NOTE: The solutions presented here are not the full model solutions as published. There is no commentary and where there is more than one correct response, only one such response is presented. Some questions for QFI QF will be answered on traditional paper/pen. Examples of such questions are not provided here as they will be similar to those asked in the past.

QFI QF Sample CBT Question

(*6 points*) You are assigned to develop the following applications of an Economic Scenario Generator (ESG) framework:

Application 1 - Determine the fair value of the investment guarantees embedded in the company's variable annuity products.

Application 2 – Measure the effectiveness of the hedging strategies for the economic capital calculation purposes (where nested simulation is needed).

Application 3 - Price a traded put option on a major equity index with early exercise, where closed-form formula for its price is not available.

(a) (*1.5 points*) Determine the type(s) of ESG simulation needed for each of the three applications above, and justify your choices.

ANSWER:

Using a set of parameters, you have simulated the risk-free interest rates and equity returns for 5 years. The discount factors from the end of each year to time zero are as below:

Scenario (i)	Year 1	Year 2	Year 3	Year 4	Year 5
1	93.70%	93.60%	90.40%	85.70%	82.70%
2	95.90%	92.10%	86.50%	82.30%	78.70%
3	97.50%	94.40%	88.30%	85.60%	84.40%
4	96.80%	90.70%	85.60%	84.50%	83.00%
5	97.10%	95.60%	91.90%	91.90%	86.07%

The equity accumulation factors $g_i(T)$ from time zero to the end of each projection year are as below, where T is the projection year (accumulation factor is the future value of \$1 invested at time zero):

Scenario (i)	Year 1	Year 2	Year 3	Year 4	Year 5
1	95.20%	106.00%	125.00%	141.90%	116.40%
2	100.80%	91.70%	111.60%	100.70%	125.40%
3	119.50%	103.10%	117.20%	99.80%	98.20%
4	75.50%	57.00%	61.20%	75.50%	90.20%
5	90.20%	70.50%	88.00%	105.70%	93.00%

The average present values of the equity accumulation factors are as below:

Year 1	Year 2	Year 3	Year 4	Year 5
k	80.02%	89.26%	90.17%	86.55%

(b) (*0.5 points*) Calculate k in the table above.

ANSWER:

Assume that the time horizon is sufficiently long and the size of this scenario set is sufficiently large to perform the martingale test.

(c) (*1 point*) Assess whether this scenario set passes the martingale test.

ANSWER:

A new set of equity returns is simulated and the following average present values of the equity accumulation factors are calculated (using the same discount factors provided above):

Year 1	Year 2	Year 3	Year 4	Year 5
99.2%	98.0%	103.3%	100.2%	100%

The accumulation factors $g_i'(T)$ of the newly simulated equity returns can be obtained by applying an adjustment $s(T)$ to the old accumulation factors $g_i(T)$ (for scenarios $i = 1, 2, 3, 4, 5$)

$$g_i^{'}(T) = e^{s(T)T} g_i(T)$$

(d) (*1 point*) Determine $s(5)$ and explain its purpose.

ANSWER:

An exotic 5-year European Asian-style call equity option with fixed strike of K has the following payoff at the end of the 5-year term (where X_4 and X_5 are the prices of underlying at the end of year 4 and 5 respectively):

$$Payoff = \text{Max}(\text{average } (X_4, X_5) - K, 0)$$

(e) (*2 points*) Determine the time zero price of the option with $K = 1$ using the appropriate scenario data above.

ANSWER:

QFI QF Sample CBT Solution

(*6 points*) You are assigned to develop the following applications of an Economic Scenario Generator (ESG) framework:

Application 1 - Determine the fair value of the investment guarantees embedded in the company's variable annuity products.

Application 2 – Measure the effectiveness of the hedging strategies for the economic capital calculation purposes (where nested simulation is needed).

Application 3 - Price a traded put option on a major equity index with early exercise, where closed-form formula for its price is not available.

(a) (*1.5 points*) Determine the type(s) of ESG simulation needed for each of the three applications above, and justify your choices.

