

ERM-RET Model Solutions

Spring 2020

1. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

- (2a) Demonstrate how each of the financial and non-financial risks faced by an entity can be amenable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, and scenario analysis.
- (2e) Evaluate the theory and applications of extreme value theory in the measuring and modeling of risk.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3d) Analyze risks that are not easily quantifiable, such as operational and liquidity risks.

Sources:

ERM-102-12: Value at Risk: Evolution, Deficiencies, and Alternative

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 12 Extreme Value Theory

ERM-105-12: Coherent Measures of Risk - An Exposition for the Lay Actuary -Glenn Meyers

Article: Modeling Tail Behaviour with Extreme Value Theory, Risk Management, Sept 2009

Embedding Cyber Risk in Risk Management: An Insurer's Perspective By Kailan Shang (pp.12 -15 of Cybersecurity: Impact on Insurance Business and Operations)

1. Continued

Commentary on Question:

Candidates are tested on their ability to understand the EVT and different risk measures. One of the EVT distribution calculations was tested. The question was designed to be relatively straightforward with minimal area for subjectivity. It is expected that candidates are able to discuss/elaborate different risk measures and EVT.

Solution:

(a) Your junior analyst, Dan, has identified the following risk metrics:

- VaR
- Total Exposure
- Standard Deviation
- CTE
- Average

Discuss advantages and disadvantages of using each of these risk metrics.

Commentary on Question:

Overall, candidates did well on part (a) by describing the advantages and disadvantages of different risk measures and showed that they understood the concepts well. One advantage and one disadvantage for each risk measures would achieve full credit.

Advantages -

VaR: industry standard, methodology well accepted, based on Normal curve, risk control tool, maximum possible loss for a given confidence level.

Total Exposure: simple to calculate, understand, used to control exposure, ex. credit risk exposure, maximum possible loss

Standard Deviation: can be used for capital purpose, moves beyond average, expected deviation from average, no assumption of normality required

CTE: considers only extreme losses, pertinent for cyber risk, average value above confidence level.

Average: simple to estimate, use for management purpose, pricing, expected value.

1. Continued

Disadvantages -

VaR as the industry standard relies on normal distribution, but in reality many risks have heavier tails than a normal distribution. Only information below confidence level. Also relies on past data to predict future performance. Risks: not occurred in the past will not be captured by a VaR calculation."

Total Exposure: not a meaningful risk metric, as this is simply an indicator for the size/volume of the operations, and does not credit/penalize risk management practices, no probability of loss, not show diversification benefit.

Standard Deviation: no tail risk included. No coherent.

CTE: may be difficult to obtain full distribution and calculate it but coherent risk measure, not a standard measure used by industry

Average: doesn't take into account extreme events, large instances of losses around mean may mask losses in the extreme scenarios.

- (b) Upon further research, Dan suggests that Extreme Value Theory (EVT) might be a more appropriate solution to model cyber risks. For that proposal, Dan drafts the following presentation slide outlining certain EVT aspects for the client.

Extreme Value Theory (EVT):

- EVT is a tool to help measure tail risk better than VaR.
- VaR assumes normal distribution
- Specifically, EVT can help quantify both the maximum of your experience data, and the frequency of values above the maximum.
- EVT uses historical data; VaR does not.
- This type of modeling can respond to fat-tailed distributions better than VaR, which is based on a normal distribution, and therefore can be a better predictor of extreme values.
- EVT accentuates agency issues compared to traditional VaR.
- EVT is a coherent risk measure unlike VaR.

Critique the statements made by Dan.

Commentary on Question:

Overall the performance on this part was mediocre. To achieve full credit, candidates needed to answer in the form of a critique, with strong justification to show understanding of the differences between EVT and VaR, instead of simply responding "yes" or "no."

1. Continued

- The slide is correct that EVT measures tail risk. VaR as the industry standard assumes Normality of risks, less relevant for cyber risk.
 - EVT quantifies the distribution from the sample maximum chosen, not the maximum of all your experience data. And EVT quantifies the sample maximum or the distribution of values above a given threshold. It's a conditional calculation.
 - VaR relies on past data to predict future performance. As a result, risks that have not occurred in the past will not be captured by a VaR calculation. Your client may have experienced unanticipated large losses simply because losses of that size were not in their experience data.
 - EVT can respond to fat-tailed distributions better than VaR, which is based on a normal distribution. VaR relies on a normal distribution, but in reality many risks have heavier tails than a normal distribution quantifies. As a result VaR underestimates the probability of an extreme event, possibly including the large cyber losses your client has experienced. EVT can more appropriately measure tail risk incidences.
 - Traditional VaR is exposed to agency issues. Risk managers can manipulate VaR by “stuffing risks into tail”. For example, if a manager is evaluated on VaR(90), they can appear to have improved their risk position by taking on risks which have a probability of less than 10% without regard to magnitude. As a result, even though the total risk may have increased, VaR(90) shows less risk is present. It is possible those in charge of managing your client's cyber risk have been manipulating the tools used to measure them without truly mitigating the risk. Less so for EVT to have such situations.
 - VaR is not a coherent risk measures. EVT is more of a conditional distribution.
- (c) The CDF of the GPD distribution is $G(x) = 1 - (1 - kx/s)^{1/k}$.
- (i) Identify the GPD threshold percentile you should use for your analysis. Explain your choice.
 - (ii) Calculate the probability of a total loss greater than 700. Show all work.
 - (iii) Explain how fuzzy logic might be a suitable alternative method to assess cyber risk exposure.

1. Continued

Commentary on Question:

Overall, candidates performed well on subpart c(i), showing their understanding of GPD distribution and threshold selection. However, the performance for c(ii) - calculating the conditional CDF of the GPD was not as strong. Many candidates missed the conditional loss above the threshold part. Most candidates were able to explain the fuzzy logic well and linked back to cyber risks.

- (i) Use 89th percentile parameter values because s^* and k^* stabilize at that point and after. Want to balance need for stability in estimators with desire to include as many data points as possible, so take very first observation where estimators stabilize.

- (ii) $G(x) = 1 - (1 - kx/s)^{1/k}$; Generalized Pareto Distribution
 $G(x) = 1 - (1 - (-0.135 * X)/120)^{(1/-0.135)} = 1 - (1 + .00112500 * X)^{-7.4074}$

Want 700 total loss, and threshold is 323, thus CPD is calculated at $X \Rightarrow 700 - 323 = 377$

$$1 - (1 + .00112500 * 377)^{-7.4074} = 0.9271$$

Conditional probability of less than this is $1 - 0.9271 = 0.0729$

Probability of a total loss greater than 700 is calculated as the Unconditional probability is $0.0729 * (1 - .89\%, \text{the percentile of the threshold, that is } 0.11) = 0.0080$

- (iii) Fuzzy logic is a valid approach as it takes into account experience data as well as expert judgment. It integrates KRI for many aspects of a company that may be vulnerable to a cyber-attack. There is a feedback loop that can be used to update the management effort by the organization to face this risk. EVT calculation can be part to measure direct impact but less so for damage reputation. It complements this other method.

