-1} r_{i\Delta} \right) \\
\hat{\sigma^{*2}} &= \frac{1}{n-2} \sum_{i=1}^n \left(r_{i\Delta} - \hat{\alpha} - \hat{\beta} r_{(i-1)\Delta} \right)^2
\end{aligned}
$$

$$\gamma = -\frac{\ln(\hat{\beta}^*)}{\Delta}$$

$$\bar{r} = \frac{\hat{\alpha}^*}{1-\hat{\beta}^*}$$

$$\sigma = \sqrt{\frac{2\hat{\gamma}\hat{\sigma}^{*2}}{1-\hat{\beta}^{*2}}}$$

(page 26)

$$\gamma = 2\left(\frac{1.96\sigma}{\hat{q}_{0.975}-\hat{q}_{0.025}}\right)^2$$

$$\bar{r} = \frac{\hat{q}_{0.025}+\hat{q}_{0.975}}{2}$$

(page 27)

$$Z(r,t;T) = e^{A(t;T)-B(t;T)\times r} \quad (15.28)$$

$$B(t;T) = \frac{1}{\gamma^*}\left(1-e^{-\gamma^*(T-t)}\right) \quad (15.29)$$

$$A(t;T) = (B(t;T)-(T-t))\left(\bar{r}^*-\frac{\sigma^2}{2(\gamma^*)^2}\right)-\frac{\sigma^2 B(t;T)^2}{4\gamma^*} \quad (15.30)$$

(page 33)

$$r(i) = \alpha_1+\beta_1 r(i-1)+\sqrt{r(i-1)}\epsilon_i, \quad i=1,2,\ldots$$

$$\frac{r(i)}{\sqrt{r(i-1)}} = \alpha_1\left(\frac{1}{\sqrt{r(i-1)}}\right)+\beta_1\sqrt{r(i-1)}+\epsilon_i$$

$$y_i = \frac{r(i)}{\sqrt{r(i-1)}}$$

$$x_{1i} = \frac{1}{\sqrt{r(i-1)}}$$

$$x_{2i} = \sqrt{r(i-1)}$$

$$y_i = \alpha_1 x_{1i}+\beta_1 x_{2i}+\epsilon_i, i=1,2,\ldots$$

$$\gamma = \frac{1-\beta_1}{\Delta}$$

$$\bar{r} = \frac{\alpha_1}{1-\beta_1}$$

$$\alpha = \frac{\sigma^2}{\Delta}$$

(page 39)

(15.70) $$Z(r,t;T) = e^{A(t;T)-B(t;T)\times r}$$

(15.71) $$B(t;T) = \frac{2\left(e^{\psi_1(T-t)}-1\right)}{(\gamma^*+\psi_1)\left(e^{\psi_1(T-t)}-1\right)+2\psi_1}$$

(15.72) $$A(t;T) = 2\frac{\bar{r}^*\gamma^*}{\alpha}\log\left(\frac{2\psi_1 e^{(\psi_1+\gamma^*)\frac{(T-t)}{2}}}{(\gamma^*+\psi_1)\left(e^{\psi_1(T-t)}-1\right)+2\psi_1}\right), \text{ and } \psi_1 = \sqrt{(\gamma^*)^2+2\alpha}$$

(page 53)

$$\begin{aligned} r_{t+s} &= r_t\exp(-\gamma^* s) + \exp(-\gamma^*(t+s))\int_t^{t+s}\theta_u\exp(\gamma^* u)du + \sigma\exp(-\gamma^* s)\int_0^s \exp(\gamma^* u)dX_u \\ E[r_{t+s}|r_t] &= r_t\exp(-\gamma^* s) + \exp(-\gamma^*(t+s))\int_t^{t+s}\theta_u\exp(\gamma^* u)du \\ Var[r_{t+s}|r_t] &= \frac{\sigma^2}{2\gamma^*}(1-e^{-2\gamma^* s}) \end{aligned}$$

(19.25) $$Z(r,0;T) = e^{A(0;T)-B(0;T)\times r}$$

(19.26) $$B(0;T) = \frac{1}{\gamma^*}\left(1-e^{-\gamma^* T}\right)$$

(19.27) $$A(0;T) = -\int_0^T B(0;T-t)\theta_t dt + \frac{\sigma^2}{2(\gamma^*)^2}\left(T + \frac{1-e^{-2\gamma^* T}}{2\gamma^*} - 2B(0;T)\right)$$

(page 54)

$$\theta_T = \frac{\partial f(0,T)}{\partial T} + \gamma^* f(0,T) + \frac{\sigma^2}{2\gamma^*}(1-\exp(-2\gamma^* T)$$

$$\begin{aligned} E[r_{t+s}|r_t] = {} & r_t\exp(-\gamma^* s) + f(0,s+t) - f(0,t)\exp(-\gamma^* s) + \\ & \frac{\sigma^2}{2(\gamma^*)^2}\left[1-\exp(-\gamma^* s)+\exp(-2\gamma^*(t+s)) - \exp(-\gamma^*(2t+s))\right] \end{aligned}$$

(page 57)

$$\begin{aligned} r(0,t) &= \sum_{i=0}^n a_i t^i \\ f(0,t) &= \sum_{i=0}^n a_i(i+1)t^i \\ \frac{\partial f(0,t)}{\partial t} &= \sum_{i=1}^n a_i i(i+1)t^{i-1} \\ \theta_t &= \sum_{i=1}^n a_i i(i+1)t^{i-1} + \sum_{i=0}^n \gamma^* a_i(i+1)t^i + \frac{\sigma^2}{2\gamma^*}\left(1-e^{-2\gamma^* t}\right) \end{aligned}$$

(page 59)

$$f(0,t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau}\right) + \frac{\beta_2 t}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

$$r(0,t) = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}} + \beta_2 \frac{1 - \exp\left(-\frac{t}{\tau}\right) - \frac{t}{\tau}\exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}}$$

$$\frac{\partial}{\partial t} f(0,t) = -\frac{\beta_1}{\tau} \exp\left(-\frac{t}{\tau}\right) + \frac{\beta_2}{\tau} \exp\left(-\frac{t}{\tau}\right) - \frac{\beta_2 t}{\tau^2} \exp\left(-\frac{t}{\tau}\right)$$

(page 62)

(19.33) $$S_Z(T_O;T_B)^2 = B(T_O;T_B)^2 \frac{\sigma^2}{2\gamma^*}\left(1 - e^{-2\gamma^* T_O}\right)$$

(19.41) $$A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + B(t;T)f(0,t) - \frac{\sigma^2}{4\gamma^*}B(t;T)^2\left(1 - e^{-2\gamma^* t}\right)$$

(19.42) $$A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + (T-t)f(0,t) - \frac{\sigma^2}{2}(T-t)^2 t$$

(19.??) $$B(t;T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^*(T-t)}\right)$$

(19.??) $$M = N(1 + r_K \Delta)$$

(19.??) $$K = \frac{1}{1 + r_K \Delta}$$

(19.44) $$V(r_0, 0) = M \times \left(K Z(r_0,0;T-\Delta)\mathcal{N}(-d_2) - Z(r_0,0;T)\mathcal{N}(-d_1)\right)$$

(19.45) $$d_1 = \frac{1}{S_Z(T-\Delta;T)} \log\left(\frac{Z(r_0,0;T)}{K Z(r_0,0;T-\Delta)}\right) + \frac{S_Z(T-\Delta;T)}{2}$$

(19.46) $$d_2 = d_1 - S_Z(T-\Delta;T)$$