CP 351 ALM Assessment Formula Package

2025 - 2026

Exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black- Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes. In sources where some equations are numbered and others are not, the page number is provided instead.

Quantitative Enterprise Risk Management, Hardy & Saunders, 2022

Chapter 6 Copulas

P. 198 $F_j(x_j) = \Pr\left[X_j \le x_j\right]$

$C(F_1(x_1),\ldots, F_m(x_m)) = \Pr\left[X_1 \le x_1,\ldots, X_m \le x_m\right]$ for j=1, ….., m

P. 211

$$C(u_1,u_2,\ldots,u_m) = u_1 * u_2 * \ldots * u_m$$

$$C(u_1,u_2,\ldots,u_m) = \min(u_1,u_2,\ldots,u_m)$$

$$C(u,v) = \max(u+v-1,0)$$

$$C(u_1,u_2,\ldots,u_m) = (u_1^{-\theta} + u_2^{-\theta} + \ldots + u_m^{-\theta} - (m-1))^{-\frac{1}{\theta}}, \theta > 0$$

$$C(u_1,u_2,\ldots,u_m) = Exp\{-((-Ln(u_1))^{\theta} + (-Ln(u_2))^{\theta} + \ldots + (-Ln(u_m))^{\theta})^{\frac{1}{\theta}}\} \text{ , } \theta > 0$$

$$C(u_1,u_2,\ldots,u_m) = \Phi_P(z_{u_1}, z_{u_2},\ldots,z_{u_m}), \quad -1 \le \rho \le 1, z_\alpha = \Phi^{-1}(\alpha)$$

P. 212

$$1 = u + v - C(u,v) + \overline{C}(u,v)$$

$$\overline{C}(u,v) = \Pr[F_x(X) > u, F_y(Y) > v] \text{ , } \hat{C}(u,v) = \overline{C}(1-u,1-v)$$

P. 214

$$\rho(X,Y) = \frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}} = \frac{E[XY] - E[X]E[Y]}{\sqrt{Var[X]Var[Y]}}$$

P. 216

$$\rho_S(X,Y) = \rho(U,V) = \frac{E[UV] - E[U]E[V]}{\sqrt{Var[U]Var[V]}}$$

$$r = \frac{1}{n-1}\sum_{i=1}^{n}(\frac{r_{x_i} - \overline{r}_x}{s_{r_x}})(\frac{r_{y_i} - \overline{r}_y}{s_{r_y}})$$

P. 218 - 219

$$\begin{aligned} r_k &= \Pr[(X - X^*)(Y - Y^*) > 0] - \Pr[(X - X^*)(Y - Y^*) < 0] \\ &= E[sign((X - X^*)(Y - Y^*))] \end{aligned} \quad (6.14, 6.15)$$

Where $sign(z) = \{-1, z < 0, 0, z = 0, 1, z > 0\}$

$$t_k = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} sign((x_i - x_j)(y_i - y_j)) \quad (6.16)$$

P. 224

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v))$$

P. 225

$$\phi(u) = \frac{(u^{-\theta} - 1)^{\delta}}{\theta^{\delta}}$$

Chapter 8: Market Risk Models

Pg 272 $$E_{t-1}\left[\sigma_t^2\right] = \sigma_t^2 \quad (8.4)$$

Pg 272

$$E_{t-2}\left[\sigma_t^2\right] = a_0 + (a_1 + b)\sigma_{t-1}^2 \quad (8.5)$$

Pg 273

$$\lim_{x\to\infty} a_0\left(\frac{1-(a_1+b)^{t-1}}{1-(a_1+b)}\right) + (a_1+b)^{t-1}\sigma_1^2 = \frac{a_0}{1-(a_1+b)} \quad (8.6)$$

Pg 275

TGARCH variance: $\sigma_t^2 = a_0 + (a_1 + \gamma \min(Y_{t-1}, 0))(Y_{t-1} - \mu)^2 + b\sigma_{t-1}^2$ (8.7)

Pg 275

$$\begin{aligned} Y_t \mid F_{t-1} &= r - \tfrac{\sigma_t^2}{2} + \sigma_t \varepsilon_t \\ \sigma_t^2 &= a_0 + a_1 (Y_{t-1} - (r - \frac{\sigma_t^2}{2}))^2 + b\sigma_{t-1}^2 \end{aligned} \tag{8.8}$$

Pg 281

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}} \quad \text{and} \quad \pi_2 = \frac{p_{12}}{p_{12} + p_{21}} \tag{8.12}$$

Pg 285

$$l(\theta) = \sum_{i=1}^{n} \ln f(x_i \mid x_1, ..., x_{i-1}; \theta) \tag{8.13}$$

Pg 288

$$\begin{aligned} f(y_2 \mid y_1) = & f(y_2, \rho_1 = 1, \rho_2 = 1 \mid y_1) + f(y_2, \rho_1 = 1, \rho_2 = 2 \mid y_1) \\ & + f(y_2, \rho_1 = 2, \rho_2 = 1 \mid y_1) + f(y_2, \rho_1 = 2, \rho_2 = 2 \mid y_1) \end{aligned} \tag{8.14}$$

