Pricing and Risk Management of

Variable Annuities

with Multiple Guaranteed Minimum Benefits

Feng Sun

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Abstract

Sales of variable annuities (VAs) are driven predominately by the performance and volatility of the equity markets. Since guaranteed minimum benefits (GMB) riders were introduced, especially the rich living benefits, the previous slow-paced sales began to surge again.

During the last few years, almost all VA carriers have introduced or updated their living benefit riders in addition to guaranteed minimum death benefits (GMDB). Several products offer more than one type of guaranteed living benefit (GLB) in order to boost sales.

The objective of the project outlined in thispaper is to develop a financial model for VA products that have various combinations of GMB riders, assess the risks associated with them, analyze risk and reward tradeoffs and address the implications to risk management.

By using stochastic modeling techniques on a ***MoSes*** platform, it can be demonstrated that, on average, profitability (measured as present value of distributable profits) is improved by adding more GMB riders to the existing VA products (with or without GMB riders). Meanwhile, the stability of earnings (measured as the volatility of profitability) is deteriorating. The dispersion of both risk and profitability are substantially amplified as a result. This fits perfectly into risk/return tradeoff framework. Insurance companies, who want to offer VAs with multiple GMB riders to boost their sales and pursue possibly more profits, need to ensure that the risks they assumed or retained are within acceptable tolerance levels by using various risk management tools.

Offering multiple GMB riders on VAs can be beneficial to policyholders and insurance companies, but is not without its risks. Because GMB riders are attracting customers, no matter whether insurance companies are profit-oriented, sales-oriented, or competition-oriented, we are expecting to see more companies add multiple GMB riders to their VA products.

INTRODUCTION

Variable annuities have become one of the top success stories for the life insurance industry in the last decade. As the tax advantage of VAs over competing investment vehicles such as mutual funds diminish, guarantees have played an increasingly important role in differentiating VA contracts. In particular, relatively rich living benefits available at attractive prices helped drive sales during the past several years.

While focusing on sales, VA issuers need to realize that the resulting risk stemmed from the addition of benefit guarantees on VA products. Understanding the risk associated with multiple guarantees and having a way to manage it becomes the key to success for VA writers.

GMB riders on VA have complex option-like characteristics. Traditional deterministic modeling could not capture the risks profiles of the guarantees and meet the modeling needs. In order to better understand the risk/return tradeoffs, as well as anticipated changes in regulatory requirements for reserves and capital, more sophisticated financial modeling capabilities, such as enhanced stochastic scenario modeling, became necessities.

By implementing stochastic modeling using ***MoSes*** software, this project analyzes the risk profiles and profitability of VA products that contain various combinations of multiple guarantee riders, which may give issuers an advantage on sales. This project also discusses the major risk factors associated with these guarantees and a way to manage them.

Six sections follow this introduction. The Background section describes and presents the features and specifications of various individual guarantee minimum benefits on Vas currently in the market place. The Modeling Process section gives an overview of the stochastic scenario modeling process and details on the VA guarantees (including multiple GMB riders) modeled. The Results and Analysis section uses the model to produce relevant quantities to analyze the risks and profitability related VAs multiple guarantee riders. The Conclusions and Recommendations section reports on the various issues related to these multiple guarantee riders and the implication to risk management. The Limitations and Restrictions section reminds readers to use caution when making an inference based on the stochastic model used in this paper At the end of the report is an appendix in which the product specifications and assumptions used in this project are provided in greater detail.

BACKGROUND

Variable annuities typically contain guarantees that expose the carrier to equity market risks. These guarantees are either payable on death, such as Guaranteed Minimum Death Benefits (GMDB); or upon policy withdrawal, such as GMWB; or during the payout phase, such as GMIB; or by the end of the accumulation phase, such as return of premium after year five to year 10 (GMAB).

The Features of Different Guarantees

TABLE 1

Typical VA Guaranteed Minimum Benefits (GMBs) in the Market

Type	Nature of Guaranteed Minimum Benefit	Standard Feature	Rich Feature	Annual Charges
GMDB	Lump Sum on death	Annual Ratchet or 5% roll-up to age 80	Combination, or 7% rollup	15-35 bps
GMIB	Account value at Annuitization	5% roll-up	6% roll-up and ratchet	Up to 75 bps
GMAB	Lump Sum at the end of specified period (5-10 years)	Generally return of premium	Available on equity funds	25-75 bps
GMWB	Return of premium via annual withdrawal	Withdraw 7% of premium annually	Reset Provision	35-75 bps

The GMDB guarantees a minimum lump sum payout upon death. A variety of GMDBs are offered today, from relatively low risk return of premium benefits to rich combination benefits that provide a guaranteed death benefit equal to the greater of an accumulation of premium at 5 percent per annum and an annual ratchet. Charges for the GMDB generally range from 0 to 35 basis points (bps,) depending on the richness of the benefit. GMDBs are offered by virtually all VA writers today.

