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Mr. C. Smith
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Dear Court:

Recently, as an assignment I had the students in our Risk Theory class extend the method of Tröbliger* to three levels of risk. Both the data of Tröbliger and the data of Thyrion** which can be found on pages 27 and 16 of Seal's Stochastic Theory of a Risk Business', were fitted.

Tröbliger's original model assumed a 2 step function for the Poisson generating function for the number of claims. The model was:

$$P(k) = .9403\,\frac{e^{-.1089}(.1089)^k}{k!} + \frac{.0597e^{-.7}(.7)^k}{k!}$$

The model produced by our students was

$$P(k) = .8911\,\frac{e^{-.0977}(.0977)^k}{k!} + .1081\,\frac{e^{-.5131}(.5131)^k}{k!} + .0008\,\frac{e^{-2.1343}(2.1343)^k}{k!}$$

The fit used a generalized Newton Raphson technique. There are several interesting points between the two models.

a) The relative stability in the average number of claims parameter for the "normal" group.

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b) The relatively large shift in the average number of claims parameter for the second class.

c) The sizable shift in the numbers from the "normal" class to the first substandard class.

d) The extremely small number in the third class and the relatively high average number of claims parameter.

Fitting the data of Thyrion produced the following model:

$$P(k) = .447\,\frac{e^{-.0090}(.0090)^k}{k!} + .545\,\frac{e^{-.347}(.347)^k}{k!} + .008\,\frac{e^{-2.615}(2.615)^k}{k!}$$

There are several interesting points:

a) The relative equality of the size of the first two classes.

b) The relative stability of the average number of claims parameters when compared to the previous example, particularly for the best class.

Unfortunately, the small number of classes prevented proper tests of significance for the parameters and of the fit.

Yours sincerely,

F.G. Reynolds

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* "Mathematisch Untersuchungen zur Beitragructgewahr in der Kraftfahversicherung" Bul Deuts. Stesell. Versich. Math vol. 5, 327-348.

** "Contribution à l'étude du bonus pour non semistre en assurance automobile" Astin Bull. Vol. 1,142-162.