2. **Learning Objectives:**

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.
- (4b) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.
- (4c) Demonstrate means for reducing risk without transferring it.

Sources:

ERM-112-12: Revisiting the Role of Insurance Company ALM within a Risk Management Framework

ERM-123-14: S&P Enterprise Risk Management Criteria (#1-71, 86-88)

ERM-106-12: Economic Capital-Practical Considerations-Milliman

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis (pp. 1-6 and 14-17)

Financial Enterprise Risk Management, Sweeting, 2011 Ch. 16 Responses to Risk

Commentary on Question:

The purpose of this question is to test the candidate's understanding of ALM and Risk Management as they relate to an insurance company's product portfolio. Candidates were tested on their knowledge of ERM best practices corresponding to both new product launches and asset management.

Solution:

- (a) Evaluate each of KRW's four current ERM practices given its current product portfolio.

Commentary on Question:

Candidates generally scored well on this part of the question.

2. Continued

I – Risk metrics should be reported more frequently (e.g. monthly) to provide senior management more opportunities to respond to current and potentially new risks.

II – KRW should define their own economic capital framework and risk measures and overlay specific regulatory and/or rating agency capital constraints.

III – Cash flow matching may not be an optimal strategy and might generate lower returns which will affect the competitiveness of the company's product(s). Also, in some cases cash flow matching may not be possible.

IV – Sensitivities should be reported with the same timing as the risk metrics from (I) and should include sensitivities which test policyholder behavior (e.g. disintermediation risk for the deferred annuity product).

(b)

(i) Outline the steps in the process of implementing strategic asset allocation.

(ii) Explain the trade-offs associated with changing the allocation to each of the following asset classes:

- Government Bonds
- Corporate Bonds
- Equity
- Real Estate

(iii) Recommend how each component of the current portfolio should change, considering the new asset allocation objectives. Justify your response.

Commentary on Question:

Candidates did not generally score well on this part of the question. With respect to subpart (ii), many candidates did not provide trade-offs for all asset classes or they did not include more than one trade-off. With respect to subpart (iii), many candidates did not make a recommendation, or the recommendation was contradictory to their answer in subpart (ii).

- (i) Steps in the process of implementing strategic asset allocation:
- a. Set the investment objective and constraints.
 - b. Establish the asset universe and assumptions.
 - c. Design a replicating portfolio based on the liability profile.
 - d. Establish risk metrics to assess the risk/return trade-off.

2. Continued

- (ii) Government Bonds – Increasing the allocation will increase the duration of the asset portfolio, which will help close the liability duration gap. However, these are the highest quality bonds and have the lowest return (and risk).

Corporate Bonds – Adding more AAA corporate bonds will increase the credit quality of the portfolio but brings with that lower return due to lower risk of default relative to BBB corporate bonds.

Equity – Increasing the allocation to equity will increase the portfolio's expected return. However, equity requires more RBC than government or corporate bonds to capital would also increase.

Real Estate – Increasing the allocation to real estate will increase the portfolio's expected return. However, Real Estate can be very illiquid.

- (iii) Cash – Maintain current level of cash to balance short-term liquidity needs.

Government Bonds – Increase allocation to close the liability duration gap.

Corporate Bonds – Decrease allocation to BBB corporate bonds to reduce duration mismatch and improve credit quality of the portfolio.

Equity – Increase allocation to increase excess yield.

Real Estate – Limit or avoid. While this increases excess yield, may be too illiquid.

- (c) The CFO suggests that aggregate capital levels would be determined by summing the resultant capital values produced by each shock.
- (i) Evaluate the appropriateness of the CFO's recommendations for the entire portfolio of KRW's existing products.
- (ii) Describe key considerations related to the implementation of the following alternative modeling methods:
- Stochastic Analysis
 - Sensitivity Analysis

2. Continued

- (iii) Additionally, KRW is looking into introducing a new traditional life product.

Evaluate whether the shocks recommended by the CFO would be appropriate for the portfolio after adding the new product line.

Commentary on Question:

Candidates did not generally score well on this part of the question. Many responses did not follow the verb in each stem of the question (e.g. “Evaluate” or “Describe”).

- (i) The mortality shock is directionally appropriate, but increased longevity is unlikely to occur as an instantaneous change. Should look to add mortality improvement.

The interest rate shock is too simplistic. Multiple shocks should be run in order to capture changes in asset duration and potential policyholder behavior.

The lapse rate shock may not serve the intended purpose. Fixed deferred annuities may benefit from no lapses and single premium immediate annuities do not give the policyholder the option to lapse.

The aggregation method will not capture any diversification between the assets and liabilities and will overstate capital at the firm level.

- (ii) Stochastic Analysis
- Determine how to define the yield curve; and
 - Need to incorporate policyholder behavior into the projection

Sensitivity Analysis

- Need to identify the key assumptions to be included; and
- Consider potential correlation between risks in the aggregation process.

- (iii) The mortality shock will not be appropriate as it will have a favorable impact to the traditional life product. Need to look at an increase in mortality.

The discount rate shock is not appropriate due to its oversimplification. However, the same shock can be applied to the traditional life product as is applied to the annuity products.

2. Continued

In general, the lapse rate shocks between the deferred annuity product and the traditional life product should be different, as each product has a different lapse risk profile.

- (d) Recommend how KRW could mitigate financial risks using product design and asset allocation.

Commentary on Question:

Candidates generally did not score well on this part of the question. No credit was awarded for responses that included asset allocation concepts previously discussed in part (c).

Product Design

- Add a surrender charge and/or market value adjustment to the deferred annuity. This will effectively modify customer behavior in a manner than benefits KRW, or at least will minimize losses in the case of early lapsation.

Asset Allocation

- Enter into a longevity swap for the single premium deferred annuity block. This strategy might allow KRW to free up capital and write more annuity business and may reduce asset volatility.

3. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.

Sources:

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion
Ch. 7 Portfolio Risk: Analytical Methods

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion
Ch. 13 Liquidity Risk

Commentary on Question:

This question tests the candidate's knowledge on the different VaR methods and how they are calculated. The question then asks about what to do with the liquidity risk that is not included in VaR. Finally, the candidate is asked which investment should be selected considering the liabilities that the premiums support.

