RISKS and REWARDS

The Newsletter of the Investment Section of the Society of Actuaries

NUMBER 31 OCTOBER 1998

Call for Authors!

Risks and Rewards needs your help. We have some ideas for future articles but are in dire need of authors. Some ideas we have are:

- Review of Professor Richard Thaler's speech (author of "The Winner's Circle") at the Investment Section breakfast at the Annual Meeting. If you are going to be at the breakfast, please volunteer.
- Financial Patents. There have been new patents issued on financial topics, including one to an actuary and former Section officer, Meyer Melnikoff. This article would discuss financial patents in general and review the specifics of one or more patents. Richard Wendt, co-editor of *Risks and Rewards*, has the patent details from the U.S. Patent Office. If you are interested in authoring such as an article, please contact Mr. Wendt at his *Directory* address.
- Review of *Financial Economics*, the new text published by the Actuarial Foundation. Two articles would be appropriate to cover the breadth of this topic.

If you would like to volunteer for one of these topics, or have other ideas for articles, please contact one of the co-editors listed on page 2.

Synthetic GIC and Guaranteed Separate Account Model Regulations

by Victor Modugno

Two new model regulations—one covering synthetic GICs and the other guaranteed separate accounts—are working their way through the regulatory process. Both were on the agenda of the 1998 NAIC summer meeting held in Boston in June. A working group under the Life Insurance (A) Committee is developing the Synthetic GIC regulation. Action on adapting a proposed model regulation was deferred until the fall meeting due to controversy over several provisions in the latest draft. A working group under the Accounting Practices and Procedures (EX4) Task Force is developing the guaranteed separate account regulation. A revised model regulation was exposed for comment. Both regulations are available on the NAIC web site (naic.org). The web site also has contact persons for comments.

Most regulators in the working groups and industry representatives of the interested parties are the same for these two regulations. The main difference between these two products is whether the assets subject to guarantees are held in a trust (synthetic GIC) or in an insurance company separate account. The reason for different working groups is that separate accounts are subject to an accounting standard (*SSAP 89*), which governs accounting and reserving for separate accounts. There is no accounting standard for synthetics.

Industry groups developed initial drafts of these model regulations based on existing separate account and synthetic GIC regulations—New York Regulation 128, California Insurance Code Sections 10506.4 and 10507, and Bulletins 95–8 and 95–10. Reserves were based on guaranteed values discounted at 1.05 of treasury spot rates while market value of assets were reduced by asset valuation reserve-based factors. The synthetic GIC regulation had several required contract provisions, which seemed unusual because the purchasers of these contracts are large institutions that are represented by attorneys and other experts. Mandated contract provisions are not needed for consumer protection.

These drafts then went to the regulatory task forces. There were several conference calls and redrafts. Many of the ideas developed by the synthetic group were incorporated into the separate account regulation. The revised filing requirements were of greatest concern to the industry group. In the

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original draft, a company would need to get approval of plan of operation in its domiciliary state. So long as that state had requirements similar to the model regulation, other states would accept this determination. Previously approved contract forms would be grandfathered. In the latest draft, approval of the plan of operation would be required in each state in which the insurer wants to issue contracts and grandfathering of existing contracts is limited.

The synthetic GIC regulation was further along in development and elicited the most comments from industry representatives prior to the Boston meeting. Most letters expressed concern about the filing requirements and the actuarial opinion and memorandum. A letter from the Stable Value Association opposing the filing requirements indicated that buyers prefer efficient and effective regulation. According to that comment letter, these regulations are out of line with the current free market, deregulation trend for sophisticated purchasers. Another area of comment was competition with banks. Banks in the synthetic business can negotiate contracts with clients without getting advance approval. Bank regulators focus on internal risk management and controls of these and other businesses of the bank using sophisticated value-at-risk measurements. Others commented that only the domiciliary state had all the information needed to assess solvency. State insurance departments should use their limited resources where they can add the most value—in monitoring overall solvency of domestic insurers and protecting unsophisticated consumers from abuses.

Consider the typical separate account or synthetic GIC sale. A GIC manager, who specializes in purchasing these contracts, requests proposals for a large 401k-plan client or a pool of smaller clients. This manager has investment professionals, credit analysts, and attorneys with substantial experience in these arrangements. The insurer or bank issuing the contract must be large, double-A rated, and have appropriate staff to pass the GIC manager's due-diligence. Issuers who are allowed to bid are concerned about protecting their capital and preserving their high ratings from Moody's and S&P, which are needed to remain in this business. They also have a group of experts in this field, including attorneys, investment professionals, and underwriting and product experts. The potential abuse of this product by a small-unrated insurer or unsophisticated buyer could be controlled through the financial qualification requirements to issue or purchase these contracts.

Because of objections raised during the Boston meeting, action on adopting the synthetic GIC model regulation was deferred until the next meeting. Meanwhile an industry group is trying to organize united opposition to some of the more burdensome requirements of the proposed regulations.

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RISKS AND REWARDS

Issue Number 31 October 1998

Published by the Investment Section of the Society of Actuaries
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This newsletter is free to Section members. A subscription is $15.00 for nonmembers.
Current-year issues are available from the Communications Department.
Back issues of Section newsletters have been placed in the Society library.
Photocopies of back issues may be requested for a nominal fee.

Actuarial Principles of Asset-Liability Management

The ALM Principles Task Force of the SOA, chaired by Mike Hughes, has completed a second draft version of Actuarial Principles of Asset-Liability Management. The task force welcomes your comments and suggestions. The draft document can be downloaded from the SOA Home Page/Libraries/Finance and Investments/ALM Principles. You may also request a hard copy by contacting Cherie Harrold at 847–706–3598.

Please provide any comments or edits on the draft Actuarial Principles of Asset-Liability Management to Kevin Long via e-mail at klong@soa.org or fax at 847–706–3599.

October's Market Demons: The '87 Stock Market Crash and Likelihood of a Recurrence

by Vinod Chandrashekaran

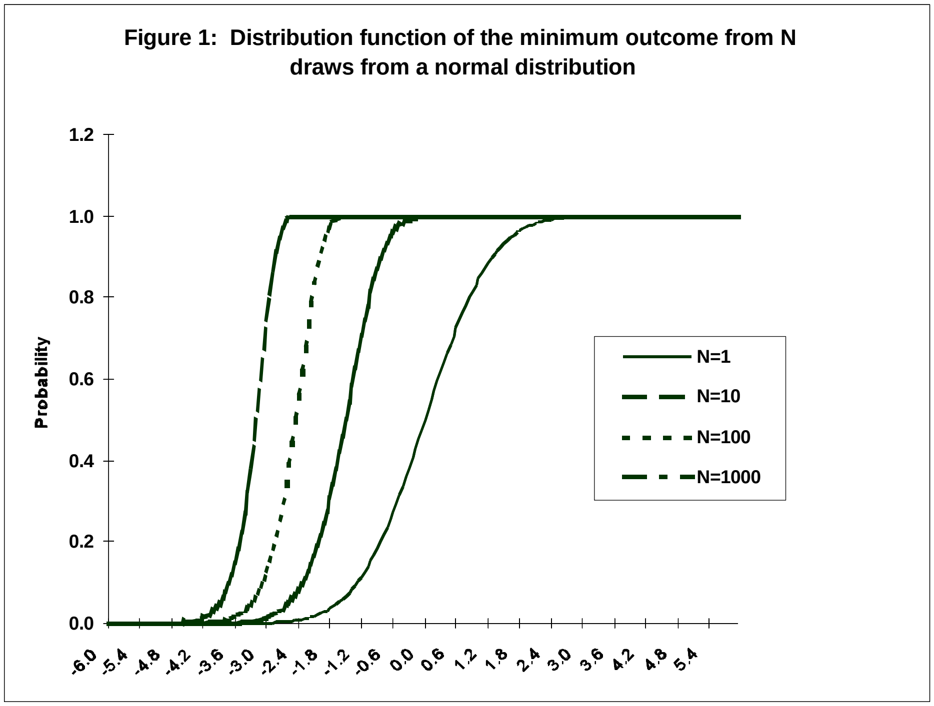
Editor's Note: *The following article originally appeared in the Winter 1998 issue of The BARRA Newsletter, Horizon, and is reprinted here with permission.*

Monday, October 27, 1997, witnessed a drop of 554 points in the Dow Jones Industrial Average, the largest one-day point drop in the history of the market. Dramatic as it was, this drop was only the twelfth-largest fall in percentage terms. The largest percentage drop in the history of the stock market occurred on Monday, October 19, 1987, when the S&P 500 Index declined by 20.5%. The recent sharp movements witnessed in global markets raise an important question: What is the likelihood of market crashes? This article seeks to provide an answer to this and related questions by focusing mainly on the crash of October 1987. Specifically, we shall seek answers to three questions:

1. Given the history of market returns, was the crash of '87 unusual?
2. How do conditional variance models (such as GARCH) behave around periods of extreme moves in the market?
3. What is the impact of the crash on backtesting and performance evaluation?

I. Was the Crash of '87 Unusual?

The average daily return and the standard deviation of the daily return on the S&P 500 Index over the last two decades have been about 0.066% and 0.96%, respectively. On October 19, 1987, the index had a return of −20.5%, which is approximately a 20-sigma event. If we make the simplifying assumption that daily returns follow a lognormal distribution, then the probability of observing a 20-sigma event is approximately equal to 2.75×10^{-89}. Based on this analysis, we would conclude that the crash of '87 was a rare and unusual event.



Effects of Repeated Draws from One Distribution

We can learn a bit more about the likelihood of a crash by taking a slightly different perspective. The return of −20.5% does not represent a single draw from a lognormal distribution. The history of publicly available daily returns on the U.S. stock market goes back over 100 years, and the random return on October 19, 1987, represents but one of the over 25,000 daily returns that have been observed over the last century. A more appropriate question to ask is: Given that we have observed 100 years of returns, what is the probability that one of the observed returns is −20.5%? Since the return on the S&P 500 Index on October 19, 1987, is the lowest on record, we can ask this question slightly differently as well: Given that we have observed 100 years of returns, what is the probability that the minimum daily return we will observe is −20.5%?

To see how much difference this perspective makes, let us consider a simple example. Let $X1, \ldots, X_N$ denote N independent draws from a normal distribution with mean zero and standard deviation 1. Define a new random variable Y as follows: $Y = \min(X_1, \ldots, X_N)$. Figure 1 graphs the cumulative distribution function of Y for $N = 1, 10, 100, 1000$. As we would intuitively expect, the figure shows that the distribution of Y shifts to the left as the value of N increases. Table 1 lists the probability that Y is less than −2 (a 2-sigma

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TABLE 1
Probability That the Minimum Outcome from *N* Draws Is a 2-Sigma or a 3-Sigma Event

N	Prob ($Y < -2$)	Prob ($Y < -3$)
1	0.023	0.001
10	0.214	0.013
100	0.911	0.126
1000	1.000	0.741

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event) and the probability that Y is less than -3 (a 3-sigma event).

Table 1 clearly shows that events that may be viewed as very unlikely to occur become much more likely to occur when we take into account the fact that we are making repeated draws from the same distribution. For example, the likelihood of a 3-sigma event when we make a single draw is 0.1%. In contrast, if we sample 1000 times, the likelihood that the minimum draw is less than -3 sigma is 74.1%. An inspection of the numbers in the table reveals another interesting fact: For small values of N (e.g., $N = 1$, 10), the probabilities within each column increase linearly with N. For example, the probability that Y is less than -2 when $N = 10$ (0.214) is approximately ten times the probability that Y is less than -2 when $N = 1$ (0.023). It can be shown that this approximate linear relationship holds for small probability events (such as 2-sigma events under a lognormal distribution) and small values of N.

The Effect of Increasing Observation Horizons

We now evaluate the likelihood of a crash, using this slightly different perspective. Assume that daily returns on the S&P 500 Index are drawn from a lognormal distribution with mean and standard deviation equal to the sample mean (0.066%) and sample standard deviation (0.96%) observed over the last two decades [1]. Using these assumptions, we can construct the theoretical cumulative probability distribution function for the minimum daily return observed over horizons ranging from one day to 100 years (see Figure 2). Figure 2 shows that tail events become much more likely as we increase the observation horizon. For example, the likelihood that the minimum negative daily return is -4% or lower is 0.0012% over a given day but increases to 26.27% over a 100-year horizon. However, in spite of the increase in likelihood of tail events due to an increase in the number of observations, it is clear from the figure that a minimum return of -20.5% is still virtually impossible to explain using data on daily returns.

It is clear from the preceding discussion that increasing the observation horizon will increase the likelihood of tail events. Since the lognormal distribution assigns positive probabilities to returns in the range (-100% ∞) there surely must be an observation horizon over which a minimum daily return of -20.5% is likely. But this line of inquiry is not very satisfying. For example, we would not find it very comforting to know that a minimum daily return of -20.5% is very likely to happen over a million years! Instead of increasing the observation horizon, we investigate two other avenues of research:

- How does the analysis above change if we increase the return horizon from daily to monthly?
- How would a change in the distributional assumption affect our conclusions?

Figure 2: Distribution function of the minimum daily return over horizon of length T (assuming lognormal distribution)

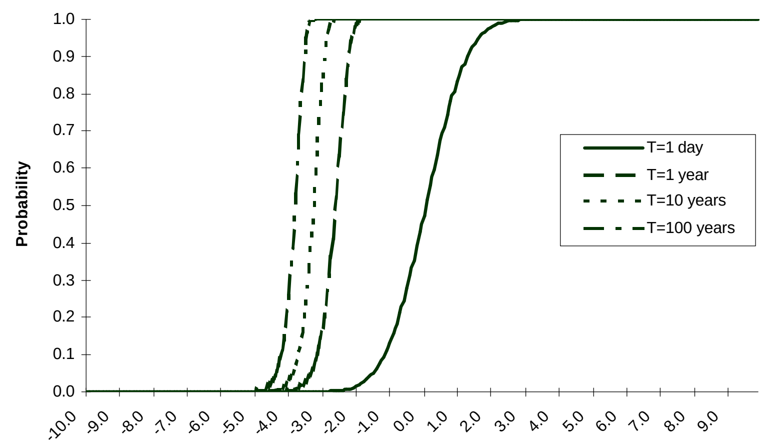
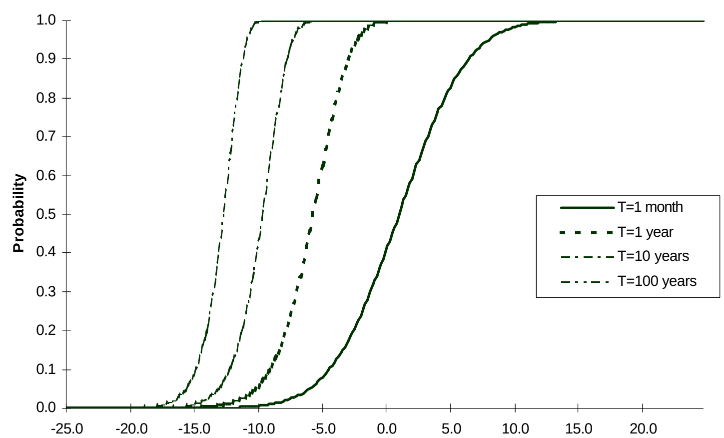


Figure 3: Distribution function of the minimum monthly return over horizon of length T (assuming lognomal distribution)



The Effect of Changing Return Horizons

First, we examine the effect of a change in the return horizon on our conclusions. The monthly mean return on the S&P 500 Index is about 1%, with a

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standard deviation of about 4%. Using these statistics, Figure 3 graphs the cumulative probability distribution function for the minimum monthly return over observation horizons ranging from one month to 100 years. The probability that the minimum monthly return over a 100-year observation period is less than −21.5% (which was the return on the S&P 500 Index over the month of October 1987) is approximately 0.0067%. These numbers suggest that even when we look at monthly returns, the market crash represents a very unlikely event.

The Effect of Changing Distribution Assumptions

Next we examine the effect of a change in the distributional assumption on our results. It is well-known that the unconditional distribution of stock returns is characterized by the presence of fat tails. A direct implication of this is that tail events are more likely than the lognormal distribution would predict. This line of inquiry has a long history. Fama [2] concluded that stock returns appeared to be drawn from a member of the stable Paretian family of distributions with infinite variance. The normal distribution belongs to the stable Paretian class and is the only member of this class with finite variance. Subsequent researchers have shown that if the time series of market returns is drawn from normal distributions with time-varying variances, then the unconditional distribution of market returns would have fat tails.