Pg 288

$$f(y_2, \rho_1 = 1, \rho_2 = 1 \mid y_1) = f(y_2 \mid y_1, \rho_1 = 1, \rho_2 = 1) * p(\rho_2 = 1 \mid y_1, \rho_1 = 1) * p(\rho_1 = 1 \mid y_1) \tag{8.15}$$

Chapter 15: Risk mitigation using options and derivatives

Pg 423 K, priced at P_0, with delta of $-\Phi(-d_1)$. Let ω_1 denote the number of units of stock purchased, and ω_2 denote the number of put options. Then we can find a delta-neutral portfolio by solving the following equations for ω_1 and ω_2 :

$V(0) = \omega_1 S(0) + \omega_2 \mathrm{P}_0$ (portfolio value),

$0 = \omega_1 - \omega_2 \Phi(-d_1)$ (portfolio delta).

Pg 428

$$V(0)=\omega_1 S(0)+\omega_2 \mathrm{P}_0-\omega_3 c_0$$
$$0=\omega_1-\omega_2\Phi(-d_1^{\mathrm{P}})-\omega_3\Phi(d_1^c)$$
$$0=\omega_2(\frac{\phi(d_1^{\mathrm{P}})}{S(0)\sigma\sqrt{T^{\mathrm{P}}}})-\omega_3(\frac{\phi(d_1^c)}{S(0)\sigma\sqrt{T^c}})$$

Pg 434 market value at time 0 of 1 due at t is $v(t)$. The swap rate c is

$$c=\frac{1-v(n)}{\sum_{k=1}^{n} v(k)}$$

Pg 437

$$x=\frac{v(1)+v(2)+.....+v(n)}{F_1 v(1)+F_2 v(2)+.....+F_n v(n)}$$

$$x=\frac{F_0-v(n)}{F_1 v(1)+F_2 v(2)+.....+F_n v(n)}$$

Black-Scholes formula for the price of a T-year put option, on a non-dividend paying stock with price S_t at time t, and with strike price K:

$$p_0=Ke^{-rT}\Phi(-d_2(0,T))-S_0\Phi(-d_1(0,T))$$
$$p_t=Ke^{-r(T-t)}\Phi(-d_2(t,T))-S_t\Phi(-d_1(t,T))$$
$$d_1(t_1,t_2)=\frac{\log(\frac{S_{t_1}}{K})+(r+\frac{\sigma^2}{2})(t_2-t_1)}{\sigma\sqrt{t_2-t_1}}$$
$$d_2(t_1,t_2)=d_1(t_1,t_2)-\sigma\sqrt{t_2-t_1}$$

$$0\le t_1<t_2, 0<t<T$$

Fixed Income Securities: Tools for Today's Markets, Third Edition. Tuckman and Serrat

Chapter 6: Empirical Approaches to Risk Metrics and Hedging

Pg 175 $$\Delta y_t^N=\alpha+\beta\Delta y_t^R+\epsilon_t \qquad (6.3)$$

Pg 175 $$\sum_t \widehat{\epsilon}_t^2=\sum_t\left(\Delta y_t^N-\widehat{\alpha}-\widehat{\beta}\Delta y_t^R\right)^2 \qquad (6.7)$$

Pg 177 $$F^R = -F^N \times \frac{DV01^N}{DV01^R} \times \widehat{\beta} \quad (6.9)$$

Pg 177 $$-F^R \times \frac{DV01^R}{100} \Delta y_t^R - F^N \times \frac{DV01^N}{100} \Delta y_t^N \quad (6.10)$$

Pg 178 $$F^N \times \frac{DV01^N}{100} \times \widehat{\sigma} \quad (6.13)$$

Pg 196 $$2F^R \left(\frac{DV01^R}{100}\right)^2 V\left(\Delta y_t^R\right) + 2F^N \frac{DV01^R}{100} \frac{DV01^N}{100} Cov\left(\Delta y_t^R, \Delta y_t^N\right) = 0 \quad (6.36)$$

Pg 196 $$F^N \times DV01^N \times \frac{Cov\left(\Delta y_t^R, \Delta y_t^N\right)}{V\left(\Delta y_t^R\right)} = -F^R \times DV01^R \quad (6.37)$$

Pg 196 $$\widehat{\beta} \equiv \frac{Cov\left(\Delta y_t^R, \Delta y_t^N\right)}{V\left(\Delta y_t^R\right)} \quad (6.38)$$