3. Continued

Solution:

(a) Consider the following VaR determination methods:

- Marginal VaR
- Incremental VaR
- Component VaR

- (i) Explain conceptually how to calculate the impact that an additional investment would have on the portfolio VaR under each method.
- (ii) Compare and contrast the use of these methods for XYZ.

Commentary on Question:

Part (a) was well performed in general. Many candidates were able to obtain credit for knowing the concepts for each VaR calculation method. Some candidates lost credit for not knowing how to compare and contrast, reversing the definitions of incremental and component VaR, or not pointing out that Incremental VaR is the most 'meaningful method' in the context of XYZ.

Solution: (a)(i)

Marginal VaR - Calculate the change in VaR for the portfolio resulting from taking an additional dollar of exposure to a given asset. It is the partial derivative with respect to that asset.

Incremental VaR - Calculate the VaR for the current portfolio, then the VaR after adding the new asset. The Incremental VaR is the difference between those two calculations.

Component VaR - A partition of the portfolio VaR that indicates how much the portfolio VaR would change approximately if the given component were deleted. To calculate this, we first need to calculate the VaR of the portfolio with the new investment and take the difference between this new value and the portfolio VaR before adding the new investment.

Solution: (a)(ii)

The Marginal VaR is the easiest to calculate but is only valid over small incremental changes for a given asset. Marginal VaR is also limiting in that it is linear.

Incremental VaR can be calculated based on the specific trade size that XYZ is making. Unlike Marginal VaR, the amount added or subtracted can be large.

3. Continued

The individual component VaRs add up to the total VaR, which helps to allocate several risks already within an organization in a meaningful way. However, this situation is involving adding a single risk, therefore the incremental VaR will be more meaningful and get you to the total portfolio VaR after adding the security.

(b)

- (i) Calculate the Incremental VaR for each of the two investment options at a 95% confidence level. Show all work.
- (ii) Explain how the answer from part (i) does not accurately reflect liquidity risk and why that should be a consideration for XYZ's investment decision.
- (iii) Identify and describe three approaches that a company can use to adjust its calculation of VaR to account for liquidity risk.

Commentary on Question:

Many candidates could calculate the before-trade VaR, but most could not handle the complexity of the math for the VaR with the additional investment. There was often confusion with what values to use in the trade sizes (W) for each of the calculation questions.

Solution (b)(i)

Before Trade: VaR (prior to trade) = $1.645 * 0.07 * 30 = 3.4545$

After Trade: w/ Corporate Bonds VaR (After trade): $1.645 * \sqrt{(30^2 * 0.07^2) + (10^2 * 0.08^2) + (2 * 0.6 * 0.07 * 0.08 * 30 * 10)} = 4.3727$

After Trade: w/ Private Placements VaR (After trade): $1.645 * \sqrt{(30^2 * 0.07^2) + (10^2 * 0.06^2) + (2 * 0.4 * 0.07 * 0.06 * 30 * 10)} = 3.9542$

Corporate Bond Incremental VaR = $4.3727 - 3.4545 = 0.9182$

Private Placements Incremental VaR = $3.9542 - 3.4545 = 0.4997$

Solution (b)(ii)

Liquidity risk represents the inability to unload an asset at a fair price due to a limited market- this is not reflected in VaR. The returns used to measure volatility are typically measured on mid-market prices, therefore this VaR calculation will not account for bid-ask spread or large volume trades. So, while the VaR is lower for the private placements, there is additional liquidity risk that is not included in this measurement that is particularly relevant given that the trade volume is beyond market depth.

3. Continued

Solution (b)(iii)

- Ensuring the time horizon that it is calculated on is longer than an “orderly” liquidation period.
- Artificially increase the volatility. This is simply a buffer that is applied to VaR rather than calculating liquidity directly.
- Add fixed spread or variable spread to the calculation to account for liquidity risk – known as LVaR.

(c)

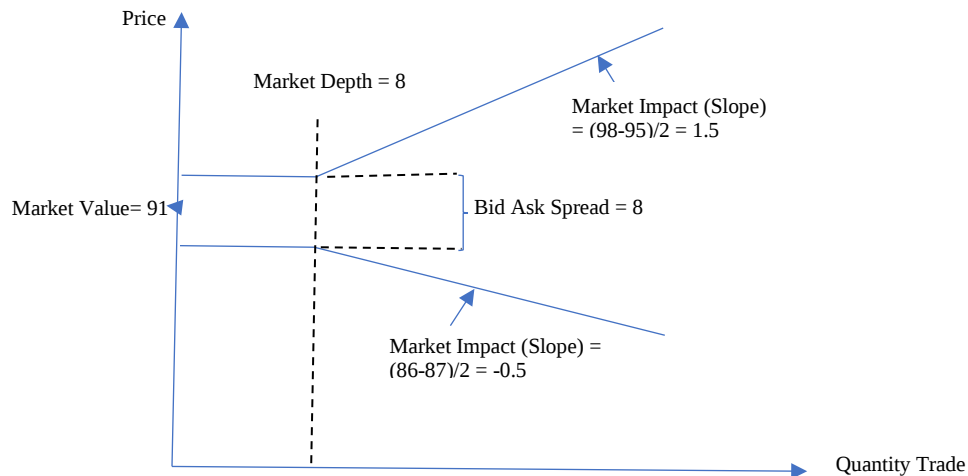
- (i) Draw a chart to show how the size of a transaction can affect the price at which it is executed by identifying:
 - Normal market size (depth)
 - Market impact
 - Market value
 - Bid-ask spread.
- (ii) Label each item in the chart from (i) with the value that applies to the Private Placement Bonds.
- (iii) Calculate the liquidity-adjusted VaR (LVaR) for each of the two investment options prior to XYZ’s investment in either of them. Show all work.

Commentary on Question:

Many candidates did not understand which prices (at quantity of 8 vs. 10) to use for the market value and bid-ask spread calculations. Some candidates drew the chart correctly but failed to label each item with the correct value. Few candidates were able to quantify the Market Impact on the bid and asked prices. For subpart (iii), the most common error was in the calculation of the spread. Some candidates used only the difference in prices and did not divide by the mid-market price.