One popular alternative to the lognormal assumption is to assume that the unconditional distribution of stock returns is log-*t*. The *log-t* distribution arises when stock returns for each period are lognormally distributed, with each period's variance being drawn from an inverted gamma distribution. If a random variable *U* has a log-*t* distribution with v degrees of freedom, then $\log(U) \equiv t_v$, where t_v follows a *t*-distribution with v degrees of freedom. The expected value of log (U) is zero, and the variance of log (U) is equal to:

$$\frac{v}{v-2} \, .$$

Let: $$\sigma_v = \left(\frac{v}{v-2}\right)^{1/2} \, .$$

Figure 4: Distribution function of the minimum daily return over horizon of length T (assuming t-distribution with 5 df)

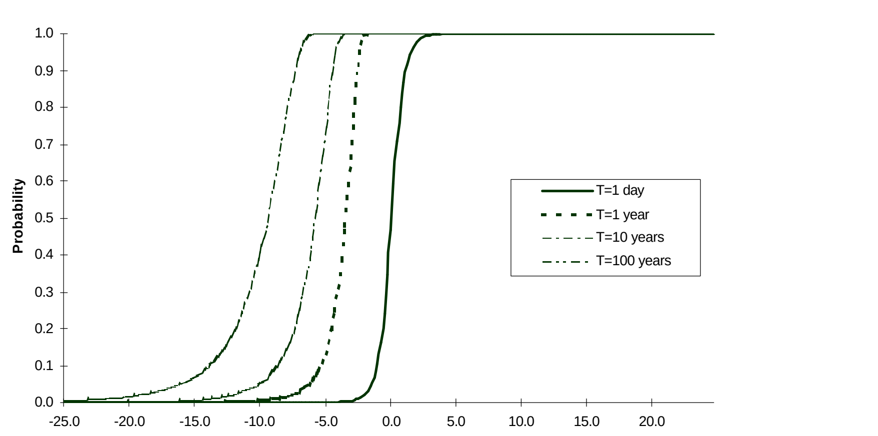
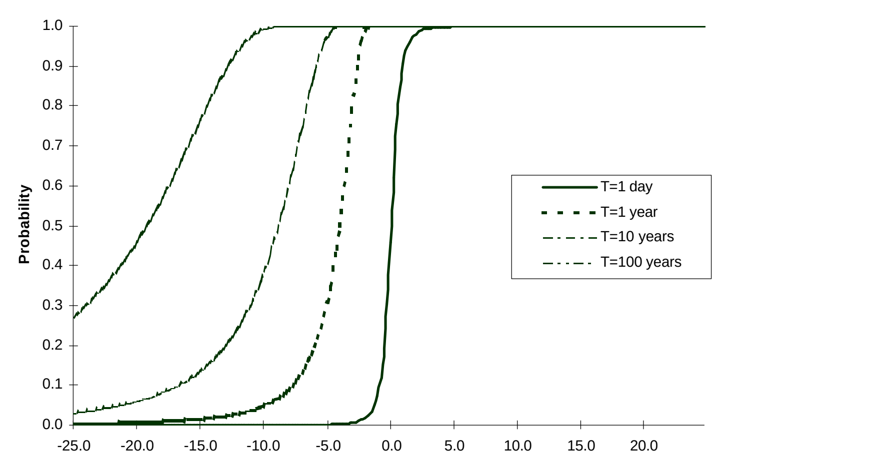


Figure 5: Distribution function of the minimum daily return over horizon of length T (assuming t-distribution with 3 df)



Let r denote the log of 1 plus the rate of return on the market. The mean and standard deviation of r are denoted by μ and σ respectively.

In our study, we assume that:

$$\sigma_v \left(\frac{r-\mu}{\sigma}\right) \, .$$

follows a *t*-distribution with v degrees of freedom. We will present results for the cases $v=5$ and $v=3$. A point worth noting about *t*-distributions is that all even moments of orders equal to or greater than the v^{th} moment are infinite. So, for example, when $v=5$, even moments of order 6 and above are infinite; and when $v=3$, even moments of order 4 and above are infinite. (Note that in the latter case the distribution has infinite kurtosis.)

Figures 4 and 5 display the cumulative probability distribution function for the minimum daily return for observation horizons ranging from one day to 100 years for $v=5$ and $v=3$, respectively. An examination of the graphs reveals that, as anticipated, tail events

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are now much more likely than they were under the lognormal distribution. When the *t*-distribution has 5 degrees of freedom, the probability that the minimum daily return observed over 100 years is less than −20.5% is about 1.52%—still a relatively unlikely event. For a *t*-distribution with 4 degrees of freedom (not graphed), the crash probability is 8.93%, still a low number. In contrast, when the *t*-distribution has 3 degrees of freedom, the same probability jumps to 43.41%—nearly even odds of a crash over 100 years!

Our answer to the question "Was the crash of '87 unusual?" is thus somewhat tentative. When the crash is viewed not in isolation but as the worst outcome of a number of draws from the same lognormal distribution, then its likelihood increases—but not to a level that makes it very likely to happen. On the other hand, when we consider fat-tailed distributions (e.g., the log-*t* distribution), we see that a market crash becomes more likely to occur.

We have confined ourselves to a study of the distribution of the minimum daily return over various horizons, using a variety of assumptions regarding the distribution of daily stock returns. In principle, this analysis can be extended to a study of the likelihood of the worst K returns ($K = 1,2, \ldots$) over the past 100 years. For example, the second smallest daily return on the S&P 500 Index over the past 100 years was −12.3%, on October 28, 1929. Using the distribution function of order statistics, we can extend the above analysis to study the likelihood that the two smallest returns on the S&P 500 are −20.5% and −12.3%

Alternative Return Distributions Compared

Given our results, a natural question to ask is: What is the true unconditional distribution of stock returns? Table 2 sheds some light on this question by tabulating selected theoretical and empirically observed percentile points for daily returns on the S&P 500 Index. The theoretical distributions have been calibrated to have the same mean and standard deviation as the sample mean (0.066%) and sample standard deviation (0.96%).

TABLE 2
Theoretical and Empirical Percentile Points for the Distribution of Daily Returns on the S&P 500 index (Returns Expressed in %)

Percentile	Empirical	Lognormal	Log-*t* with 5 Degrees of Freedom	Log-*t* with 3 Degrees of Freedom
1	−2.29	−2.17	−2.44	−2.45
5	−1.33	−1.51	−1.43	−1.24
10	−0.92	−1.17	−1.03	−0.84
25	−0.38	−0.58	−0.48	−0.36
50	0.07	0.07	0.07	0.07
75	0.54	0.71	0.61	0.49
90	1.07	1.30	1.16	0.97
95	1.49	1.65	1.57	1.37
99	2.35	2.30	2.57	2.58

An inspection of Table 2 reveals that the empirically observed mid-range percentile points (e.g., the 25th and 75th percentiles) are closer to the theoretical values for the two *t*-distributions, while the extreme percentiles (e.g., the first and 99th percentiles) are closer to those of the lognormal distribution. The values in Table 2 do not offer clear evidence on the appropriate distributional form for index returns. It would be useful to look at higher order moments to get some more clues. Table 3 presents the skewness and excess kurtosis coefficients for the observed time series of returns and the values implied by the theoretical distributions considered above.

Table 3 shows the dramatic effect of the crash on the sample skewness and excess kurtosis coefficients. When the crash is included, it is clear that it is

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TABLE 3
Theoretical and Empirically Observed Skewness and Excess Kurtosis Coefficients for Daily Returns on the S&P 500 Index

Statistics	Empirical (Including 10/19/87)	Empirical (Excluding 10/19/87)	Lognormal	Log-*t* with 5 Degrees of Freedom	Log-*t* with 3 Degrees of Freedom
Skewness	−3.30	−0.24	0	0	0
Excess Kurtosis	79.67	7.53	0	6	∞

TABLE 4
Kolmogorov-Smirnov (KS) and Kuiper (KP) Test Statistics for Hypotheses Regarding the Distribution of Daily Stock Returns

Distribution	KS Test Statistic	*P*-Value of KS Test Statistic	KP Test Statistic	*P*-Value of KP Test Statistic
Lognormal	0.083	0.0001	0.153	0.0001
Log-*t* w/5 Degrees of Freedom	0.049	0.0001	0.086	0.0001
Log-*t* w/3 Degrees of Freedom	0.018	0.1306	0.028	0.0310

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difficult to reconcile the sample higher order moments with the theoretical moments of any single distribution considered above. On the other hand, when the crash is excluded, the log-*t* distribution with 5 degrees of freedom appears to have predicted moments that match the empirically observed moments closely. However, since the crash did occur, it is debatable whether it should be dropped from the analysis simply because it represents an inconvenient data point!

We now turn to more formal tests of the distribution of daily stock returns—namely, the Kolmogorov- Smirnov test and the Kuiper test. Table 4 presents the test statistics and the associated significance levels.

The test statistics in Table 4 strongly reject the null hypotheses that daily returns arise from a lognormal distribution or from a log-*t* distribution with 5 degrees of freedom. The null hypothesis of log-*t* with 3 degrees of freedom fails to be rejected by both tests at the 1% level but is rejected by the Kuiper test at the 5% level. The results of the formal tests are thus consistent with our earlier findings and strongly suggest that the unconditional distribution of daily returns is fat-tailed with very large (possibly infinite) higher-order moments.

TABLE 5
Daily GARCH-Forecast-Standardized Residuals for the Days Surrounding the October 1987 Crash

Date	Return on S&P 500 Index (%)	GARCH-Predicted Standard Deviation (%)	GARCH-Forecast-Standardized Residual
10/13/87	1.66	1.06	1.51
10/14/87	–2.95	1.08	–2.78
10/15/87	–2.34	1.20	–2.01
10/16/87	–5.16	1.25	–4.17
10/19/87	–20.47	1.55	–13.25
10/20/87	5.33	4.01	1.31
10/21/87	9.10	4.04	2.24
10/22/87	–3.92	4.28	–0.93
10/23/87	–0.01	4.25	–0.02
10/26/87	–8.28	4.15	–2.01

TABLE 6
Monthly GARCH-Forecast-Standardized Residuals for the Months Surrounding the Crash in October 1987

Month	Return on S&P 500 Index (%)	GARCH-Predicted Standard Deviation (%)	GARCH-Forecast-Standardized Residual
September 1987	–2.20	4.36	–0.75
October 1987	–21.52	4.34	–5.20
November 1987	–8.16	7.41	–1.25
December 1987	7.35	6.93	0.91

II. GARCH Forecasts Around Periods of Extreme Market Movements

In Part I of this article we studied the unconditional distribution of stock returns over the last 100 years with special focus on the likelihood of a market crash. Our conclusion was that a market crash has nearly even odds of occurring over a period of 100 years if the unconditional distribution of daily stock returns arises from a fat-tailed distribution with very large (possibly infinite) higher-order moments. It has been widely documented that such an unconditional distribution is consistent with each period's returns being conditionally lognormally distributed with time-varying conditional variances. In this section, we focus on a particular parameterization of the conditional variance structure—namely, the GARCH (1,1) model—and study the behavior of this model around periods of extreme market movements.

GARCH Applied to October 1987

To perform this study, we estimated separate GARCH(1,1) models using daily and monthly returns on the S&P 500 Index. The daily model was estimated using returns over the period March 1980 through September 1987 (1,906 days), and the monthly model used data from January 1973 through September 1987 (177 months). Table 5 presents the GARCH-forecast-standardized residuals and other numbers of interest for the days surrounding the crash in October 1987. As the estimation period for the models excluded October 1987, our reported results are out-of-sample.

Table 5 documents a number of interesting facts:

- In response to sharp market moves in the days immediately preceding the crash, the GARCH forecast of the standard deviation for October 19, 1987, was about 50% higher than it had been about a week before the crash.
- The crash return constitutes a 13-sigma event relative to the GARCH forecast volatility for October 19, 1987, in contrast to the 20-sigma characterization of the crash using unconditional moments of the distribution of daily returns.
- After the crash, GARCH forecast volatility rises to a level of over 4% per day, which causes many of the sharp post-crash market movements to be classified as "normal" events that are plausible even if daily returns are conditionally lognormally distributed.

A look at the time series of GARCH forecasts shows that the predicted volatility continues to be very high for several weeks after the crash. For example, the daily GARCH forecast as of the end of December 1987 (using data through December 1987 to estimate

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the GARCH parameters) was 1.60%—a number that is about 50% higher than pre-crash forecasts. This is a manifestation of the well-known high degree of persistence in daily GARCH forecasts. Table 6 shows GARCH-standardized residuals using the monthly GARCH model. The monthly GARCH forecasts also rise sharply following the month of the crash and continue to remain high for a few months after the crash.

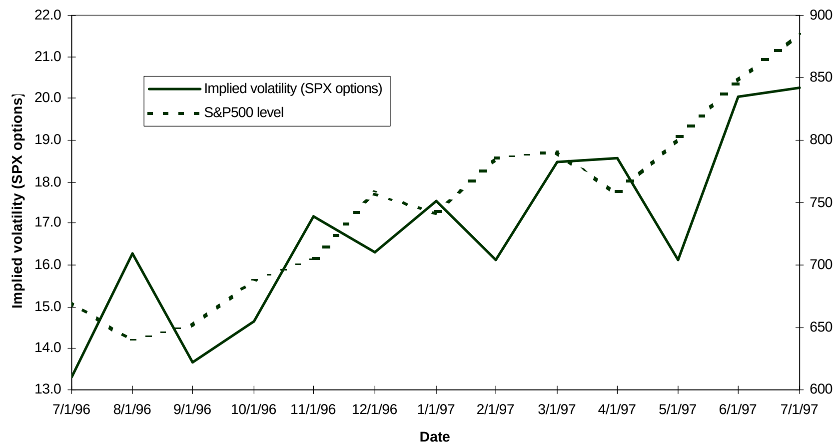
The fundamental intuition built into GARCH models is the notion of volatility clustering—i.e., periods of high volatility are likely to be followed by more periods of high volatility. If historical volatility is low, then GARCH models will continue to forecast low volatility. Although "outliers" are not ruled out even when using GARCH forecasts, the distinguishing feature of an accurate GARCH model is that these outliers would be randomly distributed in time, in contrast to the forecasts of naïve models where outliers would appear clustered together. Judged by this metric, Tables 5 and 6 provide anecdotal evidence that, although the crash itself appears as an outlier, GARCH models are at least partially successful in explaining the sharp movements around the period of the market crash.

Can We Predict an Abrupt Market Transition?

Since GARCH models use a weighted average of historical realized volatility to predict future volatility, in times of a transition from a low-volatility regime to a high-volatility regime the first few sharp movements may appear as outliers that are unanticipated by the GARCH model. An interesting question we might ask is: Are there other techniques that might be used to predict extreme market movements? This question is of clear interest in the current regime since popular debate in the weeks leading up to October 27, 1997, centered on comparisons with October 1987 and on the likelihood of another market crash.

One obvious answer is to look at S&P 500 Index option-implied volatility forecasts. Figure 6 shows the time-series evolution of the S&P 500 Index level over the past one year and the S&P 500 Index (SPX) option-implied volatility at the beginning of each month from July 1996 through July 1997. The annualized average implied volatility using near-term

Figure 6: S&P500 level and SPX-option-implied volatility over the last one year



(less than one month to maturity), near-the-money options has risen from approximately 13.29% on July 1, 1996, to about 20.25% on July 1, 1997. Over the same time period, the S&P 500 Index has risen from 670 to 885. Somewhat surprisingly, over a number of months (e.g., May and June 1997) increases in the S&P 500 Index have been accompanied by increases in option-implied volatility, an observation which is at odds with the "leverage effect" (i.e., the usually negative relationship between price movements and volatility).

One explanation for Figure 6 is that options market participants expected the S&P 500 Index to have higher short-term volatility in the coming weeks and months. In contrast to the high implied volatility forecasts, the conditional variance prediction of GARCH models ranges from approximately 14.10% as of July 1, 1996, to approximately 15.55% as of July 1, 1997. Since the sharp movements that were anticipated by options market participants were realized in October 1997, we would expect that GARCH forecasts would have also risen subsequent to the first few sharp movements in the market.

Our study of option-implied volatility over the past one year suggests that we can incorporate "forward-looking" information in volatility forecasts by combining option-implied volatility with GARCH forecasts. For example, we could estimate a GARCH model using option-implied volatility as one of the variables in the conditional variance equation. Studies by Day and Lewis [3] and Lamoureux and Lastrapes [4] suggest that these two sources of information are complementary.