ANSWER:
Application 1:
• The fair value of the embedded investment guarantee is the market-consistent value of a derivative (usually a put type option) instrument
• So risk-neutral/market-consistent ESG is required
Application 2:
• The measurement of the effectiveness of a hedging program will often involve the application of both real-world and market-consistent simulations. In the outer loop of a nested stochastic model for the economic capital calculation, the hedging strategy needs to be modeled along each real-world path which aims to represent the state of the world. While the inner loop of the nested stochastic model, cash flows of hedging are priced in that state of the world using risk-neutral scenarios projected forward from the current node of the simulation
In this case, the real-world simulation is used to assess the overall risk of the variable annuity book and to measure the effectiveness of the hedging strategy
Application 3:
• A traded American put option is a financial derivatives that has investment guarantees and its historical and current market prices are observables.
• So, a risk-neutral ESG is required.

Using a set of parameters, you have simulated the risk-free interest rates and equity returns for 5 years. The discount factors from the end of each year to time zero are as below:

Scenario (i)	Year 1	Year 2	Year 3	Year 4	Year 5
1	93.70%	93.60%	90.40%	85.70%	82.70%
2	95.90%	92.10%	86.50%	82.30%	78.70%
3	97.50%	94.40%	88.30%	85.60%	84.40%
4	96.80%	90.70%	85.60%	84.50%	83.00%
5	97.10%	95.60%	91.90%	91.90%	86.07%

The equity accumulation factors $g_i(T)$ from time zero to the end of each projection year are as below, where T is the projection year (accumulation factor is the future value of $1 invested at time zero):

Scenario (i)	Year 1	Year 2	Year 3	Year 4	Year 5
1	95.20%	106.00%	125.00%	141.90%	116.40%
2	100.80%	91.70%	111.60%	100.70%	125.40%
3	119.50%	103.10%	117.20%	99.80%	98.20%
4	75.50%	57.00%	61.20%	75.50%	90.20%
5	90.20%	70.50%	88.00%	105.70%	93.00%

The average present values of the equity accumulation factors are as below:

Year 1	Year 2	Year 3	Year 4	Year 5
k	80.02%	89.26%	90.17%	86.55%

(b) (*0.5 points*) Calculate k in the table above.

ANSWER:
K is the average of the present value of asset price at end of year 1 discounted back to time zero, across all the scenarios

$K = 1/M * \text{sum(i from 1 to M)} [X_T(i)v_T(i)]$

= (95.2% * 93.7% + 100.8% * 95.9% + 119.5% * 97.5% + 75.5% * 96.8% + 90.2% * 97.1%) / 5

= 92.61%

Assume that the time horizon is sufficiently long and the size of this scenario set is sufficiently large to perform the martingale test.

(c) (*1 point*) Assess whether this scenario set passes the martingale test.

ANSWER:
The series of expected time zero asset price is as below: 92.61%, 80.02%, 89.26%, 90.17%, 86.55%

- If the series is martingale, the expected present value of the asset prices should be equal to the price at time zero
- The time zero price of the asset is 1.
- Since the martingale test is based on random sampling, it is acceptable that there is random fluctuation, but the average of the discounted asset prices at all periods should be close to one
- Therefore the scenario is not martingale (or it fails the martingale test).

A new set of equity returns is simulated and the following average present values of the equity accumulation factors are calculated (using the same discount factors provided above):

Year 1	Year 2	Year 3	Year 4	Year 5
99.2%	98.0%	103.3%	100.2%	100%

The accumulation factors $g_i'(T)$ of the newly simulated equity returns can be obtained by applying an adjustment $s(T)$ to the old accumulation factors $g_i(T)$ (for scenarios $i = 1, 2, 3, 4, 5$)

$$g_i^{'}(T) = e^{s(T)T} g_i(T)$$

(d) (*1 point*) Determine $s(5)$ and explain its purpose.

