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Scenario Generation: Valuation versus Strategy Development

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More and more, we are seeing Monte Carlo methods being used for interest rate scenario generation. The scenarios are then used for valuation or strategy development with respect to life and annuity blocks of insurance liabilities. These methods are also being used generally in the investment community and with respect to pension funding. In applying their technology, practitioners need to create a set of scenarios that are appropriate for the application being considered. In generating such scenarios, one must make the choice of whether to use the true probability distribution or the risk-neutral distribution. The true distribution reflects the modeler's subjective views about the type and likelihood of future scenarios and history is usually a starting point for this. For the risk-neutral probability distribution, the true scenario probabilities are adjusted in order to reflect the market's pricing for risk.

We should not use the true distribution for valuation since it does not incorporate the market's pricing for risk, but it is appropriate to use this distribution for strategy development since it reflects the practioner's hopefully realistic view of the likelihood of future events. Conversely, we should not use the risk-neutral distribution for strategy development, but it is appropriate to use this distribution for valuation. Furthermore, assuming that cash flows are correctly modeled under a set of risk-neutral scenarios, the valuation can proceed by simply discounting the future cash flows by the one-period risk-free interest rates and then by weighting the pathwise values by the risk-neutral probabilities. This relatively simple procedure is referred to as risk-neutral valuation and is a consequence of the Fundamental Theorem of Asset Pricing.

The linkage between the risk-neutral distribution and the true asset return distribution is often lost sight of. We emphasize this linkage in this article. Risk-neutral valuation and thus risk-neutral scenarios are a calculational tool to answer the following question. Given a set of base assets, and the true return distribution of returns for these assets, what is the amount of assets that needs to be held to defease a liability, given that any dynamic strategy, including short selling, of the assets is allowed?

We can attempt to answer this question by first describing the following process to defease the liability. Given an asset return distribution, we define the strategy contingent cost of a liability to be the market value of the starting assets which will meet all the obligations of the liability under any scenario for the given dynamic investment strategy. We restrict the dynamic investment strategies to be ones that do not look ahead, i.e., for a given scenario, only information up to the current time in the scenario can be used to determine the investment strategy. We define the dynamic immunization value (DIV) to be the minimum of the strategy contingent costs over the set of all possible dynamic investment strategies.

If the model is not arbitrage free, then it may be possible to create portfolios of assets and liabilities that are riskless but earn a return in excess of the risk-free rate. In the most bizarre situation, the DIV can be as low as zero, or even negative. If the model is arbitrage free and there are no transaction costs or taxes, then the DIV equals the cost calculated using risk-neutral valuation. In fact, risk-neutral valuation is simply the calculational tool used to determine the DIV when the assumptions in the model allow us to do this (i.e., arbitrage free with no transaction costs or taxes). An example is the Black-Scholes option pricing formula which gives the dynamic immunization price under the assumptions of their model. Furthermore, the notion of dynamic immunization value generalizes the concept of arbitrage free pricing to the case involving market imperfections such as transaction costs. It is well defined with taxes.

For arbitrage free models, the dynamic immunization value is the same for different asset return distributions which have the same risk-neutral asset return distribution. In particular, the dynamic immunization value for the risk-neutral asset return distribution is the same as the dynamic immunization value for the original true asset return distribution. This is true because these distributions differ only by the probabilities assigned to the scenarios in the universe of all possible scenarios.

Much of the finance literature emphasizes the cases where different true asset return distributions lead to the same risk-neutral distribution, and therefore the same price and portfolio holdings for the immunizing portfolio. While this is of practical importance in many cases, what remains critical is the dynamic immunization value implied by the true asset return distribution. No matter how many true asset return distributions different imply a given risk-neutral distribution, this is irrelevant if the one true asset return distribution that is used for strategy development is inconsistent with the risk-neutral scenarios used in pricing.

For an insurance liability, the dynamic immunization value is one possible approach to determining the market value of liabilities. This is the cost of purchasing a portfolio of assets that dynamically defease the liability. If the allowed investments include below investment grade bonds, we can subtract out a default cost for each quality level. In principle we should model the spreads for default, quality and sector as stochastic. If we do not, we can use a blended yield curve, with appropriate charges for the market's pricing for default risk subtracted out.

Adjustments to the risk-neutral pricing algorithm, or the assumptions that go into it, are determined by the true asset return distribution, and conversely their validity is judged by the asset return distribution that they imply. For example, calibration to initial market prices is important in obtaining the correct answer in calculating the DIP that is used for valuation. However, if an interest rate model is calibrated to market prices by altering its deep fundamental parameters, and this results in unrealistic distributions of the yield curve, under the realistic probability measure, then the calibration process is rendered suspect. If the true asset return distribution is affected by this calibration to make it unrealistic, then inappropriate strategies may be obtained if these scenarios are then used for this purpose.

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Generalizing DIV to Reserves

We can also consider the dynamic immunization reserve where there is an allowed deviation in meeting the obligations, i.e., we relax the constraint that the hedging strategy defease the liability in all scenarios. This is a natural way of thinking about reserving that actuaries are already familiar with. Using the true distribution, suppose that we allow a percentage P of cases in which there is a shortfall in meeting the obligations, then we define the reserve to the P level. For arbitrary dynamic strategies, there does not exist an algorithm for calculating this reserve level (for example, a risk-neutral valuation type algorithm does not exist). If we parameterize the investment strategies allowed, we can use brute force methods to calculate an upper bound on this reserve. We can also consider restrictions in the investment strategy in setting this reserve level.

In a market with transaction costs, imperfect information, and a failure of transparency in information, a reserve may be more relevant than the price from an arbitrage free model. However, a dynamic immunization value that includes these imperfections can still be relevant. Unrealistic strategies can be pared by making appropriate assumptions concerning transaction costs, uncertainty in parameters, and institutional restrictions.

If we set the transaction costs to zero and assume no taxes, then the risk-neutral value from an arbitrage free model is a lower bound to the dynamic immunization value with transaction costs.

Simulation for Asset-Liability Management

Below is a general outline for creating true probability distributions of interest rates, stock, bond and other asset prices and other economic variables.

i) First, develop the stochastic process for the core state variables in the true probability measure.

ii) Then, assume a formula for risk premia and from this obtain the risk-neutral pricing measure for all asset returns.

iii) Generate scenarios in the true probability measure in the core state variables.

iv) Generate the prices at each scenario point given the core state variables at that point, and the risk premia formula, by generating risk-neutral scenarios from that new starting point, and using these to price securities. Do this by generating the cash flows of the securities in these risk-neutral scenarios and discounting them at the one-period risk-free rate in those scenarios and then averaging these prices over scenarios. Note that the set of all risk-neutral scenarios is the same as the set of all true scenarios, but the probabilities are different.

v) Perform a calibration process of the parameters of the core state variable process and the risk premia formula to fit the modeler's subjective view of the distribution which may be based on historical information. We also use the starting yield curve and other market information to determine the initial values of the core state variables. This can also be combined with the historical calibration. We may also introduce a residual process to achieve a final reconciliation with initial market prices. For example, small deviations in the model yields and treasury yields can be modeled as residuals that decay over the course of the next year.

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