The GMIB guarantees a minimum account value upon annuitization, generally based on the initial principal accumulated at a rate of 3 percent to 6 percent per annum, an annual ratchet or a combination of both. The minimum account value is used to purchase a payout annuity at conservative guaranteed purchase rates. The charge for this feature ranges from 15 to 75 bps per annum. It is offered by many of the larger variable annuity writers as a rider.

The GMAB guarantees that the policy surrender value will be a minimum amount at a given point in time (for example, the later of 10 years or attained age 70). This feature typically has a charge of 25 to 75 bps per annum and requires policyholders to follow certain asset allocation strategies. Funds with high volatility are typically not eligible. A limited number of companies currently offer this type of benefit.

The GMWB is a relatively new guarantee feature that is attracting significant attention and sales. It typically guarantees that investors will receive their money back over a period of 15 years at a rate not to exceed 7 percent per annum, regardless of market conditions. Current charges for this benefit range from 35 to 75 bps of assets per annum. GMWB feature becomes more and more popular in the market place; some companies even offer GMWB for life on VA.

GMBs are essentially setting up floors for policyholders on annuity payout contingency on certain events such as death or other decisions (such as lapse or withdrawal) policyholders made. These guarantees can be thought of as put options in the sense that its exercise price is minimum guarantee amounts based on the design (e.g., equal to premium for return of premium, equal to maximum anniversary fund value for ratchet) and exercising these options is triggered by certain pre-specified events such as death, lapse, withdrawals, etc.

GMWB, GMAB and GMIB are commonly referred to as Guaranteed Living Benefits, or GLBs. Although they share similar option-like characteristics with GMDB, GLBs differ from GMDB in the way the options are utilized. While the utilization of a GMDB would be expected to follow a mortality table, the utilization of a GLB is much harder to predict because it is driven by policyholder behavior. In other word, to collect the GMDB benefits, the insured has to die if the GMDB is in-the-moneyness (ITM). Other policyholder behaviors such as lapse and withdrawals may have a similar impact on GMDB as well as GLBs. Experience in policyholders' behaviors is still limited and needs to be analyzed under a variety of market scenarios.

We have seen VAs with more than one guarantee rider in current market, and we expect to see various combinations of these four types of guarantee riders added to one VA policy, and policyholders can have several bottom line protections against adverse equity market should the account value fall below the guaranteed level. For instance, a VA policy with both GMDB and GMIB riders protects policyholder against unfavorable equity market in case of death or annuitization.

Because no more than one guarantee may be exercised in general, it seems plausible that for VAs with multiple GMB riders, the GMB fees should give the insurer an advantage to cancel out some of guarantee related risk exposures and eventually make more profit. On the other hand, under the worst-case scenarios, where funds underperform, those guarantees may all be in-the-money, and fees that are a fixed percentage of account value are reduced. At the same time, the more fees are deducted from account value, the more account value goes down, and the higher the guarantee related risk exposures become. This could eventually make the already-lowered profits even lower.

The impact on profitability of offering multiple GMB riders on VAs is not intuitive. By using stochastic scenario modeling technique, we expect to have a better understanding of the risk and reward of writing this type of business and further improve the risk management practice.

MODELING PROCESS

The challenge for most VA issuers is to develop a financial model for variable products and their guarantees. Not only the interest rate, but also the equity market drives the profit of a variable product, and some of the guarantees are new to market. The traditional actuarial software may not be able to keep up with the latest product innovations.

VA guarantees have complex optionalities. Traditional deterministic modeling could not capture the risk profiles of the guarantees, whereas stochastic scenario modeling is able to deal with skewed risk distributions and significant volatility in the underlying variables, it can reveal the distributions of claim costs and earnings and provide sufficient information to quantify the risk and offer insight into the volatility of earnings. In particular, it is capable of modeling “tail” or extreme events and it provides a range of possible outcomes, which can be superior to a point estimate provided by a deterministic approach.

The Model Office

A variable annuity application on ***MoSes*** platform was developed to handle stochastic modeling of VA with GMBs. Various tests were conducted under different asset and liability assumptions, modeling methodologies, etc., to ensure the accuracy of modeling. Product specifications and assumptions were chosen based on typical variable annuities and associated guarantees available in the marketplace (see Appendix for details.) The assumption set uses equity scenarios generated by the regime-switching lognormal model. For each scenario, the ***MoSes*** model calculates the present value of guarantee claim costs and Present Value of Distributable Earnings (PVDE). The present value of distributable earnings was the primary metric, with the probability distribution of PVDE representing the variable annuity’s profit and risk profile.

VA Guarantees Modeled

Four key single GMBs with enriched features were selected, with the thought that these guarantees may be more representative of their class, more attractive to policyholders, and may represent a future trend. The descriptions and fees are listed in Table 2 below.