3. Continued

Solutions (c)(i) and (c)(ii)



Solutions (c) (iii)

$$LVAR = VAR + [(Ask-Bid)/Mid]/2 * Investment$$

$$\text{Corporate Bond LVAR} = 1.645 * 0.08 * 10 + [(96.2 - 95.8) / 96] / 2 * 10 = 1.3368$$

$$\text{Private Placement LVAR} = 1.645 * 0.06 * 10 + [(98.0 - 86.0) / 92] / 2 * 10 = 1.6392$$

(d)

- (i) Recommend which investment should be made based on the results obtained in (b) and (c). Justify your recommendation.
- (ii) Provide additional liquidity considerations that would affect a recommendation in the following situations:
 - I. XYZ is using the assets to back annuities with surrender options
 - II. XYZ plans to invest only 5 instead of 10
 - III. XYZ's risk appetite statement includes the following: "...We are more concerned with default and credit spread risk than liquidity risk as we typically carry enough cash to meet our expected benefit payments..."

3. Continued

Commentary on Question:

Some candidates could not make a valid recommendation due to incorrect calculation results in (b) and (c). Several candidates recommended Corporate Bonds but never considered that the liabilities are highly structured and the need for asset liquidity is not relevant. Candidates received partial credit if they recommended the wrong investment with valid justifications. Another area where candidates lost credit was providing too general of considerations in subpart (d)(ii) instead of providing considerations specifically related to liquidity.

Solutions (d)(i)

The private placements generate a higher yield but have lower liquidity and a lower Incremental VaR. They also have a higher LVaR because the asset liquidity risk is higher due to a wider bid-ask spread.

The Corporate bonds generate a higher incremental VaR but have a lower LVaR since they are publicly traded and liquid assets.

In XYZ's situation, they are using these assets to back highly structured liabilities with very predictable cash flows that is unlikely to require them to sell significant assets unexpectedly. Since they are better able to manage the higher LVaR, XYZ should invest in the private placements.

Solutions (d)(ii)

- I. Given the liabilities would be more liquid, XYZ would have to put more weight on the liquidity/LVaR in their decision making as there is higher likelihood of the need to liquidate assets at some point
- II. XYZ could put less weight on liquidity/LVaR because the trade would be within the market depth of the Private Placements
- III. This would put less weight on liquidity/LVaR as XYZ would not expect to need to liquidate assets unexpectedly.

4. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (5e) Demonstrate the ability to develop a capital model for a representative financial firm.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 14 Quantifying Particular Risks

Quantitative Enterprise Risk Management, Hardy, Ch. 2 Risk Taxonomy

Risk Appetite: Linkage with Strategic Planning Report

ERM-101-12: Measurement and Modeling of Dependencies in Economic Capital (Ch 3-5)

ERM-119-14: Aggregation of risks and Allocation of Capital (Sections 4-7 Excluding 6.3)

Commentary on Question:

This question was composed of many open ended and critical thinking subcomponents. While knowledge of the subject was required to receive full credit, so was careful consideration of what was being asked and what was being given. Many candidates missed points for not clearly answering what was asked and for not properly considering items such as years and targets that were provided in the question.

4. Continued

Solution:

- (a) Big Ben's CFO proposes enhancing the model by adding a liquidity risk component:
- (i) Explain why liquidity risk could be difficult for Big Ben to quantify.
 - (ii) Design a scenario that Big Ben could use to measure its liquidity exposure.

Commentary on Question:

For subpart (i), most candidates were able to briefly name reasons quantifying why liquidity risk is difficult and received partial credit. However, many did not explain why it would be difficult, which was required to receive full credit. For subpart (ii), results were similar, with students mentioning things to put in the scenario but not providing enough detail or justification of the scenario for full credit.

- (i) It is difficult to quantify because liquidity risk is often systemic in nature, highly correlated to other risks, and occurs uniquely in every organization. As such, data on actual liquidity crises is limited, so even if data is available, there is a good chance that Big Ben could not actually use it. Also, all organizations have some inherent liquidity risk, and its significance differs according to the type of organization.
 - (ii) Big Ben experiences a major security breach that severely impacts the reputation of the bank. This causes a run on the bank where a third of the deposit liabilities are claimed by their customers. This is more than the cash they have on hand, thereby requiring them to fund the gap by borrowing additional funds or selling assets. The resulting loss due to the stress would be the capital loss on the assets sold, and the investment income that is foregone because they don't own the asset anymore.
- (b) Big Ben's CFO states that the company should target an internal capital adequacy ratio of 150%.
- (i) Describe the process of using copulas to aggregate risk in an EC model.
 - (ii) Your firm's analysis suggests that 10% would be a reasonable fixed diversification percentage for 2018.
- Determine whether 10% would be sufficient to maintain a capital ratio above the CFO's suggested target. Show all work.

4. Continued

- (iii) Critique each of the three diversification methods being considered by Big Ben. Provide support using information from the case study.

Commentary on Question:

For subpart (i) students did well for the first steps, but most left out the need to repeat the process across many simulations. In subpart (ii), even though the year to calculate the diversification percentage was clearly given with the last word of the sentence as 2018, many students used 2019 data. Partial credit was still given for these answers if they were done correctly using the 2019 numbers, but they did not receive full credit. Students generally did well on subpart (iii). Many candidates used a different capital adequacy ratio, the one provided in the Case Study. Full credit was not awarded for using 140% instead of 150%.

- (i) To use copulas to aggregate risk in an EC model, first you must create a distribution for each component. Second you simulate a uniform random variable from $[0,1]$ that represents the marginal distributions of the copula. The cumulative distribution of each risk is then calculated. Next you take the inverse of the marginal distributions and the simulated loss represents the sum of the inverses. Finally you repeat the process across many simulations to get a simulated total loss distribution.
- (ii) The total required economic capital is:
 $(370+471+277+160) \times (1 - .10) = 1,150.2$
TAC ratio = Available EC/Required EC = $1,642 / 1,150.2 = 142.8\%$.
This is less than 150% therefore 10% is not sufficient to maintain the capital ratio.
- (iii) Variance Co-variance Matrix (correlation): Although the calculation is straightforward and easy to explain, Big Ben is concerned with how the correlations were obtained therefore the diversification benefit may not be appropriate. Even if they felt good about how they were obtained, the correlations can change over time, particularly in stress scenarios, and may need adjusted.

Fixed Diversification Percentage: This method is similar to the simple summation method, and is straightforward to calculate and explain. That simplicity fails to capture the interaction between risks, which is key for aggregating them. It is also a fixed percentage, which would not capture the diversification benefit changing over time.

4. Continued

Copula: Copulas do very well at reflecting tail dependency and it is highly likely that Big Ben has risks that are tail dependent. This comes with the offset of being difficult to specify the structure of the copula. Big Ben would need to have expertise in house to perform the copula calculations, which involve complicated simulations.

- (c) Big Ben's CFO has expressed concern that the EC produced by the current model is greater than he would expect.

Evaluate whether each of the following factors that determine Big Ben's EC level support the CFO's assertion.