Fixed Income Securities: Valuation, Risk, and Risk Management, Veronesi, 2010

Chapter 5 Interest Rate Derivatives: Forwards and Swaps

P. 157

$$F(t,T_1,T_2) = \frac{1}{(1+\frac{f_n(t,T_1,T_2)}{n})^{n(T_2-T_1)}} \quad (5.5)$$

$$F(t,T_1,T_2) = e^{-f(t,T_1,T_2)(T_2-T_1)} \quad (5.6)$$

P. 160

$$f(0,T,T+\Delta) = r(0,T) + (T+\Delta)\frac{r(0,T+\Delta)-r(0,T)}{\Delta} \quad (5.14)$$

P. 161

$$r(0,T_n)T_n = (r(0,T_1)\Delta + f(0,T_1,T_2)\Delta + \ldots + f(0,T_{n-1},T_n)\Delta) \quad (5.21)$$

P. 166

Value of FRA at $t = V^{FRA}(t) = V^{fixed}(t) - V^{floating}(t)$

$$= N * [\tfrac{Z(0,T_1)}{Z(0,T_2)} * Z(t,T_2) - Z(t,T_1)] \quad (5.25)$$

P. 170

$$P_c^{fwd}(0,T,T^*) = \frac{c}{2}\sum_{i=1}^{n} P_z^{fwd}(0,T,T_i) + P_z^{fwd}(0,T,T_n) \quad (5.34)$$

P. 175

$$V^{swap}(T_i;c,T) = 100 - (\frac{c}{2}100\sum_{j=i+1}^{M} Z(T_i,T_j) + Z(T_i,T_m)100) \quad (5.40)$$

P. 176

$$c = n(\frac{1-Z(0,T_M)}{\sum_{j=1}^{M} Z(0,T_j)}) \quad (5.43)$$

P. 181

$$f_n^s(0,T,T^*) = n\frac{1-F(0,T,T^*)}{\sum_{j=1}^{M} F(0,T,T_j)}$$ (5.51, This is a correction to the formual in the text.)

Chapter 6 Interest Rate Derivatives: Futures and Options

P. 203

P&L from futures at t=k*contract size*[$P^{fut}(t,T) - P^{fut}(t-dt,T)$] (6.4)

Handbook of Asset and Liability Management Vol 2. Applications and Case Studies

Chapter 13

None

Chapter 18

P839

$r(t) = \beta_0 + (\beta_2 + \beta_2) * (1 - e^{-t/\tau})/(-t/\tau) - \beta_2 e^{-t/\tau}$

Nelson Siegel yield curve

$\lim_{t\to\infty} r(t) = \beta_0$ Long interest rate

$\lim_{t\downarrow 0} r(t) = \beta_0 + \beta_1$ Short interest rate

β_2 and τ Curvature & scaling parameters

ALM for Life, Annuities, and Pensions, Hatfield (2024) – Section 5

P52

$$D_{lower} \le Dur_A + \frac{L}{E}(Dur_A - Dur_L) \le D_{upper}$$
$$D_{lower} + \frac{L}{E}Dur_L \le \left(1 + \frac{L}{E}\right) Dur_A \le D_{upper} + \frac{L}{E}Dur_L$$
$$\frac{D_{lower} + \frac{L}{E}Dur_L}{1 + \frac{L}{E}} \le Dur_A \le \frac{D_{upper} + \frac{L}{E}Dur_L}{1 + \frac{L}{E}}$$
$$\frac{E \cdot D_{lower} + L \cdot Dur_L}{E + L} \le Dur_A \le \frac{E \cdot D_{upper} + L \cdot Dur_L}{E + L}$$
$$\frac{E \cdot D_{lower} + L \cdot Dur_L}{A} \le Dur_A \le \frac{E \cdot D_{upper} + L \cdot Dur_L}{A}$$
$$\frac{E \cdot D_{lower} + DDur_L}{A} \le Dur_A \le \frac{E \cdot D_{upper} + DDur_L}{A}. \tag{5.3.2}$$

P53

$$C_{lower} \le Conv_A + \frac{L}{E}(Conv_A - Conv_L) \le C_{upper}$$
$$C_{lower} + \frac{L}{E}Conv_L \le \left(1 + \frac{L}{E}\right) Conv_A \le C_{upper} + \frac{L}{E}Conv_L$$
$$\frac{C_{lower} + \frac{L}{E}Conv_L}{1 + \frac{L}{E}} \le Conv_A \le \frac{C_{upper} + \frac{L}{E}Conv_L}{1 + \frac{L}{E}}$$
$$\frac{E \cdot C_{lower} + L \cdot Conv_L}{E + L} \le Conv_A \le \frac{E \cdot C_{upper} + L \cdot Conv_L}{E + L}$$
$$\frac{E \cdot C_{lower} + L \cdot Conv_L}{A} \le Conv_A \le \frac{E \cdot C_{upper} + L \cdot Conv_L}{A}$$
$$\frac{E \cdot D_{lower} + DConv_L}{A} \le Conv_A \le \frac{E \cdot D_{upper} + DConv_L}{A}. \tag{5.4.1}$$

LDI Explained – BMO Global Asset Management

None