SOCIETY OF ACTUARIES

Article from
Risks & Rewards
August 2019
Issue 75

Some Key Insights for Computing Credit Solvency Capital Requirements

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Editor's note: A full version of the research paper that inspired this contribution was published in the European Actuarial Journal.

According to the Solvency II regulation, insurers need to be able to assess the capital needs that cover the risk of annual losses due to credit risk. The applications can be for own risk and solvency assessments as well as for computing internal model solvency capital requirements (SCRs). Being able to measure credit risk is also an important precondition for the asset management of insurers. This short article describes a framework for the computation of credit capital requirements under the constant position paradigm, taking into account recovery rates. Although the framework described was originally derived under the Solvency II regulation, it can also prove useful under other international regulations. Four important steps should be performed to compute credit SCRs: First, relationships linking risk premium adjustment factors (factors that relate realistic and market-consistent probabilities) should be established consistently with the Jarrow-Lando-Turnbull approach. Then, a procedure for reconstructing constant position market-consistent histories of credit portfolios from quoted Merrill Lynch indices should be established. These reconstructed historical credit values can be modeled via mixed empirical-generalized Pareto distribution (GPD) dynamics, which require a thorough parameter estimation and validation. Finally, credit SCRs can be computed as a result of the previous three steps. The solution shown here makes explicit use of recovery rates, in contrast with the standard formula of the current Solvency II framework.

It is usually impossible to directly build an aggregate index that perfectly reflects the risk profile of the credit portfolio of any given investor. Indeed, the recovery rates of the assets constituting a credit market index are usually quite homogeneous by construction, whereas investors build up credit portfolios by selecting assets with nonstandard recovery rates. For instance, investors can select bonds of low rating and high recovery and bonds of high rating and low recovery. Such a strategy cannot be directly replicated using existing market credit indices. However, to quantify spread risk, it is important to start from credit market indices. We suggest using past available index data to construct pseudo-indices that mimic target credit portfolios in all aspects except recovery risk. These pseudo-indices then constitute an important step toward the reconstruction of market-consistent credit observations, where a final adjustment for recovery risk is made. Using a one-year mixed GPD distribution to model reconstructed credit observations allows one to achieve a quantization of spread risk and to compute SCRs and similar indicators.

In a first step, we provide pricing formulas for portfolios made of bonds with varying rating and maturity classes. For simplicity, we assume that all the bonds of such portfolios pay coupons over the same discrete set of dates (typically at the end of each year or semester). However, these bonds may naturally have differing numbers of coupons.

Let K be the number of rating classes and let M_k be the number of maturity classes for each rating class k, where k ranges from 1 to K. We assume that all the bonds from the rating class k and from the maturity class j are identical and have $N^{j,k}$ cash flows $C_i^{j,k}$ indexed by i and occurring at times T_i. Let also $R^{j,k}$ and $\tau^{j,k}$ be the recovery rate and default time of these bonds, respectively.

Then, a bond portfolio can be valued as follows:

$$V = \sum_{k=1}^{K}\sum_{j=1}^{M_k}\sum_{i=1}^{N^{j,k}} C_i^{j,k} P(0,T_i)\Big(R^{j,k} + \big(1-R^{j,k}\big)\big(1-Q\big(\tau^{j,k}<T_i\big)\big)\Big),$$

where Q is the market-consistent or risk-neutral probability measure. We also have:

$$V = \sum_{k=1}^{K}\sum_{j=1}^{M_k}\sum_{i=1}^{N^{j,k}} C_i^{j,k} P(0,T_i)\Big(R^{j,k} + \big(1-R^{j,k}\big)\big(1-\psi_{k,K+1}(0,T_i)P\big(\tau^{j,k}<T_i\big)\big)\Big),$$

where $\psi_{k,K+1}(0,T_i)$ is a risk premium adjustment factor and P is the realistic or historical probability measure. Then:

$$V = \sum_{k=1}^{K}\sum_{j=1}^{M_k}\sum_{i=1}^{N^{j,k}} C_i^{j,k} P(0,T_i) - \sum_{j=1}^{M_k}\big(1-R^{j,k}\big)\sum_{i=1}^{N^{j,k}} C_i^{j,k} P(0,T_i)\Big[e^{diag(\pi_1,\ldots,\pi_K,1)\Lambda^P T_i}\Big]_{k,K+1},$$

where $(\pi_1,\ldots,\pi_K)$ are alternative risk premium adjustment factors and Λ^P is the generator driving the historical rating transitions. These formulas allow us to express bond portfolio values as a function of risk premium adjustment factors and also to derive relationships between the different types of factors introduced.