TABLE 2 VA with Single GMB modeled		
GMB Name	**Description**	**Fees**
GMDB (D)	Max (5% Rollup, Ratchet) payable upon death	35 bps
GMIB (I)	Max (5% Rollup, Ratchet) payable upon annuitization	45 bps
GMAB (A)	Return of premium upon surrender at the end of 5 years	50 bps
GMWB (W)	Allow withdrawal up to 7% annual after 3-year waiting period	75 bps

In addition, we tested all possible hypothetical combinations of the key GMB riders with charges being the straight summation of the charges of the single GMBs they are composed of. No fee reductions are assumed. VAs with multiple GMB riders modeled, along with their corresponding acronyms are listed in Table 3.

TABLE 3 VA with multiple GMBs modeled			
	Abbreviations	**Combinations**	**Fees**
Combination of two Guarantees	**DI**	GMDB+GMIB	80 bps
	DA	GMDB+GMAB	85 bps
	DW	GMDB+GMWB	110 bps
	IA	GMIB+GMAB	95 bps
	IW	GMIB+GMWB	120 bps
	AW	GMAB+GMWB	125 bps
Combination of three Guarantees	**DIA**	GMDB+GMIB+GMAB	130 bps
	DIW	GMDB+GMIB+GMWB	155 bps
	DAW	GMDB+GMAB+GMWB	160 bps
	IAW	GMIB+GMAB+GMWB	170 bps
Combination of four Guarantees	**DIAW**	GMDB+GMIB+GMAB+GMWB	205 bps

Scenarios Used

There is no specific requirement on scenarios used for projections. However, we used 1000 scenarios selected from the American Actuary Academy’s 10,000 prescribed scenarios, which were generated using the regime-switching lognormal model. It reflects tails that are “fat enough,” as required by the proposed C-3 Phase II regulation.

Investment Allocation of Account Value

Initial deposits are invested in separate account equity and bond funds. The assumed customer allocation is 90 percent in the separate accounts, wherein rebalancing maintains 80 percent in an

S&P index fund and 10 percent in an intermediate bond fund. The remaining 10 percent is invested in the general account fixed interest guarantee.

Model Points: Ages, Gender, Sales, and Initial Deposits

The New Business file containing the model points is summarized in Table 4.

TABLE 4

Sample Model Points

Issue Age	Gender Mix (% of Male)	Premiums	Number of Policies
45	60	11,788,000	251
55	60	26,814,000	335
65	60	20,378,000	236
73	60	4,968,000	63
78	60	4,872,000	64
83	60	3,179,000	39
88	60	1,130,000	12
Total		**73,129,000**	**1000**

Policyholder Behaviors

Dynamic policyholder behavior assumption such as lapse rate, annuitization rate and withdrawal rate were assumed to vary based on the in-the-moneyness of the guaranteed amount over account value. For instance, low lapse rates and high withdrawal rates are assumed if the guarantee is deep in-the-money.

RESULTS AND ANALYSIS

One of advantages of stochastic modeling is that one can calculate the present values of the costs of GMBs under each scenario and be able to investigate how the GMBs behave under each scenario.

Pricing VA with Multiple GMB Riders

I. The Claim Costs of GMBs

1. Cost Structure

In order to test the claim costs of GMBs, we set GMB charges to zero and calculate the present value of GMB claim costs discounted at 11.5 percent and convert them into annual cost.

TABLE 5

Claims Costs of GMBs on VA

	GMDB	GMIB	GMAB	GMWB
Claim Frequency	100%	100%	36%	14%
Annual Costs	30 bps	75 bps	33 bps	10 bps

Different guarantees have different cost structures. GMDB and GMIB have high frequency, whereas GMAB and GMWB have relatively lower frequency. The annual costs differ significantly from the market pricing costs listed in Table 1, except for GMDB.

There are a variety of reasons that might cause these differences. First of all, the guarantee feature we chose for GMDB and GMIB are richer ones. Secondly, the scenarios used may differ materially. For example, the return and the volatility of the scenarios used here could be different from the one used by insurers in pricing model. Thirdly, a different mix of model points could result in different outcomes. If only issue age 55 were modeled, the cost could be much lower than policies that have issue ages 78 and 83. Fourthly, because policyholder behavior drives the GLB cost, pricing actuaries might have their own way of modeling policyholder behavior. Besides, there are other variables such as competitor’s price or competitive position that might cause the difference in pricing. For example, there was less competition for GMWB than for GMDB several years ago. A writer might charge more and still have a decent amount of sales. Finally, capital requirements also play a role in pricing. As we will see later, GMWBs require much higher required capital, and this could make the writer increase the charges to boost their return on equity (ROE.)

2. Correlations between Present Value of Claim Costs of GMBs on VA

Correlations between single GMB riders could indicate if there is a risk cancellation when bundling GMBs. Present values of claim costs of GMBs were calculated. The correlation coefficients were calculated based on 1000 scenarios runs and the results are listed in Table 6 below.