- I. Regulatory requirements
- II. Risk measure components
- III. Modeling method

Commentary on Question:

Many candidates skipped parts or all of part (c). Some provided adequate explanations but left out whether or not their answer would support the CFO's assertion, which is needed to demonstrate their grasp and ability to relate the material to a real-world situation. Most students failed to distinguish that the question was asking about the Risk measure components and not Risk components, which received no credit.

Regulatory Requirements: The components of the EC model should meet regulatory standards if it's being used for required capital calculations. A confidence level of 99.5% for VaR is consistent with regulatory standards, as well as one of the most popular in banking and insurance, so it seems appropriate. This does not support the CFO's assertion.

Risk Measure Components - VaR is the risk measure being used in the model. It represents the loss at a given percentile, and doesn't consider any values beyond that point. VaR is appropriate at higher confidence levels and is used in regulatory requirements. If Big Ben were to switch to a TVaR/CTE risk measure at the same confidence level, the capital coming out of the model would be higher. This does not support the CRO's assertion.

4. Continued

Modeling Method – Big Ben uses various internal models, such as Joint normal distribution, Stochastic, scenario analysis, and methods to measure the different risk types in its economic capital model. If a different method were chosen for a particular risk, a different level of capital will most likely be calculated. This would support the CFO's assertion if the models were overstating the capital amounts, but could also not support the assertion if the opposite were true.

5. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

- (2g) Analyze and evaluate model and parameter risk.
- (2h) Construct approaches to modeling various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.
- (5b) Define the basic elements and explain the uses of economic capital. Explain the challenges and limits of economic capital calculations and explain how economic capital may differ from external requirements of rating agencies and regulators.
- (5c) Apply risk measures and demonstrate how to use them in capital assessment. Contrast regulatory, accounting, statutory and economic capital.

Sources:

Economic Scenario Generators: A Practical Guide (pp. 7-17)

ERM-101-12: Measurement and Modeling of Dependencies in Economic Capital (Ch 5)

ERM-106-12: Economic Capital-Practical Considerations-Milliman

ERM-102-12: Value-at-Risk: Evolution, Deficiencies, and Alternatives

Commentary on Question:

This question crossed multiple learning objectives and asked for a variety of levels of analysis in looking at the use and evaluation of economic scenarios and ESGs for variable annuities, as well as the effect of risk metric choices, and risk diversification between life insurance and annuities.

Most candidates were able to attempt at least parts of this question, and many were able to get credit on many of the parts. Some candidates answered questions with recalled statements and did not apply it to the specific situation.

A common issue was candidates not using the information in the stem of the question, especially once they got to the last part of the question.

5. Continued

Solution:

- (a) You have been asked to apply the new standard to the company's variable annuity products.
- (i) Explain why stochastic scenarios would be appropriate to value the variable annuity liabilities.
 - (ii) Assess whether risk-neutral scenarios are the appropriate type to use.

Commentary on Question:

On subpart (i), many candidates were able to answer with the need for stochastic simulation to capture the value of embedded options in VAs. Some candidates discussed real-world vs. risk-neutral simulation, which is not what was asked for in this part of the question.

For subpart (ii), some candidates missed that a new valuation standard was requiring market consistent valuation, which is a big part of the reason why risk-neutral scenarios would be appropriate.

(i) Stochastic simulation overcomes many of the restrictions that are imposed by the use of an analytical formula or deterministic set of scenarios. It allows for the evaluation of complex dynamic financial systems, including policyholder behavior.

The liability cash flow of variable annuity products can be significantly different depending on the economic condition which requires the projection of multiple economic conditions.

Variable annuity guarantees can show varying degrees of in-the-moneyness depending on the economic scenario. In a typical deterministic scenario, many variable annuity guarantees never produce a non-zero value at any time horizon. Stochastic scenarios will ensure a range of economic conditions will be seen, including those with very low probability.

(ii) Risk-neutral scenarios assume there is no arbitrage, a complete market and are calibrated to reproduce current market prices of assets.

Risk-neutral scenarios are therefore better suited for a market consistent valuation, which is the new standard mentioned in the problem.

5. Continued

- (b) Evaluate the calibration and parameterization of the ESG.

Commentary on Question:

Many candidates were able to see the deficiency in the real-world scenario parameterization. Some candidates responded that calibration or parameterization was good without providing a reason and did not receive full credit.

Risk-neutral scenarios

Calibration: Given that the scenarios are supposed to be market consistent, it is good that the model is calibrated to current market prices.

Parameterization: The inputted yield curve is consistent with the asset prices which is needed to get back to the appropriate market values.

Real-world scenarios

Calibration: Real-world scenarios should be calibrated to reproduce realistic market dynamics, so the first point is good.

Parameterization: The parameterization is based on current market data, whereas real-world scenarios would generally be calibrated to historical data.

- (c)
- (i) Outline the steps involved in performing a martingale test.
 - (ii) Determine which scenario generator is more appropriate to use for the new valuation standard based on the martingale test. Show all work.

Commentary on Question:

For subpart (i), some candidates supplied the definition of a martingale instead of explaining the martingale test. Because this did not answer the question being asked, it did not receive credit.

For subpart (ii), there were many equivalent mathematical approaches, and both continuous and effective interest was acceptable for calculation. Some candidates did not show all their work, which prevented them from receiving full credit.

- (i) The martingale test is used to confirm a scenario set is risk-neutral.

Risk-neutral prices are generated for assets, and the risk-neutral short rate of interest also is generated. A time horizon is selected.

For a non-cash-flow-generating asset, take the average of the present value of the ratios of the risk-neutral prices and the values of the asset from the scenarios. If this average diverges from the time zero market value, the scenarios being used are not risk-neutral.

5. Continued

(ii) Using continuous interest:

Generator 1:

$$T = 1, X_0 = \exp(-2\% \cdot 1) \cdot 1.020 = 1.000$$

$$T = 5, X_0 = \exp(-2\% \cdot 5) \cdot 1.091 = 0.987$$

$$T = 10, X_0 = \exp(-2\% \cdot 10) \cdot 1.133 = 0.928$$

$$T = 20, X_0 = \exp(-2\% \cdot 20) \cdot 1.315 = 0.881$$

$$T = 30, X_0 = \exp(-2\% \cdot 30) \cdot 1.517 = 0.833$$

Generator 2:

$$T = 1, X_0 = \exp(-2\% \cdot 1) \cdot 1.021 = 1.001$$

$$T = 5, X_0 = \exp(-2\% \cdot 5) \cdot 1.132 = 1.024$$

$$T = 10, X_0 = \exp(-2\% \cdot 10) \cdot 1.315 = 1.077$$

$$T = 20, X_0 = \exp(-2\% \cdot 20) \cdot 1.515 = 1.016$$

$$T = 30, X_0 = \exp(-2\% \cdot 30) \cdot 1.825 = 1.002$$

While both generators diverge from 1 at various points, overall Generator 2 is closer to 1 for the T we're given. Generator 1 diverges more and more from 1 as T increases, which clearly fails the martingale test. Thus, Generator 2 is more appropriate to use for market-consistent valuation.