ANSWER:
The accumulation factor can be expressed as
$g_i(T) = \exp [mu_i(T)T]$
$= \exp[mu_i(T) + s(T)T]$
$= \exp [s(T)T) g_i(T)]$

The present value of asset price at the end of year 5 is
$\exp [s(5)*5*86.55\%]$

$s(5) = 1/5 \times \ln[1/86.55\%] = 2.89\%$

The purpose of S(5) is to adjust the equity accumulation factors so that the new factors pass the Martingale test, and can be used for pricing in a risk neutral fashion

Note to candidates: So long as the candidate shows the formula they are using, the exact notation is not required. For example, e^T or exp[T] can be used in place of e^T

An exotic 5-year European Asian-style call equity option with fixed strike of K has the following payoff at the end of the 5-year term (where X_4 and X_5 are the prices of underlying at the end of year 4 and 5 respectively):

$$Payoff = \text{Max}(\text{average } (X_4, X_5) - K, 0)$$

(e) (*2 points*) Determine the time zero price of the option with $K = 1$ using the appropriate scenario data above.

ANSWER:
The option pricing should be using risk-neutral scenario, which passes the martingale test
• The new equity accumulation factors should be used
• The adjustment on the accumulation factor for year 4 is 100.20% / 90.17% = 1.1112
• The adjusted accumulation factors for year 4 are calculated as:
o 141.90% * 1.1112 = 157.68%,
o 100.70% * 1.1112 = 111.90%,
o 99.80% * 1.1112 = 110.90%,
o 75.50% * 1.1112 = 83.90%,
o 105.70% * 1.1112 = 117.46%
The adjustment on the accumulation factor for year 5 is 100% / 86.55% = 1.1554
• The adjusted accumulation factors for year 5 are calculated as:
o 116.40% * 1.1554 = 134.49%,
o 125.40%* 1.1554 = 144.89%,
o 98.20%* 1.1554 = 113.46%,
o 90.2% * 1.1554 = 104.22%,
o 93.00% * 1.1554 = 107.45%
• The payoffs are calculated as 46.09%,28.40%,12.18%, 0%, 12.46%
• The present value of the payoff is thus calculated as
C_0 = 1/5 (0.46087473 ∗ 82.70% + 0.283953678 ∗ 78.70% + 0.121816511 ∗ 84.40% + 0 ∗ 83.00% + 0.124557176 ∗ 86.07%)
= 0.1629

QFI PM Sample CBT Question

(6 *points*) EcL issued a zero-coupon bond to fund its corporate project in the past. Except for this zero-coupon bond, EcL has no other debt or liabilities.

You are given the following:

- The remaining time-to-maturity of EcL's zero-coupon bond = 16 years
- The face value of the zero-coupon bond = $100 million.
- The market value of the zero-coupon bond = $63.89 million
- The continuously compounded risk-free interest rate = 2% for all maturities

(a) (*1 point*) Calculate the continuously compounded credit spread of the zero-coupon bond

ANSWER:

The loss-given-default of this zero coupon bond is 24.01% of the face value

(b) (*1 point*) Calculate the risk-neutral probability of default of this zero-coupon bond

ANSWER :

You are using Merton's model to estimate the total asset value. You decide to calibrate the model using the risk-neutral probability calculated in (b), and you assume an asset volatility of 10%.

(c) (*2 points*) Calculate the implied total asset value of EcL.

ANSWER :

(d) (*2 points*) Calculate the volatility of EcL's equity value

ANSWER :

QFI PM Sample CBT Solution

This solution contains an illustration of what would be acceptable syntax in a Computer Based Testing(|CBT) environment when the question asks for a solution to be provided in Microsoft Word.

The intent of CBT is not to test your ability to write formulas out in Microsoft Word.

If a formula or description calls for a special symbol, please do not replicate the symbol, rather write it out in words or use something that is clearly representative, in some cases this could be as you would write a formula in EXCEL.

For example $\sum$ could be written as sum(), and if there is a range write it out as sum from 1 to n.

In the below example subscripts were denoted by having an underscore in front of them with superscripts written in lower case. You are free to adjust the font size of use word's functionality for subscripts and superscripts; however, be mindful of time if doing so.

The intent of the below solutions is to illustrate different ways of writing out the same answer under exam conditions in Microsoft Word.

Aim to be consistent in terms of how you write formulas out, if a formula is on the formula sheet, just writing out the formula number will be acceptable. Subsequent steps though will need to show how the formula was used/manipulated.