TABLE 6

Correlations between Present Values of the Costs of GMBs on VA

	GMDB	GMIB	GMAB	GMWB
GMDB	100%			
GMIB	99%	100%		
GMAB	37%	34%	100%	
GMWB	40%	41%	31%	100%

Not surprisingly, the four types of guarantees are positively correlated when setting fees to zero. This is because these options are all in-the-money at the same time when account values drop below a certain level. GMDB and GMIB are almost perfectly correlated. This implies that there is hardly any natural hedging possibility by bundling GMDB and GMIB. However, GMAB and GMWB are not highly correlated (the coefficient is 31 percent). This is due to the low frequency of GMWB claims. This implies that there might be some cost reduction benefit by bundling GMAB with GMWB.

TABLE 7

Correlations Between Present Values of the Costs of GMBs on VA with fees

	GMDB	GMIB	GMAB	GMWB
GMDB	100%			
GMIB	98%	100%		
GMAB	54%	44%	100%	
GMWB	62%	54%	73%	100%

In reality, these guarantees cannot be offered for free. We recalculate the correlations between GMBs with fees and find that the correlations coefficients increase. Charges of GMBs reduce the account value and cause guarantee exposure to increase for all types of GMB.

3. Cost Reductions by bundling GMBs

The guarantee claim costs were tested against the sum of corresponding guarantees for both with fees and without fees.

TABLE 8

Cost Reductions for VA with Multiple GMBs without Fees (in bps)

GMBs	Combinations	Sum of single GMBs	Cost Reduction / (Increment)
DI	105	105	0
DA	63	63	0
DW	37	40	3
IA	108	108	0
IW	83	85	2
AW	29	43	14
DIA	130	138	8
DIW	113	115	2
DAW	66	73	7
IAW	114	118	4
DIAW	144	148	4

When setting fees to zero, by combining GMBs, we compare the total cost of combined GMBs and that of the sum of single GMBs that makes up these combinations. For instance, for DIA, we compare the total of claim costs of 130 bps against the sum of claim costs of single GMBs (e.g., GMDB, GMIB, and GMAB) listed in Table 2, which is 138 bps (=30bps+75bps+33bps).

As Table 8 shows, there is barely any cost savings for DI, DA and IA. There are some cost savings for DW, IW and AW. Interestingly, most of the reductions happen to be with GMWB. This is due to the fact that GMWB has relatively low frequency of claims and therefore lowers correlations with other GMBs. For three or four GMBs combined, there are consistent cost reductions. And the reductions are all less than or equal to 14 bps of account value. The highest savings of 14 bps occurs with AW.

Cost reduction is sensitive to the fee charged. Table 9 below shows that charges wiped out almost all the cost reduction except for AW (e.g., combination of GMAB and GMWB). By combining more than one guarantee rider in one policy, the account value goes down much faster regardless of fund performances, and this eventually translates into higher claim costs of guarantees under unfavorable scenarios.

TABLE 9 Cost Reductions for VA withMultiple GMBs with Fees			
GMBs	**Combinations**	**Sum of single GMBs**	**Cost Reduction / (Increment)**
DI	116	106	(11)
DA	68	63	(5)
DW	42	40	(2)
IA	117	109	(8)
IW	93	86	(7)
AW	41	43	2
DIA	162	139	(23)
DIW	131	116	(16)
DAW	80	73	(7)
IAW	134	119	(15)
DIAW	177	149	(29)

II. The Present Values of Distributable Earnings

In order to analyze the profitability of offering multiple GMB riders on VA, we investigate distribution of the PVDEs of combinations of GMBs with corresponding individual GMB components and the subsets of combinations of GMB riders that make up the target combination of GMB riders produced by stochastic modeling. Here we take DI, DIA and DIAW as examples to show the risk profiles of combinations of two, three and four GMBs. Other combinations follow similar patterns.

TABLE 10 Statistics of PVDEs of VA with Multiple GMBs After Fees under 1000 Scenarios							
Percentiles	**GMDB**	**GMIB**	**GMAB**	**GMWB**	**DI**	**DIA**	**DIAW**
Average	0.16%	-0.01%	0.63%	0.99%	0.28%	1.01%	1.45%
Standard Deviation	1.04%	1.35%	0.97%	1.33%	1.75%	2.06%	2.49%

Table 10 shows that by adding different GMB riders to existing VA product (regardless of existing GMB riders), PVDEs improve across the board on average of 1000 scenarios tested. PVDE reaches its highest point at DIAW. However, this higher PVDE comes at sacrificing of earning stability; standard deviations of PVDE increase as more GMB riders are added in.

The percentile of PVDEs under 1000 scenarios (see Table 11) tells the same story, but in greater detail. The range of the profits becomes wider as more riders are added in. At 50th percentile and above, profits go up as the number of riders increases, whereas, things turn to the opposite way at 25th percentile and below. The tail gets worse and worse as more riders are added in.

