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PRACTICAL APPLICATION

Practical Application of "Do Jumps Matter in the Long Term? A Tale of Two Horizons"

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Introduction and Summary of the Paper

In their paper "Do Jumps Matter in the Long Term? A Tale of Two Horizons," Bégin and Boudreault (2021, BB hereafter) investigate equity market jumps in the context of economic scenario generators (ESGs). ESGs are commonly used to generate simulations used to drive decision-making in a variety of actuarial applications.[1]

The problem is that many actuarial applications are long-term in nature, but equity market jumps tend to happen sporadically—in the much shorter term. For example, daily equity returns have much heavier tails and more asymmetry than annual returns. As such, annual returns are much closer to being normally distributed than daily returns.[2] This raises the issue of capturing asset return dynamics over different horizons and the relevance of jumps for ESGs in light of what BB refer to as "horizon duality."

BB study three research questions, with the third question specifically related to applications.

1. How can we replicate dynamics at different timescales? Is it a model building problem or an estimation problem?
2. How do jump-diffusion models compare with typical econometric models found in the ESG literature?
3. What are the impacts of jumps on actuarial applications?

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[1] For a good overview on ESGs, see the three-part series published by the Society of Actuaries in "The Modeling Platform" by Rahat Jain, Dean Kerr and Matthew Zhang: https://www.soa.org/sections/modeling/modeling-newsletter/2021/july/mp-2021-07-jain/

[2] See Campbell, Lo, and Mackinlay (1997) and Cont (2001) for more details. Cont coined the term "aggregational Gaussanity" to describe the scaling phenomenon.

The authors find the following:

1. Equity market dynamics (i.e., key moments such as mean, variance, skewness, kurtosis, and the full distribution) can indeed be replicated at different timescales (e.g., daily, monthly, quarterly, and annually) by using a jump-diffusion generating model.[3] This result is, however, dependent on the estimation strategy used to find the model parameters. The authors fit the model parameters using a generalized method of moments-based approach that relies on multiple moments over different horizons instead of the more commonly used maximum likelihood estimation approach. This allows them to conclude that replicating equity market dynamics is both a modelling and an estimation problem. In terms of choosing a model, BB conclude that incorporating *both* return jumps and stochastic volatility is necessary to get an adequate fit to past dynamics (i.e., SVYJ, SVCJ, or SVCJSI).

2. BB consider eight additional equity return models commonly used in the actuarial arena.[4] It turns out that, out of the eight other models, only the regime-switching lognormal model with three regimes—RSLN(3)—can adequately replicate the return dynamics at different time horizons.

3. BB analyze three different actuarial applications by simulating one million paths of daily returns over 30 years from the jump-diffusion models, the regime-switching lognormal models, and the standard lognormal models (a total of eight models).

    a. Accumulation analysis—BB simulate the growth of $100 over different time horizons. This has broad applications to any situation where capital accumulation is relevant. For example, simulating the build-up of defined contribution plan assets or the growth of an account subject to a guaranteed minimum maturity benefit. The authors demonstrate that while the mean and standard deviation of the accumulated values are similar across models, this is not the case in the left tail. The jump-diffusion models (i.e., SVYJ, SVCJ, and SVCJSI), as well as the RSLN(3) model, produce lower accumulation results, especially at the 0.1st and first percentiles. These models also give the highest probability of observing long periods of equity drawdowns.

    b. Solvency analysis—BB simulate the value of $100 over 10 and 30 years and define an insolvency event occurring if the value reaches $70 over the projection horizon. Such application is relevant as it resembles a mature defined benefit pension plan or a guaranteed minimum withdrawal benefit whose subaccount balance could be insufficient to pay for future withdrawals. Again, the jump-diffusion models and the RSLN(3) model produce higher probabilities of insolvency and greater deficits, should insolvency happen.

    c. Dynamic portfolio problems—BB analyze a hedging portfolio that attempts to replicate the payoff of $100 worth of 10-year at-the-money put options. The hedging portfolio relies on the Black and Scholes' delta, and the hedging error is assessed over the 10-year horizon. Without any surprises, the jump-diffusion models and the RSLN(3) model yield worse hedging errors overall.