(6 *points*) EcL issued a zero-coupon bond to fund its corporate project in the past. Except for this zero-coupon bond, EcL has no other debt or liabilities.

You are given the following:

- The remaining time-to-maturity of EcL's zero-coupon bond = 16 years
- The face value of the zero-coupon bond = $100 million.
- The market value of the zero-coupon bond = $63.89 million
- The continuously compounded risk-free interest rate = 2% for all maturities

(a) (*1 point*) Calculate the continuously compounded credit spread of the zero-coupon bond

ANSWER:
B=F * exp(-(r+s)T) or B = F * e^(-(r+s)T)

S=-(1/T) ln (B/F) – r

S=-(1/16) ln(63.89/100)-2% = 80%

Notes;
e can be written as exp or e^

The loss-given-default of this zero coupon bond is 24.01% of the face value

(b) (*1 point*) Calculate the risk-neutral probability of default of this zero-coupon bond

ANSWER :
B=(1- PDrn * LGD) * F * exp(-rT)

PDrn=(1/LGD) (1-B/F*exp(rT)

PDrn=(1/24.01%)(1-63.89/100 * exp(2%+16)
=50%

Notes:
Superscripts were written out in lower case letters.

You are using Merton's model to estimate the total asset value. You decide to calibrate the model using the risk-neutral probability calculated in (b), and you assume an asset volatility of 10%.

(c) (*2 points*) Calculate the implied total asset value of EcL.

ANSWER :
<u>Method 1</u>

PDrn = N(-d2) = 50% from (b), which leads to d2=0.

d_2=(ln(A/F) + (r-(sigma_a ^2)/2)T

or

d_2=(ln(A/F) + (r-(s_A ^2)/2)T

A=F*exp(d_2 * s_A * sqrt (T) – (r – (s_A^2)/2)*T)

A=100 exp (0-(2%-(10%^2)/2)*16)

= 78.66 million

Method 2

A = B + E where E = EcL's equity value

E = A N(d_1) – F * exp(-rT) * N(d_2)

= (B+E) N(d_1) – F * exp(-rT)*N(d_2)

E = (B N(d_1) – F exp (-rT) N(d_2))/(1-N(d_1)

d_1=d_2 + s_A sqrt (T)
= 0 +10% * sqrt (16)
= 0.4

N(d_1) = 0.6554

E = (63.89*0.6554 – 100 exp (-2% * 16) * 0.5)/(1-0.6554)
= 16.15

A = B+E = 63.89 + 16.15
= 80.04

Notes
For square roots, the notation of sqrt or ^.5 are both acceptable.

(d) (*2 points*) Calculate the volatility of EcL's equity value

ANSWER :

s_E = s_A *A/E (d E/ d A)
= s_A * A/E * N(d_1)

d_1 = d_2 + s_A * (T)^.5
= 0 + 10% * sqrt(16)
= 0.4

N(d_1) = 0.6554

If candidate used A = 78.66 from Method 1 of part (c) and solved for E by

E = A – B = 78.66 – 63.89 = 14.77

s_E = 10% * 78.66/14.77) * 0.6554
= 34.90%

Notes:
For σ_E writing out as s_E or Sigma_E is acceptable

QFI IRM Sample CBT Question

Part a should be answered in the provided Excel workbook. Parts b, c, d and e should be answered in Word.

(*7 points*) You are a market risk analyst at BCD Company, a U.S. based investment firm and have noted the following information about the firm's asset portfolio:

- The portfolio has a market value of USD 225 million and is fully allocated to equity
- The expected return on equity is 9%, and the standard deviation is 15%
- There is a cash inflow of USD 75 million that will be fully allocated to bonds
- The expected return on bonds is 3.5%, and the standard deviation is 4%
- The correlation between the two asset classes is 0.35
- All figures are annual

(a) (*1.5 points*) Use the analytical method to complete the following:

(i) Calculate a 1% weekly VaR of the new USD 300 million portfolio.

(ii) Calculate the change in the weekly VaR due to addition of bonds in the portfolio.

(iii) Interpret your results.

The response for this part is to be provided in the Excel spreadsheet.