TABLE 11 Percentiles of PVDEs of VA with GMBs After Fees							
Percentiles	**GMDB**	**GMIB**	**GMAB**	**GMWB**	**DI**	**DIA**	**DIAW**
99th	2.55%	2.73%	3.01%	3.61%	3.58%	4.85%	5.77%
95th	1.76%	1.85%	2.16%	2.75%	2.56%	3.67%	4.48%
90th	1.38%	1.48%	1.81%	2.32%	2.15%	3.18%	3.93%
85th	1.19%	1.27%	1.61%	2.10%	1.88%	2.88%	3.55%
75th	0.87%	0.93%	1.28%	1.74%	1.48%	2.42%	3.04%
50th	0.21%	0.19%	0.63%	1.10%	0.62%	1.42%	1.96%
25th	-0.49%	-0.78%	0.00%	0.47%	-0.68%	-0.05%	0.39%
20th	-0.69%	-1.10%	-0.16%	0.31%	-1.10%	-0.54%	-0.10%
15th	-0.92%	-1.39%	-0.33%	0.14%	-1.50%	-1.04%	-0.68%
10th	-1.18%	-1.81%	-0.54%	-0.22%	-2.05%	-1.65%	-1.35%
5th	-1.62%	-2.49%	-0.88%	-1.03%	-3.06%	-2.79%	-3.12%
1st	-2.57%	-3.85%	-1.87%	-4.25%	-5.12%	-5.58%	-7.53%

We also calculated Conditional Tail Expectation (CTE) as tail risk measure. For instance, CTE90 of PVDEs is the average of worse 10 percent of PVDEs under 1000 scenarios. The results in Table 12 reinforce the fact that tails get worse for multiple GMBs. However, we also notice that there is one exception at CTE90 between DI and DIA.

TABLE 12 CTE Measures of PVDEs of VA with multiple GMBs after fees							
Percentiles	**GMDB**	**GMIB**	**GMAB**	**GMWB**	**DI**	**DIA**	**DIAW**
CTE90	-1.80%	-2.75%	-1.10%	-1.75%	-3.41%	-3.38%	-4.04%
CTE95	-2.19%	-3.34%	-1.51%	-2.92%	-4.25%	-4.44%	-5.77%
CTE99	-2.96%	-4.41%	-2.42%	-6.19%	-5.94%	-6.74%	-9.91%

This finding fits into high risk/high return framework. We plot PVDEs of DI against those of VA with GMDB and VA with GMIB.

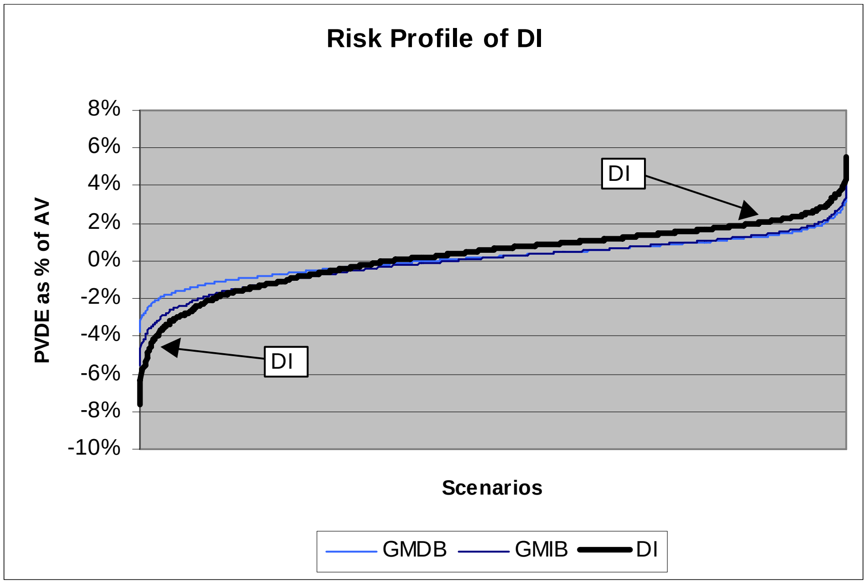

FIGURE 1. RISK PROFILE OF VA WITH GMDB AND GMIB RIDERS (DI)

Figure 1 shows that the volatility of PVDEs of DI increases tremendously even through PVDE improves on average. Combining GMDB with GMIB makes the best case better and makes the worst case worse. Under favorable scenarios, all guarantees are out-of-the-money even with GMB fees deducted from account value. Insurers collect more fees and pay potentially lower guarantee benefits. However, under adverse scenarios, most if not all guarantees are in-the-money, and fees collected may not be able to offset the losses; as a result, profits plummet. This reinforces the conclusion that adding more GMBs on VA creates a high risk/high return scheme. The results are very similar for three GMB riders combination (DIA) and four GMB riders combination (DIAW).