Past portfolio values cannot be used for regulatory and calibration purposes. This is because weights could have markedly changed in the history of the portfolio. Therefore, it is necessary to recompute past portfolio values coherently with existing market indexes, but noting that the recovery rates of those indexes may differ a lot from the actual recovery rates of the bond portfolio of the insurer. We suggest the following algorithm for reconstructing benchmarked past values of the portfolio and of the factors $\pi_{k=1,..,K}$:

- Extraction of historical monthly subindex data.
- Computation of the current value of the portfolio and its components.
- Computation of initial portfolio weights.
- Computation at time 0 of the risk premium adjustment factors $\pi_{k=1,..,K}(0)$ that relate the historical and risk-neutral measures.
- Reconstruction of past portfolio returns based on subindex data and initial portfolio weights.
- Estimation of the average recovery rate of the index. Then, computation of an initial pseudo-portfolio value whose recovery rate is that of the index.
- Computation of the past values of the pseudo-portfolio.
- Use of the pseudo-portfolio to compute at any time t the risk premium adjustment factors $\pi_{k=1,..,K}(t), t \leq 0$.
- Computation of the reconstructed historical portfolio values using actual recovery rates.

After conducting this procedure, we obtain a database of historical reconstructed portfolio values and risk premium adjustment factors. However, this database may not be sufficiently long, and some smoothing of extreme values may be required. Beyond that, the question arises as to which is the best dynamic representation of bond portfolios and risk premium adjustment factors. In a common approach, the evolution of risk premium adjustment factors is modeled with Cox-Ingersoll-Ross processes. If we look at the profile of monthly benchmarked bond portfolio increment autocorrelations (Figure 1), we see that while monthly increments display some degree of autocorrelation, annual bond portfolio increments can be assumed i.i.d. This visual deduction can be confirmed by performing ad hoc statistical tests. The main idea here is that if we are interested only in yearly simulations of balance sheets, then we can neglect autocorrelation effects; therefore, it is not necessary to use mean-reverting processes, such as the CIR process. Thinking now in terms of probability distribution, we suggest using mixed GPD, where the core of the empirical probability distribution is kept and the tails are smoothed using the GPD approach.

We construct the bond portfolio in Table 1 for conducting our illustration. Note that we purposely choose low-rated bonds with high recovery rates. This is in contrast to classic portfolios for which the recovery rate usually decreases when the rating worsens. So, our illustrative portfolio differs from quoted bond indexes in terms of recovery behavior. We are interested in seeing the impact of such a choice on SCRs. For the tails of this