In summary, our study of GARCH-standardized residuals around the period of the crash of October 1987 shows that, while the crash itself was an outlier, most of the market volatility subsequent to the crash can be fully accounted for using GARCH forecasts. GARCH models use the presence or absence of outliers to predict subsequent increases or decreases in volatility. Hence, while outliers may exist even when using GARCH forecasts, these outliers are likely to be randomly dispersed through time. In the current regime, we saw that option-implied volatility as of July 1, 1997, appeared to be much higher than GARCH forecasts. One explanation for this finding is that options market participants expected to see higher volatility in the coming weeks/months for the S&P 500 Index. Since the expected increase in volatility has been realized, we would expect that GARCH forecasts will also respond.

III. Impact of the Crash on Backtesting and Performance Evaluation

In the previous sections we have studied issues relating to the likelihood of a crash (unconditional study) and the behavior of GARCH forecasts of the S&P 500 Index volatility around the period of the crash (conditional study). In this section, we provide some thoughts on

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the influence of the crash on backtesting investment strategies and on performance evaluation.

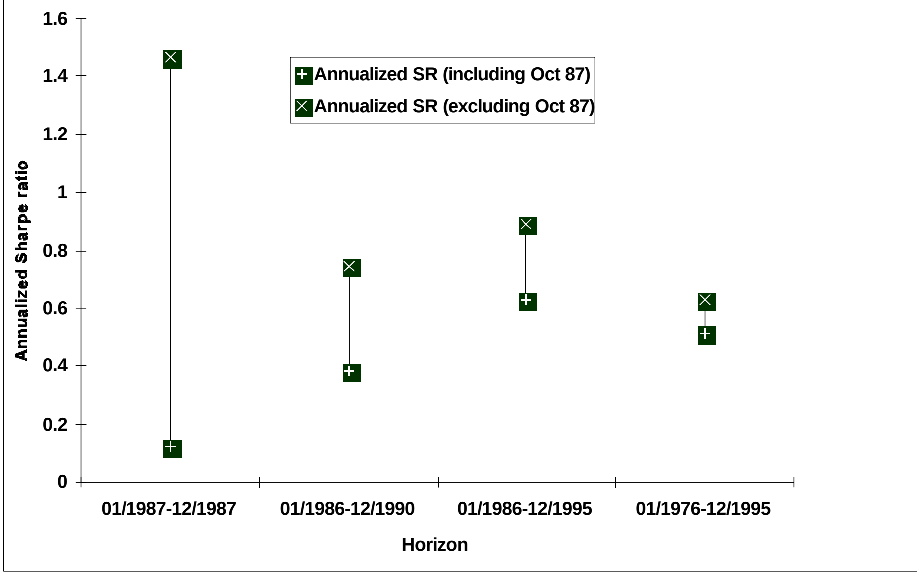
The Importance of Time Period Choice

The first point to note is that the time horizon over which backtesting and/or performance evaluation are conducted will determine the extent to which excluding the crash will affect the reported results. Figure 7 plots the ex-post Sharpe ratio on the S&P 500 Index over horizons of 1, 5, 10, and 20 years, including and excluding the crash. As the horizon lengthens, we see that the Sharpe ratio when the crash is included gradually approaches the Sharpe ratio excluding the crash. We should point out that because we need very large sample sizes to estimate mean returns accurately, the two sets of Sharpe ratios are not statistically distinguishable from one another (i.e., they are within two standard errors of each other).

The second point is that excluding the crash can have dramatic implications for the profitability of certain types of strategies. For example, Sheikh [5] demonstrates that a strategy of buying the S&P 500 Index plus writing out-of-the-money puts on the index was a profitable strategy (relative to buying the S&P 500 Index) over periods strictly before and strictly after the crash. The post-crash period that Sheikh studied was August 1988 through February 1995. In contrast, a similar strategy that was put in place starting in September 1987 lagged the cumulative return on the S&P 500 Index, even after over seven years (as of February 1995). In other words, the loss suffered in the month of the crash was more than the gains made by the strategy over the next seven years!

The third point is that it is a good idea to run backtests over historical periods that represent different regimes—e.g., bull and bear markets, periods of low volatility and high volatility, etc. Figure 8 shows the cumulative return on the S&P 500 Index over the 10-year period January 1987 through December 1996. It is evident from the figure that there have not been too many bad months, especially over the past five years. The crash represents a useful observation precisely because it was a particularly bad month. Including this observation in backtests serves as a check on the robustness of proposed investment strategies.

Figure 7: Annualized Sharpe ratio for S&P500 index (over various horizons) including and excluding October 1987



Should the Crash Be Included in Performance Studies?

Finally, we consider performance evaluation in the presence of the crash. As the above analysis of the Sharpe ratio suggests, the total risk/return picture, especially over smaller horizons, differs significantly depending on whether or not the crash is included in the sample. For an active manager who is usually fully invested in equities, including the crash does not bias performance results since the active manager is evaluated based on his or her active risk/return profile (i.e., risk and return net of the market).

Let us consider the more difficult question of an active manager who aims to achieve superior returns by forecasting the returns on the S&P 500 Index (that is, by timing the market). Let $r_{B,t}$ be the excess return on the S&P 500 Index in period t and let μ_B denote the per-period long-run expected excess return on the index. Each period, the market timer has a forecast of the excess return on the index over its long-run average. In symbols, for each period the market timer has a forecast Δf_B of the value of $r_{B,t} - \mu_B$. Let λ_{BT} denote the risk aversion coefficient of the investor for benchmark *timing* and let $\hat{\sigma}^2_{B,t}$ be the investor's forecast of the variance on the index over period t. Then, the optimal active beta position for the investor is given by:

$$\beta^*_{p,A,t} = \frac{\Delta f_{B,t}}{2\lambda_{BT}\hat{\sigma}^2_{B,t}}$$

Grinold and Kahn [6] discuss the appropriate objective function for an active manager and derive the optimal active beta policy stated above. We conducted a simulation study using the actual history of realized market returns over the period January 1987 through December 1996. The market timer is assumed to make monthly forecasts of the index return. Each month, the market timer receives a signal $g_{B,t}$ as follows:

$$g_{B,t} = IC\left(r_{B,t} - m_B\right) + s\sqrt{1 - IC^2}u_t$$

- IC = the information coefficient of the manager
- m_B = the average excess return on the index over the 10-year sample period
- s = the sample standard deviation of the excess return on the index, and
- u_t = a random number drawn from a distribution with zero mean and unit standard deviation.

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October Market Demons

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$r_{B,t}$ = is set equal to the observed excess return on the S&P 500 Index in month t.

Given this signal, the manager constructs an optimal forecast of the excess return on the index as follows:

$$\Delta f_{B,t} = E\left(r_{B,t} - \mu_B \mid g_{B,t}\right) = (IC)g_{B,t}$$

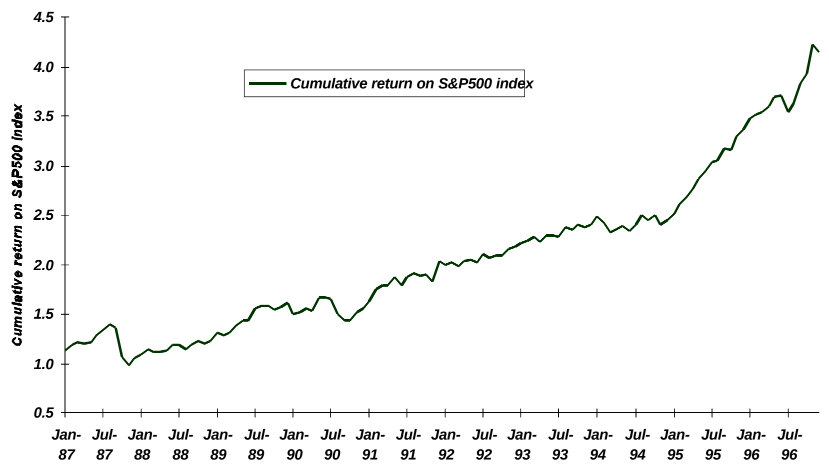
For simplicity, we assume that the investor's forecast of the variance of the index return in month t, $\hat{\sigma}^2_{B,t}$ is equal to s^2 for all months. For a given sequence of signal realizations, we can derive the corresponding time series of active beta positions. Using the actual history of market returns, we can then compute the ex-post information ratio for the investor. We considered three different *IC levels; 0.05, 0.10,* and *0.15*. For each *IC*, we ran 100 simulations of the entire 10-year history from January 1987 through December 1996. Table 7 reports the average ex-post information ratios across these simulations.

Table 7 shows that there are no significant differences between the two columns of information ratios. In other words, including the crash does not appear to make a difference for the performance evaluation of a market timer.

Summary

In this article, we have presented some perspectives on the crash of October 1987. We found that the likelihood of a market crash increases dramatically if the unconditional distribution of stock returns is fat-tailed with very large (possibly infinite) higher-order moments. Our study of GARCH forecasts showed that, with the exception of the crash itself, these forecasts were at least partially successful in capturing sharp movements around the period of the crash. We found that option-implied volatility has increased dramatically over the past one year, suggesting that the market expects higher volatility in the weeks/months ahead. Finally, we offered some thoughts on the impact of the crash on backtesting and performance evaluation. We showed via a simulation study that including the month of the crash does not have a significant effect on the ex-post information ratios of a market timer.

Figure 8: Cumulative return on the S&P500 index since January 1st, 1987



End Notes

1. Throughout this section, we assume that the time series of daily returns *iid* (independent and identically distributed) draws from the specified unconditional distribution. In Part II of this article we will explore the behavior of conditional variances around the period of the crash.

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TABLE 7
Ex-post Information Ratios (IR) for Market Timing
Over the Period January 1987 through December 1996
(Average over 100 Simulations)

Information Coefficient	Average Ex-post IR (including October 1987)	Average Ex-post IR (excluding October 1987)
0.05	0.191	0.195
0.10	0.396	0.377
0.15	0.467	0.448

Taking Stock: No Pain No Gain, or What They Did Not Tell You about Goldilocks

by Nino Boezio

If you haven't heard it already, it's time that someone finally told you the complete story about *Goldilocks and the Three Bear Markets*.

Goldilocks is a soup connoisseur and loves to try various soups. The amount and variety of soups (equities) Goldilocks (today's equity investors) eats depends on how the temperature (the economy) of the soup is—too hot, too cold, or just right. The soups overall were not much to her liking in the early 1990s because the temperature was "too cold" (recession), "too hot" in 1994 (economic overheating), but "just right" in the last few years (hence the name Goldilocks economy).

While is search for soups, Goldilocks happened to enter into the lair of the bears (overvaluation) but was not too concerned about where she arrived. She had never seen bears before and only heard rumors of past rampages in foregone times. She was told that bears these days are kept away in zoos (the new era), and if one ever were to escape, the zookeepers (Alan Greenspan, the Fed, the Bundesbank, the International Monetary Fund etc.) would quickly go after the bear and put it back into captivity. Hence there was nothing to fear. She did not know that there were still bears running free.

The bears were not at home (lost all credibility) when Goldilocks finally showed up. Goldilocks, having tried the various soups, liked the ones that in her opinion were not too hot or too cold, and gorged herself. Needless to say, all that eating made her sleepy and she did not hear the rumblings outside (Asia, Russia, South America), sounds made by the returning bears. She went to sleep.

Since Goldilocks had never seen live bears, she did not know what to fear. When Goldilocks later awoke and saw the first bear, she did not pay any attention to it. She was still hungry and thus went back to eating soup, for now some of the hot ones cooled off (a buying opportunity), not realizing that the circumstances had now changed (Phase I of a bear market—*denial*). When the second bear appears (Phase II of a bear market—*realization*), she may realize that eating more soup in the presence of the bears will put her in danger and her trepidation may result in her hunger being replaced by hesitation and concern. She will stop eating and the hot soups (equities) will get even cooler (cheaper). When the third bear appears (Phase III of a bear market— *capitulation*), she may finally acknowledge her plight and run for cover (and will even start vomiting what she has already eaten), and all the soups will become freezingly cold. In this latter phase, she should probably realize that if the bears have not already done her harm, then they probably will not do so and she should probably heat up the soups and start eating once again. However, by then she probably has run out and missed out on the soupfest (Phase I of a bull market) that could ultimately ensue if she had only stuck around longer. Or perhaps the bears' stove is broken (a depression) and she will not get a chance to eat more soup for a very long time.

> ***"The downfall of the Soviet Union in 1989 leading to only one superpower, peace in the Middle East, positive demographics, and global capitalism may have lulled Joe and Lucy Public into believing that nothing can go wrong."***

No Pain, No Gain: Understanding Goldilocks

Humanity has been searching for the Holy Grail for thousands of years—in modern-day terminology this is called the "free lunch" (or should we call it the "free soup").

There has always been a quest by the average person to find the ultimate happiness in personal life, the perfect balance between recreation and work (for some that would mean no work), and for sufficient wealth so that one may guide his own destiny. For the latter, wealth generation became the equivalent to investing as much as one could in the stock market.

The stock market, as it became in the 1920s (Japan in the 1980s), was increasingly seen as the "no pain always gain" approach to increase one's financial health. It would grant a person ultimate financial independence ("deliverance from serfdom") and allow one to retire early to a life of luxury. The downfall of the Soviet Union in 1989 leading to only one superpower, peace in the Middle East, positive demographics, and global capitalism may have lulled Joe and Lucy Public into believing that nothing can go wrong. Initially it was a good bet, until too many began to believe it and drove valuations skyward all over the world.

No one in the world community wants to suffer pain, even though their behavior may at times create pain for others and ultimately themselves. When the pain comes, there is often a mad scramble to find a "cure" rather than to accept the hurt, learn from it, take the lumps, and move on. The cure in the economic world has become known as the International Monetary Fund, a strong U.S. economy which potentially can suck up the excess supply of goods, the actions of this or that politician or leader, emerging Western Europe, or sometimes just simply the use of money from one group to pay for the misgivings or "irrational exuberance" of another group. As was pointed out in my article "It's Different This Time" in the March 1998 issue of *Risks and Rewards,* traditional solutions may not work this time, and the problems are substantially different than they initially appear on the surface. It is now said that about 50% of the global economy is in or on the verge of entering a recession because of falling world demand from the international financial crisis.

It is sad to see a person's hopes dashed or rattled as a result of any

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The Baby Boom, the Baby Bust, and Asset Markets

by Timothy Cogley and Heather Royer

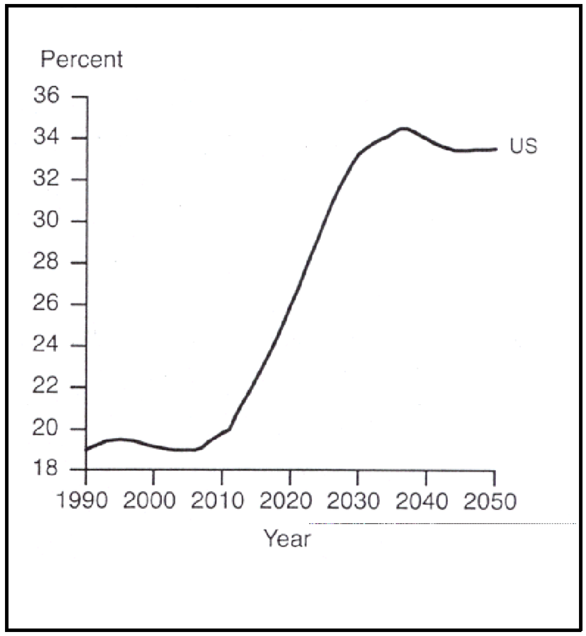
Editor's Note: *Reprinted from the Federal Reserve Bank of San Francisco* Economic Letter, *Number 98–20, June 26, 1998. The opinions expressed in this article do not necessarily reflect the views of the management of the Federal Reserve Bank of San Francisco, or of the Board of Governors of the Federal Reserve System.*

In about 10 to 15 years, the first wave of post-war baby boomers will begin to retire, and we will start to see a large generational shift from young to old. This generational shift is illustrated in Figure 1, which shows the expected path of the so-called "old-age dependency ratio," which is defined as the number of people aged 65 and older divided by the working population (those aged 20 to 64). This ratio will begin to increase sharply when the baby boomers first begin to retire and will climb from 20% currently to nearly 35% by the year 2035. In other words, when the last of the baby boomers reaches retirement age, there will be only three workers for each elderly person, compared with five workers now. This shift will be brought on not only by the sheer number of boomers but also by their increasing longevity. At the same time, we can expect to see a thinning in the ranks of the young, as many boomers delayed childbearing or chose to have fewer children or none at all.