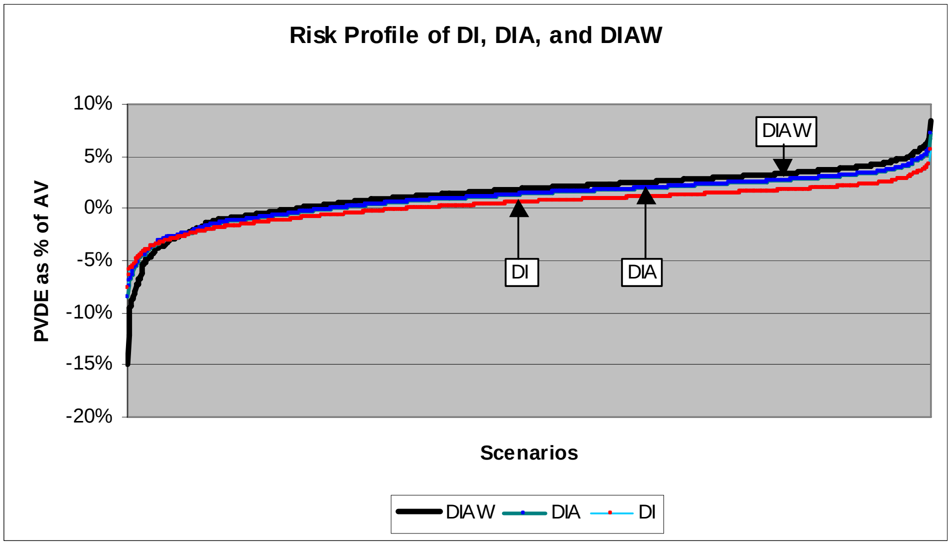

FIGURE 2. RISK PROFILE OF VA WITH DI, DIA, AND DIAW

Figure 2 also shows that DIAW reaches the 0 percent line sooner. In other words, even through bundling GMBs may make worse case worst, it may break even quicker than otherwise.

In summary, by adding more GMB riders to the existing VA product, VA insurers assume more risk. The volatility of earnings goes up and so does the profitability.

Risk Management of VA with Multiple GMB Riders

Risks associated with variable annuity products can be classified into three categories: insurance risk, market risk and policyholder behavior risk. Insurance risks (such as mortality and morbidity risks) follow the large number theorem. This type of risk is more predictable as pools get large. Market risk is unlike insurance risk. Aggregating the exposure does not help to mitigate the risks because individual account value is almost correlated with market. Risk associated with policyholder behavior is very difficult to predict. It may be compounded with market conditions. As a result, market risk and policyholder behavior risk could create a significant challenge to VA GMB writers.

Our analysis shows that cost reduction effect is diminishing as rider fees are assessed when offering multiple GMB riders in one VA policy. Therefore, it increases the concentration of market risk.

Even if adverse scenarios have a low probability of occurring, managers need to be prepared to mitigate the simultaneous impact resulted from revenue loss due to lower fund balances, declining sales due to bad market conditions, and increase in risk-based-capital requirement under newly adopted RBC C3 Phase II regulation in case it does.

One way to manage policyholder behavior risk and equity market exposure is via product innovation or revisiting the pricing. Some companies have also successfully increased fees for GMB features without any adverse marketing consequences to date. This pricing flexibility has empowered companies to more properly price for the risks of such features, including the cost of hedging the risks. However, from this analysis, we noticed that the ability to raise fees on the pricing and product development side can be limited and this strategy may not work in a long run. Even though increasing charges may help the earnings and therefore reduce volatility of earnings, it works better under favorable scenarios. Under adverse scenarios where risks are exposed, increasing fees does not cut the tail very much; on the contrary, it might make tail even worse. In addition, increasing fees may not reduce market shares at initial stage where there is less competition. As competition in the marketplace heats up, fee increases becomes less an option.

There are three other major approaches to managing VA risk: reinsurance, self-insurance and capital markets hedging. Historically, self-insurance and reinsurance have been the risk management strategies of choice. Internal product-based liability hedging and capital market-based hedging have become more and more popular for their ability to limit the tail exposure, and furthermore generate savings on Risk-Based-Capital under C3 Phase II.

The simplest and most common form of hedging is Delta hedging. It is achieved by offsetting the Delta on variable annuities with options or futures on publicly traded indices. Hedging Delta is a first order attempt to limit exposure to market movements. A more sophisticated (and more costly) dynamic hedging program will seek to mitigate volatility risk by also matching the Vega of a block of business by buying options. Exchange-traded options are generally available with terms to expiry lasting up to one or two years. Customized longer-term options are available from brokers on an over-the-counter (OTC) basis.