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[3] BB actually look at four different jump-diffusion models: stochastic volatility (SV) only, SV with jumps in returns only (SVYJ), SV with correlated jumps in return and variance (SVCJ), SV with correlated jumps in return and variance along with stochastic jump intensity that allows for jump clustering (SVCJSI).

[4] Lognormal (LN), lognormal with jumps, regime-switching lognormal with two or three regimes, GARCH, nonlinear GARCH, GARCH with jumps, and nonlinear GARCH with jumps.

Additional Application

In this essay, we consider an additional application. Using the utility framework described by Warren (2019), we examine the impact of using one of BB's fitted jump-diffusion models on a pension plan sponsor's long-term asset allocation decision. We want to compare asset allocation results to those using the standard finance workhorse model of a geometric Brownian motion (i.e., lognormal return generating process or LN hereafter). Below we outline the approach which was intentionally kept simple for the sake of space and convenience:

- Stocks and bonds are used as core portfolio building blocks.
  - Equity return: mean is 9.4%, and average volatility is 15.7% (note that SVCJSI uses the GMM calibration results from BB). Returns are nominal.
  - Bond return: mean is 4.9%, and volatility is 7.9% (Warren, 2019). Returns are nominal.
  - Correlation between stocks and bonds: it is assumed to be 0.045 (Warren, 2019).
- Pension liabilities are assumed to behave like bonds but with a 1% tracking error.
- Annual benefit payments are assumed to be equal to 5% of the initial liability amount.
- No pension contributions are assumed.
- We use a reference-dependent utility function with parameters specified in Warren (2019).
- We run two simulations and compute the optimal portfolio under each. In keeping with Warren's framework, the optimal portfolio combines stocks and bonds that maximize a utility function score.
  - Simulation 1—uses LN for stocks and bonds using the means, volatilities and correlations noted above. We simulate 1,000 paths.
  - Simulation 2—replaces the LN results for equities with those provided by BB for their SVCJSI model. BB provide the parameters for the jump-diffusion models in Table 2 of their paper. This is helpful as the user does not have to parameterize the model using the intensive generalized method of moments approach. Using the given parameters, we coded the model in Python, supplemented with a correlated LN model for bond fund return, and simulated 1,000 paths of equity returns and bond fund returns.

Results and Observations

Table 1.

EQUITY ALLOCATION AS A FUNCTION OF THE FUNDED RATIO FOR TIME HORIZONS OF 3 AND 10 YEARS

	3 Years		10 Years	
Funded Ratio	**LN**	**SVCJSI**	**LN**	**SVCJSI**
0.70	1.00	1.00	1.00	1.00
0.75	1.00	1.00	1.00	1.00
0.80	1.00	1.00	1.00	1.00
0.85	0.81	0.80	1.00	1.00
0.90	0.54	0.46	1.00	1.00
0.95	0.33	0.27	0.95	1.00
1.00	0.23	0.19	0.96	1.00
1.05	0.20	0.15	0.88	0.99
1.10	0.16	0.10	0.80	0.97
1.15	0.25	0.14	0.86	0.85
1.20	0.35	0.21	0.88	0.75

We show a 3-year horizon to align with Warren (2019). He argues that even though a pension fund may have a long horizon, a plan sponsor may look to evaluate their decision over a shorter time period.

Both models suggest a 100% equity allocation with a 3-year time horizon when the initial funded ratio is less than 80%. As soon as the funded ratio is higher than 80%, the allocations exhibit a U-shape, with the SVCJSI model being systematically smaller than that of the LN model. The jumps clearly make the equity riskier, reducing the allocation when the funded ratio is large enough compared to the LN model. This is consistent with our agent being risk-averse and not wanting to be impacted by such jumps.