The prolonged graying of America, with an escalating ratio of elderly to young people, will have severe consequences for pay-as-you-go public retirement programs such as Medicare and Social Security. Simply put, if there are no changes in these programs, there won't be enough working Americans available to put money into them to support the ever growing populace of retirees. Intermediate projections suggest that Social Security will face bankruptcy in about the year 2030, and some figures place insolvency about a decade sooner. Cuts in benefits, tax increases, massive borrowing, lower cost-of-living adjustments, later retirement ages, or a combination of these elements will be necessary to sustain the programs.

Because of doubts about the future of Social Security, it may be prudent for households to prepare for retirement by increasing their own personal savings. Venti and Wise (1996) report that older Americans already have begun to do so. For example, they report that the personal retirement assets of those aged 65 to 69 is already significantly larger than what previous generations had set aside for retirement, and that the average has more than tripled since 1984. In addition, Venti and Wise project that the personal financial assets of those who will be 76 in 2025 will be roughly double that of those who were 76 in 1991. Research by economists in the Congressional Budget Office (1993) also reports that baby boomers have begun to accumulate more assets than prior generations.

FIGURE 1
Old-Age Dependency Ratio in the United States



Because baby boomers will have to rely more heavily on personal savings to prepare for their retirement, they have an interest in how capital markets will fare as they approach their golden years. While strains on public retirement programs are well-known and much documented, the growing ratio of old people to young also has implications for returns on private savings. In particular, the imbalance in the ratio of generational cohorts may also adversely affect returns on private savings.

Implications for Baby Boomers' Retirement Plans

To understand the relation between demography and capital markets, it is useful to think about the Life Cycle model of consumption and saving. Roughly speaking, the Life Cycle model states that people work and save when they are young and live off the proceeds when they retire. A typical Life Cycle profile is illustrated in Figure 2, which plots wealth as a function of age. In this figure, a person starts to work and save at age 25. His initial income is normalized to 1 (i.e., the units of wealth are a year's income), and we assume that real income grows at a rate of 1.8% per year (the average annual growth rate of per capita income over the last 120 years). We also assume that our hypothetical consumer saves 10% of his income and invests it in a mix of stocks and bonds that earns a real return of 5% per year. He works until age 65, at which time he begins to sell off his assets and live off the proceeds. The key feature of the figure is that wealth has a hump shape over the life cycle. It peaks at retirement age and then begins to decline. In other words, older people tend to be net sellers of financial assets.

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In an economy with a stable age distribution, this would have no effect on capital markets. When each cohort reached retirement age, it would sell its assets to younger cohorts who were accumulating wealth, and with steady population growth there would always be enough of the latter to absorb the sales of the former. But what happens when population growth isn't steady and the economy's age distribution isn't stable? In particular, what happens when the old-age dependency ratio rises, and there are proportionally fewer young savers to buy up the assets of the older retirees? In this case, by the law of supply and demand, one would expect the price of assets to fall. As aging baby boomers begin to sell their financial assets, they will presumably be selling to the next waves of savers, the so-called Generation Xers and Yers, which are significantly smaller population cohorts. With relatively fewer buyers than in the past, boomers may find themselves selling into a weak market when they retire.

Is there any empirical support for this prediction? Long-run forecasting is extremely difficult, and we won't know for sure until baby boomers actually begin to retire. But baby boomers have affected the economy at every stage of their life cycle, in ways more or less in accordance with the Life Cycle Hypothesis, and its success in other contexts lends some credence to our conjecture about retirement.

For example, some versions of the Life Cycle model predict that people will invest differently at different stages in their lives. When people are young and starting families, one would expect them to invest heavily in housing, and the arrival of a large cohort at that stage of their life cycle should raise house prices. Mankiw and Weil (1989) and Bakshi and Chen (1994) studied this implication of the model and reported that there was an increase in housing prices between 1970 and 1980, when the first wave of baby boomers were in their 20s and early 30s.

Similarly, when people grow a bit older and begin to think about retirement, one would expect that they would begin investing more in financial assets. The arrival of a large cohort at that stage of the life cycle should raise the price of financial securities. The first wave of baby boomers reached age 35 in 1981, which coincides roughly with the beginning of the long bull market in stocks (again, see Bakshi and Chen). This may reflect (at least in part) the predicted Life Cycle effects.

FIGURE 2
A Typical Life Cycle Wealth Profile

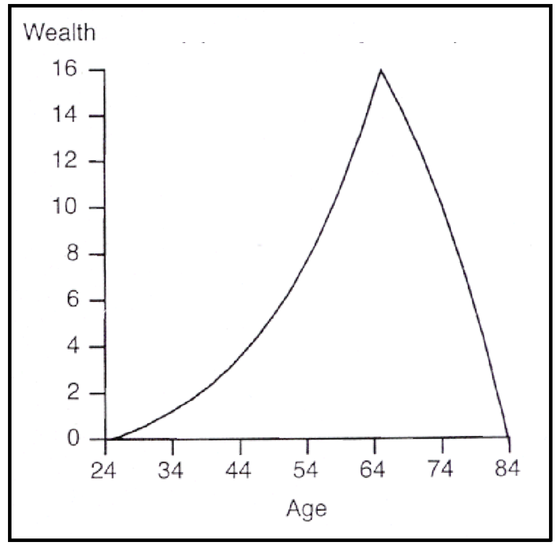
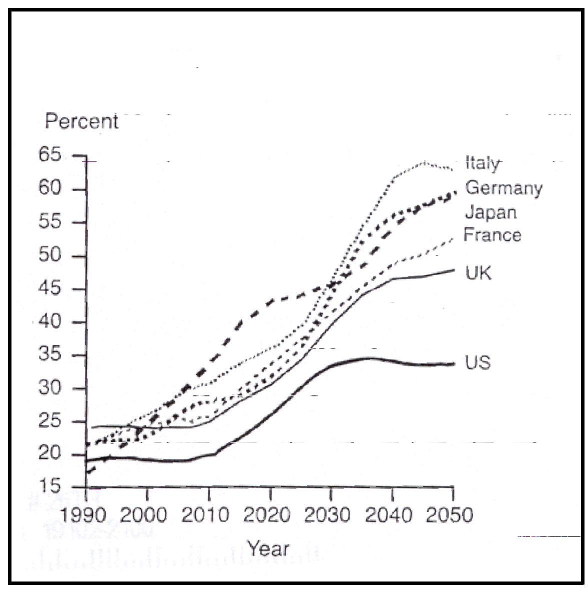


FIGURE 3
Old-Age Dependency Ratios in Various Countries



International Diversification?

There is a possible way out. Capital markets are integrated internationally, and it may be possible for aging boomers to avoid losses if large numbers of young investors can be found elsewhere in the world. That is, aging boomers in the U.S. needn't sell exclusively to young people in the United States. They can sell to anyone throughout the world. Thus, U.S. demographics aren't necessarily decisive; world demography matters more. The key issue concerns the extent to which aging patterns are synchronized or asynchronized across countries. U.S. demographics can be diversified internationally if the aging patterns are asynchronized, so that some other country's boomers are young when our boomers are old, but they can't be diversified if all populations are graying simultaneously.

Unfortunately, demographic trends in industrialized nations suggest a synchronization across countries. For example, Figure 3 superimposes old-age dependency ratios for Germany, Japan, France, Italy, and the U.K. on that for the U.S. Populations are aging in all these countries, and, in fact, all will have far greater dependancy ratios than the United States. This may seem surprising, because unlike the United States these countries did not experience large increases in fertility in the 1950s and early 1960s. Why then are their populations aging? In Japan there was an increase in fertility, but it peaked earlier than in the U.S., and their boomers are now older than ours. In other countries, such as France and Germany, the population is aging because there was a sharp decline in fertility from the 1970s through the 1990s. In any case,

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because the demographic profiles are synchronized, it seems unlikely that investors in these countries will be net buyers of capital when aging Americans begin to sell. If anything, this figure suggests that international linkages among developed countries are likely to amplify life cycle effects in the United States.

What about developing countries? Demographers project that their old-age dependency ratios will also rise, but expect the increase to occur roughly 50 years later than in the industrialized world. Since their demographic profiles differ from the developed world's, perhaps aging boomers in the latter can sell to younger boomers in the former. But will they have the means to buy? Capital tends to be scarce in developing countries, and unless they can grow rich in the next 25 years, it seems unlikely that they will be in a position to become net lenders to the developed world.

Other Considerations

The looming crunch might be slightly eased under several scenarios. For example, educated baby boomers may choose to stay in their careers longer, working past the traditional age of retirement; they need not sell their assets if they earn steady paychecks. In addition, the period over which the Baby Boom generation is expected to retire spans about 30 years. Capital markets might have time to adjust to the gradual decline in supply of funds for capital investment. For example, if Gen-Xers, Yers, and Zers were to anticipate further cuts in Social Security benefits, they might save a higher fraction of their incomes, and this would compensate for the fact that there are relatively few of them. Despite such possibilities, the surging old-age dependency ratio remains a significant generational challenge, not just for Social Security, but perhaps for private retirement plans as well, in ways we are just beginning to explore.

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Subjective Value at Risk

by Glyn Holton

Editor's Note: *The following article originally appeared in the August 1997 issue of* Financial Engineering News *and is reprinted with permission.*

Value-at-Risk (VaR) is becoming somewhat of a revolution. Around the globe, organizations are racing to implement the new technology. Pundits propose extending VaR to other risks, including credit risk and operational risk [1]. Some even suppose that all the risks of an organization should be summarized with a single risk measure [2].

It is the nature of revolutions that there be a backlash. One has begun. Critics suggest that VaR may be ineffective for assessing risks other than market risks [3]—or that it fails even with market risk [4]. Others have noted disturbing inconsistencies between risk estimates produced by different implementations of VaR [5].

If the VaR revolution is to succeed, it must be tempered by such concerns. After all, VaR is only a tool. All tools have limitations. For example, a hammer can drive nails, but it cannot drive screws. Saying that the hammer is limited is different from saying it is flawed.

To understand the limits of VaR, we need to explore what it means to "quantify" risk. Let's start by defining risk. Risk is exposure to uncertainty. Accordingly, risk has two components: (1) uncertainty; and (2) exposure to that uncertainty.

A synonym for uncertainty is ignorance. We face risk because we are ignorant about the future—after all, if we were omniscient, there would be no risk. Because ignorance is a personal experience, risk is necessarily subjective. When we put a number on risk, that number says as much about us—how little we know—as it says about the world around us.

Suppose you are in a casino. A man rolls a die behind a screen. If the result is a 6, you are going to lose $100. Behind the screen, the man sees the result of the die toss, but you have not yet seen it. In this example, the outcome is certain. It has already been determined. Uncertainty exists only in your head—but the risk is real until you see the die.

Let's try to quantify your risk in this example. To characterize the risk, we need to describe the uncertainty as well as your exposure to that uncertainty. Obviously, your exposure is $100. That is the amount you stand to lose. But what is your uncertainty— what is the probability that you will lose $100?

If you say it is one chance in six, I am sorry. You are wrong. I forgot to mention that the die is 10-sided. This illustrates an important point. Whenever we try to quantify risk, we are describing our own understanding of a situation. Often, there will be aspects

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of a situation that we are simply unaware of. It is one thing to not know the answer to a question. It is another matter to not even know the question exists.

Returning to our casino example, we still don't know your probability of losing $100. It is not one in ten. After all, the man throwing the die may be cheating. We are aware of the possibility, but it is difficult to place a number on that risk.

Would it help if I told you the man is unshaven and smells of whiskey? Maybe your opinion would change if I told you instead that he is a kindly grandfather wearing a boy scout cap. Changing the description may sway some peoples' opinions. It may not sway others'. Risk is subjective.

So what does this mean if we want to measure the financial risks of an organization? To find out, let's look at how risks are quantified. It is a four-step process:

- Define the risk to be measured
- Agree on a model for that risk
- Specify a risk measure that is compatible with that model
- Estimate the value of that measure implied by the model.

For example, the process might be as follows:

- Risk: market risk of a specified portfolio
- Risk model: market variables are assumed to be jointly normally distributed with specified volatilities and correlations
- Risk measure: one-day 90% VaR
- Risk estimate: achieved with Monte Carol simulation using 5,000 quasi-randomly generated scenarios.

It is the second step of the process that is pivotal. It is at this point that we take the subjective notion risk and describe it in an objective manner. However, a group of individuals may agree on a model, but retain their own subjective opinions about the risk. In this sense, the model does not make risk an objective notion, it merely makes the measure of risk an objective notion.

Let's continue with the example of market risk. Suppose a trading operation has implemented the above VaR system. One day a trader takes on a sizable long position in the Japanese yen, exceeding her risk limit. She knows the markets and is aware of a combination of market factors—perhaps central banks are intervening in the markets—that are going to drive the yen up in the short-term. She considers the position appropriate.

Her risk manager disagrees. He doesn't know about central bank intervention—and he doesn't care. All he knows is that the trader has exceeded her limit, and he calls her on it.

Reviewing the VaR number that indicates her limit violation, the trader retorts: "The model is wrong. I know the markets. I know what the central banks are doing. I'm on the phone with FX professionals all day long. This VaR model is just a bunch of formulas. It doesn't know the yen is going up, but I do. There is zero risk in my long position because any other market position, under these circumstances, would be ridiculous."

Who is right, and who is wrong? The trader knows the markets. It's her job. By the same token, what is the point in having a risk manager who is going to be overruled by every trader with a market view?

Some might perceive that the answer is to build a better VaR model— one that somehow captures the trader's intuitive understanding of central bank intervention. Others may cling to the existing VaR model, claiming that efficient markets and no-arbitrage conditions ensure its ultimate validity.

In fact, neither approach can possibly work. They both make a supposition that there is a "right" model—if only we can identify it. Markets, however, are too complex and ever-changing for any model to fully describe. Selecting a model is a subjective process.

Our FX trader and risk manager have a legitimate difference of opinion. To resolve such a situation, we have to get beyond the simplistic notion that one is right and the other is wrong. I so doing, we must challenge the idea that every risk has a number—that there is a "right" model that will find that number, and other models are "wrong." We must embrace the notion that risk is subjective.

We cannot manage market risk by having a risk manager forming—and then enforcing—his own subjective opinions about the riskiness of a trader's position. This would be unfair to the trader, and it would reduce the risk manager to being, in effect, just another trader.

Instead, we implement an objective benchmark for risk in the form of a VaR model. It may assume that market variables are normally distributed despite some observers preferring the lognormal assumption. It may not capture market leptokurtosis. It probably won't understand "sticky" volatilities. This is not important.

If we have a perfect model, it would know everything there was to know about the markets. It would eliminate the need for traders. We could trade the portfolio based upon the model—and we would be foolish not to.

A VaR model, however, is limited because it is objective whereas risk taking is subjective. If we deny that subjectivity, we deny a role for human judgment. Rather than trade portfolios based upon a model, we rely upon traders because we believe they understand things the model cannot.

This leaves us with two—potentially inconsistent—market views: that of the model; and that of the traders.

The question is: How can we use the objective VaR model to manage the risk-taking process, but not place arbitrary—or even dangerous—restrictions upon the activities of traders?

The answer is risk limits. These represent explicit authority for traders to take positions that differ from the model's perception of the markets. Risk limits enable an organization to manage risk by limiting traders to taking positions within a specified range. The role of the VaR model is to objectively define what that range is. The trader's role is to select the optimal position within the range.

In this context, VaR is just a tool for delimiting a set of acceptable portfolios. We can call it a "risk measure" if we like, but we don't have to.

Like any tool, VaR has limitations. It will be useful for performing some tasks, but not others. For example, other possible applications of VaR

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Subjective Value at Risk
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include determining capital requirements, capital allocation, or performance-based compensation.

Each process entails risk assessment. Accordingly, each is subjective. If we wish to apply the objective tool VaR to any of these, we must first ask what role VaR is to play. In each case, some mechanism must be found that will enable VaR to support subjective human judgment—without replacing it. For market risk management, the answer was risk limits. For other possible applications, the question remains open.

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END NOTES

1. See the J.P. Morgan CreditMetrics Technical Document and "VaR in Operation" by Duncan Wilson, *Risk*, December 1995.
2. See "CIBC Gets Commercial," *Risk*, August 1996.
3. See "Modeling of Operations Risk," by M. Yone, et al., in the Financial Risk Management Discussion Group (March 1997).
4. See "The World According to Nassim Taleb," *Derivatives Strategy*, December/January 1997.
5. See "Value at Risk: Implementing a Risk Management Standard," by Chris Marshall and Michael Siegel, *Journal of Derivatives*, Spring 1997.