Figure 1
Autocorrelations of Bond Portfolio Increments

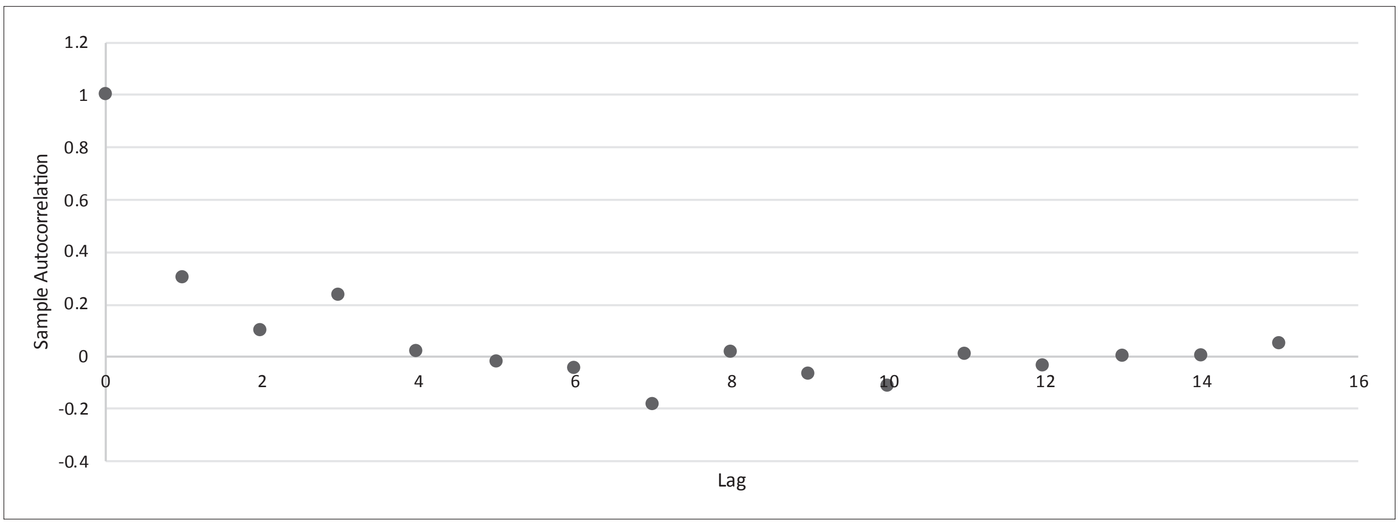

Table 1
Bond Data set as of 12/31/14

Issuer	Ranking	Recovery Rate	Maturity	Coupon	Dirty Price	Mod. Duration
BEI	AAA	Full	11/10/2016	8%	115.49	1.7
FINANCEMENT FONCIER	AAA	Full	29/12/2021	5.62%	121.62	5.9
KFW	AAA	Full	21/01/2019	3.875%	119.08	3.7
GERMANY	AAA	Full	15/08/2023	2%	114.46	8
OAT	AA	Full	25/10/2019	3.75%	117.96	4.5
PROCTER	AA	44%	24/10/2017	5.125%	114.91	2.7
STATOIL	AA	40%	10/09/2025	2.875%	117.38	9.3
COMMONWEALTH	AA	40%	10/11/2016	4.25%	108.09	1.8
AIRBUS GP FIN.	A	55%	12/08/2016	4.625%	108.47	1.6
AIRBUS GROUP FIN.	A	55%	25/09/2018	5.5%	120.48	3.4
AIR LIQ.FIN	A	56%	15/10/2021	2.125%	110.02	6.3
CREDIT AGRICOLE	A	61%	22/12/2024	3%	101.01	8.4
PIRELLI INTER	BBB	64%	18/11/2019	1.75%	101.10	4.4
SEB	BBB	65%	03/06/2016	4.5%	107.71	1.4
VEOLIA	BBB	65%	24/05/2022	5.125%	131.83	6.3
URENCO FINANCE	BBB	60%	02/12/2024	2.375%	101.33	8.6

portfolio, and after applying the eight-step algorithm described earlier, we estimated the parameters of the well-known GPD:

$$C_{u,\xi,\sigma}(x) = 1 - \left(1 + \frac{\xi}{\sigma}(x-u)\right)^{-\frac{1}{\xi}},\ x > u$$

using an improved Hill method. It is possible to check the values estimated using a maximum likelihood approach, Lorenz curves, Gini coefficients, POT graphs and a Kolmogorov-Smirnov test, to cite only a few methods. The latter test yields the results in Table 2.

Table 2
Kolmogorov-Smirnov Statistics and P-Values

θ^n	θ_c^n	$p-value^n$	θ^p	θ_c^p	$p-value^p$
0.07	0.14	0.80	0.17	0.35	0.74

Here the statistics θ^n and θ^p (n stands for the negative tail and p for the positive one) are always inferior to their respective critical values and the p-values are high. Using classic statistical vocabulary, this test says that we cannot exclude that the GPD is appropriate for modeling bond portfolio tails.

We obtain Table 3, whichcompares the SCRs of the total portfolio and of the subportfolios of given ratings using the standard Solvency II formula and the GPD smoothed model.

Table 3
Credit SCRs

	Standard Formula	GPD-Smoothed Model
Total Portfolio	5.5%	7.2%
Subportfolio AAA	1.4%	0%
Subportfolio AA	3.2%	8.5%
Subportfolio A	6.1%	10.6%
Subportfolio BBB	11.8%	10.4%

Note that the numbers shown in Table 3 are, in fact, simplifications of SCRs because we did not make any assumptions on the contracts issued by the firm, and we did not incorporate retroaction effects of credit risk on liabilities. We see that for this portfolio, the model most often predicts higher SCRs than the standard formula. However, when top-quality bonds are public or semipublic and present virtually no credit risk, the model

consistently predicts a null SCR. Also, when low-rated bonds have high recovery rates, the model predicts lower SCRs than the standard formula. Indeed, the current standard formula does not take into account recovery effects; therefore, the framework suggested in this article permits extension of the standard formula at least in terms of recovery risk. Finally, observe that credit SCRs cannot be straightforwardly approximated by ratings. This is a confirmation that credit risk is polymorphic in essence and cannot be captured by one or two proxy variables. ■

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