BCD is looking to further reduce its equity exposure and is considering two options:

1. Reallocate a portion of the equity position into cash with a standard deviation of 0%
2. Purchase equity put options

(b) (*1 point*) For each option above:

(i) Describe the impact on the 1% weekly VaR of the portfolio.

(ii) Explain whether the analytical VaR method is appropriate.

ANSWER: (i) (ii)

Management expresses two main concerns with the performance of BCD's asset portfolio:

1. Loss in excess of that of a 1-in-100 year event
2. Exposure to a very unlikely market decline, such as the 2008 financial crisis

(c) (*1.5 points*) For each of the above concerns:

(i) Evaluate the use of VaR to quantify the risk.

(ii) Recommend an alternate risk measure to supplement VaR.

ANSWER:

(i)

(ii)

BCD is considering investing in Company XYZ. XYZ is issuing its first and only bond in the form of a zero-coupon bond to finance its expansion. The bond is due in one year and is the firm's only liability. The face value of the bond is USD 250 million.

Let A_1 represent the market value of XYZ's assets in 1 year. You construct the table below to illustrate the payoff structure for XYZ's bondholders and stockholders:

Payoff	**A_1 < \$250M**	**$A_1 \geq$ \$250M**
Bondholders		
Stockholders		

(d) (*1.5 points*)

(i) Determine the payoff structure by completing the table above.

(ii) Explain the implicit options that each capital supplier faces.

(iii) Identify the supplier that is more exposed to credit risk. Justify your response.

ANSWER:

(i)

Payoff	**$A_1 < \$250M$**	**$A_1 \geq \$250M$**
Bondholders		
Stockholders		

(ii)

(iii)

Rather than issuing this bond in its home country (the U.S.), XYZ decides to issue the bond in Japanese Yen (JPY). XYZ also enters a one-year forward contract (buy yen, sell dollars) at a rate of JPY 110 per USD 1 on a notional of JPY 27,500 million.

BCD decides to purchase the bond.

(e) (*1.5 points*) Explain how BCD and XYZ are each exposed to the following risks with respect to these transactions:

(i) Credit risk

(ii) Liquidity risk

(iii) Exchange rate risk

ANSWER:

(i)

(ii)

(iii)

QFI IRM Sample CBT Solution

Part a should be answered in the provided Excel workbook. Parts b, c, d and e should be answered in Word.

(*7 points*) You are a market risk analyst at BCD Company, a U.S. based investment firm and have noted the following information about the firm's asset portfolio:

- The portfolio has a market value of USD 225 million and is fully allocated to equity
- The expected return on equity is 9%, and the standard deviation is 15%
- There is a cash inflow of USD 75 million that will be fully allocated to bonds
- The expected return on bonds is 3.5%, and the standard deviation is 4%
- The correlation between the two asset classes is 0.35
- All figures are annual

(a) (*1.5 points*) Use the analytical method to complete the following:

(i) Calculate a 1% weekly VaR of the new USD 300 million portfolio.

(ii) Calculate the change in the weekly VaR due to addition of bonds in the portfolio.

(iii) Interpret your results.

The response for this part is to be provided in the Excel spreadsheet.

BCD is looking to further reduce its equity exposure and is considering two options:

1. Reallocate a portion of the equity position into cash with a standard deviation of 0%
2. Purchase equity put options

(b) (*1 point*) For each option above:

(i) Describe the impact on the 1% weekly VaR of the portfolio.

(ii) Explain whether the analytical VaR method is appropriate.

ANSWER: (i) Cash: • There would be a reduction in VaR.

• The volatility of the portfolio will decrease.
• Even if cash holdings produce 0% return, this will still reduce the losses in the worst scenarios.

Put Options:
• There would be a reduction in VaR.
• Put options have a positive skew as they will produce many small negative returns (premium payments) and relatively few large returns (during large market downturns).
• Because we are looking at the largest 1% of losses, the put options will act as a hedge to large market downturns.

(ii)
Cash:
• We can still use the analytical method.
• Cash is another asset class, so simple return/volatility/correlation assumptions are sufficient.

Put Options:
• The analytical method is no longer applicable.
• This method assumes a normal distribution of returns, but options do not follow a normal distribution.