It is worth noting that hedging comes at a cost. The present value of future profits (e.g., PVDE) will be reduced when including cost and benefits of hedging. Frequent hedging will further limit the tail exposure, but could result in an overall negative present value of future profits. Besides, The effectiveness of many VA hedging programs has yet to be tested, especially in times of severe market stress. It takes time to know the true usefulness of different hedging programs.

In summary, there are a number of risk management tools that can mitigate the risk level and increase the stability of companies' earnings for VA issuers. As far as being able to keep the risks within acceptable tolerance levels, it is up to senior management to decide whether to run the risk naked or use risk mitigation tools in exchange for a possible reduction in profit.

CONCLUSIONS AND RECOMMENDATIONS

Pricing VA with Multiple GMB Riders

Base on stochastic scenario analysis, profitability, measured as present value of distributable profits, is improved for every combination of GMBs on average. However, the volatility increases at the same time. This is consistent with the rule of high-risk-requires-high-return framework. This higher return is at the expense of taking more risk and additional capital allocation.

We found that the GMB claim costs are not in line with pricing cost in the market for various reasons. The claim costs of four single GMBs are positively correlated without fees and charges make the correlations even higher. Combining GMBs offers some cost reductions when fees are not charged. Cost reductions diminish when GMB fees were charged against account value.

Risk Management of VA with Multiple GMB Riders

In light of reduced reinsurance capacity, the proposed changes to capital requirements and pressure from rating agencies, companies are increasingly focusing their attention on hedging the risk from VA guarantees. Capital market hedging can be achieved by offsetting the volatility of earnings on variable annuities with options or futures on publicly traded indices. The effective hedging program cannot only help VA carriers control the risk within a certain level, but also provide a tangible reduction to required capital.

For VA carriers offering multiple GMB riders, it is up to senior management to decide whether to pursue additional profit by assuming additional risks, or sacrifice some profit to keep the risks within acceptable tolerance levels by using hedging or other risk management tools.

LIMITATIONS AND RESTRICTIONS

In general, a model, at best, is a proxy but useful. A perfect model is unattainable. One should be cautious when making inferences out of the model.

Model Office

MoSes is open-box financial modeling software with flexibility. Users are able to modify the code to develop customized tasks. However, like any other model office, more flexibility means more chances of generating errors. This project made every effort possible to ensure the quality of modeling and accuracy of the calculations. However, errors may still exist because like any other software, to test all possible cases is simply not possible.

Scenarios used

Even through there is no specific scenarios requirement in the project, it is widely recognized that the choices or standards used to create a scenario set can dramatically affect the results of stochastic modeling. The scenarios used in this project are 1000 scenarios selected from the American Actuary Academy’s 10,000 prescribed scenarios. Questions can be raised with regard to the number of scenarios, method of selecting scenarios, type of scenarios (risk-neutral versus real-world,) regime-switching lognormal model used to generate the scenarios, etc.

Policyholder behaviors

What is not so widely recognized is that policyholder behavior due to variations in economic scenarios can affect results just as dramatically, or more so. Many of these variations in policyholder behavior are often modeled insufficiently, if at all. Even though dynamic assumptions are used to model policyholder behaviors, they are based on in-the-moneyness of guarantees. This may not be appropriate because human behaviors are not simply driven by in-the-moneyness. Other factors such as policyholders’ age, economic status and knowledge of VA guarantees may have strong impact on policyholders’ decision-making process regarding their policies. Besides, there might be a range of possible policyholder behavior patterns and not all policyholders react “reasonably” or “logically” in the tail scenarios.

Products modeled

The four GMB riders we chose to model have relatively rich features and are expensive as well. These may not be representative of the marketplace. Assuming the rich features are more attractive to policyholders and policyholders intend to elect a combination of those may not be realistic.

Mix of Business

The model points chosen were intended to simulate the mix of business for a typical insurance company. However, issue ages, gender mix and premiums may not reflect reality in testing the risks associated with multiple GMB riders on VA. For instance, younger policyholders may choose different GMB riders and may utilize the guarantees differently from elder policyholders.

Interactions among Factors Listed Above

There might be some interactions among the factors listed above that cause the results to differ materially. For instance, policyholder behavior may vary by issue age as we mentioned before. Or policyholders who bought rich features of guarantees on VAs may tend to be wealthy and may have personal financial advisors. And those professionals may keep their eyes on the guarantees and make recommendations to help policyholders make the most out of their guarantees. In this case, anti-selection reaches its highest level. Insurance companies' profitability suffers the most. The model used in this project did not capture this interaction.

In summary, financial models are never precise, but often very useful. One needs to interpret the results with caution and make reasonable inferences.