(d) Basic Life's current economic capital (EC) calculation uses deterministic scenarios. The CFO is concerned that the EC does not appropriately quantify the impact of a market downturn on guaranteed payouts associated with the variable annuity products. He tells you the company is considering using the new ESG to calculate EC to address this concern.

(i) Explain whether the new risk-neutral ESG is appropriate to use for calculating EC for variable annuities.

In addition, the CFO proposes that the following risk metrics be considered for quantifying the market risk:

- Value at Risk
- Standard Deviation
- Tail Value at Risk (CTE)
- Maximum Loss

(ii) Recommend which of the proposed metrics is the most appropriate to quantify EC for market risk. Justify your choice.

5. Continued

Commentary on Question:

Candidates often indicated that risk-neutral scenarios were not appropriate in subpart (i), and that real-world scenarios were better. In order to receive full credit, an explanation as to why this was the case was required.

Candidates, in general, did well on subpart (ii). Some candidates recommended VaR, which was considered an acceptable response if supported by appropriate arguments.

(i) To be able to calculate an EC risk measure (such as VaR99.9), the probability distribution of risk-neutral scenarios would need to be converted to real-world probabilities.

It is not straightforward how one could use the market price of securities to construct this probability distribution. Calibrating the probability in this way would make results difficult to interpret as well. Therefore, the new risk-neutral ESG would not be appropriate to use for economic capital.

(ii) I recommend Tail-VaR

Coherence is a desirable trait for EC risk metrics, and Tail-VaR & maximum loss are coherent; Standard deviation and VaR are not coherent.

Maximum loss would probably be too extreme, as market results are fat-tailed and the guarantees may have very high value at extremely low probabilities

Standard deviation by itself does not provide information on capital needs; it would require knowing the mean.

VaR could be used, but due to not having subadditivity, one could get an underestimate of EC when aggregating market risks.

- (e) Basic Life is considering launching a single premium immediate annuity (SPIA) product.
 - (i) Describe how launching a SPIA product could result in a diversification benefit for Basic Life.
 - (ii) Compare and contrast the use of a correlation matrix versus a copula to reflect this diversification benefit in the EC calculation.

5. Continued

Commentary on Question:

Strong responses to subpart (i) discussed the mortality/longevity offsets of life insurance and income annuities. Some candidates instead discussed the diversification benefit between SPIAs and VAs, which is not as clear and only received partial credit.

For subpart (ii), many candidates were able to state aspects of copulas and correlation matrices, but did not mention how this pertained to capturing the specific diversification benefit mentioned in subpart (i).

(i) Launching a payout annuity product could result in a diversification benefit because Basic Life sells life insurance products currently and there is a natural hedge between life insurance and annuity products in mortality and longevity.

(ii) Modelling: Copulas require marginal distribution functions and sometimes stochastic simulation, while a correlation matrix approach only needs the amount of economic capital associated with each risk and correlations between risks. Mortality/longevity risk isn't commonly modelled stochastically so the correlation matrix may be more appropriate.

Tail dependence: Correlation matrices generally understate tail dependence of risks but copulas are able to reflect varying degrees of tail dependence depending on the copula. Mortality/longevity risk doesn't normally have fat tails and hence the tail dependence may not be significant to capture.

6. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (2g) Analyze and evaluate model and parameter risk.
- (2h) Construct approaches to modeling various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.

Sources:

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis

ERM-118-14: Model Validation Principles Applied to Risk and Capital Models in the Insurance Industry

ERM-119-14: Aggregation of risks and Allocation of Capital

ERM-101-12: Measurement and Modeling of Dependencies in Economic Capital

Commentary on Question:

This question focused on qualitative assessment methods for external risk based on the Energetix context from the core case study. The question also covered principles of model risk and potential aggregation methods.

Solution:

- (a) ExRM must include the following types of scenarios in the model:

- Global Scenario
- Multi-Event Scenario
- Synthetic Scenario

Identify the type of scenario for each of the Scenarios 1, 2 and 3. Justify your response.

Commentary on Question:

Most candidates did well on this part. Most candidates assumed they had to choose one of the listed types for each scenario. Partial credit was awarded if the question was interpreted this way.

6. Continued

Scenario 1 is a Synthetic Scenario. It describes a hypothetical condition that has not been observed.

Scenario 2 is a Single-Event scenario. It focuses on a single specific event.

Scenario 3 is a Global Scenario. It will affect all electrical companies on a global level; these scenarios affect a whole industry, unfold over time, test interdependency, systemic event, etc.

- (b) You are asked to evaluate the ExRM using Model Validation Principles Applied to Risk and Capital Models as identified by the North American CRO Council.
- (i) Identify two Principles that Energetix follows. Justify your response.
 - (ii) Identify two Principles that Energetix is clearly not following. Justify your response.

Commentary on Question:

Candidates appeared to struggle on this part. Several candidates listed the same principle for both (i) and (ii). In this case points were only awarded for it being listed once.

- (i) Model design and build need to be consistent with the model's intended purpose. The model was built specifically for this purpose.

Establish an owner of model validation. The owner is the group responsible for attesting, resolving and reporting on model validation aspects. The owner is established (even though it is the wrong owner, the owner is clearly established)

- (ii) Ensure the model validation is an independent process. This isn't occurring as the group that validates is also the group that built it.

Model Documentation is scarce / non-existent

- (c) Energetix plans to look at the external risks in aggregate for its three main operating businesses. Energetix instructed its other businesses to begin using the same ExRM and to include Scenario 2 and Scenario 3 in each of their models.

Evaluate Scenario 2 and Scenario 3 for use in CR and EUI.

6. Continued

Commentary on Question:

Candidates appeared to struggle on this part, especially for Scenario 2 as it directly impacts GUI, but they should be opining on the potential indirect impact on EUI and CR. Partial credit was awarded if the candidate was able to demonstrate limited justifications for including / excluding the scenario.

Scenario 2

EUI - Energetix is headquartered in CO, but we don't specifically know which 7 states they do business in, and it only operates GUI in 5 of those states. We don't have enough information to know the impact on EUI

CR – CR is regulated at a federal level and wouldn't be impacted by this, or potentially could be positively impacted. Not a good scenario for ExRM.