With a longer horizon of 10 years, both models suggest 100% equity allocation for initial funded ratio smaller than 90%. They differ when it is higher than 90%. On the one hand, we have a higher equity allocation for SVCJSI when the initial funded ratio is between 95% and 110%. When the initial funded ratio is larger than 115%, on the other hand, the equity allocation is smaller for SVCJSI. The higher optimal equity allocation suggested by SVCJSI for the initial funded ratio between 95% and 110% is likely related to the runoff of the pension plan. At the end of the 10th year, 50% of initial liability is paid off. With potential return jumps, tail events can lead to much worse funded ratios, but the long horizon justifies higher equity allocation to recover from these tail events and capture the equity risk premium (ERP). This benefit wears off with a higher initial funded ratio as the chance of getting below the target funded ratio (i.e., 100%) is much lower and is not penalized significantly by the utility function.

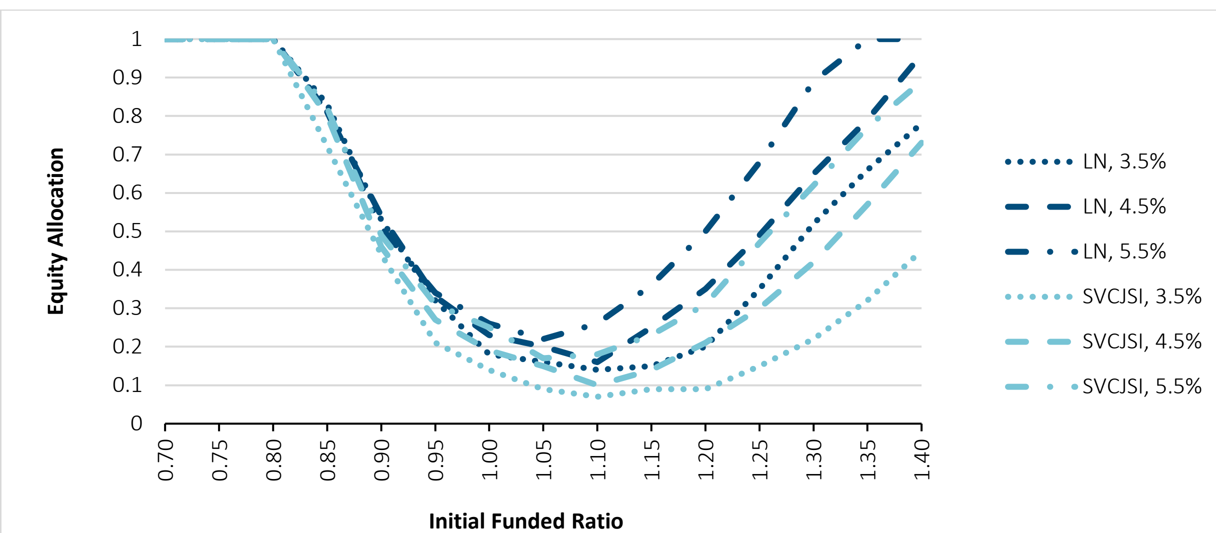
Sensitivity Tests

Two additional sensitivity tests are performed to evaluate whether this pattern persists with different ERPs and degrees of risk aversion.

EQUITY RISK PREMIUM

We assumed an ERP of 4.5% in our base case—the difference between the expected equity and bond fund returns. We test two additional ERP assumptions (i.e., 3.5% and 5.5%) by adjusting the expected bond fund return; correlation is left the same. We only consider the 3-year horizon for the sake of space.

Figure 1
IMPACT OF ERP ON THE EQUITY ALLOCATION FOR A 3-YEAR HORIZON.



It is clear that the equity allocation suggested by both the LN and SVCJSI models increases with the ERP, as expected. The differences between the LN and SVCJSI models observed under the base case are also present under the alternative ERP assumptions.

