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V/réf N/ref Lausanne. May 2, 1984

Dear Sir:

I have only just received my copy of ARCH 1983.1 so please excuse my delay in answering the five letters appearing therein addressed to my views. I can answer these letters, excepting only the encouraging remarks of John Beekman, by enunciating four basic principles (BP):

BP1 A random variable has an expected value.

BP2 A random variable has one or more parameters.

BP3 The expected value cannot include any of the realizations of the random variable.

BP4 Such realizations are the only means of estimating the values of the parameters of the random variable.

The random variable we are concerned with is the number of deaths observed in a mortality study. The parameter is a function of the random variable, namely q_x. The writer is aware of three different expressions representing the expected value of the number of deaths between exact ages x and x+1:

Wittstein's (1862) $s_x q_x + (N_x - W_x)\int_0^1 q(x+k,x+1)dk$

where s_x is the number of "beginners" at age x, N_x is the number of new entrants during the year, W_x is the number of withdrawals and q(x+k,x+1) is the probability of dying between ages x+k and x+1.

Wolfenden's (1942) $s_x q_x + \int_0^1 q(x+k,x+1)d(N_{x+k} - W_{x+k})$

$N_{x+k} - W_{x+k}$ being an up-and-down staircase with steps wherever an individual enters or exits.

Seal's (1954) $q_x \sum_{j=1}^{N} (t_a^{(j)} - t_b^{(j)})$

where $t_a^{(j)}$ and $t_b^{(j)}$ denote the dates of entry and exit of the jth individual.

The third expression can be obtained from the second by writing $q(x+k,x+1) = (1-k)q_x$, as Wolfenden did; the first assumes a uniform distribution of entrants and exits over the year. Notice that any actual deaths must be ignored in arriving at a b-value. Apart from the representation of q(x+k,x+1) in terms of q_x all we have to do is substitute the actual number of deaths for the "expected" under any of these formulas.

Yours sincerely,

H L Seal

Hilary L. Seal

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