Scenario 3

EUI – This would be a good scenario for EUI to use as natural gas is something they rely on.

CR – this would either have no impact or a potential positive impact on CR. Not a good scenario for ExRM.

- (d) Energetix's CRO has proposed that in order to determine the total exposure across the three main operating businesses, each business would identify its own top five risks, and Energetix would simply sum the resultant exposures.
 - (i) Evaluate the CRO's proposed aggregation process.
 - (ii) Propose an alternative aggregation approach.

Commentary on Question:

Most candidates did well on this part. Alternative responses included any approach mentioned in the source reading. Minimal points were earned when a candidate provided an alternative approach without providing support for their choice.

6. Continued

(i)

The CRO's proposal is focused only on summing exposures - that is potential severity - not taking into account probability of occurrence of each risk. This would simply overstate overall total exposure for Energetix, would not consider potential diversification between the units and would not take into account the probability of occurrence. It would be a simple measure to implement but a crude one for Energetix. Business units may have inconsistent methods for defining their top 5 risks.

(ii)

One approach would be to consolidate by risk - frequency and severity - for the whole Energetix. A top-down approach. Identify top risks at the enterprise level. Correlations between the business units would be indirectly considered by this process. Scenarios could be used for this purpose to assess overall impact on Energetix.

7. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.

Sources:

ERM – 332-20: Longevity Risk Management

Commentary on Question:

This question tested the candidates' understanding of capital management, risk transfer techniques, and risk management tools for pension plans.

Solution:

- (a) Mr. Lyon suggests that adding a cost-of-living-adjustment to the SLIC pension plan would mitigate longevity risk.

Critique Mr. Lyon's statement.

Commentary on Question:

Most candidates correctly identified that this would increase longevity risk for the plan sponsor but did not consider that this would mitigate risk from the participant perspective.

7. Continued

This would mitigate longevity risk from the participant's perspective, not the plan sponsor's. It would actually increase the longevity risk to a plan sponsor.

- (b) Compare the longevity risk exposures faced by the SLIC and AHA pension plan sponsors with respect to the following:
- Demographic Factors
 - Benefit Structure

Commentary on Question:

In order to receive full credit candidates had to identify whether SLIC or AHA had higher longevity risk than the other plan for both demographic factors and benefit structure along with appropriate reasoning.

Demographic:

Number of members – SLIC has a 50/50 active/inactive split while AHA has a 90/10 split. AHA has more longevity risk due to relatively more active members.

Age profile – SLIC's active members have an average age of 52 while AHA's are 41. SLIC's inactive members have an average age of 73 while AHA's are 63. AHA has more longevity risk due to relatively younger population.

Benefit structure:

SLIC has an annuity form of payment and does not offer lump sums while AHA has both a lump sum and annuity form of payment. AHA's longevity risk is lower than SLIC as active members can elect a lump sum.

Although, AHA has retirees so not everyone takes a lump sum therefore the longevity risk exposure on the remaining retirees is higher due to anti-selection.

- (c)
- (i) Identify the factors that would cause longevity risk to be relatively more significant than other risk sources for a pension plan.
 - (ii) Evaluate whether the SLIC or the AHA pension plan is likely to have more relative exposure to longevity risk based on each of the factors identified in (i).

7. Continued

Commentary on Question:

To receive full credit for (i) it was only necessary to list factors that generally pertain to pension plans and not relate them to the case study. While for (ii) it was necessary to relate the identified factors to both the SLIC and AHA plans. Many candidates did poorly on this question for both subparts (i) and (ii).

(i)

Longevity risk is relatively higher if:

- The plan has a low funded status
- Has de-risked by reducing equity allocation
- Has hedged a significant amount of its liability interest rate risk

(ii)

	SLIC	AHA
Accounting Funded Status	76%	76%
Fixed Income %	39%	24%
Interest Rate Risk Hedge	None	None

SLIC plan has relatively more exposure to longevity risk due to these factors.

(d) Caerus has asked you to present the following risk management solutions to SLIC and AHA:

- Annuity Buy-out
- q-Forward
- Out-of-the-money longevity swap

(i) Describe each of the risk management solutions with respect to each of the following:

- Implementation
- Risks transferred or hedged
- End holder of the risks

(ii) Rank the three risk management solutions for the SLIC pension plan based on cost and the considerations in (i). Justify your answer.

Commentary on Question:

(i) Most candidates were able to describe an annuity buy-out but struggled with q-forwards and out-of-the-money longevity swaps.

(ii) Different rankings were accepted provided that appropriate justifications were provided.

7. Continued

(i)

Strategy	Implementation	Risks transferred	Nature of end holder
Annuity Buy-out	Purchase annuities from insurer; payments made to participants directly from insurer	Longevity and all other financial and demographic risks	Insurer
q-Forward	Exchange of amount proportional to actual, realized mortality rate for amount proportional to assumed mortality rate	Longevity risk	Capital market participants
Out-of-the-money longevity swap	Pension plan makes fixed payments representing assumed longevity, and receives variable payments representing actual longevity. Differs from regular longevity swap due to attachment/exhaustion points (i.e. floors/caps)	Portion of longevity risk	Capital market participant

(ii)

- 1) Annuity buy-out. Assuming that it is priced well and the sponsor can afford it, a buy-out also helps remove other risks. Prior analysis showed that risks other than longevity are relatively significant.
- 2) Out-of-the-money longevity swap. It would be cheaper than the q-forward and mitigate more extreme longevity risk.
- 3) q-forward

- (e) Dr. Graham likes the concept of a pension buy-in for AHA, but wants to consider alternatives to buying annuities from insurance companies.

Describe the process by which AHA could mimic a pension buy-in through the capital markets.

Commentary on Question:

In order to receive full credit candidates had to correctly identify the products used to mimic a buy-in and describe how AHA could use those products.

Enter longevity and interest rate swaps with a counterparty.

Longevity swap – AHA pays a pre-determined amount each year which reflects fixed longevity and receives a floating amount which reflects actual longevity.

Interest rate swap – AHA pays a fixed amount and receives a floating amount.

A total return swap can be used to reduce the funding costs.

8. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (2g) Analyze and evaluate model and parameter risk.
- (4b) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.

Sources:

Embedded Options in Pension Plans – pg. 6-7, 28-60 (RET)

Pension Risk Transfer – pg. 1-7, 11-46 (RET)

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Tim notices that the BBI yields have been trending lower in the past several quarters, approaching the minimum crediting rate, and he has become concerned with the plan's valuation.