DEGREE OF RISK AVERSION

The reference-dependent utility function used in the base case follows that used in Warren (2019):

$$\text{Utility}(\text{FR}_t) = \begin{cases} \gamma\left[\left(\frac{\text{FR}_t}{\text{TFR}}\right)^{\alpha} - 1\right] & \text{if } \text{FR}_t \geq \text{TFR} \\ \lambda\left[\left(\frac{\text{FR}_t}{\text{TFR}}\right)^{\beta} - 1\right] & \text{if } \text{FR}_t < \text{TFR} \end{cases}$$

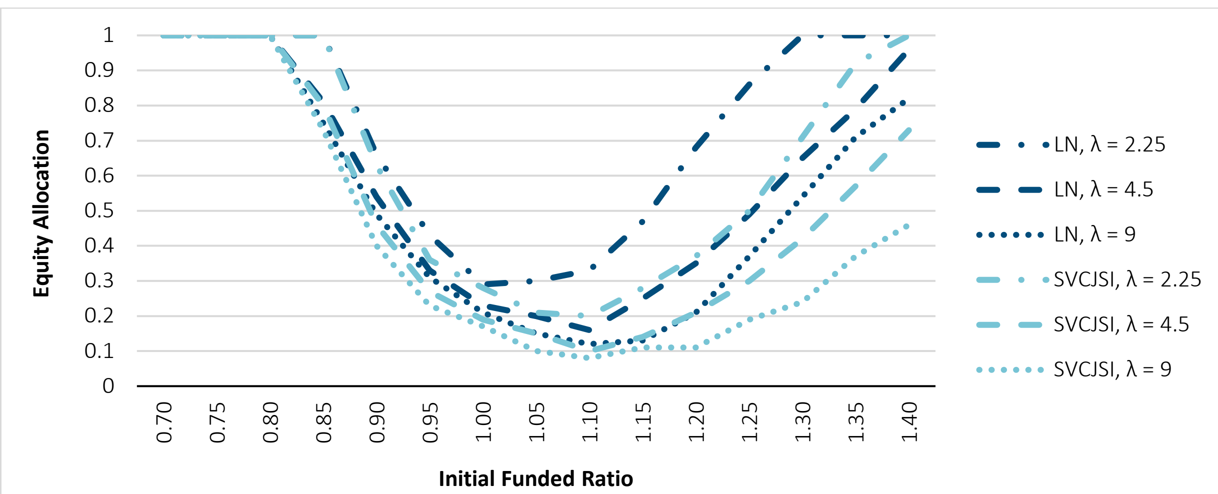
where

FR_t: funded ratio at time t;
TFR: target funded ratio, which is assumed to be 1 in this example,
α: curvature parameter on overfunding (assumed to be 0.44),
β: curvature parameter on underfunding (assumed to be 0.88),
γ: weighting parameter on overfunding (assumed to be 1),
λ: weighting parameter on underfunding (assumed to be 4.5).

By adjusting the weighting parameter on underfunding λ, the degree of risk aversion can be changed. In other words, a higher λ leads to higher risk aversion. Two additional cases are thus tested by halving and doubling the value of λ in the base case.

Figure 2

IMPACT OF RISK AVERSION ON EQUITY ALLOCATION FOR A 3-YEAR HORIZON



It is clear that a higher degree of risk aversion leads to lower equity allocations for both ESGs. The differences between the LN and SVCJSI models observed under the base case are still present under the alternative utility function assumptions. Our results are thus robust to this assumption.

Conclusions

BB's paper is important because simulations are very commonly used in many actuarial applications to aid with effective decision-making. As BB show, equity return simulations should capture *both* short- and long-term dynamics if users want to have confidence in the efficacy of the simulation model to replicate market features. A model that better reflects these time-horizon-dependent stylized facts will lead to better decisions and, therefore, better outcomes for all stakeholders.

References

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