APPENDIX

Product Specifications and Assumptions

Policy Duration:	Duration 0
ITM% (In-The-Moneyness):	0%
Mix of Business:	See Table 4 for details
Plan Type:	Non-Qualified
General / Separate Account Mix:	90% Separate Account / 10% General Account
Fund Class:	80% S&P 500 Total Return and 10% Bond Return
Scenarios:	Regime switch lognormal, and meet NAIC calibration points
Projection Period:	30 Years
Annuitization Rate:	3% annually
Dynamic assumptions:	R=The guaranteed benefit divided by Account Value. For R<= 70%, multiple=1; at R=100%, multiple=0.75; at R=120%, multiple=0.50, If R exceed 130%, multiple=0. The multiple is linearly interpolated for R-values between breakpoints. Lapses are dynamically reduced based on increasing value in the guaranteed benefit amount where applicable.
Front End Load:	$35 per policy administrative fee deducted monthly; waived if gross premium or account value is greater than $50,000. Annual charge is never greater than 2%.

Back End Load:

Year	% of Net Premium*
1	8%
2	8
3	7
4	6
5	5
6	4
7	3
8+	0

*Net premium equals total premium paid less reductions for withdrawals in excess of earnings. For surrender charge, net premium is additionally reduced for any free amount remaining in excess of earnings.

Maximum Annual Penalty

Free Withdrawal Amount:	Larger of 10% of gross premium and earnings, defined as policy value less net premium, defined as above
Free Partial Withdrawal:	7% of account value per annum, if no rider. Available in all years and taken monthly. If living benefit rider modeled, 7% of total withdrawal base is taken. Utilization rates: 80%

Renewal Premium:	Permitted, but only single premium modeled.
Death Benefit:	Max of ratchet and rollup reduced pro-rata upon GMDB withdrawal.
Investment Advisory Fee:	122 bps per annum, deducted monthly
Revenue Sharing:	59 bps per annum, deducted monthly
M&E Charges:	130 bps of account value
Commissions:	% of premium: 8.60% for age 0-80; 5.35% for age 81-85; 4.20% for age 86-90% of account value: 1%
Commission Chargeback:	None
DCA:	Not modeled
Acquisition Expense:	$180 per policy
	55 bps of premium
GMB Charges:	GMB Charges as basis points of account value

GMB	Designs	Charges
GMDB	MAX of Roll up and ratchet	35bps
GMIB	MAX of Roll up and ratchet	45bps
GMAB	Return of Premium at year 5	50bps
GMWB	7% maximum withdrawal	75bps

Mortality:	60%/40% male/female blend of Annuity 2000 table
Lapse Rates:	Base lapse rates:

Year	Lapse Rate
1	2%
2	3
3	4
4	6
5	9
6	12
7	15
8	30
9	20
10+	15

Maintenance Expense:	$59.68 per policy
	6 bps of account value
Termination Expenses:	$66.85 per death
	$22.28 per surrender
	$36.79 per partial withdrawal
Premium Tax:	0.05%

Maintenance Expense Inflation:	4%
General Account Earned Rate:	5.25%
Statutory Reserves:	Base/GMDB reserve calculated per CARVM Living Benefit Reserve: Greatest of 1. Accumulation of living benefit fees with interest (5%) 2. PV (at 5%) of future benefits from guaranteed value less current account value. 3. Reserve table factors based on % in or out of the money
Tax Reserves:	Equal statutory reserve
FIT:	35% of taxable income; DAC tax based upon 1.40% capitalizations, amortized over 10 years. DAC capitalization assumes 80% of business non-qualified
Target Surplus:	60 basis points of base stat reserves, plus 7% of GMDB reserves, plus 2.05%/10.25% (if AV above or below premium) of CARVM offset, plus 1.65% of AV if living benefit reserve out of the money, or 3.30% of AV if living benefit reserve in the money
Earned Rate on Surplus:	5%
Reinsurance:	None
Hedging:	None
GMDB:	GMDB guarantees that the policy value will be at least as high as the maximum of 5% rollup (RLP) and ratchet (MAV).
GMIB:	GMIB guarantees that the policy value will be at least as high as the maximum of annually ratchet (MAV) and 5% rollup (RLP).
GMAB:	GMAB guarantees that the policy value will be at least as high as the Guaranteed Future Value ("GFV") at the end of the waiting period (10 years). Withdrawals will reduce the GFV. If policy value at reduction is less than the GFV, the reduced amount is on a pro-rata basis, otherwise on a dollar-to-dollar basis.
GMWB:	GMWB guarantees a withdrawal amount regardless of policy value, even after policy has gone to zero, until the Minimum Remaining Withdrawal Amount ("MRWA") runs out. Policyholder is allowed to take per annum, without penalty, 7% of Total Withdrawal Base ("TWB"), which is initially set to premiums paid and increases by additional premiums after the rider, if any, and decreases by a pro-rata adjustment for withdrawals in excess of the 7% annual free amount. MRWA is the total amount policyholders can withdraw under GMWB, and is initially set to policy value, increases by additional premiums, and decreases by withdrawals on a dollar-to-dollar basis for the 7% annual free amount and by a pro-rata adjustment for withdrawals in excess of the 7% annual fee amount.