The asset investment manager, Eric Morton, suggests to Tim "Let's value the annual guarantee using simulation. We can use existing stochastic BBI scenarios from my team to simulate payoffs. You can just calculate the average of the present values of the simulated payoffs for each path using a risk-free discount rate of 3%."

Critique Eric's suggestion.

8. Continued

Commentary on Question:

The solution requires a basic understanding of risk-neutral simulation. However, very few candidates provided a strong enough answer to receive full credit on this part.

- Use of stochastic simulation to value the annual guarantee is a good suggestion since the liability is complex and not easily determined with a closed form solution.
- Not a good idea to use the average of the present values using a single risk-free rate of 3% since each payoff is path-dependent.
- Instead, use the risk-free short rate in each scenario to obtain the present values which can then be averaged.

(b) Describe how the following changes would affect the outcome of the valuation of the guarantee.

- Increasing the expected return of the BBI
- Increasing the volatility of the BBI
- Delaying the assumed benefit commencement date

Commentary on Question:

Most candidates scored reasonably well on part (b).

- Increasing returns of the BBI would make it less likely the guarantee would be in the money, and so reduce the value of the guarantee.
- Increasing volatility increases the price of options. Since the guarantee is akin to a put option, increasing volatility would also increase the value of the guarantee.
- Delaying commencement date would create additional periods where the guarantee could come into the money. Additionally, the increased time horizon would drive up the value of the implicit put option. The guarantee value would increase.

(c)

(i) Eric wants to consider hedging the plan with either static moving notional interest rate hedges or dynamic moving notional interest rate hedges.

Explain how to implement each of the two hedges.

(ii) Explain why a static hedge will not be a perfect hedge.

8. Continued

(iii) Tim's and Eric's teams gathered the following information on the pension plan and asset market data:

- The account balance value of the plan for the actives and the terminated vesteds is at \$600 million as of January 1, 2020
- Benefit commencement for all non-retired participants is assumed to happen at January 1, 2030

Zero Coupon Treasury Rates by Tenor at January 1, 2020

1 Year	5 Year	10 Year
1.0%	1.5%	3.0%

Assumed BBI by Projection Year

Years 1-4	Years 5-9	Years 10+
3.0%	1.0%	2.5%

Price of Instruments as of January 1, 2020 (per 1 dollar notional)

BBI 1-year Call with 2% Strike	BBI 1-year Put with 2% Strike	10-year Interest Rate Swap BBI with 2% fixed
0.14	0.10	0.60

They want to evaluate the financial impact of implementing a dynamic moving notional interest rate hedging strategy to be implemented on January 1, 2020.

Calculate the following:

- The cost of the instruments to be purchased on January 1, 2020
- The estimated payoff of the hedge as of January 1, 2021

Show your work.

Commentary on Question:

No candidates scored well on this part of the question. It was designed to be fairly straightforward, and the correct answer can be found directly in the source material.

c(i)

Static moving notional interest rate hedge:

- Plan sponsor enters into an interest rate swap at time 0
- Sponsor pays a fixed rate
- Counterparty pays max of 2%, BBI
- Exchange swap payments each year in cash
- Notional is stochastic variable, based on crediting rate

Dynamic moving notional interest rate hedge:

- Purchase risk free bonds at time 0 in the amount needed to protect against the minimum guaranteed amount at assumed benefit commencement date
- Every year, purchase a call option to hedge against excess interest rate returns

8. Continued

c(ii)

Not a perfect hedge because:

- Notional can only be guessed at
- When interest rates are higher than predicted, sponsor has to pay for additional notional plus higher priced options. Interest on floating rate payoffs offset these higher amounts.
- When interest rates are lower than predicted, sponsor overpays on notional

c(iii)

Initial instruments:

- Minimum balance at 1/1/2030 = $\$600 \times 1.02^{10} = \731
- Buy a 10-year zero coupon bond for $\$731 / 1.03^{10} = \544
- Buy a 1 year call option that pays max of $[BBI - 2\%, 0]$ on projected year end balance = $\$600 \times 1.03 \times 0.14 = \86.5

Estimated payoff of hedge

- 1% of $1.03 \times \$600 = \6

(d)

- (i) Identify the issues with the asset allocation that might cause Benson International to reject an in-kind transfer from Castel.
- (ii) Castel provided the following information for a group annuity buyout for the retirees.
 - The economic liability for longevity improvement is estimated to be an additional 1% of the total PBO.
 - The NPV of future administrative expenses and investment fees is estimated to be 3.5% of the total PBO.
 - The NPV of future PBGC Premiums is estimated to be \$0.2 million.
 - The settlement premium based on deals of similar size is estimated to be 1.5bps of the total economic liability.

Estimate the cost of entering into a group annuity buyout deal with Benson International as of December 31, 2019. Show your work.

- (iii) Zack has decided against using a dynamic moving notional interest rate hedge due to uncertainty of the ultimate cost.

Recommend whether Castel should implement the static moving notional interest rate hedge or the group annuity buyout, assuming the costs are comparable. Justify your recommendation.

8. Continued

Commentary on Question:

Candidates scored reasonably well on this part of the question. On subpart (ii), several candidates mistakenly priced a full buyout vs. a retiree only buyout (which was clearly stated in the question), for which they received reduced credit. Also, as there is some debate as to whether PBGC premiums should be considered an obligation of the insurer, candidates who excluded these expenses in the determination of the economic liability received full credit. Finally, the risk transfer premium was clearly stated as 1.5 bps, even though 150 bps better aligns with current market trends. For this reason, candidates who used 150 were given substantial credit, or potentially full credit if they provided an explanation of why they used it instead of the 1.5 bps given in the question.

Answer:

d(i)

- There is a large asset-liability duration gap
 - PBO of retirees is 2,900 but mid to long term assets are only 850
- Assets include large portion of risky bonds. Benson is likely wary of default risk
- Almost 1/3 of assets are cash already and another 1/3 in short term bonds; little gain to Benson in accepting these / loss to Castel on liquidation

d(ii)

- Retiree PBO = 2,900
- Longevity improvement = $2,900 \times .01 = 29$
- NPV admin expenses = $2,900 \times .035 = 101.5$
- PBGC = 0.2
- Economic Liability = $2,900 + 29 + 101.5 + 0.2 = 3,030.7$
- Settlement premium = $3,030.7 \times .00015 = 0.5$
- Cost of buyout = $3,030.7 + .5 = 3,031.2$

d(iii)

Given comparable costs, Castel should implement the group annuity buyout:

- Removes all risks versus hedge, wherein Castel carries longevity, financial and other risks
- No ongoing administrative costs, including PBGC premiums
- Benson has high S&P rating, so fiduciary obligation can be deemed satisfied