Management expresses two main concerns with the performance of BCD's asset portfolio:

1. Loss in excess of that of a 1-in-100 year event
2. Exposure to a very unlikely market decline, such as the 2008 financial crisis

(c) (*1.5 points*) For each of the above concerns:

(i) Evaluate the use of VaR to quantify the risk.

(ii) Recommend an alternate risk measure to supplement VaR.

ANSWER:
(i)
Loss in excess of a 1 in 100 annual event:
• Recommend Conditional Tail Expectation (CTE)
• CTE provides additional information beyond the VaR measure, capturing the severity of the portfolio's value above a particular confidence level.
• CTE captures the tail risk and poor scenarios better than other metrics,

incorporating a level of conservatism.

2008 Financial Crisis:
• Recommend scenario analysis or stress testing
• Scenario analysis can be used to test situations outside the normal range of probability and can be designed to simulate actual extreme events.

(ii)
Loss in excess of a 1 in 100 annual event:
• Recommend Conditional Tail Expectation (CTE)
• CTE provides additional information beyond the VaR measure, capturing the severity of the portfolio's value above a particular confidence level.
• CTE captures the tail risk and poor scenarios better than other metrics, incorporating a level of conservatism.

2008 Financial Crisis:
• Recommend scenario analysis or stress testing
• Scenario analysis can be used to test situations outside the normal range of probability and can be designed to simulate actual extreme events.

BCD is considering investing in Company XYZ. XYZ is issuing its first and only bond in the form of a zero-coupon bond to finance its expansion. The bond is due in one year and is the firm's only liability. The face value of the bond is USD 250 million.

Let A_1 represent the market value of XYZ's assets in 1 year. You construct the table below to illustrate the payoff structure for XYZ's bondholders and stockholders:

Payoff	**A_1 < \$250M**	**$A_1 \geq$ \$250M**
Bondholders		
Stockholders		

(d) (*1.5 points*)

(i) Determine the payoff structure by completing the table above.

(ii) Explain the implicit options that each capital supplier faces.

(iii) Identify the supplier that is more exposed to credit risk. Justify your response.

ANSWER:

(i)

Payoff	**$A_1 < \$250M$**	**$A_1 \geq \$250M$**
Bondholders	A1*	$250M
Stockholders	$0	A1* - $250M

**Please note that A1 would be an acceptable notation (use of subscript and superscript is not equired)*

(ii)
- Stockholders effectively hold a long call option on the assets of the firm with the face value of the liabilities as the strike price.
- Bondholders have implicitly written the stockholders a put on the assets (i.e. short put).

(iii)
- The bondholders are more exposed to credit risk.
- By effectively writing a put option, the potential losses to bondholders are much larger than those of the stockholders.

Rather than issuing this bond in its home country (the U.S.), XYZ decides to issue the bond in Japanese Yen (JPY). XYZ also enters a one-year forward contract (buy yen, sell dollars) at a rate of JPY 110 per USD 1 on a notional of JPY 27,500 million.

BCD decides to purchase the bond.

(e) (*1.5 points*) Explain how BCD and XYZ are each exposed to the following risks with respect to these transactions:

(i) Credit risk

(ii) Liquidity risk

(iii) Exchange rate risk

ANSWER:

(i)
BCD:
• The purchase of the bond exposes BCD to the credit risk that XYZ defaults on its payment.

XYZ:
• The forward contract exposes XYZ to credit risk.
• There is a chance that the counterparty to the forward contract will not be able to pay the amount owed.

(ii)
BCD:
• Based on how the bond is trading in the market, the firm may not be able to sell it at market price.

XYZ:
• The firm's commitment to paying the face value of the bond exposes the firm to liquidity risk.
• The market value of its assets might exceed $250M, but they need to be able to sell these assets and produce cash to pay for the liability.

(iii)
BCD:
• The firm is exposed to FX risk because the redemption value of the bond is Yen denominated.
• A weakening value of the Yen will result in losses for BCD.

XYZ:
• Although the debt was issued in yen, the forward contract has effectively hedged all FX exposure and replaced it with credit risk.
• In one year, XYZ will be able to exchange $250M for ¥27.5B, regardless of how the foreign exchange rates move during that time period.