Applying Insurance Company Quantitative Techniques for Improved Capital Budgeting

by Tony Dardis and Andrew Berry

The insurance industry has always used sophisticated quantitative techniques for appraising capital investment. The same, however, cannot always be said of other industries. In a 1994 study, the Confederation of British Industry found that only about one quarter of manufacturing companies use quantitative methods to assess project risk, with the majority relying on subjective judgment. It is generally thought that manufacturers in the United States have similarly been slow to adopt quantitative techniques in appraising projects. So, could some of these insurance industry techniques be applied to help organizations in other fields? In particular, should consideration be given to the use of these techniques for appraisals of capital projects?

This article recognizes and acknowledges the work of both the U.K. Institute of Actuaries and the Society of Actuaries in this area, in particular the important paper authored by a working party set up by the U.K. Institute entitled "Capital Projects," published in the *British Actuarial Journal* (Volume 1, Part II, 1995, pages 155–300). Many of the definitions used in the introductory sections of what follows are taken directly from the Institute paper. We take the discussion somewhat further, however, in looking at some of the more state-of-the-art techniques currently in use today within the insurance industry. A similar SOA working party is in its formative stages in the United States.

We have defined a capital project in the same fashion as the Institute working party, that is, "any project where the investment has significant physical, social, or organizational consequences and is not merely to secure a transfer of ownership of an existing asset [such as portfolio investment]." This definition therefore includes such schemes as:

- Physical construction, such as building a factory, bridge, or road
- Starting a new business producing goods or services, or a new product line in an existing business
- Taking over and modernizing an existing business or physical asset
- Developing a new asset for an existing business
- Repairing or renewing an existing asset.

Current Capital Budgeting Techniques

Capital projects are most commonly evaluated using pay-back period, net present value, or internal rate of return. Again, using the Institute paper definitions:

- ***Pay-back Period Technique***: A project is accepted if the number of years of projected cash flow required to return the initial investment is less than a pre-set maximum cut-off period (no account taken of the time value of money).
- ***Internal Rate of Return***: Find the interest rate (IRR) that equates the present value of expected future cash flows with initial costs and accept the project if the IRR exceeds the opportunity cost of capital.
- ***Net Present Value***: Find the present value (NPV) of the expected future cash flows of a project discounted at the opportunity cost of capital and accept the project if the NPV is greater than zero.

IRR and NPV incorporate the time value of money through discounting to present values and try to incorporate the notion of risk through the use of the relevant discount rate. Risk in this context means that actual returns from the project (revenues less costs) may be

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different than expected. This volatility of returns will be different between different projects.

The relative riskiness of the project is incorporated into the discount rate by adding a risk premium to the "risk-free" interest rate as reflected by a Treasury bill. This risk premium is necessary to compensate investors for the risk they are taking by providing higher returns. The key questions are how large should the risk premium be and how do we calculate it? The answer to these questions requires an assessment of the risks in the project. In most cases this assessment is arbitrary.

Rather than try to estimate the risk costs inherent in a capital project, organizations will use their cost of capital (that is, the rate at which they can raise capital) as the discount rate. The reason for this is that this is the rate of return that the financial markets require to compensate them for taking the risk of investing in the organization. This macro approach to estimating risk premiums assumes that the financial markets are efficient in estimating risk. Applying this rate to new capital projects also assumes that the capital project will have the exact same risk profile as the organization's existing risk profile, an assumption that is clearly unreliable for investments in new areas or operations such as new product lines or major construction projects. In these situations, appropriate discount/cost of capital rates may be obtained from two sources:

- ***Comparative Data***. Use a cost of capital figure from a comparable organization or project. For large capital projects, comparable data simply isn't available. Projects such as Boston's Central Artery project are of such a size and unique nature that there are no historical indicators of their risk profiles. Entering new markets also presents problems. Should a company with no expertise in a particular industry expect to have a risk profile similar to the industry as a whole?
- ***Subjectively apply a loading to the organization's own cost of capital.*** Launching new product lines or acquisitions may have a required discount rate above the organization's existing cost of capital, as both revenue and cost projections are subject to volatility. Cost reduction projects may have a required discount rate below the existing cost of capital.

Even incorporating appropriate risk premiums, most NPV or IRR estimates focus on single deterministic point estimates for making investment decisions. A simple extension of this is to use some sort of scenario analysis to include a number of different potential outcomes (for example, optimistic, pessimistic and most likely). In the absence of any information about the probability of each scenario occurring, investments are chosen according to some decision rules which usually involve minimizing the possible losses from the pessimistic scenarios.

"Where DFA is especially useful is in allowing the user to build a sophisticated model that incorporates the interrelationship between variables."

Introducing Probability Distributions

Scenario analysis can be extended to assign probability estimates against the different scenarios to develop an expected outcome and standard deviations for each result. Although scenario analysis begins to include probability estimates, it is still a macro top-down approach to estimating risk. Rather than relying exclusively on this approach, an organization should also be building a bottom-up risk profile. This will identify the potential sources of risk (risk factors), the impact they will have on potential cash flows, and develop a probability distribution for each of the variables. Risk factors can have an impact on both the cost and revenue side of the project financials—demand is lower than expected, project delays increase the cost of the initial investment, and so on. Some of these variables may be related. For example, a new product failing field trials may increase the R&D costs in launching it, but it will also reduce the potential sales revenues if the delay causes the company to miss a product season or a competitor beats it to market. Similarly, an economic downturn could increase financing costs in construction of a new sports stadium and reduce demand for tickets. Other variables may be independent or act as natural hedges.

With faster computer run times, simulation of potential net returns should be easier. These techniques are being used in insurance settings by actuaries and could be adapted to capital budgeting. These simulation techniques make bottom-up risk profiling possible and recognize the volatility of individual risk factors, their impact on returns, and the degree to which they are interrelated.

Covariance is not the only consideration in developing a project risk profile: investment decisions are not static. In many cases, management has some options over the future direction of the investment. It can abandon the project, increase its investment, or have an option to revise the project at a later date. This situation calls for a dynamic analysis.

Dynamic Financial Analysis (DFA) is a sophisticated simulation model developed in an insurance industry setting. Multiple scenarios are performed to examine the fortunes of a company enabling a thorough understanding of the impact of the risks to which the organization is exposed. In particular, by looking at "extreme point" results, the analysis may tell us a great deal about our organization's susceptibility to a "disaster-type" situation which may be hidden in normal mean/variance type analysis. DFA also incorporates future management decisions, or options, by building certain decision rules into the simulation. For example, if returns are below x, we abandon the project, or if field trials show y demand, we increase or reduce the investment in launching a product.

Where DFA is especially useful is in allowing the user to build a sophisticated model that incorporates the

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interrelationship between variables. A model used for capital project purposes could incorporate a myriad of interrelationships.

As an extension of the DFA approach, insurance practitioners are also looking at the banking concept of "value-at-risk" (VAR). The basic idea of VAR is to look at the extreme points in a loss distribution and to determine essentially what is the most amount of money that can be lost. One definition might be "VAR is the maximum amount of money by which the value of my portfolio may decline in the next week, or time *t*, with 95% confidence." Clearly, the tools of DFA could be used to determine a value for VAR.

The DFA and VAR techniques could be of use in a construction project context. For example, consider a capital construction project, requiring $100 up-front costs, which has an expected payback of approximately $110, but with some uncertainty in this return. The distribution of potential income is shown below. There is a small probability of there being a negative return, and an extremely small probability of us losing all our money:

Income	Probability
0	0.0001
80	0.0099
102	0.2400
110	0.5000
118	0.2500

The standard deviation of returns does not really bring out the fact that there is a small possibility of a very large loss as shown in Table 1.

DFA and VAR allow us to hone in on the extreme points and analyze further the scenarios that produce large losses. Therefore, we could define the VAR as the most amount (in money or percentage terms) that could possibly be lost, with a 99% confidence level (that is, at the loss level where cumulatively 99% of returns are above). In this instance it would be 20%. This then might be defined as our level of "risk" for the project. We may then say that this project has an expected return of 9.77% with a risk level of −20%.

Choosing Between Investments

One of the problems with probabilistic models is the interpretation of the data for decision making. Unlike the single deterministic point estimates, there is no simple decision rule. DFA tells us our expected return is 9.77% with risk of –20%, but what is it telling us to do? Should we accept the project or not? The answer is to establish the expected "risk" and "return" of all possible investment opportunities open to us, and to see how the particular project under consideration fits in with this complete picture. This is essentially an extension of the Markowitz portfolio selection model, a "classical" concept in financial economics, and its by-product, the efficient frontier. The original Markowitz idea is that for a given level of risk, defined as the standard deviation of "returns" on a portfolio of assets, there is a combination of assets that will maximize expected return. The generalized version of the Markowitz model is that for any given level of risk, there is a strategy, or project that will maximize return.

By plotting the risk/return point of our construction project on a chart with all other potential investment opportunities—including the risk free Treasury bill return—we can see whether our project falls on the efficient frontier. If it does, then the project might be accepted; if not, then we might wish to look at other project opportunities.

One additional problem needs to be addressed. Even if the project lies on

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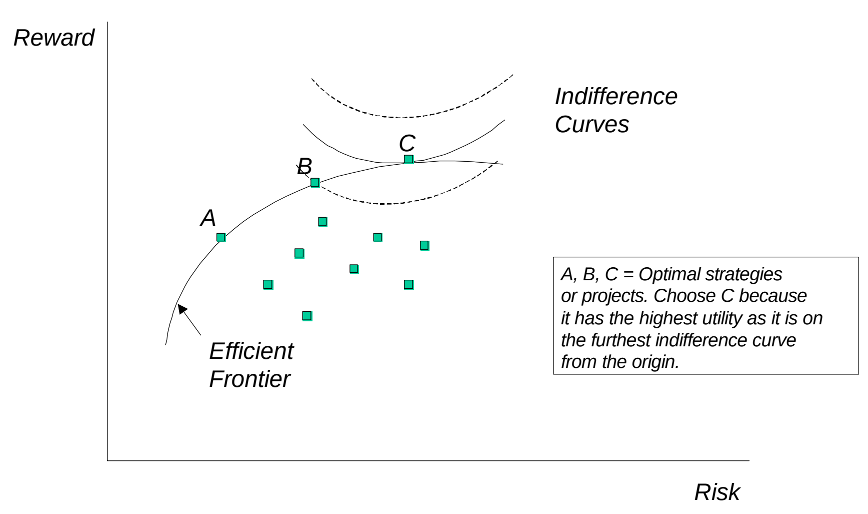
TABLE 1

Returns (X)	Probability (p)	pX	pX^2
−100%	0.0001	−0.01	1.00
−20%	0.0099	−0.198	3.96
2%	0.2400	0.48	0.96
10%	0.5000	5.00	50.00
18%	0.2500	4.50	81.00
	1.0000	9.77	136.92

Mean = 9.77%
Variance = 136.92 − 9.77 × 9.77 = 41.47%
SD = 6.44%

Exhibit 1: Utility and the Efficient Frontier



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the efficient frontier, the question remains as to whether the riskiness of the project is acceptable. In this respect, there is another curve that needs to be drawn, representing the investor's "utility." If the project lies sufficiently close to the utility curve, then the project may be deemed acceptable.

The concepts involved in the above discussion are illustrated graphically in Exhibit 1. The utility curve is the most subjective element of all, and in practice may indeed be assessed purely on the basis of judgement.

Insurance practitioners have been using the efficient frontier in the area of asset-liability management for some years now, particularly to assist in establishing asset allocation in the context of a certain liability profile. Moreover, the insurance industry has taken the whole process a few steps further, defining an efficient frontier as being any business strategy the practitioner wishes to test out (for example, to test the introduction of a new product line or using a new marketing outlet). In this way, reward can be defined as any performance objective that is most relevant to the successful management of the organization and is basically what we wish to maximize. Risk then represents what we wish to minimize (or at least control) and might be defined as the probability of insolvency over the next five years. Under the new definitions, strategies that might previously have been thought of as not-so-risky may have some element of risk that might concern us (Treasury bills, for example, may indeed be quite "risky" in certain instances).

In this way, the assessment of a capital project need not just be in terms of expected returns and standard deviation of returns, but may use much more sophisticated definitions to really get to the heart of the nature of the practitioner's business. For example, in the context of building a sports stadium with taxpayers' money, perhaps return needs to consider the many possible spin-off effects in terms of employment and other benefits to the community; the risks might incorporate the potential collapse of neighborhoods and additional traffic congestion. In this way, the true impact to all stakeholders of the organization can be considered. For government-sponsored capital projects these social costs and benefits can be as important as the financial measures of risk and return.

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Capital Projects Working Party— Recruitment Drive

The Society of Actuaries Finance Practice Area is in the process of resurrecting the Capital Projects Working Party. Tony Dardis is acting as chairperson and is keen to hear from anyone who would like to join the group.

Tony and Andrew Berry have an article "Applying Insurance Company Quantitative Techniques for Improved Capital Budgeting" in this edition of *Risks and Rewards* that gives some background to the subject matter covered under the banner of "capital projects." We think this is an exciting potential growth area for actuaries in the United States.

The initial work of the group will focus on the following areas:

- Identification of actuaries working in the capital projects area interviews with those actuaries about the nature of their work
- Making contact with the business schools about quantitative techniques currently used by U.S. industry for capital budgeting purposes
- Preparation of a Capital Projects Specialty Guide, or reading list.

If you are interested in joining Tony's group, you can contact him at 972–701–2739, 972–701–2575 (fax) or dardist@tillinghast.com (e-mail).

It's Time to Give More Focus to Risk Control

by Patrick Reinkemeyer

Editor's Note: *This article originally appeared in the September 14, 1998 issue of* The National Underwriter, *a publication of the National Underwriter Company, and is reprinted here with permission.*

Variable life and variable universal life policies have a powerful marketing advantage over traditional life—the lure of stock market returns.

As the equity market has soared through the 1990s, individual investors have become enamored of the riches promised by compounding double-digit returns over a long time horizon, and that variables are the only insurance vehicles that can offer such largess.

The market's recent downside tremors may have made investors aware, however, that the stock market goes down as well as up. And baby boomers nearing retirement may start growing a bit more cautious in order to consolidate their gains and ensure their quality of life in the golden years.

These trends suggest that risk control may become an increasingly important marketing feature for all financial products.

Fortunately, variable policies have the ability to offer a well-designed set of investment options. As a result, variable policyholders can use one of the best risk control tools in existence—diversification.

Through sensible diversification, variable policyholders can be taught how to obtain their individual target rate of return with the minimum level of risk. This kind of analysis can be done by applying a "mean-variance optimizer" to the investment options in a variable policy.

An "optimizer" is a financial tool, used by analysts, that seeks to maximize investment return while minimizing risk.

Professionals employ the tool to help find the fund combination that gives investors the most bang for the buck. (Of course, doing so is subject to the inevitable caveat of all investment analysis: The analysis is based on historical data and there is no guarantee that history will repeat itself. Nevertheless, analysis of this kind is a terrific tool for explaining and proving the benefit of taking advantage of the diverse investment options available in most variable life policies.)

To illustrate the potential of this type of analysis, we conducted a mean-variance optimization analysis on the three VL policies using the past three years of their actual monthly performance.

Since the number of fund options is an important element in policy design, we picked our policies to cover the range available in today's marketplace: MML Bay State Variable Life Plus has four options. Phoenix Flex Edge Variable Universal Life offers 17 funds, and Nationwide's Best of America Flexible Premium VUL offers 41 choices.

Our analysis suggests that bigger is better, as far as obtaining optimal risk reduction is concerned. As the accompanying table shows, at each of three levels of target return—8%, 14%, and 20%—the Best of America policy offered the lowest risk in its optimal portfolio, and MML Bay State had the highest risk level in its optimal portfolio. This simply results from the optimizer being able to find funds within the larger Best of America policy that have low correlations and, hence, more effective diversification.

However, there are disadvantages to having a large lineup. A large number of funds makes choosing between a wide range of funds, without the benefit of the optimizer's perfect hindsight, a more challenging task.

Therefore, in order to see how close investors came to an optimal risk-minimizing allocation, we also estimated what the actual allocation of all investors in these policies was three years ago. In this analysis, the smaller policy, MML Bay State, came out on top.

For that policy, the clients' actual choices were quite close to the ideal, primarily because they put most of their money in the only domestic-equity option, Equity, which turned out to be the optimal investment in that policy. In the Phoenix Flex Edge contract, however, investors also put most of their money in the domestic-equity option, Growth, but they could have had lower risk with the same level of return with a blend that included the International and MultiSector Fixed Income options.

Overall, our estimates suggest that the investors in MML Bay State may have had slightly higher return and lower risk than did investors in Phoenix, even though they had a smaller, and less optimal set of investment options. In short, investors had more opportunities to make mistakes away from optimality in the larger policy.

This analysis suggest the range of challenge and opportunity facing variable insurance product designers. They have the opportunity to tailor fund lineups and marketing to emphasize the power of diversification to reduce risk.

On the other hand, if they offer huge fund lineups to enable the most diversification possible, they face the challenge of trying to educate policyholders about how to choose among the funds to create a portfolio appropriate for their risk tolerances.

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Minimum Risk at Three Levels of Target Return

Policy	No. of Options	% Risk at Target Return		
		8%	14%	20%
MML Bay State Variable Life Plus	4	0.61	1.75	2.91
Phoenix Flex Edge VUL	17	0.56	1.45	2.56
Best of America PPVUL	41	0.43	1.15	1.87

Risk is measured by standard deviation of monthly returns.
Source: *Morningstar Variable Annuities/Life, Chicago*

A 99-Year Prospective Test of an Interest-Rate Theory

by Daniel F. Case

Editor's Note: *This article describes a theory of long-term interest-rate trends that was propounded in 1899 by an actuary, Charlton T. Lewis. The article then presents a preliminary examination of how well that theory stands up in the light of 20th-century experience. The purpose is to invite more thorough evaluation by any experts who find the theory possibly useful.*

If one wishes to develop, interpret, or evaluate a 100-year term structure of interest rates, one needs a notion of how interest rates may behave over the next 100 years or so. In forming such a notion one may take a look at the ability of some theories, including perhaps some mathematical models, to explain interest-rate trends that have occurred in the past.

In addition to seeing how well a theory explains the past, it is helpful to see how well it has predicted the future. We cannot observe a l00-year future, of course, in the case of theories that were propounded only recently. In the case of a theory that was published 100 years ago, however, we can observe its predictive success over a period of 100 years.

This article will discuss a theory that was published in 1899. The approach of this paper will, accordingly, differ from that of a typical research paper. Rather than develop a theory on the basis of observed facts, this paper will briefly describe the previously developed theory and then discuss how well it appears to have stood up since it was published. For a better understanding of the theory than can be gained from this paper, readers can consult the 1899 paper.

The paper in question is "The Normal Rate of Interest" by Charlton T. Lewis, a member of the Actuarial Society of America [1]. That paper, besides presenting a theory, gave both a medium-term and a long-term prediction of interest-rate trends. The medium-term prediction was evaluated thus at a 1919 meeting of the American Institute of Actuaries:

> You will perhaps recall that just about 20 years ago, one of the large life insurance companies published letters from distinguished financiers on the future course of the interest rate. Those letters were almost unanimous, if I read the matter correctly, in predicting further decline in the interest rate, as it had been declining for some 30 years. The one man who stood out against that view was Mr. Charlton T. Lewis, in his very scholarly paper. Anyone interested in this subject should certainly read and re-read that paper. You know the facts are that Mr. Lewis was right [2].

Lewis elaborated on (or clarified) his theory a bit in a second paper, published in 1904 [3]. In addition to the two papers, the discussions of them, and the above-cited 1919 comments, extensive comments on Lewis' work were made at a 1920 meeting of the Actuarial Society [4]. There is a further brief reference dating from 1934 [5].

Materials and Methods

The materials for this study are the two Lewis papers cited above (which will be briefly summarized here) and some interest-rate and other data from years following their publication. The method will be first to compare Lewis' interest-rate predictions with rates and trends subsequently experienced. Then a preliminary test of the central assertion of Lewis' theory will be presented.

LEWIS' THEORY

Lewis' 1899 paper sets forth his theory and contains his predictions. The theory pertains to long-terms trends and may be summed up by the following two sentences: "On the contrary, all experience proves that the demand for capital finds its supreme stimulus in the expectation of productiveness. This expectation is excited chiefly by discovery and invention."[6]

In the foregoing excerpt, "On the contrary" refers to assertions by many economists (six of whom Lewis named) that increased wealth and economic progress of themselves lower interest rates. Lewis found no evidence, in the historical trends outlined in his paper, that increases in wealth (bringing increases in the supply of capital) are not accompanied by corresponding increases in the demand for capital.

Lewis also challenged, with respect to long-term trends, the theory that "abundance of money in itself makes interest low."[7] To refute that theory he offered an example of a marked, sustained rise in interest rates following dramatic discoveries of gold in California and Australia.

Lewis analyzed interest-rate trends in terms of forces of two types (not sharply distinguishable from each other): "wave" forces, "which act within definite and often narrow limits of space or time," and "tidal" forces, "which act for long periods and upon the markets of the world."[8] Among "wave" forces, he mentioned government manipulations of the supply and value of money and "substitutes for money," wars and rumors of wars, changes in government spending, seasonal demands for money or credit, and the like.

In order to identify the "tidal" forces, Lewis examined long-term interest-rate trends during the 19th century in the light of economic developments. He found that long periods of rising interest rates were associated with periods of discovery and invention, while long periods of falling interest rates were associated with relative stagnation. He found such periods, each 20 to 30 years long, alternating during the century. He acknowledged that it might be impossible to explain completely what caused "the alternations of enterprise and stagnation in the world of industry and trade," but, clearly, considered it possible to detect evidence of the beginning of a new cycle.[9]

Lewis had in mind a "normal" level about which interest rates oscillate under both wave and tidal forces.

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Following a discussion of capital transactions he wrote, "Since enterprise always stands ready to use capital productively, it is always willing to exchange for the capital available at the earlier date a larger capital of a later date, the addition being limited only by the expected increase of value."[10] Thus he related the normal rate of interest to the average rate of productiveness of capital during a time interval.

Some of Lewis' discussion of the average rate of productiveness seems questionable and was challenged by one or more discussants of his work. Lewis did not, however, use that aspect of his theory in arriving at his estimate of the normal rate of interest. Before turning to that estimate, however, let us look at two elaborations, or clarifications, of the theory that appear in Lewis' 1904 paper.

First, Lewis distinguished capital from property. "Property consists of individual things, each of which can be seen and handled, used and enjoyed; each with its own distinct features of utility. ... Capital consists of dollars or other ideal units of an infinite mass, every one of which is absolutely identical with every other."[11]

Second, Lewis emphasized that the expectation of productiveness of capital "rests upon the nature of man in a progressive society, and not upon the nature of property. Were the spirit of enterprise destroyed and the speculative hazards of fortune ended, the demand for capital in industry would be limited to the amounts needed under old and tried methods of production."[12] Further:

> "Economists have long perceived that periods of invention, discovery and enterprise are those in which the demand for capital is effective and interest high. But the connection between the spirit of enterprise and the increased demand has always been sought in the slow process of absorbing capital in new enterprises, converting the floating supply into fixed forms, and reducing the available stock in the markets. In reality the connection is far closer and the effect upon the rate of interest is much quicker, than this process can explain.
>
> "The reason is that the demand is determined, not by the experience of past productiveness, but by the hope of future profit."[13]

LEWIS' PREDICTIONS

Lewis used a two-step procedure in developing the medium-term prediction that won praise 20 years later. First, he provided evidence that interest rates were beginning to head upwards from a presumed low point. He then wrote:

> "The forces which have turned the great tidal movement are obvious, and are as wide as the civilized world. Invention and enterprise have taken new life everywhere. ... The rapid development of steam navigation, of railway improvement, of ship canals, of electrical art in a thousand forms, the increase of buildings, furnaces, mills, machinery, the opening of new colonies, in short, the conversion of floating into fixed capital, goes on at an accelerating pace. ... Whenever hitherto such an epoch of invention and enterprise has checked a long-continued accumulation of idle capital and turned the great tidal wave of interest from ebb to flow, the process has been progressive for many years, and has continued to gain force and rapidity long after it had first become conspicuous.
>
> ... If the world's peace is maintained, there is not in prospect any check to the gradual rise of interest, at least until the average rate shall fully reflect the average yield of productive capital."[14]

Lewis then addressed the question of what that average yield—the normal interest rate—is. He wrote: "The question what that average yield is demands the actuary's methods applied to the data of the economist. My object is to stimulate inquiry, not to dogmatize on its results."[15] What follows in Lewis' paper was, then, perhaps acknowledged by him to be only an expedient.

Lewis wrote: "There could be no better measure of the true normal yield of invested capital than the average percentage of interest realized by life insurance companies upon their invested assets."[16] He observed that a little more than 20 years previously that rate had been a full 6% and that it now appeared to have bottomed out at slightly below 5%. He pointed out, in a footnote, that the published rates included realized capital gains and losses, which caused an overstatement of the overall yield when interest yields were falling and an understatement when they were rising. He also, of course, pointed out that the published rates were portfolio, not new-money, yields. He wrote:

> "These considerations must be taken into account, and the effect of each estimated in detail to reach the true average rate of interest. Such an examination would probably prove that the true rate in 1897 was considerably below the apparent rate of 4.92%, and possibly somewhat below 4.5%, but that the average rate for the whole period of declining interest from 1872 to 1897 was above 5.4%.
>
> "It seems reasonable to believe that this last-named average, taken through a period of declining rates, fairly represents the permanent average income from safe investments." [17]

We may regard 5.4% as Lewis' estimate of the normal interest rate by his selected measure. We may regard his medium-term prediction as being that; in the absence of war, the rate realized by life insurers on their invested assets would rise to at least 5.4%.

Lewis acknowledged that further examination was needed; he regarded his conclusions as a "working hypothesis."[18]

METHOD OF COMPARING PREDICTED WITH ACTUAL

Lewis' medium-term interest-rate prediction will be compared with various published rates of interest relating to the

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period 1899–1921. His estimate of the "permanent average income from safe investments" will be compared with a 99-year average taken from insurers' portfolio yields during the period 1899– 1919 and Moody's Aaa corporate bond yields during the period 1920–97. Lewis' underlying theory—that the primary "tidal" force is the expectation of productiveness—will be tested by comparing year-by-year new-money yields with the yearly percent changes in the numbers of U.S. patents and trademarks issued and in the gross domestic product, measured in constant dollars.

Discussion

A good test of Lewis' estimate of the "average yield of productive capital" might be a comparison of 5.4% with life insurers' average new-money investment yields during the twentieth century. As an approximation, portfolio yields might be used instead. Of course, it might be difficult or impossible to ascertain precisely how such published numbers were derived at each time during the century. Also, as the mix of investments in insurers' portfolios changed over time, a history of their overall yields might lose meaning as an indicator of interest-rate trends. The corporate-bond yields used in this article have the advantages of being new-money rates and being, presumably, consistent from year to year.

As for Lewis' theory, one may ask to what extent Lewis' 1899 predictions represent an application of his theory as such. His paper, presented in October 1899, was evidently written that year. By that time, according to the paper, new-money interest rates had already started to rise, and economic expansion was already well underway. Presumably, the then current period of discovery and invention had been going on for some time. One might say, therefore, that Lewis' paper in effect states that there was evidence that a period of discovery and invention had begun at some recent time and that his theory predicted that the period would continue for a good while longer and carry with it a continuing rise in interest rates. When that period began, how its beginning might have been detected, and how to determine whether it was still in progress are not stated.

A good test of Lewis' underlying theory would analyze trends in the level of expectation of productiveness. As indicated previously, Lewis asserted that this expectation is excited chiefly by discovery and invention. Certainly, the numbers of patents and trademarks issued each year, discussed later in this paper, are at best a crude measure of the level of discovery and invention. The present writer, however, is not knowledgeable in the matters that would have to be analyzed in order to get a better measure. The writer hopes that any experts who are interested in Lewis' theory will look for ways to test it more soundly and thoroughly.

The numbers on gross domestic product are included below as a possible indication of the productiveness that may have been expected some years before. Gross domestic rather than gross national product is used because it focuses on capital located in the U.S., rather than on capital owned by U.S. interests.

By 1920 it was being suggested that there is a strong connection between inflation and interest rates. In a discussion of Lewis' papers in 1920, R.W. Huntington remarked, "Mr. Lewis did not have in his mind any clear idea of inflation as a cause of increasing the interest rates."[19] Lewis' 1899 paper did mention inflation, as follows: "Each large issue of such currency causes violent fluctuations, first for a very short time in rates of interest on temporary loans, and then more lastingly in the nominal prices of goods…"[20] The issuing of currency was classed by Lewis as one type of wave force. Accordingly, Lewis treated inflation as a product of a wave force. He may or may not have regarded inflation as itself a force that acts upon interest rates. In any case, since he did not include inflation in his discussion of tidal forces, the following tests of his predictions and theory have not been designed to reflect inflation.

Results

LEWIS' INTEREST-RATE PREDICTIONS

The following was stated by Douglas H. Rose in the 1920 discussion mentioned above:

> "The Spectator Company is in the habit of publishing annually in its *Year Book* the rate of interest earned on mean invested funds of a limited number of life companies. Going back 40 years, the averages for five-year periods are as follows:
>
> | | |
> |---|---|
> | 1880–1884 | 5.50% |
> | 1885–1889 | 5.37% |
> | 1890–1894 | 5.15% |
> | 1895–1899 | 4.88% |
> | 1900–1904 | 4.66% |
> | 1905–1909 | 4.77% |
> | 1910–1914 | 4.80% |
> | 1915–1919 | 4.87% [21] |

Yields on Moody's Aaa-rated corporate bonds for the period 1919 (the earliest year for which such a figure was found) through 1997 are shown in Table A.

We can calculate a 99-year average interest rate from the data shown above and in Table A by using for 1899 the Spectator Company's number for 1895–1899, using the 1900–1919 Spectator numbers as if they were new-money rates for those years, and using the Moody yields for the years 1920–97. The justification for using portfolio rates for 1899-1919 is that new-money rates began that period somewhat below the portfolio level and ended the period somewhat above it.

The resulting 99-year average is 5.65%, a rate slightly above Lewis' estimated normal interest rate of 5.4%. It must be noted again, of course, that the Moody yields do not represent the measure that Lewis had in mind. Also, interest rates were considerably higher in 1997 than in 1895–99. If interest rates do not decline significantly during the 10 years following 1997, an average over the years 1909–2007 will be higher than the above 5.65% average for 1899–1997.

The Moody's Aaa yield for 1919, as shown in Table A, is 5.49%, while the insurers' portfolio yield for the years 1915–19, shown above, is 4.87%. In order to guess what the insurers' new-money yield was in 1919, we may note that the yield on municipal high-grade bonds rose quite steadily from 3.12% in 1900 (the earliest year for which such a figure was found) to

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A 99-Year Prospective Test
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4.50% in 1918, dropping to 4.46% in 1919 and rising again to 4.98% in 1920.[22] One may guess that a portfolio of high-grade municipals would have earned somewhat under 4% in 1919—that is, a bit more than 50 basis points less than the 1919 new-money rate on those bonds. One may, correspondingly, guess that the insurers' overall new-money rate in 1919 was a bit more than 50 basis points above their overall portfolio rate—hence in the neighbor-hood of the Moody's Aaa rate of 5.49% for that year. Accordingly, the Moody's Aaa yields may be a reasonable proxy for insurers' overall new-money yields of that time.

We cannot, of course, guess from the data presented here what differences there were between Moody's Aaa yields and the insurers' overall yields in years subsequent to 1919. To the present writer, however, the closeness of Lewis' 5.4% estimate to the 5.65% 99-year average calculated here is remarkable.

Lewis estimated the normal rate of interest, but did not attempt to estimate a likely range of fluctuation. He wrote, "All fluctuations are governed by the familiar law of marginal utility; so that, as soon as an actual deficiency of capital is revealed, extreme needs begin to assert themselves in violent competition, and the rate may rise indefinitely."[23]

As for Lewis' medium-term prediction, it called for insurers' new-money yields to rise to at least 5.4% during the tidal period then underway. Moody's Aaa yields topped 6.1% in 1920, but they dropped below 5.2% in 1922 and remained below that level for over three decades. We have the question of to what extent the high yields of 1919– 1921 were the result of wave forces, such as war and inflation, and not the culmination of a tidal movement.

Not knowing the Moody's Aaa yields for years before 1919, we cannot judge from them the size of the wave. We may note, however, that the yields on high-grade municipals for the years 1916–1922 were 3.94%, 4.20%, 4.50%, 4.46%, 4.98%, 5.09%, and 4.23%, and the unadjusted index of yields of American railroad bonds for the same years was 4.49%, 4.79%, 5.23%, 5.29%, 5.81%, 5.57%, and 4.85%.[24] From those numbers we may guess that wave forces increased interest rates by more than a percentage point above what the tidal forces alone would have produced. It appears that Lewis' medium-term prediction was not genuinely fulfilled.

LEWIS' THEORY

Finally, how does Lewis' theory look in the light of 20th century experience to date? As a preliminary inquiry into that question, we can try to identify patterns in the accompanying Figure 1, which plots bond yields and the percent changes in three other measures: the number of U.S. patents issued each year for inventions (which constitute the overwhelming majority of total U.S. patents issued), the number of trademarks registered each year, and the gross domestic product (GDP) as measured in constant dollars (in "chained" dollars in recent years). Since the numbers of patents issued and trademarks registered have been highly volatile, the percent changes shown for them in Figure 1 are equal to 1/10 of the actual percent changes. The interest-rate numbers for years before 1919 are derived from the unadjusted index of yields of American railroad bonds by ratioing those numbers up so that the number for 1919 equals the Moody's corporate Aaa rate for that year. The rates of change for the GDP for years before 1920 are derived from published five-year groupings; the writer does not know how volatile from year to year those rates were in fact.

The sources for the numbers in Figure 1 are:

- U.S. patents issued for inventions and trademarks registered—for the years through 1970, U.S. Dept. of Commerce, Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 2*, 1975, 957–9, Washington, D.C.; for later years, U.S. Dept. of Commerce, Bureau of the Census, *Statistical Abstract of the United States*, various years, Washington, D.C.
- Bond yields—the same sources as were used for Table A, with the first source listed there being used for years prior to 1919.

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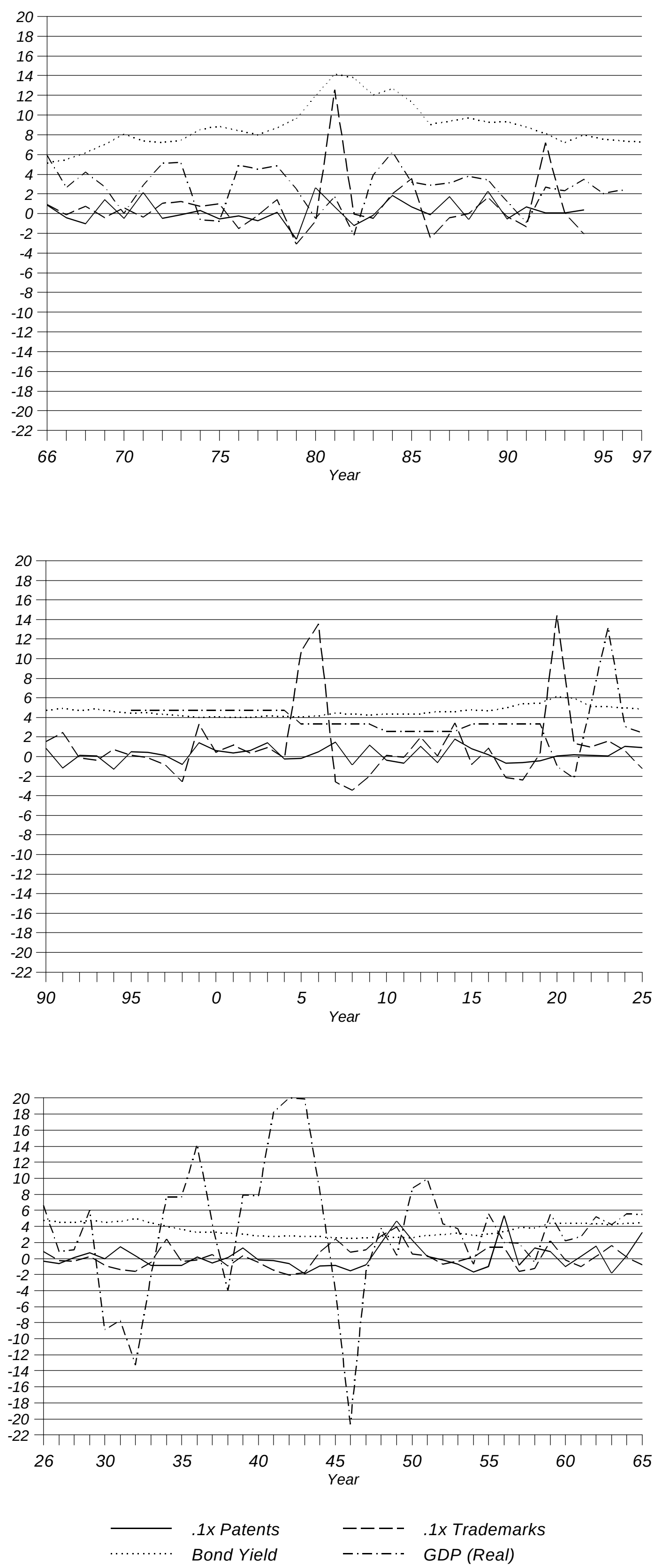
TABLE A
Corporate Aaa
(Moody's Seasoned Bond Yields
1919 to 1997
(Percent per Annum)

Year	Yield	Year	Yield
1919	5.49		
1920	6.12		
1921	5.97	1961	4.35
1922	5.10	1962	4.33
1923	5.12	1963	4.26
1924	5.00	1964	4.40
1925	4.88	1965	4.49
1926	4.73	1966	5.13
1927	4.57	1967	5.51
1928	4.55	1968	6.18
1929	4.73	1969	7.03
1930	4.55	1970	8.04
1931	4.58	1971	7.39
1932	5.01	1972	7.21
1933	4.49	1973	7.44
1934	4.00	1974	8.57
1935	3.60	1975	8.83
1936	3.24	1976	8.43
1937	3.26	1977	8.02
1938	3.19	1978	8.73
1939	3.01	1979	9.63
1940	2.84	1980	11.94
1941	2.77	1981	14.17
1942	2.83	1982	13.79
1943	2.73	1983	12.04
1944	2.72	1984	12.71
1945	2.62	1985	11.37
1946	2.53	1986	9.02
1947	2.61	1987	9.38
1948	2.82	1988	9.71
1949	2.66	1989	9.26
1950	2.62	1990	9.32
1951	2.86	1991	8.77
1952	2.96	1992	8.14
1953	3.20	1993	7.22
1954	2.90	1994	7.97
1955	3.06	1995	7.59
1956	3.36	1996	7.37
1957	3.89	1997	7.26
1958	3.79		
1959	4.38		
1960	4.41		

Sources: For years 1919–1970: U.S. Dept. Of Commerce, Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 2,* 1975, 1003, Washington, D.C. For years 1971–1966, U.S. Dept. Of Commerce, Bureau of the Census, *Statistical abstract of the United States,* various years and pages, Washington, D.C. For 1997, Moody's Investors Service, *Moody's Bond Record,* February 1998, Vol. 65, No. 2, 38.

FIGURE 1—Bond Yields vs. Changes in Other Measures



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- GDP—for years through 1928, *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 1*, 232; for the years 1929 through 1958, U.S. Dept. of Commerce, Bureau of Economic Analysis, *National Income and Product Accounts of the United States, Volume 1, 1929-58*, 1993, 3, and *Volume 2, 1959–88*, 1992, 4, Washington, D.C.; for subsequent years, *Statistical Abstract of the United States*, various years.

We may first observe the rising interest-rate trend heralded by Lewis, which lasted through 1920. We see that in 1899 the levels of patent and trademark approvals improved slightly, and if the upsurge of trademark approvals in 1905 and 1906 represented in part an effort to reduce a heavy backlog of applications, trademark activity must have been lively during the first few years of the century. A similar backlog of trademark applications may have developed during World War I. Overall, patent issues for inventions increased from 20,377 in 1898 to 43,892 in 1916, and trademark registrations increased from 1,238 in 1898 to 6,791 over the same period.[25] If patent and trademark approvals are a good indication of the level of discovery and invention and hence of expectations of productiveness, that pattern offers support for Lewis' theory.

On the other hand, real GDP was increasing at a lower rate during the last 15 years of Lewis' upward tidal period than its rate for many years preceding 1905. If that pattern reflects people's expectations of productiveness during the period of the interest-rate increase, it is evidence against Lewis' theory.

A downward interest-rate trend began in about 1922 and continued through 1946. We see that patent approvals for inventions were relatively flat from 1921 (37,798 issues) through 1941 (41,109 issues),[26] decreasing from 1942 through 1947, and then increasing fairly vigorously in 1948–50. Presumably, the war influenced the pattern from 1942–50. The long, rather flat period through 1941, however, seems to support Lewis' theory. As for trademark activity, registrations continued to increase through 1923 (14,834 registrations), were flat from then through 1930 (13,246 registrations), and then declined through 1941 (8,530 registrations). [27] That pattern seems to offer additional support.

Real GDP was highly volatile during most of the 1923–1951 period, and the present writer hazards no speculations about it.

An upward interest-rate trend began in 1951 and lasted until about 1982. Both patent issues for inventions and trademark registrations were on the upswing during that period. The patent issues increased from 43,040 in 1950 to 65,800 in 1981. Over the same period, trademark registrations increased from 16,817 to 42,700.[28] Those patterns seem to support Lewis' theory quite strongly.

There were reasonably healthy increases in GDP during most of the 1951–1982 period.

A downward interest-rate trend began in about 1983 and may or may not be still in progress as of 1998. The trend in patents has been from 65,800 in 1981 to 101,700 in 1994, and the trend in trademarks over the same period has been from 42,700 to 63,900.[29] Those patterns offer evidence against Lewis' theory.

Also with respect to real GDP, the trend from 1981 to 1996 looks not much different from the trend during the preceding period.

Conclusions

The present writer has not attempted a thorough investigation of whether Lewis' interest-rate theory holds up under 20th century conditions. Preliminary findings seem, however, somewhat encouraging. Interest rates (Moody's Aaa corporate bond yields) have continued to follow Lewis' observed pattern of "tidal" trends. The lengths of those trends (excluding the downward trend that began in 1982 or 1983 and may or may not still be continuing) have been within or close to Lewis' observed 19th century lengths of 20–30 years. As for Lewis' theory that the primary tidal force influencing interest rates is the expectation of productiveness, the evidence shown in this paper with regard to patent and trademark approvals seems to support the theory, in varying degrees, with regard to three of the four tidal interest-rate trends discussed here, but definitely not with regard to the trend that began in about 1983.

Lewis' estimate of the normal rate of interest seems, on the basis of 99 years of subsequent experience, to have come quite close to the mark. His medium-term prediction regarding a tidal trend beginning in about 1898 seems to have been fulfilled with regard to its duration, but not genuinely with regard to the level it would reach.

The present writer hopes that Lewis' papers will kindle an interest in further investigations along the lines of his theory, with modifications and/or refinements as may appear appropriate in the light of 20th century experience and thought.

End Notes

1. Lewis, Charlton T., "The Normal Rate of Interest," Actuarial Society of America *Papers and Transactions*, 1899–1900, Volume VI, Nos. 21, 22, 23, and 24, pp. 158–171.
2. American Institute of Actuaries *Record*, 1919, Volume VIII, pp. 309–311.
3. Lewis, Charlton T., "Notes on a Factor, Hitherto Overlooked, of the Rate of Interest," Actuarial Society of America *Transactions*, 1904, Volume VIII, Nos. 31, 32, pp. 8–16.
4. "Abstract of Discussion of ('Some Influences Affecting the Interest Rate,' by Wendell M. Strong)," Actuarial Society of America *Transactions*, May 20 and 21, 1920, Vol. XXI, Part One, No. 63, pp. 437–451.
5. American Institute of Actuaries *Record*, 1934, Volume XXIII, p. 134.
6. Lewis, Charlton T., "The Normal Rate of Interest," op. cit., 165.
7. Ibid., 164.
8. Ibid., 161.
9. Ibid., 166.
10. Ibid., 160.
11. Lewis, Charlton T., "Notes on a Factor, Hitherto Overlooked, of the Rate of Interest," op. cit, 9.

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12. Ibid., 12.
13. Ibid., 15–16.
14. Lewis, Charlton T., "The Normal Rate of Interest," op. cit., 168.
15. Ibid., 169.
16. Ibid., 169.
17. Ibid., 170.
18. Ibid., 171.
19. "Abstract of Discussion of … ('Some Influences Affecting the Interest Rate,' by Wendell M. Strong)," op. cit., 441.
20. Lewis, Charlton T., "The Normal Rate of Interest," op. cit., 161.
21. "Abstract of Discussion of … ('Some Influences Affecting the Interest Rate,' by Wendell M. Strong)." op. cit., 440.
22. U.S. Dept. Of Commerce, Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 2,* 1975, 1003, Washington, D.C.
23. Lewis, Charlton T., "The Normal Rate of Interest," op. cit., 161.
24. U.S. Dept. Of Commerce, Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 2,* op. cit., 1003.
25. Sources cited for Figure 1.
26. Ibid.
27. Ibid.
28. Sources cited for Figure 1.
29. Ibid.

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Review of Financial Journals

Reviewed by Edwin A. Martin

Term Structure and Interest Rates

The low Treasury rates and widening spreads that we've experienced lately have many of us thinking about the yield curve. We reviewed several articles related to interest rates and the yield curve.

- **"Recovery and Implied Default in Brady Bonds" by Karan Bhanot, *Journal of Fixed Income,* June 1998**

  The author demonstrates that implied default probabilities in Brady bonds are significantly higher than a zero-recovery model would suggest. The analysis has an impact on the valuation of spreads on foreign debt and duration calculations. The numerical example on Argentine bonds supports the author's conclusion.

- **"Term Premium Estimates from Zero-Coupon Bonds: New Evidence on the Expectations Hypothesis" by Upinder S. Dhillon and Dennis J. Lasser, *Journal of Fixed Income*, June 1998**

  This article is very interesting because of its inconsistency with prior research. The authors use zero-coupon Treasuries to provide strong evidence for liquidity premiums in the term structure and show that the liquidity premiums increase with maturity. In additional, they find that current forward rates can be used to forecast quarterly interest rates.

- **"Rewards to Extending Maturity" by Dale L. Domian, Terry S. Maness, and William Reichenstein, *Journal of Portfolio Management,* Spring 1998**

  This article discusses the risks and benefits of extending the maturity of fixed-income investments to increase yield as well as support for different term-structure theories and might be of use to actuaries developing interest rate crediting strategies.

- **"An Approach to Scenario Hedging" by Charles F. Hill and Simon Vaysman, *Journal of Portfolio Management,* Winter 1998**

  This article discusses a method of optimizing a bond portfolio versus fixed-rate liabilities using only a handful of scenarios selected using principal components analysis. Factors are developed based on three yield curve shape changes (shift, twist, and butterfly) and permutations of those basic shape changes. They are used to optimize the portfolio with better results than duration matching or key-rate matching.

- **"What Really Happened to U.S. Bond Yields" by Peter Best, Alistair Byrne, and Antti Ilmanen, *Financial Analysts Journal,* May/June 1998**

  The authors study the fixed-income yields over the last 15 years as well as several explanatory factors: bond risk premium, expected inflation, and real short-term rates. The study finds that all three factors have contributed to the decline in interest rates over the study period.

Equity-Indexed Annuities

Two articles on option valuation may be of interest to those involved with equity-indexed annuities. The first is "A Frequency Distribution Method for Valuing Averaging Options," *ASTIN Bulletin*, November 1997. The author, Edwin H. Neave, finds pay-off frequency distributions to value American and European averaging options. The author uses a discrete time, recombining binomial asset price process. Both geometric and arithmetic averaging options are analyzed.

The second article is "A Closed-Form Approximation for Valuing Basket Options," *Journal of Derivatives*,

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Review of Financial Journals

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Summer 1998. The authors, Moshe Arye Milevsky, and Steven E. Posner, demonstrate a technique that approximates the sum of the lognormals by an inverted gamma distribution. The results show that the use of the inverted gamma produces very reasonable valuations for Asian and basket options.

Actuaries interested in learning more about low-discrepancy sequences as a method of reducing the number of simulations needed to accurately price financial instruments might find value in "Low-Discrepancy Sequences: Monte Carlo Simulation of Option Prices" by Silvio Galanti and Alan Jung in the Fall 1997 *Journal of Derivatives*. The authors explain the concept of low-discrepancy sequences and describe some of the popular algorithms in the context of valuing stock options, as well as potential problems with the technique.

Portfolio Management

An interesting view of fixed-income portfolio risk and value-added return is found in "Bond Managers Need to Take More Risk" by Ronald N. Kahn in the Spring 1998 *Journal of Portfolio Management*. Optimal value added is described as a function of a manager's ratio of active return-to-risk as well as an investor's risk tolerance. The article considers different types of fixed-income strategies and seeks to improve the odds of success by taking more of "the right kind of risk." The article discusses how some market calls, such as interest rate bets, are difficult to apply successfully on a frequent basis.

Interesting Web Pages

In this section, we thought we might try highlighting a web page oriented toward risk management. A site that would be of interest to actuaries interested in financial risk management is the Contingency Analysis web site at www.contingencyanalysis.com. The site is an excellent source for information and papers on risk analysis and has a helpful glossary of terms, a list of publications on risk management and links to other web pages on risk management.

J.P. Morgan, purveyors of the RiskMetrics®, CreditMetrics® and related software products have published reference material on its value-at-risk models on its web page at www.jpmorgan.com. RiskMetrics® is a portfolio value-at-risk model while CreditMetrics® is a credit-risk based VAR model for bond portfolios. J.P. Morgan started an on line magazine, the *CreditMetrics® Monitor*, which can be downloaded from www.jpmorgan.com/RiskManagement/CreditMetrics/CreditMetrics.htm. It includes an article on credit derivatives, a bank loan recovery study, and an article entitled "Uses and Abuses of Bond Default Rates" by Stephen Kealhofer, Sherry Kwok and Wenlong Weng, that analyzes the statistical properties of default rates based on discrete bond rating categories (AAA, AA, and so on) and compares them to a continuous scale developed by the authors' company.

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Taking Stock

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financial downturn as in one's personal portfolio. It takes guts to follow the philosophy of investing for the long-term when prices keep falling. Fundamentalists contend that securities' prices are becoming "quite attractive" while market technicians warn that stock markets (despite short-term blips) are headed much lower, and the bear market will last until at least the middle of 1999. Either way, we must remember that the art of investing is as much psychological as it is a financial decision. And there is no free lunch.

It takes pain to do the research, there will always be mistakes, and it involves patience. But there is also is no guarantee that certain results will always be achieved, even over the long-term. That is why many are stimulated by working in the investment industry or studying the financial markets. It is a puzzle that has too many pieces, one never knows if all the pieces are on the table, and even so, one may only find only a few that fit together.

The End Of Globalization?

It is peculiar that the 1920s was characterized by inflating stock markets, deflation, and globalization. When financial markets collapsed, globalization was replaced by protectionism and nationalism. Deflation was already being seen in our economic cycle in 1996 (a year before the Asian crisis) as commodity prices started a downtrend. And despite strong U.S. growth, inflation measures continued to remain low, which bewildered officials at the Federal Reserve. Perhaps like the 1920s, we were already in a situation before the problems surfaced in Asia, where gains from production were outstripping increases in demand. When deflation occurs, corporate earnings and profits become squeezed, leading to less investment and expansion down the road.

Americans are probably willing to tolerate greater imports as long as the U.S. economy remains strong. But when the U.S. economy slows, we could be hearing calls for import "restrictions" and protection for U.S. workers from foreign competition and trade, as we may recall occurred in the late 1980s. If history repeats itself, then we will all be in for a very rough ride. And we may find that capitalism, which has been fought for by Western powers so vehemently since World War II, will not be the economic strategy of choice for many important countries of the world because they have now tried it and, for them, it does not work.

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What One Can Learn from the Bank of Canada

by Nino Boezio

Canada, like many countries, was considered to be a dramatically rising star in early 1997. Even though it, like most of the Western world, was a debt-laden country, it took major steps to cut spending and bring its fiscal house in order. And as with most, falling interest rates helped alleviate its debt problem as less of its fiscal budget was required for debt financing. Canada, like Europe, was being seen as a late bloomer from the severe recession of the early 1990s and, hence, it had a lot of fast catching up to do to the United States and the United Kingdom. Even though taxes were still high, this was not seen as a major impediment in a world where Canadian industry and exports would soon be in full gear. High unemployment and weak consumer demand in preceding years meant that wage pressures and inflation would not materialize for a very long time. This combination of fundamentals overall suggested that Canada would have economic growth possibly higher than any of the other G-7 nations in the coming years. It was also increasingly being suggested that the Canadian dollar ($Can) could eventually rise above 80¢ U.S. by 1998 or 1999.

The Asian crisis helped to change all that, as it has for most of the world. Even though still in trade surplus, the Canadian surplus fell greatly because of falling commodity demand. The fundamentals for Canadian companies no longer looked as good, particularly when approximately 40% of Canadian exports are commodity-related.

Because fundamentals were no longer as attractive (as can also be said for the Asian and South American economies, Australia and Eastern Europe) less foreign investment took place, leading to a falling currency. And even though the falling currency was mainly tied to weakening fundamentals (and strong fundamentals underlying the U.S. dollar driving up that currency) the Bank of Canada tried to fix the problem through monetary mechanisms.

Short-term interest rates in Canada were much lower than that of the United States in early 1997. Call money was at 3% p.a. whereas the equivalent U.S. Fed funds rate was at 5.5% p.a.. This rate was lower because the Canadian economy was still perceived to be in a much weaker state of growth than its U.S. counterpart, and hence required more monetary stimulus.

When the Asian crisis took center stage in the fall of 1997, the Canadian dollar began to sag and approached the psychologically important level of 70¢ U.S. Even though all other commodity-based currencies were suffering much more (the Australian currency that was initially higher in value to the U.S. dollar than $Can but was now already trading in the 65¢ range) the Bank of Canada refused to accept this worldwide phenomena and "drew a line in the sand."

It first approached the problem by selling its foreign currency reserves to buy Canadian dollars which only provided temporary relief. Then by ill-advised advice from various economists who claimed that short-term money rates were too low relative to that of the U.S. and should be increased, the Bank raised its call money rate to 5%. It is difficult to say whether this rate rise slowed the Canadian economy down significantly, but it can be noted that major gains in economic growth and declines in unemployment halted soon after. This is also somewhat puzzling considering that the Canadian and U.S. economies have historically been closely-linked and tend to prosper and suffer together at a similar magnitude.

The currency eventually broke below 70¢ U.S. and the Bank was reluctant to raise rates further without endangering the economy. It spent billions of dollars in foreign reserves and used borrowing and the selling of securities to buy $Can, to little ultimate avail. Each successive intervention was also increasingly unsuccessful and currency speculators became no longer afraid of the Bank. Currency traders began to realize that economic fundamentals "rule" (one can read my article "Does Raising Domestic Interest Rates Strengthen A Currency?" in the Sept. 1994 issue of *Risks and Rewards*) and that any attempt to prop up the dollar by the Bank would be met by renewed selling and even lower currency values, unless economic prospects were improved. Also, the wisdom behind Bank intervention was increasingly coming under attack as outdated and inappropriate and thus the Bank was losing credibility very quickly.

It was also difficult for the Bank to change its image. It would claim that it no longer cared or would let the currency float more freely, but everyone knew that the Bank would try to strike periodic blows to short-sellers. It was also rather confusing when the Canadian Prime Minister would claim that the falling dollar was good for the economy in that it would stimulate exports and thus no further Bank intervention was necessary, only to find the Bank intervening a few days later.

"When it was finally acknowledged that the fundamentals of the Canadian economy had to be changed and that currency free-fall was just a symptom, there were some offers to the Canadian public of all-round tax cuts to stimulate consumer demand."

When it was finally acknowledged that the fundamentals of the Canadian economy had to be changed and that currency free-fall was just a symptom, there were some offers to the Canadian public of all-round tax cuts to stimulate consumer demand. High taxes were an issue of contention for years but it was suspected that the Federal government was partially unwilling to cut taxes (even though it could afford to do so) because it wanted to give such "goodies" away in a future election year. As

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Bank of Canada
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a result, the government lost the initiative and was finding itself becoming reactive rather than proactive to the currency crisis. Unfortunately, economic confidence had been so badly shaken that such an approach would not likely produce too much in the way of positive results in the near-term. The tax-cut announcement was met with a further decline in the $Can, which was a combination of scepticism, too-little-too-late, or just worries about the prospects for further debt reduction if the tax cuts were made. It would not be a quick fix. It also still has to be realized that Canadian fundamentals, like those of most countries today, are largely affected by what is occurring in the global village and any attempt to divorce an economy from such impacts is difficult, if not impossible.

Summary

Hopefully the Bank of Canada, and any other foreign central banks that have responded to currency declines by intervention or interest rates, is learning that these are only short-term solutions and are probably much more short-term than was the case in the past. The currency markets today are much larger and therefore market forces will reward and punish its players more swiftly. A financial body cannot force a certain price or currency level to be accepted without adverse consequences elsewhere. Unfortunately, these should have been lessons learned from bygone eras. As pointed out in *Grant's Interest Rate Observer* (August 14, 1998, pg. 3) "raising interest rates and tightening monetary growth in the face of an economic downturn are the very policies that the modern age was presumed to have outgrown (How many of today's policy makers read the history of the Great Depression in graduate school and shook their heads at the sheer blockheadedness of the central bankers who had put up a discount rate to defend a gold-exchange parity?) Yet such policies are the very ones being widely implemented today."

A currency is only as strong as its economy even though there may be swings in the short term. If investors want to go there, businesses want to invest and sell there, people are buying there, and the tax and politic climate is accommodating, then a strong currency will follow. That is why everyone at the time of this writing wants $US. Only when the mood changes can the currency experience a major shift in value against other currencies. The only sign of strength in $Can (after it fell to 63 cents) was apparently when commodity prices stabilized or moved up.

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Competing Education

by Zain Mohey-Deen

To perform the complex quantitative analysis in pricing derivative products, investment and commercial banks have hired from the ranks of mathematicians, physicists and actuaries—persons also deemed to be inclined towards the investment field. The commercial and investment banks ascertained that these individuals had the necessary mathematical skill, and these banks later supplemented that ability with in-house training to "convert" these professionals to the investment industry. With the mushrooming of several financial mathematics programs at major universities, firms can now hire a person who requires substantially less in-house "conversion." The programs purport to train students to be "financial engineers." Conversely, graduates of these programs have skills which are attractive to the traditional employers of actuaries. We should be aware of them as our potential competitors, primarily in our emerging practice areas, but eventually on our traditional turf. To start with, we should have a degree of familiarity with the academic programs in which they train.

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There is substantial overlap between the SOA Investment track syllabus and the curriculum for these courses. Students taking SOA courses have the distinct advantage of being able to combine study of the theory with on-the-job practice if they work in the investment department of an insurance company or are involved in asset/liability management, pricing, or cash-flow testing for investment-oriented insurance products. Even financial math programs with well-developed internship arrangements will not be able to match this traditional strength. Moreover, through job rotations, the actuary's overall knowledge of his financial institution is much broader—actuaries know how the other areas in the organization operate and fit together. Beyond the mathematics of pricing, the actuary knows marketing, underwriting, financial reporting, administration, and so on. The actuary is a better rounded financial professional—at least as to insurance companies. Another disadvantage of these school programs is the cost. In some cases tuition for the program can be as high as $30,000.

On the other hand, the financial math courses have a more interactive approach in learning. The training is laboratory intensive. Sophisticated software is licenced to these programs. In general, students are less isolated and have a better opportunity to ask and resolve questions by discussing with other students and faculty. The nature of the field is such that it is easier to develop an understanding of the material by "playing" with models. To some extent, these advantages can be overcome by the use of technology. Technological initiatives being developed by the SOA will address some of these concerns. For example, by posting questions on the web site, access to a larger group of students and practitioners is possible. The redesign of the examination system will also help bridge the gap—the new Course Seven is an on-site intensive seminar in modeling. There is a strong body of academic and practitioner actuaries that stand ready to provide this training. Some of the academic actuaries are already on the faculty in financial math programs.

The travel time involved in the SOA examination system is another disadvantage. On a full-time basis, a Master's in financial math can be obtained in nine months, and a part-timer could complete the course in anywhere from nine months to three years. Once again, the SOA exam redesign may help overcome this disadvantage.

It is worthwhile for actuaries to have some familiarity with these programs. In some cases actuaries may be able to make use of these courses to supplement traditional continuing education. It may also make sense to forge alliances with these institutions, particularly where there are actuaries on faculty, to enhance our regular education effort. Some programs are offered by the mathematics department, others within the business school or, in some cases, the engineering school. Financial Math programs are offered in the major cities in Canada and the U.S. For those interested in investigating further, a sample of the universities offering such courses and their web sites is provided below:

Carnegie Mellon University	fastweb.gsia.cmu.edu/MSCF
Columbia University	www.math.columbia.edu/department/masters_finance.shtml
Cornell University	www.orie.cornell.edu/meng/brochure/tables/financial1.html
MIT	web.mit.edu/sloan/www
New York University	www.math.nyu.edu/programs/math_fin.html
Oregon Graduate Institute	www.cse.ogi.edu/CompFin/
University of Chicago	finmath.uchicago.edu
University of Toronto	www.math.toronto.edu/finance

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Fair Value Conference

by Shirley Hwei-Chung Shao

The Society of Actuaries and New York University join forces again! A conference on "Fair Value of Insurance Business" will be held on March 18 and 19, 1999 in New York City. Please mark your calendar and inform your colleagues in the financial management community of this event of the year! We are also looking for papers to enrich this conference.

The goal of the conference is to extend and update the body of knowledge from the first conference two years ago, to highlight similarities in various theoretical developments, and to work towards resolution of differences and implementation issues. The scope of the conference has been broadened to address fair valuation efforts which consider insurance business as an integrated whole.

This conference will:

- Provide an overview and comparison of various theoretical developments
- Provide an update on various efforts in accounting and management reporting
- Suggest how the various theories may be applied to financial reporting and management uses in practice
- Discuss implementation issues and potential solutions.

A call for papers is being held in conjunction with the conference. The goal of this call for papers is to promote fresh perspectives on this challenging topic, to provide a solid foundation for the conference, and to advance the state of the art on insurance valuation. Papers should discuss fair value accounting for insurance with respect to recent developments in accounting initiatives as well as management practices.

We would particularly would like to receive papers on:

Summaries of the various uses and the common and unique needs for each user (for example, earning emergence pattern)**:** These uses/needs can come from statutory regulators (for example, the Valuation Task Force work, dynamic solvency analysis), GAAP, International Accounting Standards Committee (IASC), the investment community (for example, rating agencies, analysts), and company management (asset liability management, risk management, performance measurements, hedging strategies). Can the various uses/needs be met using a single "fair value" framework?

Discount Rates. It seems that most theoretical developments are variations of discounted cash-flow approaches. In these cases, what should be used as the discount rate/curve (that is, what should be the risk spread over then current Treasuries)? It is particularly confusing on the liability side when the risk premiums work in the opposite direction from the liability side (which is intuitively uncomfortable).

Cash-Flow Components. Can we find common ground on the following issues?

- Free cash flows versus all cash flows: What's "free" and for whose purpose is it "free"?
- How to treat policyholder dividends for mutual companies: Are they indeed "free" cash flows?
- What to do when liabilities depend on asset performance (e.g., crediting interest rate strategy, dividends)?

Stochastic Process. Although most people think of using this process only for interest rate sensitivity, it can be applied to other risk drivers, for example, mortality.

- How to develop a credible process (because it probably cannot be validated in the market)? For example, how to derive the option value?
- How to improve the speed of calculation (for example, low frequency distribution technology)?

Confidence Level. Should there be any margins (or just expected value) built in the fair values? The NAIC project refers to this at various points on the S curve (survivorship function). It attempts to fulfill multiple needs/uses under a single framework by selecting different points on the S curve.

Liability Floor. Is it necessary to have cash values as the floor since we already have that in the statutory valuations? For GAAP, some would say there is no such floor with the establishment of deferred acquisition costs. The cash-value floor does not exist for most company management uses.

Liability Selection. At the last conference, FASB seemed to be interested in knowing whether all liabilities should be "fair valued" since it selected only certain assets (for example, available for public sale) to be fair valued.

The call for papers also encourages discussions beyond insurance liabilities, including interaction with assets and/or insurance enterprise value. More information on the call for papers can be found on the SOA web site as www.soa.org\research\cfp2.html.

We hope for your participation in this conference, either as a paper presenter or an attendee, because we believe this to be a very important issue for our profession to address. So, look up more information on papers on the web site and watch for registration information in January. We promise this conference to be thought provoking!

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